

A Model Reference Adaptive Control without Strictly Positive Real Condition

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Abstract: This paper considers the model reference adaptive control problem for the system (1) with unmodeled dynamics η_n satisfying (2). Using bounded external excitation and randomly varying truncation techniques, we give design method of a model reference adaptive controller which is similar to that in [1]. It is shown that the closed-loop system is globally stable, the estimation error for parameter contained in the modeled part is of order ϵ , and the closed-loop system under the model reference adaptive control law is suboptimal in the sense of $\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^n (A_m(z)y_{i+1} - B_m(z)z_i^* - C_m(z)w_{i+1})^2 \leq O(\epsilon^2) + \gamma^2 b_1^2$, while the SPR condition usually used in other papers is replaced by a stability condition.

Key words: SPR condition; model reference adaptive control; unmodeled dynamics

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一种不需要严正实条件的模型参考自适应控制

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摘要: 对于具有未建模动态 η_n 的系统(1)和(2), 考虑了模型参考自适应控制问题. 利用有界的外部激励和随机变向截尾技术, 我们给出了一种类似于[1]的模糊参考自适应控制器的设计. 对于这种控制器, 我们证明了闭环系统是全局稳定的, 建模部分的参数估计是 $O(\epsilon)$, 且这种模型参考自适应控制器在 $\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^n (A_m(z)y_{i+1} - B_m(z)z_i^* - C_m(z)w_{i+1})^2 \leq O(\epsilon^2) + \gamma^2 b_1^2$ 意义下是次优的, 而其他文献中用到的严正实条件减弱为稳定性条件.

关键词: 严正条件; 模型参考自适应控制; 未建模动态

1 Introduction

Consider the following stochastic system with unmodeled dynamics η_n :

$$A(z)y_n = B(z)u_n + C(z)w_n + \eta_{n-1}, \quad n \geq 1, \quad (1)$$

where $\{y_n\}$, $\{u_n\}$ and $\{w_n\}$ are the system output, input, and random disturbance sequences, respectively, $y_n = u_n = w_n = 0$ for all $n < 0$, and $A(z)$, $B(z)$, and $C(z)$ are polynomials in backward shift operator z :

$$A(z) = 1 + a_1 z + \cdots + a_p z^p, \quad p \geq 0,$$

$$B(z) = b_1 z + \cdots + b_q z^q, \quad q \geq 1,$$

$$C(z) = 1 + c_1 z + \cdots + c_r z^r, \quad r \geq 0,$$

with known upper bounds p , q , and r for true orders and unknown coefficients a_i , b_j , and c_k , $i = 1, 2, \dots, p$, $j =$

$1, 2, \dots, q$, $k = 1, 2, \dots, r$. The unmodeled dynamics η_n is \mathcal{F}_n -measurable and assumed to be dominated by

$$|\eta_n| \leq \epsilon \sum_{i=0}^n a^{n-i} (|y_i| + |u_i| + |w_i| + 1), \quad (2)$$

with $a \in (0, 1)$, $\epsilon \geq 0$.

In a large number of theory results for parameter estimation and adaptive control for ARMAX model, in order to treat the correlated noise, we need to impose the following strictly positive real (SPR) condition:

$$C^{-1}(e^{i\lambda}) + C^{-1}(e^{-i\lambda}) - 1 > 0, \quad (3)$$
$$\forall \lambda \in [0, 2\pi], \quad i^2 = -1$$

on system (1) when ELS algorithm is applied. Since SPR condition is very restrictive, one has been trying to

relax it, the relative papers can be referred to [2~6].

All of these papers, however, concerns exclusively with time-invariant systems without the unmodeled dynamics η_n .

This paper will relax the SPR condition for the system (1) and (2) that contains a very large class of systems from model reference adaptive control view of point. As we know, the analysis of the problem has been recognized as difficult and complicated due to the following two reasons:

1) Even small bounded disturbances may cause adaptive control algorithms to go unstable, if other precautions are not taken, to say nothing of η_n tending to infinity as the past input, output, and noise grow.

2) An important property of $\sum_{i=0}^n (w_i - \hat{w}_i)^2$ (\hat{w}_i is an estimate of w_i) obtained by the SPR condition is difficult to be produced only by the stability condition (i.e., $C(z)$ is stable).

Just based on these reasons, we think that it is a meaningful work to analyze the influence of unmodeled dynamics contained in a system upon the behavior of model reference adaptive control systems (such as stability, control performance, and accuracy of parameter estimation for the modeled part of system (1)) under the assumption of stability.

In order to weaken the SPR condition, we use the "two-step" estimation algorithm^[7]. In the first step of the algorithm, the following important property of the noise estimates is produced under a stability condition (i.e. $C(z)$ is a stable polynomial).

$$\sum_{i=0}^n (w_i - \hat{w}_i)^2 = O(\epsilon^2)n + o(n), \quad (4)$$

where \hat{w}_i is an estimate of w_i . In the second step, we prove that the estimation error for parameter contained in the modeled part is of order ϵ . Then using bounded external excitation and randomly varying truncation techniques, we give design method of a model reference adaptive controller which is similar to that in [1], and prove that the closed-loop system is stable and the model reference adaptive control law is suboptimal in the sense of

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^n (A_m(z)y_{i+1} - B_m(z)z_i^* - C_m(z)w_{i+1})^2 \leq$$

$$O(\epsilon^2) + \gamma^2 b_1^2.$$

This paper is organized as follows: Section 2 gives design method of a robust model reference adaptive controller for stochastic systems with unmodeled dynamics η_n , and main results in this paper. Conclusion is presented in Section 3.

2 Design of a robust model reference adaptive controller

A1) $\{w_n, \mathcal{F}_n\}$ is a martingale difference sequence defined on the probability space (Ω, \mathcal{F}, P) with

$$\sup_{n \geq 0} E[w_{n+1}^2 | \mathcal{F}_n] < \infty, \text{ a.s.},$$

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n w_i^2 = \sigma^2 < \infty, \text{ a.s.}, \quad (5)$$

where $\{\mathcal{F}_n\}$ is a nondecreasing family of σ -algebras which will be defined later.

A2) $A(z) \neq 0, B(z) \neq 0$, and $C(z) \neq 0$ for all $z: |z| \leq 1$, and the upper bounds for the following quantities are available:

$$\|\theta\|, \sup_{|z| \leq 1} |A^{-1}(z)B(z)|,$$

$$\sup_{|z| \leq 1} |A^{-1}(z)C(z)|, \sup_{|z| \leq 1} |zB^{-1}(z)A(z)|,$$

$$\sup_{|z| \leq 1} |zB^{-1}(z)C(z)|, \sup_{|z| \leq 1} |A_m^{-1}(z)B_m(z)|,$$

$$\sup_{|z| \leq 1} |A_m^{-1}(z)C_m(z)|, \sup_{|z| \leq 1} |A_m^{-1}(z)b_1|,$$

where $A_m(z), B_m(z)$ and $C_m(z)$ are defined later. These bounds are denoted by $\lambda, k_{AB}, k_{AC}, k_{BA}, k_{BC}, k_{A_m B_m}, k_{A_m C_m}$, and $k_{A_m b_1}$.

A3) $A(z)$ and $B(z)$ are coprime with

$$|a_p| + |b_q| \neq 0.$$

Since $C(z)$ is stable, the following conclusion holds:

$$C^{-1}(z)A(z) = 1 + \sum_{i=1}^h f_i z^i, \quad C^{-1}(z)B(z) = \sum_{i=1}^h g_i z^i, \quad (6)$$

with $|f_i| = O(\rho^i)$ and $|g_i| = O(\rho^i), i \geq 0$ for some $\rho \in (0, 1)$.

This conclusion is established in [8]. Set

$$F(z) = 1 + \sum_{i=1}^h f_i z^i, \quad G(z) = \sum_{i=1}^h g_i z^i, \quad (7)$$

where h is taken appropriately large positive integer such that the following assumption holds

$$A4) \quad h \geq \frac{|\ln \epsilon|}{|\ln \rho|} + \frac{\ln(1 - \rho)}{\ln \rho} - 1,$$

where ϵ is determined in (2). For $F(z)$ and $G(z)$, we need the following assumption.

A5) $F(z)$ and $G(z)$ are coprime with

$$|f_h| + |g_h| \neq 0.$$

It is worth noting that A5) is not implied by A3) and vice versa.

Now we give two-step estimation algorithm. Step 1 is used to estimate random disturbance sequence $\{w_n\}$, and Step 2 is used to estimate the unknown parameter vector θ .

Step 1 The algorithm for estimating random disturbance sequence $\{w_n\}$

$$\hat{w}_n = y_n - a_n^T \psi_n, \quad (8)$$

$$a_{n+1} = a_n + b_n S_n \psi_n (\gamma_{n+1} - a_n^T \psi_n), \quad (9)$$

$$S_{n+1} = S_n - b_n S_n \psi_n \psi_n^T S_n, \quad (10)$$

$$b_n = \frac{1}{1 + \psi_n^T S_n \psi_n}, \quad (11)$$

$$\psi_n = (\gamma_n, \dots, \gamma_{n-h+1}, u_n, \dots, u_{n-h+1})^T, \quad (12)$$

$$r_n = 1 + \sum_{i=0}^n \|\psi_i\|^2, \quad (13)$$

for arbitrary initial values $S_0 > 0$ and a_0 , where

$$a_n = (-f_1(n), \dots, -f_h(n), g_1(n), \dots, g_h(n))^T$$

is the estimation of $a = (-f_1, \dots, -f_h, g_1, \dots, g_h)^T$.

Step 2 The algorithm for estimating the unknown parameter vector θ .

$$\theta_{n+1} = \theta_n + c_n P_n \psi_n (\gamma_{n+1} - \theta_n^T \psi_n), \quad (14)$$

$$P_{n+1} = P_n - c_n P_n \psi_n \psi_n^T P_n, \quad (15)$$

$$c_n = \frac{1}{1 + \psi_n^T P_n \psi_n}, \quad (16)$$

$$\varphi_n = (\gamma_n, \dots, \gamma_{n-p+1}, u_n, \dots, u_{n-q+1}, \hat{w}_n, \dots, \hat{w}_{n-r+1})^T, \quad (17)$$

$$\theta_n = (-a_1(n), \dots, -a_p(n), b_1(n), \dots, b_q(n), c_1(n), \dots, c_r(n))^T, \quad (18)$$

where the initial values $P_0 > 0$ and θ_0 are chosen arbitrarily, θ_n is an estimate of $\theta = (-a_1, \dots, -a_p, b_1, \dots, b_q, c_1, \dots, c_r)^T$, \hat{w}_n is obtained by (8).

Now let us consider the design of model reference controller. Consider the following model reference control objective: Find a sequence of control $\{u_k\} \in \gamma_k \triangleq \{\gamma_0, \dots, \gamma_k, u_0, u_{k-1}\}$, which make the resulting closed loop system become

$$A_m(z) y_n = B_m(z) z_n^* + C_m(z) w_n + C_m(z) \eta_{n-1}, \quad (19)$$

where

$$A_m(z) = 1 + \bar{a}_1 z + \dots + \bar{a}_{\bar{p}} z^{\bar{p}}, \quad \bar{p} \geq 0,$$

$$B_m(z) = \bar{b}_1 + \dots + \bar{b}_{\bar{q}} z^{\bar{q}-1}, \quad \bar{q} \geq 1,$$

$$C_m(z) = \bar{c}_0 + \bar{c}_1 z + \dots + \bar{c}_{\bar{r}} z^{\bar{r}}, \quad \bar{r} \geq 0,$$

and $\{z_n^*\}$ is a sequence of bounded reference input, i. e., there exists a constant $l > 0$ such that $|z_n^*| \leq l$, $\forall n \geq 0$.

For any polynomial $H(z) = \sum_{i=0}^n h_i z^i$, we first define

$$H^0(z) \triangleq h_0, \quad zH^1(z) \triangleq H(z) - H^0(z). \quad (20)$$

Lemma 1^[1] Suppose that a sequence of input $\{u_n\} \in \gamma_n$ exists which make the closed loop system satisfy (19). Then it is necessary that the relation

$$C_m^0(z) = A_m^0(z) \quad (21)$$

be satisfied. Conversely, if (21) is satisfied, the control law obtained by solving the following equation

$$C_m(z) (\theta^T \varphi_n^0) = (C_m(z) - A_m(z)) \gamma_{n+1} + B_m(z) z_n^*, \quad (22)$$

for u_n results in the closed loop system (19), with

$$u_n \in \gamma_n,$$

$$\varphi_n^0 = (\gamma_n, \dots, \gamma_{n-p+1}, u_n, \dots, u_{n-q+1}, w_n, \dots, w_{n-r+1})^T.$$

By Lemma 1, in order to achieve model reference control objective, $\bar{c}_0 = 1$ is required. Since θ is unknown, then the model reference adaptive control law is chosen as

$$u_n^1 = (b_1 n + \delta_n)^{-1} [-A_m^1(z) \gamma_n + C_m^1(z) \hat{w}_n + B_m(z) z_n^* + (b_1 n u_n - \theta_n^T \varphi_n)], \quad (23)$$

where

$$\delta_n \triangleq \delta (\log^{\frac{1}{2}} r_n)^{-1} \operatorname{sgn}(b_{1n}) I_{[|b_{1n}| < \delta (\log^{\frac{1}{2}} r_n)^{-1}]}, \quad (24)$$

b_{1n} is an estimate of b_1 at the time n , $\operatorname{sgn}(\cdot)$ is sign function, and

$$I_{[|x| < c]} \triangleq \begin{cases} 1, & |x| < c, \\ 0, & |x| \geq c. \end{cases}$$

$A_m^1(z)$ and $C_m^1(z)$ are defined as (20), δ is a fixed positive number.

Obviously, by (24), we can obtain the following properties:

$$(b_{1n} + \delta_n)^2 \geq \frac{\delta^2}{\log r_n}, \quad n \geq 0, \quad (25)$$

and

$$|(b_{1n} + \delta_n) - b_{1n}| \leq \frac{\delta}{\log^{\frac{1}{2}} r_n}, \quad n \geq 0. \quad (26)$$

Obviously, the control law (23) is well-defined.

Remark 1 The reason why the control law is taken as (23) instead of $C_m(z)(\theta_n^T \varphi_n) = (C_m(z) - A_m(z))\gamma_{n+1} + B_m(z)z_n^*$ defined by the certainty equivalence principle can be found in [1], and by simple operation, (23) is equivalent to (4.5) in [1] when $d = 1$ (d is time-delay).

As explained in [8], some robustness in the estimation algorithm can only be guaranteed for persistently exciting signals. Let $\{v_n\}$ be a sequence of independent and identically distributed random variables independent of $\{w_n\}$ and such that

$$E_{v_n} = 0, E_{v_n^2} = \mu, \mu > 0, |v_n| \leq \gamma, n \geq 0. \quad (27)$$

Without loss of generality, we take that

$$\mathcal{F}_n = \sigma\{w_i, v_i, i \leq n\}$$

and set

$$\mathcal{F}_n = \sigma\{w_i, v_{i-1}, i \leq n\}.$$

Let us define two sequences of stopping time $\{\tau_k\}$ and $\{\sigma_k\}$ with

$$1 = \tau_1 < \sigma_1 < \tau_2 < \sigma_2 < \dots \quad (28)$$

as follows

$$\begin{aligned} \sigma_k &= \sup\{t > \tau_k : \sum_{i=\tau_k}^{t-1} |u_i^1|^2 c \leq M_0(j-1) + |u_{\tau_k}^1|^2, \\ &\quad \forall j \in (\tau_k, t]\}, \\ \tau_{k+1} &= \inf\{t > \sigma_k : 2^k \sum_{i=\tau_k}^{t-1} |u_i^1|^2 \leq t, |u_{\tau_k}^1|^2 \leq M_1 t\}, \end{aligned} \quad (29)$$

where u_n^1 is defined as (23), and M_0 and M_1 are chosen to satisfy

$$\begin{aligned} M_0 &\geq [16((4k_{AB}^2 \|\theta_1\|^2 + (1 + \|\theta\|^2)(1 + 4k_{AB}^2)) + \\ &2)\{(2\sigma^2 + 8k_{BC}^2\sigma^2 + 1 + 16k_{BA}^2 \cdot \\ &[10(k_{AC}^2\sigma^2 + k_{AB}^2l^2 + k_{AB}^2\gamma^2) + \\ &8k_{AB}^2(\gamma^2 + 1) + 4k_{AC}^2\sigma^2 + \frac{1}{2}]) + \\ &(4k_{AC}^2 \|\theta_1\|^2 + 2\|\theta_1\|^2 + 2(1 + \|\theta\|^2)(1 + \\ &2k_{AC}^2))\sigma^2 + 2(1 + \gamma^2)\}, \end{aligned} \quad (31)$$

$$\begin{aligned} M_1 &\geq [16(4k_{AB}^2 \|\theta_1\|^2 + (1 + \|\theta\|^2)(1 + 4k_{AB}^2))(1 + \\ &\gamma^2) + (4k_{AC}^2 \|\theta_1\|^2\sigma^2 + 2\|\theta_1\|^2\sigma^2 + \\ &2(1 + \|\theta\|^2)(1 + 2k_{AC}^2\sigma^2) + 2], \end{aligned} \quad (32)$$

where $\theta_1 = (\bar{a}_1, \dots, \bar{a}_p, \bar{c}_1, \dots, \bar{c}_r)^T$, \bar{a}_i and \bar{c}_j are coefficients of $A_m(z)$ and $C_m(z)$, respectively.

The adaptive control law is defined as

$$u_n = u_n^0 + v_n, \quad (33)$$

with v_n satisfying (27) and u_n^0 is given as

$$u_n^0 = \begin{cases} u_n^1, & \text{if } u \in [\tau_k, \sigma_k), \\ 0, & \text{if } u \in [\sigma_k, \tau_{k+1}), \end{cases} \quad (34)$$

where u_n^1 is defined as (23).

Remark 2 In the presence of unmodeled dynamics, in order to obtain both the robust parameter estimation and the robust model reference adaptive control simultaneously, persistently exciting signals $\{v_n\}$ is added to the truncated input u_n^0 . In the case of $\eta_n = 0$ for all $n \geq 0$, v_n can be chosen as a random dither tending to zero. For the idea of the design of the adaptive control law, a detailed explanation is given in [8].

Now we are in the position to give main results of the paper.

Theorem 1 For the system (1) and (2), the model reference adaptive control law is defined by (23), (24), and (27) - (34) with $\{\theta_n\}$ produced by two-step algorithm. If Assumptions A1) ~ A5) hold, then there exists a constant $\epsilon^* > 0$, such that for any $\epsilon \in [0, \epsilon^*)$, the closed-loop system has the following properties:

$$\begin{aligned} 1) & \limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^n (u_i^2 + \gamma_i^2) < c, \text{ a.s.}; \\ 2) & \limsup_{n \rightarrow \infty} \|\theta_n - \theta\| < c_1 \epsilon, \text{ a.s.}; \\ 3) & \limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^n (A_m(z)\gamma_{i+1} - B_m(z)z_i^* - C_m(z)w_{i+1})^2 \leq \\ & c_2 \epsilon^2 + \gamma^2 b_1^2, \text{ a.s.}, \end{aligned}$$

where c , c_1 and c_2 are constants independent of ϵ .

Remark 3 In the absence of unmodeled dynamics $\eta_n(\epsilon = 0)$, we can adopt attenuating excitation technique^[7] such that

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^n v_i^2 = 0,$$

i.e. the model reference adaptive control law is optimal in the sense of

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n (A_m(z)\gamma_{i+1} - B_m(z)z_i^* - C_m(z)w_{i+1})^2 =$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^n (b_1 v_i)^2 = 0.$$

3 Conclusion

This paper considers the model reference adaptive

control problem for system (1) with unmodeled dynamics η_n satisfying (2). Using bounded external excitation and randomly varying truncation techniques, we give design method of a model reference adaptive controller which is similar to that in [1]. It is shown that the closed-loop system is globally stable, the estimation error for parameter contained in the modeled part is of order ϵ , and the closed-loop system under the model reference adaptive control law is suboptimal in the sense of

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^n (A_m(z)y_{i+1} - B_m(z)z_i^* - C_m(z)w_{i+1})^2$$

$\leq O(\epsilon^2) + \gamma^2 b_1^2$, while the SPR condition is replaced by a stable polynomial. Meanwhile simulation verifies the result. The result on robust adaptive pole-placement control without SPR condition can be found in [12].

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