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Robust Stability Criteria of Interval Polynomial Systems Dropping-Degrees *

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Abstract: In this paper, the "leapfrog" phenomenon mentioned by B.R.Barmish is studied. We generalized zero exclusion condition to the degenerated parameter polynomials. And we got a revision of Kharitonov's theory and its following polytope polynomials results.

Key words: degree-dropping polynomial; interval polynomial; value set

可降阶区间多项式系统的鲁棒稳定准则

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摘要: 讨论了B.R.Barmish提及的"leapfrog"现象.并给出了可降阶多项式的根对参数逐点连续性的一般性证明.之后,引入了参数集中的阶次非增路径的概念,并将排零条件推广到可降阶含参多项式系统.我们改进了Kharitonov's定理和之后的一些多面体多项式族方面的结果.

关键词: 可降阶多项式; 区间多项式; 值集

1 Introduction

There is a long-standing interest in robust control problems involving structured real parametric uncertainty. Kharitonov's paper[1] began to receive attention in the control field in 1983, which was introduced by B.R. Barmish in [2]. Important work includes Zero Exclusion Condition[1], Value Set[2], Edge Theorem[3], and Convex Direction[4], etc. All of these assumed that degrees of the character polynomials of the systems are invariant. So that the roots are continuously dependend on the parameters. If the degrees are not invariant, the "leapfrog" phenomenon mentioned by B. R. Barmish[5] may occur. In this paper we give a general proof of the continuous root dependence on its parameters of a degree dropping polynomial in the sense of pointwise continuousness. We also define the nonincreasing path of a parameter set. With these concepts we generalize zero exclusion condition to the degenerated parameter polynomials. And we get a revision of Kharitonov's Theory and its following polytope polynomials results.

2 Problem and main results

Consider the parameter polynomial

$$p(s,q) = \sum_{i=0}^{n} a_i(q)s^i, \ q \in Q \subset \mathbb{R}^l, \quad (2.1)$$

where $a_i(q)$ continuously depends on q, $i = 0,1,\cdots,n$. If $\deg(p(s,q))$ is invariant, every root $s_i(q)$ of $p(s,q)$ continuously depends on parameter q. It is the basis of the standard Zero Exclusion Condition[1]. If there exists $q^0 \in Q$ such that $a_n(q) = 0$, i.e. $\deg(p(s,q^0)) < n$, degree dropping occurs. The standard results of parameter robust control do not hold automatically. There are many counter examples, among them is the famous "leapfrog" phenomenon in B. R. Barmish[6]: if $p(s,q)$ experiences degree dropping, then as q varies, the branches of the root locus can "leapfrog" from the strict left half plane into the strict right half plane without crossing the imaginary axis.

We have found that when degree dropping occurs, the finite root $|s_i(q)| < \infty$ still continuously depends on its parameter q. Thus we can generalize the Zero Exclusion Condition to the degree-dropping polynomials. With the help of the Rouché's Lemma[6], we can generalize the root locus continuous dependence on parameter of polynomials with invariant degree to the degree-dropping case. We ought to claim that the continuity is in the point-

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wise sense.

Theorem 2.1(Pointwise continuity) Given a family of plynomials $\mathscr{P}$

$$\mathscr{P}=\{p(s,q):p(s,q)=\sum_{i=0}^{n}a_i(q)s^i,$$
$$q\in Q \text{ is a connected set in } \mathbb{R}^l\} \qquad (2.2)$$

where $a_i(q)$ continuously depends on q, for $i=0,1,\cdots,n$. Then each finite root $s_i(q^0)$ of $p(s,q^0)$ continuously depends on q at point $q^0\in Q$.

Proof Let $s_i(q^0)$ denote any finite root of $p(s,q^0)$. We can choose $\varepsilon>0$ small enough, such that in the sphere region $S_\varepsilon=\{s: |s-s_i(q^0)|<\varepsilon\}$ there is only a single root (including multiple roots) of $p(s,q^0)$ and no root on its boundary ∂S_ε. Then

$$p(s,q)=\sum_{i=0}^{n}a_i(q)s^i=$$
$$\sum_{i=0}^{n}a_i(q^0)s^i+\sum_{i=0}^{n}(a_i(q)-a_i(q^0))s^i.$$

Denotes $\sum_{i=0}^{n}a_i(q^0)s^i$ by $f(s)$, $\sum_{i=0}^{n}(a_i(q)-a_i(q^0))s^i$ by $g(s,q)$, then $f(s)$ and $g(s,q)$ are analytic on the closed region $\bar{S}_\varepsilon$. Because $f(s)$ has no root on the boundary ∂S_ε, $\sigma=\min_{s\in\partial S_\varepsilon}|f(s)|$ is strictly larger than zero. Since $a_i(q)$ continuously depends on q, there exists $\delta>0$ such that

$$|a_i(q)-a_i(q^0)|<\frac{\sigma}{(n+1)(|s_i(q^0)|+\varepsilon)^i},$$
$$i=0,1,\cdots,n \qquad (2.3)$$

holds, whenever $\|q-q^0\|<\delta$. Therefore, for any $s\in\partial S_\varepsilon$, we have

$$|g(s,q)|=\left|\sum_{i=0}^{n}(a_i(q)-a_i(q^0))s^i\right|\leqslant$$
$$\sum_{i=0}^{n}|a_i(q)-a_i(q^0)||s^i|<$$
$$\sum_{i=0}^{n}\frac{\sigma|s^i|}{(n+1)(|s_i(q^0)|+\varepsilon)^i}\leqslant$$
$$\sum_{i=0}^{n}\frac{\sigma}{n+1}=\sigma.$$

Thus, $|g(s)|<\sigma\leqslant|f(s)|$ is always valid on the boundary ∂S_ε. From Rouché's Lemma, we know that $f(s)+g(s,q)$ and $f(s)$ has same number of roots in the region S_ε. That is to say, for any $q\in\{q:\|q-q^0\|<\delta\}$, $p(s,q)$ have n_i roots (j- multiples root is looked on as j roots, n_i is the multiples of root $S_i(q^0)$) which satisfies

$$|S_i(q)-S_i(q^0)|<\varepsilon. \qquad (2.4)$$

In conclusion, each finite root $s_i(q^0)$ of $p(s,q^0)$ continuously depends on q at point $q^0\in Q$. Q.E.D.

Remark 2.1 This conclusion has no prerequisite on whether $a_n(q^0)=0$ or not. That means $p(s,q^0)$ could be a degree dropping polynomial.

Proposition 2.1 (Root locus of degree-dropping polynomials) Suppose $q(\lambda)$, $\lambda\in[-1,1,]$ is a continuous path and

$$\deg(p(s,q(\lambda)))=\begin{cases}k, & \lambda=0,\\ l, & l>k,\lambda\in[-1,0),\\ m, & m>k,\lambda\in(0,1].\end{cases} \qquad (2.5)$$

Then $(l-k)$ roots of $p(s,q(\lambda))$ vanish, when $\lambda\to 0^-$; $(m-k)$ roots vanish when $\lambda\to 0^+$. k roots continuously depend on λ at point $\lambda=0$. The vanished roots move towards infinite, $\lim_{\lambda\to 0}|s_i(\lambda)|=+\infty$.

Proof The first two conclusions can be directly derived from Theorem 2.3. In the following, we only prove the third conclusion: $s_i(\lambda)$ tend to be infinity as $\lambda\to 0$.

Proceeding by contradiction, if $s_i(\lambda)$ does not move to infinity as $\lambda\to 0^+$ (0^-), then $\exists M_0>0$, such that $\forall\lambda>0(<0)$ there exists a λ^* between 0 and λ with $|s_i(\lambda^*)|\leqslant M_0$. Therefore we can choose a sequence of λ_h converging to zero such that $|s_i(\lambda_h)|\leqslant M_0$ hold for $h=1,2,\cdots$. Since $\{s_i(\lambda_h)\}_{h=1}^{+\infty}$ are bounded, we can find a sub-sequence of λ_h which, still denoted by λ_h, satisfies $s_i(\lambda_h)\to s_i^*$ for some s_i^*. Since $p(s_i(\lambda_h), q(\lambda_h))=0$ for every $h=1,2,\cdots$, let $h\to\infty$ and we get

$$0=\lim_{h\to\infty}p(s_i(\lambda_h),q(\lambda_h))=$$
$$\lim_{h\to\infty}\sum_{j=k+1}^{n}a_j(q(\lambda_h))s_i^j(\lambda_h)+$$
$$\lim_{h\to\infty}\sum_{j=0}^{k}a_j(q(\lambda_h))s_i^j(\lambda_h)=$$
$$\lim_{h\to\infty}\sum_{j=0}^{k}a_j(q(\lambda_h))s_i^j(\lambda_h)= \qquad (*)$$
$$\sum_{j=0}^{k}a_j(q(0))(s_i^*)^j=p(s_i^*,q(0))$$

where $(*)$ holds because $\{s_i(\lambda_h)\}_{h=1}^{+\infty}$ are bounded and

$\lim_{h\to\infty} a_j(q(\lambda_h))$ is equal to zero for every $j = k+1,\cdots,n$. Hence s_i^* is a root of $p(s,q(0)) = 0$. By the proof of the Theorem 2.1, the multiple number of root s_i^* is equal to the number of roots of $p(s,q) = 0$ in a sufficiently small neighborhood of s_i^* when q is sufficiently close to $q(0)$, which means the number of the roots of $p(s,q(0)) = 0$ is equal to $p(s,q(\lambda_h)) = 0$ when h is large enough. But it is contrary to the assumption (2.5) that the degree of $p(s,q(\lambda_h))$, $m(l)$, is larger than k, the degree of $p(s,q(0))$. Therefore the required result is well verified. Q.E.D.

When λ passes through zero point, there will be some infinite roots to appear and then disappear. That is why "leapfrog" phenomenon[5] appears.

Definition 2.1 Path $q(\lambda),\lambda \in [\lambda^-,\lambda^+]$, is called a degree-nonincreasing path if for any given λ_1, $\lambda_2 \in [\lambda^-,\lambda^+]$, deg $p(s,q(\lambda_2)) \leqslant \deg(p(s,q(\lambda_1)))$ holds when $\lambda_2 \geqslant \lambda_1$.

Theorem 2.2(Generalized zero exclusion condition A) Let D be a region in the complex plane $\mathbb{C}$, and the polynomial family $\mathscr{P} = \{p(s,q):p(s,q) = \sum_{i=0}^{n} a_i(q)s^i, q \in Q\}$, where Q is a finite piecewise connected set in $\mathbb{R}^l$. And $a_i(q)$ continuously depends on q for $i = 0,1,\cdots,n$. If there exists a D- stable element $p(s,q^0)$ with the highest degree in $\mathscr{P}$, and a degree-nonincreasing path from q^0 to any $q \in Q$. Then $\mathscr{P}$ is robust D- stable iff

$$0 \notin p(z,Q), \forall z \in \partial D. \qquad (2.6)$$

Proof "$\Rightarrow$" Assuming $\mathscr{P}$ is robust D- stable and we must prove that $0 \notin p(z,Q), \forall z \in \partial D$. Proceeding by contradiction, suppose that $0 \in p(z,Q)$, for some $z^* \in \partial D$. Then $p(z^*,q) = 0$ for some $q^* \in Q$; i.e., the polynomial $p(s,q^*)$ has a root at $s = z^* \in \partial D$ which contradicts robust D -stability of $\mathscr{P}$.

"$\Leftarrow$" we assume that $0 \notin p(z,Q)$, for all $z \in \partial D$ and must show that $\mathscr{P}$ is robust D- stable. Proceeding by contradiction, $p(s,q^1)$ is not robust stable for some $q^1 \in Q$. From the prerequisite, we know that there exists a continuous function $\Phi:[0,1] \to Q$, such that $\Phi(0) = q^0, \Phi(1) = q^1$ and $\Phi(\lambda)$ is a degree-nonincreasing path. So along the path from 1 to 0, degree of $p(s,q)$ can only increase, and additional roots may appear but no root disappears. Theorem 2.3 guarantees that all $m(\leqslant n)$ roots of $p(s,q^1)$ will continuously change to the roots of $p(s,q^0)$, as $\Phi(\lambda)$ from 1 to 0, denoted as $S_i(\Phi(\lambda)), i = 1,\cdots,m$. Since $S_i^*(\Phi(1)) \notin D$ and $S_i^*(\Phi(0)) \in D$ hold, according to the continuity of $S_i^*(\Phi(\lambda))$, there must exist a $\lambda^* \in (0,1)$, such that $S_i^*(\Phi(\lambda^*))$ is on the boundary of D. Denote $q^* = \Phi(\lambda^*), z^* = S_i^*(q^*)$, we have

$$p(z^*,q^*) = 0, \ z^* \in \partial D.$$

It is the contradiction we seek. Q.E.D.

Along the degree-nonincreasing path $q(\lambda)$ in Theorem 2.2 the "leapfrog" phenomenon does not exist. That is the reason why zero exclusion condition is still valid for the family of degree-dropping polynomials.

Proposition 2.2 Suppose $\mathscr{P}$ is an interval polynomial family $\mathscr{P} = \{p(s,q):p(s,q) = \sum_{i=0}^{n} q_i s^i, q \in Q\}$, where $Q = \{q: q_i \in [q_i^-,q_i^+], q_i^- \geqslant 0, i = 0,1,\cdots,n\}$ and $\mathscr{P}$ has a stable element with the highest degree. Then $\mathscr{P}$ is robust stable iff for any nonnegative frequency ω, origin $z = 0$ does not belong to the Kharitonov rectangle, that is

$$0 \notin p(j\omega,Q), \quad \forall \omega \geqslant 0. \qquad (2.7)$$

Proof Let $p(s,q) = \sum_{i=0}^{n} [q_i^-,q_i^+]s^i$. For the reason that $p(s,q^0)$ has the highest degree, $q_n^0 \neq 0$, hence $q_n^0 > 0$. For any $q \in Q$, we choose the path from q^0 to q in the form of $q(\lambda) = (1-\lambda)q^0 + \lambda q, \lambda \in [0,1]$, the convex combination of q^0 and q, so $q(\lambda)$ will always be in the set Q. And $q_n^0(\lambda) = (1-\lambda)q_n^0 + \lambda q_n > 0$, for any $\lambda \in [0,1)$. Thus, deg $(p(s,q(\lambda))) = n$, for any $\lambda \in [0,1)$. Now, we can say the path $q(\lambda)$ is a degree-nonincreasing path. The prerequisite of Theorem 2.2 is satisfied, and the Proposition is immediate.

Q.E.D.

With the help of Proposition 2.2, we can generalize Kharitonov's Theorem[1,6] to the degree-dropping case.

Theorem 2.3(Generalized Kharitonov's theorem) An interval polynomial family $\mathscr{P}$ as Proposition 2.2 is robustly stable iff its four Kharitonov polynomials

$$\begin{cases} k_1(s) = q_0^- + q_1^- s + q_2^+ s^2 + q_3^+ s^3 + \\ \qquad q_4^- s^4 + q_5^- s^5 + q_6^+ s^6 + \cdots, \\ k_2(s) = q_0^+ + q_1^+ s + q_2^- s^2 + q_3^- s^3 + \\ \qquad q_4^+ s^4 + q_5^+ s^5 + q_6^- s^6 + \cdots, \\ k_3(s) = q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3 + \\ \qquad q_4^+ s^4 + q_5^- s^5 + q_6^- s^6 + \cdots, \\ k_4(s) = q_0^- + q_1^+ s + q_2^+ s^2 + q_3^- s^3 + \\ \qquad q_4^- s^4 + q_5^+ s^5 + q_6^+ s^6 + \cdots \end{cases} \tag{2.8}$$

are stable.

Proof The proof of necessity is obvious. To establish the sufficiency, we assume that the four Kharitonov polynomials are stable and must prove that $\mathscr{P}$ is robustly stable. There must exist a highest degree element of $\mathscr{P}$ among $k_1(s), \cdots, k_4(s)$. And with the help of Proposition 2.2, we make use of generalized zero exclusion condition A (Theorem 2.2). The proof procedure is similar to the originality Kharitonov's theorem (see [5] for details). Q.E.D.

3 Refinement and generalization

In Section 2 we have defined degree-nonincreasing path, and we have generalized zero exclusion condition A (Theorem 2.2). But the theorem requires that there exists a degree-nonincreasing path from the stable element in $\mathscr{P}$ with the highest degree to any element in $\mathscr{P}$. It could not always be satisfied. So we need the following theorem to include more general family $\mathscr{P}$.

Theorem 3.1 (Generalized zero exclusion condition B) Let D be a region in the complex plane $\mathbb{C}$, and the polynomial family $\mathscr{P} = \{p(s,q): p(s,q) = \sum_{i=0}^{n} a_i(q) s^i, q \in Q\}$, where Q is a finite set in $\mathbb{R}^l$, and $a_i(q)$ continuously depends on q for $i = 0,1,\cdots,n$. If Q could be divided into finite piecewise connected subsets $\{Q_k\}_{k=1}^{m}$ with $\bigcup_{k=1}^{m} Q_k = Q$. And if there exist a D- stable element $p(s,q^k)$ with locally highest degree in $p(s, Q_k)$, and a degree-nonincreasing path from q^k to any $q \in Q_k$. Then $\mathscr{P}$ is robust D- stable iff

$$0 \notin p(z,Q), \forall z \in \partial D.$$

Proof The necessity is similar to Theorem 2.2. We only need to prove the sufficiency. Assuming $0 \notin p(z, Q)$, for all $z \in \partial D$, then for any $k \in \{1,2,\cdots,m\}$, $0 \notin p(z,Q_k)$, and all $z \in \partial D$. From Theorem 2.2, we know $p(z,Q_k)$ is robust D- stable. And from $Q = \bigcup_{k=1}^{m} Q_K$, we know that $\mathscr{P}$ is robust D- stable. Because for any $q \in Q$, there exists a $k \in \{1,2,\cdots,m\}$ such that $q \in Q_k$, then $p(s,q^k)$ is robustly D- stable. Q.E.D.

With Theorem 3.1, we can deal with some specific type of parameter uncertainties. Here we consider

$$\mathscr{P} = \{p(s,q): p(s,q) = \sum_{i=0}^{n} (\alpha_i^\tau q + \beta_i) s^i, \ q \in Q\}, \tag{3.1}$$

where $Q \subset \mathbb{R}^l$ is a polytope, $\alpha_i \in \mathbb{R}^l$ is a column vector and β_i is a scalar, for $i = 0,1,\cdots,n$. We can also denote $\mathscr{P}$ by conv $\{p(s,q^j)\}$, $q^j, j = 1,\cdots,l$ are the vertices of polytope Q. Let $a_n(q) = \alpha_n^\tau q + \beta_n = 0$, and we get

$$\alpha_{n1}^\tau q_1 + \cdots + \alpha_{nl}^\tau q_l + \beta_n = 0, \tag{3.2}$$

which is a hypoplane in space $\mathbb{R}^l$. Let $\Pi = \{\dot{q}: \alpha_n^\tau q + \beta_n = 0, q \in \mathbb{R}^l\}$. If $\mathscr{P}$ has dropping-degree, then $\Pi \cap Q \neq \emptyset$, i.e., hypoplane Π intersects polytope Q (may only tangent). If Q is divided into two parts, each of which is still a polytope. Dropping-degree will occur on the surface cut by Π. If there exists a D- stable $p(s,q^{(k)})$, $q^{(k)} \in (Q_k/\Pi)$, for each $k = 1,2$, we can choose a path $q(\lambda) = (1-\lambda)q^{(k)} + \lambda q, \lambda \in [0,1]$, for any $q \in Q_k, k = 1,2$. It is clear that $q(\lambda), \lambda \in [0,1)$, has invariant degree. Thus $q(\lambda), \lambda \in [0,1]$, is a degree-nonincreasing path. It is concluded as the following proposition.

Proposition 3.1 (Polytope polynomial family) $\mathscr{P} = \{p(s,q): q \in Q\}$ is an polytope polynomial family, where $Q = \mathrm{conv}\{q^j, j = 1,\cdots,l\}$. D is a region on the complex plane. If the hypoplane Π, defined as (3.2), divides Q into two parts Q_1, Q_2 and there exists a D- stable element $p(s,q^{(k)}, q^{(k)} \in (Q_k/\Pi)$, for each $i = 1, 2$. Otherwise, there exists $q^{(0)} \in (Q/\Pi)$, such that $p(s,q^{(k)})$ is D- stable. Then $\mathscr{P}$ is robustly D- stable iff

$$0 \notin p(z,Q) = \mathrm{conv}\{p(z,q^j), j = 1,\cdots,l\},$$
$$\forall z \in \partial D.$$

Remark 3.1 We can not obtain Edge Theorem[3,5] only under the prerequisite of Proposition 3.1.

4 Conclusions

From the studied "leapfrog" phenomenon, we have improved the fundamental settings of robust control of systems with parameter uncertainty in some sense. Now we can deal with polynomial family with invariant and variant degree in the same framework. There are still some problems that need to be studied. One is how to divide a more general parameter set Q to satisfy the prerequisite of Theorem 3.1, and the other is under which condition can Edge Theorem or even Vertex Test Theorem be generalized to the dropping-degree polynomial family. Maybe more recent results could be reconsidered under this general settings.

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(Continued from page 436)

6 Conclusion

In this paper, a new variable structure MRAC scheme is introduced. It is shown that the new scheme is applicable in the absence of SPR assumption. It is also shown that the tracking error will converge, in a finite time, to zero if $n^* = 1$ and to a small residual set if $n^* > 1$.

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