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Disturbance Decoupling for Periodic and Multirate Systems *

SUN Zhendong

(The Seventh Research Division, Beijing University of Aeronautics and Astronautics, 100083, P.R.China)

Abstract: We address the disturbance decoupling problem of periodic and multirate discrete-time systems via synchronous controllers. A theory of multifeedback controlled invariant subspaces is developed. Based on this theory, a constructive solution for the solvability of the disturbance decoupling problem is presented. A complete characterization of the friend set of a given multifeedback controlled invariant subspace and a partial characterization of the DDP-PTD solutions are offered.

Key words: periodic and multirate systems; disturbance decoupling; multifeedback controlled invariant subspace

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周期多频采样系统的干扰解耦

孙振东

(北京航空航天大学第七研究室·北京, 100083)

摘要: 研究了利用同步控制器解决周期多频采样系统的干扰解耦问题.为此,首先提出多反馈受控不变子空间的概念,并给出其基本性质.基于这一框架,得到周期多频采样系统干扰解耦问题易验证的可解条件,并利用参数化方法刻划了解耦反馈集合.

关键词: 周期多频采样系统; 干扰解耦; 多反馈受控不变子空间

1 Introduction

Consider a periodic linear, discrete-time multirate system with disturbance

$$\begin{cases} x(t+1) = A(t+1)x(t) + B(t+1)u(t) + \\ \qquad\qquad Dv(t),\ t = 0,1,\cdots, \\ y(kT) = Cx(kT),\ k = 0,1,\cdots, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ are state variables, $u(t) \in \mathbb{R}^q$ are inputs, $v(t) \in \mathbb{R}^l$ are unknown disturbances, and $y(kT) \in \mathbb{R}^p$ are outputs. The matrices $A(t), B(t), C$ and D are of appropriate dimensions. $A(t+kT) = A(t) \overset{\text{def}}{=} A_t$, $B(t+kT) = B(t) \overset{\text{def}}{=} B_t$, $k = 1,2,\cdots$. T is the period. Note that the outputs are measured once during a period, so system (1) is named a multirate model.

The purpose of this paper is to study the disturbance decoupling problem for system (1). The controllers used here are of the form

$$\begin{aligned} &u(jT+i-1) = F_i x(jT+i-1), \\ &i = 1,\cdots,T,\ j = 0,1,\cdots. \end{aligned} \tag{2}$$

Recall that the periodically time-varying controllers like (2) were utilized to multirate control for time-invariant linear systems in numerous literature, see, References [1] and [2].

Disturbance decoupling problem for periodic and multirate discrete-time linear systems (DDP-PMD). Find, if possible, a sequence of feedback matrices $F_1,\cdots,F_T$, such that controller (2) render the outputs $y(kT)$, $k = 1,2,\cdots$, of system (1) independent of the disturbances $v(t)$, $t = 0,1,\cdots$. The corresponding ordered feedback matrices, if they exist, $(F_1,\cdots,F_m)$ will be called a solution of DDP-PMD.

For the case of $T = 1$, system (1) is time-invariant, and the corresponding disturbance decoupling problem has been completely solved in the framework of Wonham's geometric approach[3]. The solution was given in terms of the concept of (A,B)-invariant subspace which plays a key role in Wonham's theory.

The single-rate version of DDP-PMD, in which the

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outputs are sampled at every step and are required to be decoupled from the disturbances, was solved by Grasselli and Longhi in [4]. See also References [5] and [6]. Note that DDP-PMD is not a substitute for DLP of [4]. See Reference [1] for a brief discussion of a related engineering background.

Inspired by Wonham's work, we aim at extending the standard geometric framework so as to incorporate our problem. To do this, the concept of multifeedback controlled invariant subspace is formulated. A criterion and an algorithm are provided for calculationg the maximal multifeedback controlled invariant subspace contained in a given subspace. Based on these results, a solution for the DDP-PMD is obtained.

2 Multifeedback controlled invariant subspaces

In what follows, $\mathscr{B}_i = \mathrm{Im}B_i$ is the image of map B_i, $\mathbb{N}$ is the set of natural numbers, and W, V, K are certain subspaces of $\mathbb{R}^n$. Given a matrix $A \in \mathbb{R}^{n\times n}$, we have the kernel of A $\ker A = \{x \in \mathbb{R}^n : Ax = 0\}$ and $A^{-1}W = \{x \in \mathbb{R}^n : Ax \in W\}$. Define $\boldsymbol{A} = (A_1, \cdots, A_T)$, $\boldsymbol{B} = (B_1, \cdots, B_T)$.

Definition 1 Suppose V is a subspace of $\mathbb{R}^n$, we call V an $(\boldsymbol{A}, \boldsymbol{B})$-invariant subspace, if there exists a sequence of linear mapplings $F_1, \cdots, F_T$, all from $\mathbb{R}^n$ to $\mathbb{R}^q$, to satisfy

$$(A_T + B_T F_T)(A_{T-1} + B_{T-1}F_{T-1})\cdots(A_1 + B_1F_1)V \subseteq V.$$

If $T = 1$, then the $(\boldsymbol{A}, \boldsymbol{B})$-invariant subspace coincides with the standard (A, B)-invariant subspace.

Now we present a simple lemma which will play a fundamental role in the following derivations.

Lemma 1 Suppose W, V are two subspaces of $\mathbb{R}^n$. Then the necessary and sufficient condition for the existence of a matrix $F: \mathbb{R}^n \to \mathbb{R}^q$ such that $(A + BF)W \subseteq V$ is

$$AW \subseteq V + \mathscr{B}.$$

It is well known that a criterion for V to be an (A, B)-invariant subspace is $AV \subseteq V + \mathscr{B}$. A similar criterion exists for $(\boldsymbol{A}, \boldsymbol{B})$-invariant subspace

Theorem 1 V is an $(\boldsymbol{A}, \boldsymbol{B})$-invariant subspace if and only if

$$A_T A_{T-1}\cdots A_1 V \subseteq V + \mathscr{B}_T + A_T\mathscr{B}_{T-1} + \cdots + A_T A_{T-1}\cdots A_2\mathscr{B}_1.$$

Proof Necessity. Assume that the matrices $F_1, \cdots, F_T$ are taken such that

$$(A_T + B_TF_T)\cdots(A_1 + B_1F_1)V \subseteq V.$$

Expanding the above expression, the necessity follows.

Sufficiency. By using

$$(A_T\cdots A_1)V \subseteq (V + \mathscr{B}_T + \cdots + A_T\cdots A_3\mathscr{B}_2) + \mathrm{Im}(A_T\cdots A_2B_1)$$

and Lemma 1, we know that there exists a matrix $F_1: \mathbb{R}^n \to \mathbb{R}^q$, such that

$$(A_T\cdots A_1 + A_T\cdots A_2B_1F_1)V \subseteq V + \mathscr{B}_T + \cdots + \mathrm{Im}(A_T\cdots A_3B_2).$$

Let $W_1 = (A_1 + B_1F_1)V$. We further have

$$A_T\cdots A_2W_1 \subseteq (V + \mathscr{B}_T + \cdots + A_T\cdots A_4\mathscr{B}_3) + A_T\cdots A_3\mathscr{B}_2.$$

Utilizing Lemma 1, we may find a matrix $F_2: \mathbb{R}^n \to \mathbb{R}^q$, such that

$$(A_T\cdots A_2 + A_T\cdots A_3B_2F_2)W_1 \subseteq V + \mathscr{B}_T + \cdots + A_T\cdots A_4\mathscr{B}_3.$$

Define $W_2 = (A_2 + B_2F_2)W_1$.

Proceed the above discussion repeatedly. We may obtain $F_i: \mathbb{R}^n \to \mathbb{R}^q$, and $W_i \subseteq \mathbb{R}^n$, satisfying $W_i = (A_i + B_iF_i)W_{i-1}$, $i = 3, \cdots, T-1$, and

$$A_T\cdots A_{i+1}W_i \subseteq V + \mathscr{B}_T + \cdots + A_T\cdots A_{i+2}\mathscr{B}_{i+1},\quad i = 3, \cdots, T-1.$$

It is obvious that

$$W_{T-1} = (A_{T-1} + B_{T-1}F_{T-1})\cdots(A_1 + B_1F_1)V.$$

By using $A_TW_{T-1} \subseteq V + \mathscr{B}_T$, we can find a linear map $F_T: \mathbb{R}^n \to \mathbb{R}^q$, such that

$$(A_T + B_TF_T)W_{T-1} \subseteq V.$$

Therefore,

$$(A_T + B_TF_T)\cdots(A_1 + B_1F_1)V \subseteq V.$$

Denote the set of $(\boldsymbol{A}, \boldsymbol{B})$-invariant subspaces to be $\mathscr{I}(\boldsymbol{A}, \boldsymbol{B}; \mathbb{R}^n)$. Given $V \in \mathscr{I}(\boldsymbol{A}, \boldsymbol{B}; \mathbb{R}^n)$, define

$$\mathscr{F}(\boldsymbol{A}, \boldsymbol{B}; V) = \{(F_1, \cdots, F_T): F_i: \mathbb{R}^n \to \mathbb{R}^q, i = 1, \cdots, T,\ \text{s.t.}\ (A_T + B_TF_T)(A_{T-1} + B_{T-1}F_{T-1})\cdots(A_1 + B_1F_1)V \subseteq V\},$$

which is called the friend set of the $(\boldsymbol{A}, \boldsymbol{B})$-invariant subspaces V.

Using the proof of Theorem 1, we may give a complete parameterization of the friend set of a given $(\boldsymbol{A}, \boldsymbol{B})$-invariant subspaces.

Suppose $V \in \mathscr{I}(\boldsymbol{A}, \boldsymbol{B}; \mathbb{R}^n)$. Define a set of subspaces in $\mathbb{R}^n$ to be

$$W_i = (A_T \cdots A_{i+1})^{-1}(V + \mathscr{B}_T + \cdots + A_T \cdots A_{i+2}\mathscr{B}_{i+1}),\quad i = 1,\cdots,T-1.$$

Then, $(F_1,\cdots,F_T) \in \mathscr{F}(A,B;V)$ if and only if the following constraints are satisfied

$$\begin{cases} (A_1 + B_1F_1)V \subseteq W_1, \\ (A_2 + B_2F_2)(A_1 + B_1F_1)V \subseteq W_2, \\ \quad\vdots \\ (A_{T-1} + B_{T-1}F_{T-1})\cdots(A_1 + B_1F_1)V \subseteq W_{T-1}, \\ (A_T + B_TF_T)\cdots(A_1 + B_1F_1)V \subseteq V. \end{cases} \tag{3}$$

The above constraints could be transformed into the compatible matrix equations whose form is $MXN = Q$. Thus we may find the general solutions from top to bottom (see, e.g., Reference 7). All these general solutions together form a complete parameterization of $\mathscr{F}(A,B;V)$.

Similar to the time-invariant case, for an arbitrarily given subspace $K \subseteq \mathbb{R}^n$, we may construct a maximal (A,B)-invariant subspace contained in K.

Denote $\underline{\mathscr{I}}(A,B;K) = \{V \subseteq K : V \in \mathscr{I}(A,B;\mathbb{R}^n)\}$. It follows easily from Theorem 1 that the set $\underline{\mathscr{I}}(A,B;K)$ is closed under the operation of subspaces addition.

Proposition 1 The set $\underline{\mathscr{I}}(A,B;K)$ contains a unique supreme element $\sup \underline{\mathscr{I}}(A,B;K)$.

Proposition 2 Define the sequence V^i according to

$$V^0 = K,$$
$$V^i = K \cap (A_T \cdots A_1)^{-1}(\mathscr{B}_T + A_T\mathscr{B}_{T-1} + \cdots + A_T \cdots A_2\mathscr{B}_1 + V^{i-1}),\ i \in \mathbb{N}, \tag{4}$$

then $V^i \subseteq V^{i-1}$, $i = 1,2,\cdots$, and for some $l, l \leqslant \dim(K)$.

$$V^l = \sup \underline{\mathscr{I}}(A,B;K).$$

These two propositions generalize the corresponding results for the case of $T = 1$.

3 Solutions for DDP-PMD

In this section, we study the solvability condition of DDP-PMD using the theory of (A, B)-invariant subspaces.

We now analyze the data of the closed-loop system (1) and (2):

$$y((i+1)T) = C(A_T + B_TF_T)\cdots(A_1 + B_1F_1)x(iT) + \sum_{j=2}^{T} C(A_T + B_TF_T)\cdots(A_j + B_jF_j)D_v(iT + j - 2) + CDv(iT + T - 1),\ i = 1,2,\cdots.$$

Analogous to the solvability condition of DDP in [3], we can obtain a lemma from the above analysis.

Lemma 2 DDP-PMD is solvable if and only if there exists an (A,B)-invariant subspace V, and a sequence of feedback matrices $(F_1,\cdots,F_T) \in \mathscr{F}(A,B;V)$, such that

i) $V \subseteq \ker C$;

ii) $\operatorname{Im}D + (A_T + B_TF_T)\operatorname{Im}D + \cdots + (A_T + B_TF_T)\cdots(A_2 + B_2F_2)\operatorname{Im}D \subseteq V$.

Note that the above solvability condition cannot be verified in general.

Utilizing Lemma 2 and the theory of (A,B)-invariant subspace, a verifiable solvability condition can be developped.

Theorem 2 A necessary and sufficient condition for DDP-PMD to be solvable is

$$\operatorname{Im}D \subseteq V^* \cap A_T^{-1}(V^* + \mathscr{B}_T) \cap \cdots \cap (A_T \cdots A_2)^{-1}(V^* + \mathscr{B}_T + \cdots + A_T \cdots A_3\mathscr{B}_2),$$

where $V^* = \sup \underline{\mathscr{I}}(A,B;\ker C)$.

Proof Necessity. By using Lemma 2, there must exist $V \in \underline{\mathscr{I}}(A,B;\ker C)$ and $(F_1,\cdots,F_T) \in \mathscr{F}(A,B;V)$ such that

$$\operatorname{Im}D + (A_T + B_TF_T)\operatorname{Im}D + \cdots + (A_T + B_TF_T)\cdots(A_2 + B_2F_2)\operatorname{Im}D \subseteq V.$$

Then we have

$$\operatorname{Im}D \subseteq V \cap A_T^{-1}(V + \mathscr{B}_T) \cap \cdots \cap (A_T \cdots A_2)^{-1}(V + \mathscr{B}_T + \cdots + A_T \cdots A_3\mathscr{B}_2).$$

Because $V \subseteq V^*$, the necessity follows.

Sufficiency. Define

$$K_1 = A_T^{-1}(V^* + \mathscr{B}_T),$$
$$K_2 = (A_TA_{T-1})^{-1}(V^* + \mathscr{B}_T + A_T\mathscr{B}_{T-1}),$$
$$\vdots$$
$$K_{T-1} = (A_T \cdots A_2)^{-1}(V^* + \mathscr{B}_T + \cdots + A_T \cdots A_3\mathscr{B}_2),$$
$$K_T = (A_T \cdots A_1)^{-1}(V^* + \mathscr{B}_T + \cdots + A_T \cdots A_2\mathscr{B}_1).$$

Note that $A_TK_1 \subseteq V^* + \mathscr{B}_T$. By using Lemma 1, we may find a matrix $F_T : \mathbb{R}^n \to \mathbb{R}^q$ so that $(A_T + B_TF_T)K_1 \subseteq V^*$. Note also that

$$A_{T-1}K_2 \subseteq A_T^{-1}(V^* + \mathscr{B}_T) + A_TA_T^{-1}\mathscr{B}_{T-1} =$$

$$K_1 + \mathscr{B}_{T-1} + \ker A_T = K_1 + \mathscr{B}_{T-1}.$$

The last equation follows from the fact that $\ker A_T \subseteq K_1$.

Applying Lemma 1, we may find a matrix $F_{T-1}:\mathbb{R}^n \to \mathbb{R}^q$ such that $(A_{T-1} + B_{T-1}F_{T-1})K_2 \subseteq K_1$. Therefore,

$$(A_T + B_TF_T)(A_{T-1} + B_{T-1}F_{T-1})K_2 \subseteq V^*.$$

Applying Lemma 1 repeatedly, we further find a sequence of matrices $F_{T-1},\cdots,F_2:\mathbb{R}^n \to \mathbb{R}^q$ such that

$$(A_{T-i+1} + B_{T-i+1}F_{T-i+1})K_i \subseteq K_{i-1},\ i = 3,\cdots,T-1$$

and

$$(A_T + B_TF_T)\cdots(A_2 + B_2F_2)K_{T-1} \subseteq V^*.$$

It can be verified that $A_1K_T \subseteq K_{T-1} + \mathscr{B}_1$. By using Lemma 1 we can find a matrix $F_1:\mathbb{R}^n \to \mathbb{R}^q$, such that $(A_1 + B_1F_1)K_T \subseteq K_{T-1}$. Thus, a use of Theorem 1 gives

$$A_T\cdots A_1V^* \subseteq V^* + \mathscr{B}_T + \cdots + A_T\cdots A_2\mathscr{B}_1,$$

which means that $V^* \subseteq K_T$. So, we have

$$(A_T + B_TF_T)\cdots(A_1 + B_1F_1)V^* \subseteq V^*.$$

It follows from Lemma 2 that the sequence of matrices $(F_1,\cdots,F_T)$ is a solution for DDP-PMD.

Remark Theorem 2 is readily applicable to the study of multirate control of time-invariant systems. Suppose system (1) is time-invariant, and our interest is seeking a feedback control law (2) such that the outputs y at specific sampling times $y(0), y(T), y(2T),\cdots$ are decoupled from the disturbances. According to Theorem 2, this problem is solvable if and only if

$$\operatorname{Im}D \subseteq V^* \cap A^{-1}(V^* + \mathscr{B}) \cap \cdots \cap A^{-(T-1)}(V^* + \mathscr{B} + \cdots + A^{T-2}\mathscr{B}), \quad (5)$$

where $A = A(t)$, $\mathscr{B} = \operatorname{Im}B(t)$. Reference [1] has shown that if we use the asynchronous periodically time-varying controller

$$\begin{aligned} &u(jT + i - 1) = F_ix(jT),\\ &i = 1,\cdots,T,\ j = 0,1,\cdots, \end{aligned} \quad (6)$$

instead of (2), then the corresponding decoupling condition, using our notations, is

$$\operatorname{Im}D \subseteq V^* \cap A^{-1}V^* \cap \cdots \cap A^{-(T-1)}V^*. \quad (7)$$

It is obvious that condition (7) is more restrictive than (5). Therefore the synchronous periodical feedback strategy (2) has more decoupling capability than the asynchronous periodical feedback strategy (6).

If DDP-PMD for system (1) is solvable, then we may characterize a family of solutions which are independent of the disturbance matrix D.

Suppose matrices $H_1,\cdots,H_T$ and $C_1,\cdots,C_T$ are taken so that $\operatorname{Im}H_1 = \ker C_T = V^*$, and

$$\operatorname{Im}H_T = \ker C_{T-1} = A_T^{-1}(V^* + \mathscr{B}_T),$$

$$\vdots$$

$$\operatorname{Im}H_2 = \ker C_1 = (A_T\cdots A_2)^{-1}(V^* + \mathscr{B}_T + \cdots + A_T\cdots A_3\mathscr{B}_2).$$

Define

$$\mathscr{F}(A,B;V^*) = \{(F_1,\cdots,F_T):F_i = -(C_iB_i)^+C_iA_iH_iH_i^+ + Y_i - (C_iB_i)^+C_iB_iY_iH_iH_i^+,\ \forall Y_i \in \mathbb{R}^{q\times n},\ i = 1,\cdots,T\}.$$

Corollary Suppose system (1) is DDP-PMD solvable, then each element of the set $\mathscr{F}(A,B;V^*)$ is a DDP-PMD solution for (1).

Proof Define subspaces

$$K_i = (A_T\cdots A_{T-i+1})^{-1}(V^* + \mathscr{B}_T + \cdots + A_T\cdots A_{T-i+2}\mathscr{B}_{T-i+1}),\ i = 1,\cdots,T-1.$$

By using the proof of Theorem 2, any sequence of feedback matrices $(F_1,\cdots,F_T)$ satisfying

$$\begin{cases} (A_T + B_TF_T)K_1 \subseteq V^*,\\ (A_{T-1} + B_{T-1}F_{T-1})K_2 \subseteq K_1,\\ \quad\vdots\\ (A_2 + B_2F_2)K_{T-1} \subseteq K_{T-2},\\ (A_1 + B_1F_1)V^* \subseteq K_{T-1} \end{cases} \quad (8)$$

is a DDP-PMD solution for (1).

The relationships (8) could be equivalently expressed as matrix equations

$$C_i(A_i + B_iF_i)H_i = 0,\ i = 1,\cdots,T. \quad (9)$$

By using the theory of matrices (see, e.g., Reference [7]), $\mathscr{F}(A,B,V^*)$ is exactly the set of general solutions of equations (9).

4 Conclusion

In this paper, the disturbance decoupling problem of periodic and multirate discrete-time linear systems has been addressed. A new concept of multifeedback controlled invariant subspace is introduced. A verifiable criterion for a subspace to be multifeedback controlled invariant is given. Using these, a necessary and sufficient condition for the solvability of DDP-PTD is obtained. More importantly, we can completely characterize the friend set of a given multifeedback controlled invariant subspace and partially characterize the DDP-PTD solutions.

The stability issue of DDP-PTD is an important topic for further investigation. Future work should first introduce appropriate notion of (A, B)-controllability subspace, then apply it to the disturbance decoupling problem with stability for system (1).

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本文作者简介

孙振东 1968年生.1990年毕业于青岛海洋大学应用数学系.1993年厦门大学系统科学系获硕士学位.1996年于北京航空航天大学七研获博士学位,同年进入清华大学自动化系从事博士后研究.现为北京航空航天大学副教授.

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本文作者简介

胡剑波 浙江大学工业控制技术研究所博士,副教授.主要研究领域:变结构控制、工业控制和飞行控制.

褚 健 1963年生.1982年毕业于浙江大学.1986年至1989年留学日本京都大学.1989年获工学博士学位.1993年聘为浙江大学教授,博士生导师,现为工业自动化国家工程研究中心副主任.首批"长江"学者计划特聘教授,浙江大学先进控制研究所所长.主要从事时滞系统控制,非线性控制,鲁棒控制等理论与应用的研究工作.