第 16 卷第 3 期
1999 年 6 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol.16, No.3
Jun., 1999

The Realization for 2-D Singular Systems

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Abstract: This paper discusses the problems of realization of the 2-D linear shift-invariant singular general state-space models (2-D SGM) with standard boundary conditions. A new realization approach to 2-D SGM is developed.

Key words: 2-D systems; singular systems; realization; state-space models

2-D 奇异系统的实现

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摘要: 讨论了 2-D 奇异线性常系数一般模型(2-D SGM)在标准边界条件下的状态空间实现问题,提出了一种新的实现算法.

关键词: 2-D 系统; 奇异系统; 实现; 状态空间模型

1 Introduction

Two-dimensional (2-D) state-space models have been extensively studied during the past decades. During this period many 1-D state-space techniques have been generalized to their 2-D counterparts[1]~[7]. There are a few papers such as [7] that have discussed the realization problem of 2-D singular systems. However, the realization algorithms there are based on series of relatively complicated transformation and not very explicit in form. In this paper, we have improved the corresponding algorithm on the basis of some results in [3].

2 Realization

Consider the linear shift-invariant discrete 2-D systems of the descriptor form

$$\begin{aligned} &Ex(i+1,j+1) = \\ &A_0x(i,j) + A_1x(i+1,j) + \\ &A_2x(i,j+1) + B_0u(i,j) + \\ &B_1u(i+1,j) + B_2u(i,j+1), \end{aligned} \tag{1a}$$

$$y(i,j) = Cx(i,j) + Du(i,j). \tag{1b}$$

Where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^l$ and $E, A_k, B_k (k = 0,1,2)$, C and D are constant matrices of appropriate dimensions. E is singular, and $E, A_k (k = 0,1,2)$ satisfies the 2-D regular pencil condition:

$$\det(z_1z_2E - z_1A_1 - z_2A_2 - A_0) \not\equiv 0. \tag{2}$$

The system (1) is also called 2-D singular general state-space model (2-D SGM). The following standard form gives the boundary conditions for (1)

$$x(0,j), x(i,0), \text{for } i,j = 0,1,2,\cdots. \tag{3}$$

In [4] and [5] Kaczorek proposed a significant notion called the admissible boundary condition, and proved that the system (1) has a solution only if (3) is admissible. The transfer function matrix of (1) is defined in [3]:

$$\begin{aligned} G(z_1,z_2) = C[&z_1z_2E - z_1A_1 - z_2A_2 - A_0]^{-1} \cdot \\ &[B_0 + z_1B_1 + z_2B_2] + D. \end{aligned} \tag{4}$$

Definition 1 The matrix group

$$\{E, A_0, A_1, A_2, B_0, B_1, B_2, C, D\} \tag{5}$$

is called a realization of a given transfer function matrix $G(z_1, z_2)$ if they satisfy the equation (4).

Now, let

$$G(z_1,z_2) = N(z_1,z_2)/d(z_1,z_2) \in \mathbb{R}^{l\times m}(z_1,z_2) \tag{6}$$

be an arbitrary rational function matrix, where $N(z_1, z_2) \in \mathbb{R}^{l\times m}[z_1,z_2]$ is an $l \times m$ polynomial matrix in z_1, z_2 and

$$d(z_1,z_2) = \sum_{i=0}^{n_1}\sum_{j=0}^{n_2} d_{i,j}z_1^iz_2^j. \tag{7}$$

with $d_{i_0,n_2} \neq 0$ and $d_{n_1,j_0} \neq 0$ for some i_0 and j_0.

Definition 2[6] The transfer function matrix $G(z_1, z_2)$ is called proper if $d(z_1, z_2)$ is acceptable

* This project is supported by National Natural Science Foundation of China under Grant 69474001.
Manuscript received Jul.21, 1997, revised Oct.26, 1998.

(i,e., $d_{n_1,n_2} \neq 0$) and for $1 \leqslant i \leqslant l$, $1 \leqslant j \leqslant m$,

$$\deg_{z_k} n_{i,j}(z_1,z_2) \leqslant n_k,\ k = 1,2. \tag{8}$$

Here $n_{i,j}(z_1,z_2)$ are the elements of $N(z_1,z_2)$, and $\deg_{z_k} n(z_1,z_2)$ denotes the degree of $n(z_1,z_2)$ with respect to z_k, $k = 1,2$.

Obviously, the matrix (4) is proper if and only if E is nonsingular, i.e. the system (1) is a regular system. Let λ,μ be the real numbers such that

$$d(-\lambda,-\mu) \neq 0. \tag{9}$$

And let

$$\hat{G}(\omega_1,\omega_2) \triangleq G(\omega_2^{-1}-\lambda,\omega_1^{-1}-\mu) = G(z_1,z_2). \tag{10}$$

Where $\omega_1 = (z_2+\mu)^{-1}, \omega_2 = (z_1+\lambda)^{-1}$.

Lemma 1 Let $G(z_1,z_2)$ and $\hat{G}(\omega_1,\omega_2)$ be given by (6) and (10) respectively. Then if (9) holds, $\hat{G}(\omega_1,\omega_2)$ is always proper with respect to ω_1,ω_2.

Proof Note that $\hat{G}(\omega_1,\omega_2)$ can be re-written as

$$\hat{G}(\omega_1,\omega_2) = \hat{N}(\omega_1,\omega_2)/\hat{d}(\omega_1,\omega_2). \tag{11}$$

Where

$$\hat{N}(\omega_1,\omega_2) = \omega_1^{m_1}\omega_2^{m_2} N(\omega_2^{-1}-\lambda,\omega_1^{-1}-\mu) \in \mathbb{R}^{m\times l}[\omega_1,\omega_2],$$

$$\hat{d}(\omega_1,\omega_2) = \omega_1^{m_1}\omega_2^{m_2} d(\omega_2^{-1}-\lambda,\omega_1^{-1}-\mu) \in \mathbb{R}[\omega_1,\omega_2].$$

and $\omega_1^{-m_1}\omega_2^{-m_2}$ are the least common denominator of $d(\omega_2^{-1}-\lambda,\omega_1^{-1}-\mu)$ and the elements in $N(\omega_1^{-1}-\lambda,\omega_1^{-1}-\mu)$. Obviously, the highest possible degree of $\hat{N}(\omega_1,\omega_2)$ is (m_1,m_2), and the term with highest degree in $\hat{d}(\omega_1,\omega_2)$ is $d(-\lambda,-\mu)\omega_1^{m_1}\omega_1^{m_2}$. Therefore by (9) and definition 2 the proof is completed.

Thus, by Lemma 1 and the well-known realization theory of 2-D regular systems[6] there is a regular realization for $\hat{G}(\omega_1,\omega_2)$ as follows:

$$\hat{G}(\omega_1,\omega_2) = \hat{C}[\omega_1\omega_2 I - \omega_1\hat{A}_1 - \omega_2\hat{A}_2 - \hat{A}_0]^{-1}\cdot (\hat{B}_0 + \omega_1\hat{B}_1 + \omega_2\hat{B}_2) + \hat{D}. \tag{12}$$

Lemma 2 There always exists a realization (12) with $\hat{B}_0 = 0$ for $\hat{G}(\omega_1,\omega_2)$ in (10).

Proof In fact, let

$$\bar{A}_0 = 0 \text{ or } \begin{bmatrix} \hat{A}_0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{A}_1 = \begin{bmatrix} \hat{A}_1 & 0 & 0 \\ I_n & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} \hat{A}_1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} \hat{A}_2 & \hat{A}_0 & \hat{B}_0 \\ I_n & 0 & 0 \\ 0 & 0 & \end{bmatrix} \text{ or } \begin{bmatrix} \hat{A}_2 & \hat{B}_0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{B}_0 = 0,\ \bar{B}_1 = \begin{bmatrix} \hat{B}_1 \\ 0 \\ I_m \end{bmatrix} \text{ or } \begin{bmatrix} \hat{B}_1 \\ I_m \end{bmatrix},$$

$$\bar{B}_2 = \begin{bmatrix} \hat{B}_2 \\ 0 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} \hat{B}_2 \\ 0 \end{bmatrix},$$

$$\bar{C} = [\hat{C},0,0] \text{ or } \bar{C} = [\hat{C},0],\ \bar{D} = \hat{D},$$

respectively. Then by (12) we have

$$\hat{G}(\omega_1,\omega_2) = \bar{C}(\omega_1\omega_2 I - \omega_1\bar{A}_1 - \omega_2\bar{A}_2 - \bar{A}_0)^{-1}\cdot (\omega_1\bar{B}_1 + \omega_2\bar{B}_2) + \bar{D}.$$

This completes the proof.

Theorem 1 let $\Omega \in \mathbb{R}^{n\times n}$ be an arbitrary inevitable constant matrix and $\hat{A}_i, \hat{B}_j, \hat{C}$ and $\hat{D}$ be defined by (12). Note that by Lemma 2, we can always claim that $\hat{B}_0 = 0$. Then

$$\{E,A_0,A_1,A_2,B_0,B_1,C,D\} \tag{13}$$

is a realization of $G(z_1,z_2)$, where

$$E = -\Omega^{-1}\hat{A}_0, C = \hat{C}, D = \hat{D}, \tag{14a}$$

$$A_0 = \Omega^{-1}(\lambda\mu\hat{A}_0 + \lambda\hat{A}_1 + \mu\hat{A}_2 - I), \tag{14b}$$

$$A_1 = \Omega^{-1}(\hat{A}_1 + \mu\hat{A}_0), \tag{14c}$$

$$A_2 = \Omega^{-1}(\hat{A}_2 + \lambda\hat{A}_0), \tag{14d}$$

$$B_0 = \Omega^{-1}(\lambda\mu\hat{B}_0 + \lambda\hat{B}_1 + \mu\hat{B}_2), \tag{14e}$$

$$B_1 = \Omega^{-1}(\hat{B}_1 + \mu\hat{B}_0), \tag{14f}$$

$$B_2 = \Omega^{-1}(\hat{B}_2 + \lambda\hat{B}_0). \tag{14g}$$

Proof By (12) and (14) it is easy to see that

$$C[z_1z_2E - z_1A_1 - z_2A_2 - A_0]^{-1}\cdot (B_0 + z_1B_1 + z_2B_2) + D = \hat{G}(\omega_1,\omega_2). \tag{15}$$

With $\omega_1 = (z_1+\mu)^{-1}, \omega_2 = (z_1+\lambda)^{-1}$. Therefore, combining with (10) we complete the proof.

It is very interesting to note that

$$\Omega^{-1} = \lambda\mu E + \lambda A_1 + \mu A_2 - A_0. \tag{16}$$

This coincides with that in [3].

Remark Usually, it is very convenient to realize a regular transfer function matrix (11) directly as a Rouser model or FMM II[7]. In this case $A_0 = 0$ and $B_0 = 0$. Such a realization is always available for an arbitrary matrix (11)[7, Chapter 2]. As a result of the realization of (13), Theorem 1 has a special form with $E = 0$.

3 Conclusion

A new realization algorithm is proposed on the basis of the approach for computing the transfer function matrix for 2-D SGM in [3]. It is simpler and more explicit than that in [7]. The minimum realization problem is not discussed in this paper.

References

1 Lu W S. Some new results on stability and robustness of two-dimensional discrete systems. Multidimensional Systems and Signal Processing, 1994,5(2):345 – 361

2 Zou Yun and Yang Chengwu. An algorithm for computation of 2D eigenvalues. IEEE Trans. Automat. Contr., 1994,39(7):1436 – 1439

3 Zou Yun, Du Chunling and Zhou Xiangzhong. Disturbance decoupling control for 2-D linear shift-invariant systems. IFAC Youth Automation Conference, Beijing, 1995

4 Zou Yun and Yang Chengwu. Algorithms for computation of the transfer function matrix for 2D regular and singular general state space models. Automatica, 1995,31(9):1311 – 1315

5 Kaczorek T. The singular general model of 2-D systems and its solution. IEEE Trans. Automat. Contr., 1988,33(11):1060 – 1061

6 Kaczorek T. General response formula and minimum energy control for the general singular model of 2-D systems. IEEE Trans. Automat. Contr., 1990,35(4):433 – 436

7 Kaczorek T. Two-Dimensional Linear System. Berlin: Springer-Verlag, 1985

8 Kaczorek T. Realization problem for singular 2-D systems. Bulletin of the Polish A academy of Science, Tech. Sci., 1989,37(1):37 – 48

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Table 4 Average computational time of algorithms with different sizes of machines

Number of machines	MGA	GA	Rajendran	Ho & Chang
5	97.6200	114.6200	1.2200	0.3200
10	155.9800	147.6600	2.1400	0.5800
15	212.9800	179.5800	3.0800	0.8000
20	266.9600	209.3000	4.0000	1.0200

4 Conclusions

From the figures in the Table 1 to Table 4, we can see that:

1) The MGA show consistent improvement over the general GA. The average improvement of MGA is significant of 2.878% better than that of the Rajendran's heuristic.

2) The computational time of all these algorithms increases with increasing problem size. The running time of the MGA and GA is directly proportional to the number of jobs. The computational effort of the MGA is less affected by the number of machines. MGA takes similar amount of computational time with GA, while both of them take much longer time than the heuristic algorithms.

Referencess

1 Goldberg D E. Genetic Algorithms in Search, Optimization, and Machine Learning. Reading, Mass: Addison-Wesley, 1989

2 Gupta J N D. Heuristic algorithms for multistage flowshop scheduling problem. AIIE Transaction, 1972, 4(1): 11 – 18

3 Miyazaki S, Nishiyama N and Hashimoto F. An adjacent pairwise approach to the mean flowtime scheduling problem. J Operations Research Society of Japan, 1978, 21: 287 – 299

4 Ho J C and Chang Y L. A new heuristic for the n-job, M-machine flow-shop problem. European J Operational Research, 1991, 52(2): 194 – 202

5 Rajendran C. Heuristic algorithm for scheduling in a flowshop to minimize total flowtime. Int. J Production Económics, 1993, 29(1): 65 – 73

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