A Riccati Equation Approach to Stabilization of Uncertain Linear Systems with Time-Varying State Delay

Fang Yangwang and Han Chongzhao
(School of Electronic & Information Engineering, Xi'an Jiaotong University·Xi'an, 710049, P.R.China)

Abstract: This paper is concerned with the problem of quadratic stabilization based on observer for uncertain systems with time-varying state delay. Two classes of uncertainties are treated. By applying the Lyapunov stability theorem and norm inequalities, we prove several theorems about quadratic stabilization. The proposed state feedback quadratic stabilizer can be obtained by solving a Riccati equations. Finally, illustrative example is given to demonstrate the application of these criteria.

Key words: uncertain system; time-varying delay; quadratical stability; observer

不确定时变时滞系统稳定性的 Riccati 方程法

方洋旺 韩崇昭
(西安交通大学电信学院·西安,710049)

摘要: 讨论了不确定时变时滞系统的基于观测器的二次稳定性问题.有两类不确定性被讨论.利用 Lyapunov 稳定性定理及范数不等式,我们证明了几个关于二次稳定性的定理.通过求解 Riccati 方程可以获得系统可二次稳定的状态反馈控制器.最后,通过实例验证了上述结果.

关键词: 不确定系统; 时变时滞; 二次稳定性; 观测器

1 Introduction

Many techniques have been proposed to stabilize uncertain systems with state delay via state feedback in [1]~[3]. Most of the research on designing controllers to stabilize dynamic systems with state delay has been focused on the use of state feedback for the constant delay factor [4]. Variable delay was treated but some sort of decomposition (dynamic or quadatic) was employed in dealing with the uncertainies. In this paper, the problems of quadratic stabilization based on observer for continuous-time uncertain systems with time-varying state delay are discussed. Two classes of uncertainties are treated: 1) general normed bounded uncertainites and 2) satisfying matching condition uncertainities. By applying the Lyapunov stability theorem and norm inequalities, we obtain several theorems about quadratically stabilizable based on observer. Finally, typical pollution dynamic model in [1] is given to demonstrate the merit of the present schemes.

In the sequel, let W' be the matrix transpose of W and I be identity matrix. we denote $V > 0$ positive difinite square matrix of V, and B the Banach space of continuous functions $\varphi: [-d, 0] \to \mathbb{R}^n$ with $\|\varphi\|_*: = \sup\|\varphi(\alpha)\|$, where $\|\cdot\|$ is Euclidean norm. If $x: [-d, \tau] \to \mathbb{R}^n$ is continuous and $\tau > 0$, then we introduce $x_t \in B$ and $x_t(\alpha) = x(t+\alpha)$, $-d \leqslant \alpha \leqslant 0$.

2 System description

Let us consider the uncertain system

$$\begin{aligned} \dot{x}(t) = & [A + \Delta A(\theta(t))]x(t) + \\ & [A_1 + \Delta A_1(\theta(t))] \cdot x(t - d(t)) + \\ & [B + \Delta B(\theta(t))]u(t), \end{aligned} \tag{1a}$$

$$y(t) = Cx(t), \tag{1b}$$

$$x_{t_0}(t) = \phi(t),\ t \in [-d^*, 0]$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, are state vector, control vector, measure vector respectively. $\theta(t) \in \mathbb{R}^p$ is the uncertain element. $d(t)$ is any bounded function satisfying $0 \leqslant d(t) \leqslant d^* < \infty$, $\dot{d}(t) \leqslant \eta < 1$, $A_0 \in \mathbb{R}^{n\times n}$, $A_1 \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ are known constant metrices. $\Delta A(\cdot)$, $\Delta A_1(\cdot)$, $\Delta B(\cdot)$ are uncertain real function matrices, $\phi(t)$ is a continuous vector-valued initial function defined on $[-d^*, 0]$.

Manuscript received May 8, 1997, revised Sep. 8, 1998.

We now make the following assumptions.

A1 The uncertain vector $\theta(t):\mathbb{R}\to\Omega$ is continuous, where $\Omega\in\mathbb{R}^q$ is a prescribed compact subset of $\mathbb{R}^q$.

A2 The pair (A,B) is controlable.

Let
$$a:=\max_{\theta\in\Omega}\|\Delta A(\theta)\|,$$
$$b:=\max_{\theta\in\Omega}\|\Delta B(\theta)\|,$$
$$a_1:=\max_{\theta\in\Omega}\|\Delta A_1(\theta)\|.$$

3 Main results

Suppose the observer of system (1) is the following equation:

$$\dot z(t)=Az(t)=Bu(t)+rP_0C'(y-Cz(t)),\tag{2}$$

and the controller

$$u(t)=-\alpha B'Pz(t),\tag{3}$$

where $z(t)$ is the estimate of the actural state $x(t)$. Algebraic manipulation of Eqs.(1),(2),(3) together with the fact that $e(t)=x(t)-z(t)$ creates the closed-loop system:

$$\dot x=(A-\alpha BB'P)x+\alpha[B+\Delta B(\theta)]B'Pe+H(t),\tag{4a}$$

$$\dot e=(A-\gamma P_0C'C)e+\alpha\Delta B(\theta)B'Pe+H(t),\tag{4b}$$

where $H(t)=[\Delta A-\alpha\Delta B(\theta)B'P]x+[A_1+\Delta A_1(\theta)]x(t-d(t))$.

We introduce the following Lyapunov function candidate:

$$V(x,e)=e'P_0e+x'Px+\int_{t-d(t)}^{t}x(s)'Rx(s)\mathrm{d}s,\tag{5}$$

where P, P_0 and R are positive definite matrics.

Definition 3.1 System (1) is said to be quadratically stabilizable based on observer if there exists a feedback control $u(t)=-\alpha B'Pz(t)$, where $\alpha>0, P=P'>0$, and observer (2) such that the following condition is satisfied: given any admissible uncertainties $\Delta A(\cdot),\Delta A_1(\cdot)$, $\Delta B(\cdot)$, the Lyapunov derivation of $V(x,e)$ along the equations (4) satisfies the inequality

$$\dot V(x,e)=\dot x'Px+x'P\dot x+\dot e'Pe+e'P_0\dot e+x'Rx-(1-\dot d(t))x'(t-d(t))Rx(t-d(t))\leqslant-\varepsilon_c\|x\|^2-\varepsilon_0\|e\|^2,$$

where $\varepsilon_c>0$, $\varepsilon_0>0$, for all non-zero $x\in\mathbb{R}^n$ and all $t\in[0,\infty)$.

$$A'P+PA-PH_cP+T_c+\varepsilon_cI=0,\tag{6}$$

$$A'P_0+P_0A-P_0\dot H_0P_0+T_0+\varepsilon_0I=0,\tag{7}$$

where

$$H_c:=(2\alpha-2\beta b^2\alpha^2-\frac{1}{\beta})BB'-\frac{1}{\beta}(3+a^2+a_1^2),$$
$$T_0:=\alpha^2\beta(2b^2+1)PBB'P,$$
$$H_0:=2rC'C-\frac{1}{\beta}(3+a^2)I,$$
$$T_c:=2\beta I+3(1-\eta)^{-1}\beta(I+A_1'A_1),$$
$$\varepsilon_c>0,\varepsilon_0>0.$$

Lemma 3.1[5] For any matrices (or vector) X and Y with appropriate dimensions, we have $X'Y+Y'X\leqslant\beta X'X+\frac{1}{\beta}Y'Y$, where any $\beta>0$.

Theorem 3.2 Suppose system (1) satisfies assumption A1 and furthermore, there exist positive definite matrices P and P_0 which satisfy (6), (7) respectively. Then system (1) is quadratically stabilizable based on observer.

Proof To examine the stability of Eqs.(4), we introduce the following Lyapunov function candidate:

$$V(x,e)=e'Pe+x'Px+\int_{t-d(t)}^{t}x(s)'Rx(s)\mathrm{d}s,$$

where $R=3\beta(1-\eta)^{-1}(I+A_1'A_1),\beta>0$. The derivation of $V(x,e)$ along the trajectories of Eqs.(4) is given by:

$$\begin{aligned}\dot V(x,e)={}&\\ &\dot e'P_0e+e'P_0\dot e+\dot x'Px+x'P\dot x+x'Rx-\\ &(1-\dot d)x(t-d(t))'Rx(t-d(t))\leqslant\\ &e'P_0e+e'P_0e+x'Px+x'Px+x'Rx-\\ &(1-\eta)x(t-d(t))'Rx(t-d(t)).\end{aligned}$$

By lemma 3.1, we have

$$\dot V(x,e)\leqslant x'(A'P+PA-PH_cP+T_c)x+e'(A'P_0+P_0A-P_0H_0P_0+T_0)e,$$

where

$$H_c:=(2\alpha-2\beta b^2\alpha^2-\frac{1}{\beta})BB'-\frac{1}{\beta}(3+a^2+a_1^2)I,$$
$$T_c:=2\beta I+3(1-\eta)^{-1}\beta(I+A_1'A_1),$$
$$T_0:=\alpha^2\beta(2b^2+1)PBB'P,$$
$$H_0:=2rC'C-\frac{1}{\beta}(3+a^2)I.$$

If P and P_0 are the positive definite solution of the equations (6) and (7) respectively, then $\dot V(x,e)\leqslant-\varepsilon_c\|x\|-\varepsilon_0\|e\|<0$. The proof is completed.

Remark Parameters $\alpha,\beta,\gamma,\varepsilon_c$ and ε_0 can be adjusted independently. In practical use, parameters α,β are adjusted to guarantee $H_c > 0$ and γ is adjusted to guarantee $H_0 > 0$.

If system (1) satisfies the following matching coditions, we will give a sufficient condition about positive difinite solution existence of Eqs.(6) and (7) respectively. We make the following assumption:

A3 Matching conditions: $\Delta A(\theta) = BD(\theta), \Delta A_1(\theta) = BE(\theta), \Delta B(\theta) = BF(\theta), A_1 = BG$.

A4 rank $(C) = p$, where $C \in \mathbb{R}^{p\times n}$, then for any positive definite metrix P_0, there exists a matrix F, such that $FC = B'P_0$.

Let
$$d:=\max_{\theta\in\Omega}\|D(\theta)\|,$$
$$e:=\max_{\theta\in\Omega}\|E(\theta)\|,$$
$$f:=\max_{\theta\in\Omega}\|F(\theta)\|.$$

We consider the observer
$$\dot z(t) = Az + Bu + \gamma BF(y - Cz) = Az + Bu + \gamma BB'Pe, \tag{8}$$
where $e = x - z$. Algebraic manipulation of Eqs.(1), (4) together with A4 yields the following closed-loop system:
$$\dot x = (A - \alpha BB'P)x + \alpha BB'Pe + \alpha BF(\theta)B'Pe + H(t), \tag{9a}$$
$$\dot e = (A - \gamma BB'P_0)e + \alpha BF(\theta)B'Pe + H(t), \tag{9b}$$
where $H(t) = B[D(\theta) - \alpha F(\theta)B'P]x + B[G + E(\theta)]x(t - d(t))$.

Lemma 3.3 when A2 and inequality
$$f^2(4 + d^2 + e^2) < 1 \tag{10}$$
holds, then for some $\alpha > 0, \beta > 0$ and $\gamma > 0$, the following equations
$$A'P + PA - PBH_cB'P + T_c + \varepsilon_c I = 0, \tag{11}$$
$$A'P_0 + P_0A - P_0BH_0B'P_0 + T_0 + \varepsilon_0 I = 0, \tag{12}$$
where
$$H_c:=[2\alpha - \beta f^2\alpha^2 - \frac{1}{\beta}(4 + d^2 + e^2)]I,$$
$$H_0:=[2\gamma - \frac{1}{\beta}(3 + d^2 + e^2)]I,$$
$$T_c:=2\beta[I + (1-\eta)^{-1}(I + G'G)],$$
$$T_0:=\beta\alpha^2(3f^2 + 1)PBB'P,$$
and $\varepsilon_c > 0, \varepsilon_0 > 0$, have positive definite solutions P and P_0 respectively.

Proof When inequality (10) holds, the equation $2\alpha - \beta f^2\alpha^2 - \frac{1}{\beta}(4 + d^2 + e^2) = 0$, has two roots $\alpha_{1,2} = \frac{1 \pm \sqrt{1 - f^2(4 + d^2 + e^2)}}{\beta f^2} > 0$. For any $\alpha > 0$ which satisfies $\alpha_1 < \alpha < \alpha_2$, then
$$H_c:=[2\alpha - \beta f^2\alpha^2 - \frac{1}{\beta}(4 + d^2 + e^2)]I > 0.$$
In addition, when $\gamma > 1/2\beta(3 + d^2 + e^2)$, $H_0:=[2\gamma - \frac{1}{\beta}(3 + d^2 + e^2)]I > 0$. By assumption A2, $T_c + \varepsilon_c I > 0$, $T_0 + \varepsilon_0 I > 0$ and theorem[6,pp71], then the equations (11) and (12) have positive definite solution P and P_0 respectively.

Theorem 3.4 Suppose system (1) satisfies assumptions A1 ~ A4 and inequality (10), then Eqs.(11) and (12) have positive definite solutions P and P_0 respectively. So system (1) is quadratically stabilizable based on observer.

Proof To examine the stability of system (9), we consider the following Lyapunov function candidate:
$$V(x,e) = e'P_0e + x'Px + \int_{t-d(t)}^{t} x'(s)Rx(s)\mathrm{d}s,$$
where $R = 2(1-\eta)^{-1}\beta(I + G'G)$. The derivative of $V(x,e)$ along the trajectories of Eqs.(9) is given by:
$$\dot V(x,e) = \dot e'P_0e + e'P_0\dot e + \dot xPx + x'P\dot x + x'Rx - (1 - \dot d)x'(t - d(t))Rx(t - d(t)),$$
where
$$\dot e'P_0e + e'P_0\dot e = e'(A'P_0 + P_0A)e - 2\gamma e'P_0BB'P_0e + 2\alpha e'PBF(\theta)'B'P_0e + 2H(t)'P_0e,$$
$$\dot x'Px + x'P\dot x = x'(A'P + PA)x - 2\alpha x'PBB'Px + 2\alpha e'PBB'Px + 2\alpha e'PBF(\theta)'B'Px + 2H(t)'Px.$$
By Lemma 3.1, we have
$$\dot V(x,e) \leqslant x'\{AP + PA - P[2\alpha - \beta f^2\alpha^2 - \frac{1}{\beta}(4 + d^2 + e^2)]BBP + 2\beta[(I + (1-\eta)^{-1}(I + GG)]\}x + e'\{A'P_0 + P_0A - P_0[2\gamma - \frac{1}{\beta}(3 + d^2 + e^2)]BB'P_0 + \beta\alpha^2(3f^2 + 1)PBB'P\}e.$$
Let
$$H_c:=[2\alpha - \beta f^2\alpha^2 - \frac{1}{\beta}(4 + d^2 + e^2)]I,$$
$$T_c:=2\beta[I + (1-\eta)^{-1}(I + G'G)],$$
$$H_0 = [2\gamma - \frac{1}{\beta}(3 + d^2 + e^2)]I,$$

$$T_0: = \beta\alpha^2(3f^2+1)PBB'P.$$

By Lemma 3.3 and A2, the equations (11) and (12) have the positive definite solutions P and P_0 respectively, then $\dot{V}(x, e) \leqslant -\varepsilon_c \| x \|^2 - \varepsilon_0 \| e \|^2 < 0$. Proof is completed.

4 Examples

We consider the following river pollution model in [1]:

$$A = \begin{bmatrix} -2 & 0 \\ -1 & -2 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.6 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.5 \end{bmatrix},$$

$$\Delta A = \begin{bmatrix} 0.08\cos 3t & 0 \\ -0.08\sin 3t & 0.05\sin 5t \end{bmatrix},$$

$$\Delta A_1 = \begin{bmatrix} -0.01\sin 5t & 0 \\ 0.02\sin 2t & -0.01\sin t \end{bmatrix},$$

$$\Delta B = \begin{bmatrix} 0.01\sin 5t & 0 \\ 0 & 0.01\sin 5t \end{bmatrix},$$

for $\varepsilon_c = \varepsilon_0 = 1$, $\alpha = 2.4$, $\beta = 2.8$, $\gamma = 1.5$, the solutions of Eqs.(11) and (12) are

$$P = \begin{bmatrix} 3.91 & -0.16 \\ -0.16 & 2.11 \end{bmatrix}, P_0 = \begin{bmatrix} 4.69 & -0.33 \\ -0.33 & 3.24 \end{bmatrix}$$

respectively. The gains are $K_c = \begin{bmatrix} 5.63 & -0.23 \\ -0.39 & 5.06 \end{bmatrix}$,

$K_0 = \begin{bmatrix} 8.44 & -0.74 \\ -0.59 & 7.29 \end{bmatrix}$ respectively.

5 Conclusion

Design of linear high-gain controllers to stabilize uncertain time-varying delay system based on observer has been developed in this paper. The uncertainties and delay factors are considered unknown but bounded, the design approach is essentially based on the construtive use of Lyapunov functionals. The example on typical pollution model has been performed.

References

1 Mahmoud M S. Output feedback stabilization of uncertain systems with state delay. Control and Dynamic Systems, 1994, 63(2): 197-257

2 Petersen I R and Hollot C V. A Riccati equation approach to the stabilization of uncertain linear systems. Automatica, 1990, 35(3): 357-361

3 Mahmoud M S and Al-Muthairi N F. Design of robust controllers for time-delay systems. IEEE Trans. Automat. Contr., 1996, 41(3): 450-452

4 James Louisell. Absolute stability in linear delay —— differential systems: III, posedness and robustness. IEEE Automat. Contr., 1996, 41(3): 453-456

5 Feng G, Cao S G and Rees N W. An approach to H_∞ control of a class of nonlinear system. Automatica, 1996, 32(10): 1469-1474

6 吴受章.应用最优控制.西安:西安交通大学出版社,1987

本文作者简介

方洋旺 1966年生.1990年硕士毕业于陕西师范大学数学系,现为西安交通大学电信学院博士生.研究兴趣主要为非线性控制系统理论,H_∞控制,鲁棒控制等.

韩崇昭 1943年生.1968年毕业于西安交通大学电机工程系,1981年毕业于中国科学院研究生院并获硕士学位,现为西安交通大学电信学院副院长,教授,博士生导师.研究兴趣主要为随机与自适应控制理论,工业过程控制与稳态优化,非线性系统频域分析等.