

Design of a Self-Scheduled H_∞ Controller for Plants with Variable Operating Conditions

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Abstract: This is concerned with the design of self-scheduled state feedback controllers with guaranteed H_∞ performance for a class of plants with variable operating conditions. The plant is assumed to be described by a linear interpolation of proper stable coprime factorizations of the transfer functions at two representative operating points. Base on the notion of quadratic H_∞ performance, sufficient and necessary conditions for the existence of the state-feedback H_∞ controller are given as infinite algebraic Riccati inequalities that cannot be solved directly. Then sufficient condition for the solvability of these infinite algebraic Riccati inequalities is given in the form of finite LMIs. In this way, the design of the state-feedback H_∞ controller is reduced to solving a feasibility problem constrained by these LMIs.

Key words: plants with variable operating conditions; H_∞ control; state feedback; self-scheduled; LMI

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工作点可变系统的一种自调度 H_∞ 控制器设计

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摘要: 对于一种工作点可变系统, 即被控对象表达为两个典型工作点上的传递函数矩阵的正则稳定的既约分解的线性内插, 研究其 H_∞ 的控制问题, 提出了一种自调度状态反馈 H_∞ 控制器的设计算法, 并将该自调度状态反馈 H_∞ 控制器的设计问题转化为基于有限个线性矩阵不等式 (LMI) 的凸优化问题, 采用现有的凸优化工具即可对其进行求解。

关键词: 工作点可变系统; H_∞ 控制; 状态反馈; 自调度; LMI

1 Introduction

Dynamics of almost all plants in reality varies according to the condition under which the plant is operated. An idea to describe the variation of plant dynamics is interpolating a number of representative models defined at representative operating points. In [1~5], the plant is assumed to be described by a linear interpolation of proper stable co-prime factorizations of transfer functions of two representative models. Stabilization problems have been extensively considered using fixed controllers in [1, 2, 4, 5] and using interpolated controllers which are linear interpolations of co-prime factorizations of two stabilizing controllers for the two representative models in [3~5].

In this paper, the design of gain-scheduled state-feedback controllers with guaranteed H_∞ performance for

such interpolated plants is considered. It is assumed that the measurement of the linear interpolation parameter is available in real time. The resulting state-feedback H_∞ controller is time-varying and automatically 'gain scheduled' along the trajectory of linear interpolation parameter. Based on the notion of quadratic H_∞ performance, sufficient and necessary conditions for the existence of the state-feedback H_∞ controller are firstly given as infinite algebraic Riccati inequalities that cannot be solved directly. The sufficient condition for the solvability of these infinite algebraic Riccati inequalities is given in the form of finite LMIs. In this way, design of the state-feedback H_∞ controller is finally reduced to solving a feasibility problem constrained by these LMIs.

2 Problem formulation

Let G_1 and G_2 be two representative transfer function

models of G that are defined at two representative operating points. The proper stable coprime factorization of G_1 and G_2 are defined as:

$$G_i = N_i D_i^{-1}, \quad i = 1, 2, \quad (1)$$

where $N_i, D_i \in \mathbb{RH}_\infty$. The plant G is described as linear interpolation of the factorization^[1-6]:

$$\begin{cases} G = ND^{-1}, \\ N = \alpha \cdot N_1 + (1 - \alpha) \cdot N_2, \\ D = \alpha \cdot D_1 + (1 - \alpha) \cdot D_2, \end{cases} \quad (2)$$

where $\alpha (0 \leq \alpha \leq 1)$ is a parameter which represents the variation of the plant dynamics.

Lemma 1^[6] Suppose the minimal realization of $G_i (i = 1, 2)$ is $\{A_i, B_i, C_i, 0\}$, then N_i, D_i satisfying (1) are

$$\begin{aligned} N_i &= \begin{bmatrix} A_i - B_i F_i & B_i \\ C_i & 0 \end{bmatrix}, \\ D_i &= \begin{bmatrix} A_i - B_i F_i & B_i \\ -F_i & I \end{bmatrix}, \end{aligned}$$

where F_i is a matrix making $A_i - B_i F_i$ stable.

From Lemma 1, we can get the state-space description of (2).

Lemma 2 The interpolated plant (2) can be described by the following equations in state space:

$$\begin{cases} \dot{x}_p(t) = A_p(\alpha)x_p(t) + B_p(\alpha)u(t), \\ y(t) = C_p(\alpha)x_p(t), \end{cases} \quad (3)$$

where $u(t)$ is the control input, $y(t)$ is the measured output, $x_p(t)$ is the state of the plant, and

$$\begin{cases} A_p(\alpha) = \begin{bmatrix} A_1 - (1 - \alpha)B_1F_1 & \alpha B_1F_2 \\ (1 - \alpha)B_2F_1 & A_2 - \alpha B_2F_2 \end{bmatrix}, \\ B_p(\alpha) = \begin{bmatrix} \alpha B_1 \\ (1 - \alpha)B_2 \end{bmatrix}, \quad C_p(\alpha) = [C_1 \quad C_2]. \end{cases} \quad (4)$$

Proof From (2) and Lemma 1,

$$N = \alpha N_1 + (1 - \alpha)N_2 =$$

$$\begin{bmatrix} A_1 - B_1F_1 & 0 & \alpha B_1 \\ 0 & A_2 - B_2F_2 & (1 - \alpha)B_2 \\ C_1 & C_2 & 0 \end{bmatrix} = \begin{bmatrix} A_p(\alpha) - B_p(\alpha)[F_1 \quad F_2] & B_p(\alpha) \\ C_p(\alpha) & 0 \end{bmatrix}.$$

$$D = \alpha D_1 + (1 - \alpha)D_2 =$$

$$\begin{bmatrix} A_1 - B_1F_1 & 0 & \alpha B_1 \\ 0 & A_2 - B_2F_2 & (1 - \alpha)B_2 \\ -F_1 & -F_2 & I \end{bmatrix} =$$

$$\begin{bmatrix} A_p(\alpha) - B_p(\alpha)[F_1 \quad F_2] & B_p(\alpha) \\ -[F_1 \quad F_2] & I \end{bmatrix}.$$

Therefore, from Lemma 1,

$$G = ND^{-1} = \begin{bmatrix} A_p(\alpha) & B_p(\alpha) \\ C_p(\alpha) & 0 \end{bmatrix}.$$

Let the weighting system be

$$\begin{cases} \dot{x}_w(t) = A_w x_w(t) + B_{w1}w(t) + B_{w2}y(t), \\ z(t) = C_w x_w(t), \end{cases} \quad (5)$$

where $w(t)$ is the exogenous input, $z(t)$ is the controlled output, $x_w(t)$ is the state of the weighting system.

Combining (3) and (5), the overall system can be described as:

$$\begin{cases} \begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_w(t) \end{bmatrix} = \begin{bmatrix} A_p(\alpha) & 0 \\ B_{w2}C_p(\alpha) & A_w \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_w(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_{w1} \end{bmatrix} w(t) + \begin{bmatrix} B_p(\alpha) \\ 0 \end{bmatrix} u(t), \\ z(t) = [0, C_w] \begin{bmatrix} x_p(t) \\ x_w(t) \end{bmatrix}, \\ y(t) = [C_p(\alpha) \quad 0] \begin{bmatrix} x_p(t) \\ x_w(t) \end{bmatrix}. \end{cases} \quad (6)$$

Denote $x(t) = [x_p(t), x_w(t)]^T$ then (6) is transformed to:

$$\begin{cases} \dot{x}(t) = \bar{A}(\alpha)x(t) + \bar{B}_1(\alpha)w(t) + \bar{B}_2(\alpha)u(t), \\ z(t) = \bar{C}_1(\alpha)x(t), \\ y(t) = \bar{C}_2(\alpha)x(t), \end{cases} \quad (7)$$

where

$$\begin{cases} \bar{A}(\alpha) = \begin{bmatrix} A_p(\alpha) & 0 \\ B_{w2}C_p(\alpha) & A_w \end{bmatrix}, \\ \bar{B}_1(\alpha) = \begin{bmatrix} 0 \\ B_{w1} \end{bmatrix}, \quad \bar{B}_2(\alpha) = \begin{bmatrix} B_p(\alpha) \\ 0 \end{bmatrix}, \\ \bar{C}_1(\alpha) = [0 \quad C_w], \quad \bar{C}_2(\alpha) = [C_p(\alpha) \quad 0]. \end{cases} \quad (8)$$

The purpose of this paper is to design a state feedback controller $u(t) = K(\alpha)x(t)$ that makes the closed-loop system stable and satisfies $\|G_{zw}\|_\infty < \gamma$ for a given $\gamma (\gamma > 0)$ for any $\alpha(t) \in [0, 1]$. The closed-loop system is as shown in Fig. 1.

Remark 1 In this paper, the H_∞ control problem is formed by adding a weighting system. Similar results can be obtained with the same analysis in this paper for other formations of H_∞ control problem of the interpolated plant.

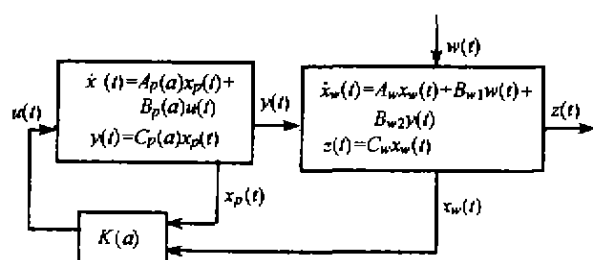


Fig. 1 The closed-loop system

3 Design of the self-scheduled state-feedback H_∞ controller

Lemma 3⁽⁷⁾ For the system

$$\begin{cases} \dot{x}(t) = \bar{A}x(t) + \bar{B}_1 w(t) + \bar{B}_2 u(t), \\ z(t) = \bar{C}_1 x(t), \\ y(t) = \bar{C}_2 x(t). \end{cases}$$

The sufficient and necessary condition for the existence of a state feedback controller $u(t) = Kx(t)$ which makes the closed-loop stable and satisfies $\|G_{zw}\|_\infty < \gamma$ is:

There exist $\epsilon (\epsilon > 0)$ and $P (P = P^T > 0)$ satisfying

$$P\bar{A} + \bar{A}^T P + P\left(\frac{1}{\epsilon^2}\bar{B}_1\bar{B}_1^T - \epsilon\bar{B}_2\bar{B}_2^T\right)P + \bar{C}_1^T\bar{C}_1 < 0,$$

K can be constructed as: $K = -\frac{\epsilon}{2}\bar{B}_2^T P$.

From Lemma 3, we can get the following lemma:

Lemma 4 For system (7), the sufficient and necessary condition for the existence of a state feedback controller $u(t) = K(a)x(t)$ which makes the closed-loop stable and satisfying $\|G_{zw}\|_\infty < \gamma$ is:

There exist $\epsilon (\epsilon > 0)$ and $P (P = P^T > 0)$ satisfying

$$P\bar{A}(a) + \bar{A}^T(a)P + P\left(\frac{1}{\epsilon^2}\bar{B}_1(a)\bar{B}_1^T(a) - \epsilon\bar{B}_2(a)\bar{B}_2^T(a)\right)P + \bar{C}_1^T(a)\bar{C}_1(a) < 0. \quad (9)$$

For any $a \in [0, 1]$, $K(a)$ can be constructed as:

$$K(a) = -\frac{\epsilon}{2}\bar{B}_2^T(a)P.$$

Multiplying P^{-1} from each side of (9), we can get:

$$\begin{aligned} &\bar{A}(a)P^{-1} + P^{-1}\bar{A}^T(a) + \frac{1}{\epsilon^2}\bar{B}_1(a)\bar{B}_1^T(a) - \\ &\epsilon\bar{B}_2(a)\bar{B}_2^T(a) + P^{-1}\bar{C}_1^T(a)\bar{C}_1(a)P^{-1} < 0. \end{aligned} \quad (10)$$

Denote: $Q = P^{-1}$, then (10) is transformed to:

$$\begin{aligned} &\bar{A}(a)Q + Q\bar{A}^T(a) + \frac{1}{\epsilon^2}\bar{B}_1(a)\bar{B}_1^T(a) - \\ &\epsilon\bar{B}_2(a)\bar{B}_2^T(a) + Q\bar{C}_1^T(a)\bar{C}_1(a)Q < 0. \end{aligned} \quad (11)$$

Lemma 5 For the system (7), the sufficient and necessary condition for the existence of a state feedback controller $u(t) = K(a)x(t)$ which makes the closed-loop stable and satisfying $\|G_{zw}\|_\infty < \gamma$ is:

There exist $\epsilon (\epsilon > 0)$ and $Q (Q = Q^T > 0)$ satisfying (11) for any $a \in [0, 1]$, $K(a)$ can be constructed as:

$$K(a) = -\frac{\epsilon}{2}\bar{B}_2^T(a)Q^{-1}.$$

From (4) and (8), we can get:

$$\begin{aligned} \bar{A}(a) &= \bar{A}_1 + a\bar{A}_2, \quad \bar{B}_1(a) = \bar{B}_1, \\ \bar{B}_2(a) &= \bar{B}_{21} + a\bar{B}_{22}, \quad \bar{C}_1(a) = \bar{C}_1, \end{aligned} \quad (12)$$

where

$$\bar{A} = \begin{bmatrix} A_1 - B_1 F_1 & 0 & 0 \\ B_2 F_1 & A_2 & 0 \\ B_{w2} C_1 & B_{w2} C_2 & A_w \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} B_1 F_1 & B_1 F_2 & 0 \\ -B_2 F_1 & -B_2 F_2 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{B}_1 = \begin{bmatrix} 0 \\ 0 \\ B_{w1} \end{bmatrix}, \quad \bar{B}_{21} = \begin{bmatrix} 0 \\ B_2 \\ 0 \end{bmatrix}, \quad \bar{B}_{22} = \begin{bmatrix} B_1 \\ -B_2 \\ 0 \end{bmatrix},$$

$$\bar{C}_1 = [0 \quad 0 \quad C_w].$$

Substitute (12) into (11), (11) is converted to:

$$F_0(Q) + aF_1(Q) + a^2F_2(Q) < 0, \quad (13)$$

$$F_0(Q) = \bar{A}_1 Q + Q\bar{A}_1^T + \frac{1}{\epsilon^2}\bar{B}_1\bar{B}_1^T - \epsilon\bar{B}_{21}\bar{B}_{21}^T + Q\bar{C}_1^T\bar{C}_1Q, \quad (14)$$

$$F_1(Q) = \bar{A}_2 Q + Q\bar{A}_2^T - \epsilon\bar{B}_{21}\bar{B}_{22}^T - \epsilon\bar{B}_{22}\bar{B}_{21}^T, \quad (15)$$

$$F_2(Q) = -\epsilon\bar{B}_{22}\bar{B}_{22}^T. \quad (16)$$

It can be observed that $F_0(Q)$, $F_1(Q)$, $F_2(Q)$ are symmetric matrices.

Theorem 1 For system (7), the sufficient and necessary condition for the existence of a state feedback controller $u(t) = K(a)x(t)$ that makes the closed-loop stable and satisfies $\|G_{zw}\|_\infty < \gamma$ is: there exist $\epsilon (\epsilon > 0)$ and $Q (Q^T > 0)$ satisfying (13) for any $a(t) \in [0, 1]$, $K(a)$ can be constructed as:

$$K(a) = -\frac{\epsilon}{2}\bar{B}_2^T(a)Q^{-1}.$$

In Theorem 1, sufficient and necessary conditions for the existence of the state-feedback H_∞ controller are given as infinite algebraic Riccati inequalities that cannot be solved directly, sufficient condition for the solvability of

these infinite algebraic Riccati inequalities will be given in the form of finite LMIs in the following. In this way, the design of the state-feedback H_∞ controller is finally reduced to solving a feasibility problem constrained by these LMIs.

Lemma 6 Let H be a 2-dimensional convex hyperpolyhedron that includes $T = \{(\alpha, \alpha^2) \mid \alpha \in [0, 1]\}$ in it. Denote m ($3 \leq m < \infty$) vertices of H are: (x_i, y_i) ($i = 1, 2, \dots, m$). If there exists Q such that:

$$F_0(Q) + x_i F_1(Q) + y_i F_2(Q) < 0, \quad i = 1, 2, \dots, m, \quad (17)$$

then Q satisfies

$$F_0(Q) + \alpha F_1(Q) + \alpha^2 F_2(Q) < 0, \quad (18)$$

for all $\alpha \in [0, 1]$. Here $F_0(Q), F_1(Q), F_2(Q)$ are assumed to be symmetric matrices.

Proof Since H is convex, and $T = \{(\alpha, \alpha^2) \mid \alpha \in [0, 1]\}$ is included in H , any point in T can be described as a convex combination of the m vertices of H . That is to say: for any $(\alpha_0, \alpha_0^2) \in T$, there must exist $\lambda_1, \dots,$

λ_m , satisfying $\sum_{i=1}^m \lambda_i = 1, \lambda_i \geq 0$. Such that:

$$\alpha_0 = \sum_{i=1}^m \lambda_i x_i, \quad \alpha_0^2 = \sum_{i=1}^m \lambda_i y_i.$$

If there exists a Q such that (17) holds, then:

$$F_0(Q) + x_1 F_1(Q) + y_1 F_2(Q) < 0, \quad (19)$$

$$F_0(Q) + x_2 F_1(Q) + y_2 F_2(Q) < 0, \quad (20)$$

\vdots

$$F_0(Q) + x_m F_1(Q) + y_m F_2(Q) < 0. \quad (21)$$

Multiply (19) by λ_1 , multiply (20) by λ_2, \dots , multiply (21) by λ_m , and take the sum of them, we can get:

$$\sum_{i=1}^m \lambda_i F_0(Q) + \left(\sum_{i=1}^m \lambda_i x_i\right) F_1(Q) + \left(\sum_{i=1}^m \lambda_i y_i\right) F_2(Q) < 0,$$

$$F_0(Q) + \alpha_0 F_1(Q) + \alpha_0^2 F_2(Q) < 0.$$

Therefore (18) holds for all $\alpha \in [0, 1]$.

Since the size of H is closely related to the conservativeness of the evaluation, it is necessary to construct a small size H for sharp evaluation.

Let the m ($3 \leq m < \infty$) vertices of H are: (x_i, y_i) ($i = 1, 2, \dots, m$). The sufficient condition for the solvability of (13) for all $\alpha \in [0, 1]$ can be described by the following condition:

$$F_0(Q) + x_i F_1(Q) + y_i F_2(Q) < 0, \quad i = 1, 2, \dots, m. \quad (22)$$

Substitute (14) ~ (16) into (22), (22) can be transformed to the following LMI using Schur complement:

$$\begin{bmatrix} A & QC_1^T \\ \bar{C}_1 Q & I \end{bmatrix} > 0, \quad i = 1, 2, \dots, m, \quad (23)$$

where

$$A = -(\bar{A}_1 + x_i \bar{A}_2) Q - Q(\bar{A}_1 + x_i \bar{A}_2)^T - \frac{1}{\rho^2} \bar{B}_1 \bar{B}_1^T + \epsilon(\bar{B}_{21} \bar{B}_{21}^T + x_i \bar{B}_{21} \bar{B}_{22}^T + x_i \bar{B}_{22} \bar{B}_{21}^T + y_i \bar{B}_{22} \bar{B}_{22}^T).$$

Theorem 2 If there exist ϵ ($\epsilon > 0$) and Q ($Q^T > 0$) satisfying (23), then the state feedback controller

$$u(t) = K(\alpha)x(t), \quad K(\alpha) = -\frac{\epsilon}{2} \bar{B}_2^T(\alpha) Q^{-1}$$

will make the closed-loop stable and satisfy $\|G_{zw}\|_\infty < \gamma$.

In Theorem 2, the design of the state feedback H_∞ controller is converted to a feasibility problem constrained by finite LMIs that can be solved by LMI control toolbox.

4 A design example

Suppose

$$G_1 = \frac{s+2}{(s-1)(s+1)}, \quad G_2 = \frac{s+4}{(s-1)(s-2)}.$$

The minimal realization of G_1 and G_2 is

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = [2 \quad 1],$$

$$A_2 = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad C_2 = [0 \quad 1],$$

and F_1, F_2 are selected as:

$$F_1 = [7 \quad 5], \quad F_2 = [1 \quad 4].$$

Suppose the weighing system is

$$\begin{cases} x_w(t) = 2.7w(t) - 2.7y(t), \\ z(t) = x_w(t). \end{cases}$$

The purpose is to design a state feedback controller that makes the closed-loop system stable and satisfies $\|G_{zw}\|_\infty < 1$.

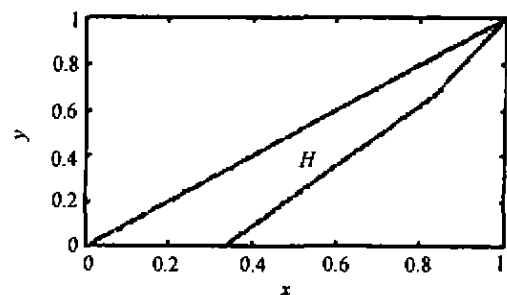


Fig. 2 Construction of H

Here H is constructed by four straight lines:

$$y = x, y = 2x - 1, y = \frac{4}{3}x - \frac{4}{9}, y = 0$$

as shown in Fig. 2. Four vertices of H are

$$(0, 0), (\frac{1}{3}, 0), (\frac{5}{6}, \frac{2}{3}), (1, 1).$$

The solution can be found by MATLAB:

$$Q = \begin{bmatrix} 79.8568 & -106.9679 & 109.3518 & -31.0099 & 6.3668 \\ -106.9679 & 175.5929 & -102.1837 & 24.4185 & -2.9403 \\ 109.3518 & -102.1837 & 797.8009 & -111.7225 & 1.2384 \\ -31.0099 & 24.4185 & -111.7225 & 34.8137 & 1.4610 \\ 6.3668 & -2.9403 & 1.2384 & 1.4610 & 5.4527 \end{bmatrix},$$

$$\epsilon = 273.1357.$$

And the self-scheduled state feedback H_∞ controller can be readily constructed:

$$u(t) = -\frac{\epsilon}{2} \bar{B}_2^T(\alpha) Q^{-1} x(t),$$

$$\bar{B}_2^T(\alpha) = [0 \quad \alpha \quad 4 - 4\alpha \quad 1 - \alpha \quad 0].$$

5 Conclusion

This paper has considered the design of self-scheduled state feedback controllers with guaranteed H^∞ performance for a class of plants with variable operating conditions. Design of such a controller is converted to solving a feasibility problem constrained by finite LMIs that can be solved by commercially available LMI control toolbox. The effectiveness of the proposed algorithm is demonstrated by a design example.

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