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Study on Stability of Nonlinear Closed-Loop System Based on Integrating Function Control of PWM Power Converter*

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Abstract: With the Buck converter as an example, an integrating function control law is proposed to realize the objective of the control strategy for PWM converters. To ensure stable operation of the nonlinear closed-loop system, stability of the system based on integrate function control is studied in this paper.

Key words: stability; integranting function control; closed-Loop System
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PWM 开关变换器积分函数非线性闭环控制系统稳定性的分析

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摘要: 以 Buck 电路为例, 提出一种积分函数闭环控制策略以实现 PWM 开关变换器控制目标. 为保证该非线性控制系统工作稳定, 本文对积分函数闭环控制系统的稳定性进行分析.

关键词: 稳定性; 积分函数控制; 闭环系统

1 Introduction

PWM DC/DC converters have many beneficial features and are widely used. It is very important to further improve the performance of PWM DC/DC switching power supplies. The closed-loop dynamic characteristics of a switching power supply are largely determined by the control strategy employed. The objective of the control strategy study is: 1) zero steady output voltage error; 2) low output impedance and fast response; 3) robustness.

In this paper, the integrating function control law for the Buck converter is proposed to realize the objective of the control strategy for PWM DC/DC converters. To ensure stable operation of the system, stability of nonlinear closed-loop system based on integrating function control is studied in this paper. The computer simulation by PSPICE show that excellent performance can be achieved by integrating function control.

2 Integrating function control law for the Buck converter

A switching power supply consists of two basic parts. One is the power stage, or the switching converter; the other is the control circuit, as shown in Fig.1, where V_{ref} is the reference voltage, x denotes the combination of the feedback, and α is the duty ratio.

The power stage controls the power absorbed from the unregulated supply voltage v_g and provides a regulated constant output voltage v_0 at the load. The main purpose

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of the control circuit is to generate a proper duty ratio according to the conditions of circuit so that the variation of the output voltage is reduced as much as possible when the supply voltage or load current changes[1-3]. For different control laws, such as direct-duty-ratio control, current-programmed control, etc., the effect of the supply voltage or load current disturbance is different.

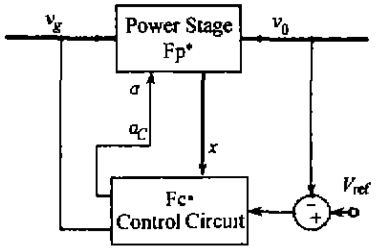

Fig. 1 Block diagram of PWM switching converter

An integrating function control principle is used to formulate the control strategy of the Buck converter to realize the above objective. Fig. 2 gives the topology of the Buck converter and Fig. 3 gives its large-signal low frequency averaged-circuit model which can be derived from the state-space-averaging method[4,5]

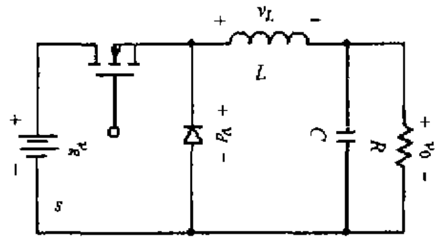

Fig. 2 Buck converter topology

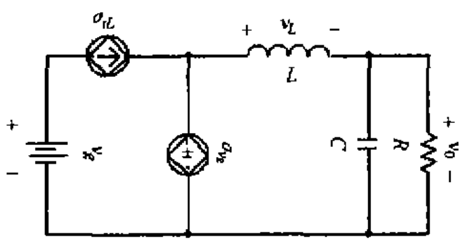

Fig. 3 Low frequency average model

In the averaged-circuit model, the active switch is modeled by a controlled current source with its value equual to the averaging current flowing through it over one switching cycle, i.e., $\bar{i}_S = \alpha i$ for Buck converter, where "i" is the averaged inductor current. The diode is modeld by a controlled voltage source with its value equal to the averaging voltage across it over one switching cycle, i.e., $\bar{v}_d = \alpha v_g$ for Buck converter.

For Fig. 3, the voltage variable relation can be expressed as:

$$\alpha v_g = \bar{v}_0 + \bar{v}_L, \tag{1}$$

where $\bar{v}_L$ is the averaged value of the inductor voltage and α is the duty ratio required for the switching converter. αv_g can be expressed as:

$$\bar{v}_d = \alpha v_g = \frac{1}{T_S}\int_0^{T_S} S(t)\, v_g \mathrm{d}t = \frac{1}{T_S}\int_0^{\alpha T_S} v_g \mathrm{d}t. \tag{2}$$

Then the duty ratio required by the Buck converter at a specific operating point of $\bar{v}_0, \bar{v}_L$ and v_g can be determined as:

$$\frac{1}{T_S}\int_0^{\alpha T_S} v_g \mathrm{d}t = \bar{v}_0 + \bar{v}_L. \tag{3}$$

The control circuit can now be constructed to generate the duty ratio. Let the input and output relation of the control circuit be:

$$\frac{1}{T_S}\int_0^{\alpha_C T_S} v_g \mathrm{d}t = P(V_{ref} - \bar{v}_0) + \bar{v}_L, \tag{4}$$

where V_{ref} is the reference voltage, P is the gain of the proportional error amplifier and α_C denotes the duty ratio generated by the control circuit.

In the practical circuit, the output of the control circuit is connected to the gate of the active switch in the power stage, making $\alpha = \alpha_C$. Therefore, the closed-loop characteristics can be obtained by Equation (3) and (4) as:

$$\frac{1}{T_S}\int_0^{\alpha T_S} v_g \mathrm{d}t = \bar{v}_0 + \bar{v}_L = \frac{1}{T_S}\int_0^{\alpha_C T_S} v_g \mathrm{d}t = P(V_{ref} - \bar{v}_0) + \bar{v}_L. \tag{5}$$

From (5), the output voltage can be found as:

$$\bar{v}_0 = \frac{P}{P+1} V_{ref}. \tag{6}$$

The reference voltage V_{ref} is constant for constant output voltage application. Equation (6), thus, shows that by the control law (4), the closed-loop averaged output voltage is forced to be proportional to a constant reference voltage. This result means that the closed-loop output voltage is independent of the supply voltage and the load current. In other words, the averaged output voltage remains unchanged even when there is disturbance from either the supply voltage or the load current. The above control objective of PWM DC/DC converters is, therefore, realized.

3 Stability analysis

The large-signal averaged-circuit model of the Buck converter is shown in Fig. 4. The stability analysis is based on the large-signal averaged-circuit model[6].

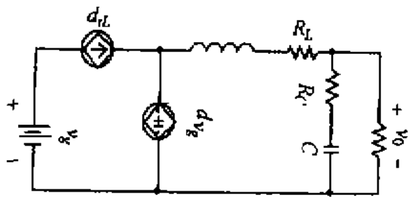

Fig. 4 Averaged-circuit model of Buck converter

As discussed in paper[1], the retrieved low frequency component of inductor voltage delays the actual one by one switching period, T_S, i.e.

$$\bar{v}_L' = \bar{v}_L(t - T_S), \tag{7}$$

where $\bar{v}_L'$ denotes the retrieved inductor voltage. The actual integrating function control law is constructed as:

$$\frac{1}{T_S}\int_0^{\sigma_c T_S} v_g \mathrm{d}t = P(V_{\mathrm{ref}} - \bar{v}_0) + \bar{v}_L' - K_\mathrm{d}\frac{\mathrm{d}\bar{v}_0}{\mathrm{d}t}. \tag{8}$$

Here the proportional and differential controller is used as what has been implemented. In (8), K_d is the time constant of the differentiator. In a practical implementation, instead of differentiating the output voltage directly, $\frac{\mathrm{d}\bar{v}_0}{\mathrm{d}t}$ can be obtained easily for the Buck converter by sensing the current in the filter capacitor so that the effect of the noise in $\bar{v}_0$ will be eliminated. From (7),(8) and the relation of the power stage (3), the closed-loop output voltage can be expressed as:

$$\bar{v}_0 + \bar{v}_L = P(V_{\mathrm{ref}} - \bar{v}_0) + \bar{v}_L' - K_\mathrm{d}\frac{\mathrm{d}\bar{v}_0}{\mathrm{d}t}, \tag{9}$$

$$\bar{v}_0 + \bar{v}_L = P(V_{\mathrm{ref}} - \bar{v}_0) + \bar{v}_L(t - T_S) - K_\mathrm{d}\frac{\mathrm{d}\bar{v}_0}{\mathrm{d}t}. \tag{10}$$

In the steady state, the average inductor voltage and the output voltage do not change and the derivative of the output voltage is zero. The closed-loop steady-state output voltage can be found as:

$$V_0 = \frac{P}{P+1}V_{\mathrm{ref}}. \tag{11}$$

The averaged output voltage $\bar{v}_0$ and inductor voltage $\bar{v}_L$ can be expressed as: $\bar{v}_0 = V_0 + \tilde{v}_0$ and $\bar{v}_L = V_L + \tilde{v}_L$, where the upper case letter denote the steady-state value and a tilde " ~ " above a variable denotes a dynamic component. Substituting the above relation into (10), the dynamic equation can be found from (10) as:

$$\tilde{v}_L + \tilde{v}_0 = \tilde{v}_L(t - T_s) - P\tilde{v}_0 - K_\mathrm{d}\frac{\mathrm{d}\tilde{v}_0}{\mathrm{d}t}. \tag{12}$$

Equation (12) is a linear differential equation with time delay. Its Laplace transformation can be found as:

$$V_L(s) + V_0(s) = E^{-sT_S}V_L(s) - PV_0(s) - sK_\mathrm{d}V_0(s), \tag{13}$$

where all the initial conditions are set to zero. Because of the low frequency assumption, e^{-sT_S} can be expressed approximately as:

$$\mathrm{e}^{-sT_S} = 1 - sT_S. \tag{14}$$

The relation between $V_0(s)$ and $V_L(s)$ can be found from Fig.4 as:

$$V_L(s) = \left(\frac{s^2LC}{1 + sR_cC} + \frac{sL}{R} + \frac{sR_LC}{1 + sR_CC} + \frac{R_L}{R}\right)V_0(s). \tag{15}$$

Substituting (15) and (14) into (13), the closed-loop dynamic output voltage can be derived as:

$$(b_0s^3 + b_1s^2 + b_2s + b_3)V_0(s) = 0, \tag{16}$$

where the coefficients are:

$$b_0 = T_SLC\left(1 + \frac{R_C}{R}\right),$$

$$b_1 = \frac{T_SL}{R} + T_SR_CC + \frac{T_SR_LR_CC}{R} + K_\mathrm{d}R_CC,$$

$$b_2 = K_\mathrm{d} + \frac{T_SR_L}{R} + (P + 1)R_CC,$$

$$b_3 = P + 1.$$

The effect of P and K_d on the stability of the system is investigated. When P changes from 1 to 100, the root-locus of (16) is plotted, as shown in Fig.5 for $K_\mathrm{d} = 0$, 0.02, 0.05, respectively, The parameters of the power stage used in the simulation are $T_S = 20\mu\mathrm{s}$, $L = 240\mu\mathrm{H}$, $C = 880\mu\mathrm{F}$, $R = 12\Omega$ and $R_L = 0.05\Omega$, $R_C = 0.15\Omega$, here T_S is the switching period.

Fig.5 gives two conjugate segments of the root-locus when $K_\mathrm{d} = 0$, which is equivalent to the proportional controller, another segments lies in the left half plane and is not plotted here. Obviously the system is not stable as two roots lie in the right half of s-plane. When the gain of $\frac{\mathrm{d}\bar{v}_0}{\mathrm{d}t}$ feedback, K_d is 0.02, all the roots of (16) become real and lie in the left half plane. The system is stable. Fig.6 gives the two segments of the root locus. The third one lies far away left of these two and is not plotted here. Further increasing K_d, Fig.7 gives one segment of root-locus when $K_d = 0.05$. In this case, the other segment of the root locus shrinks to almost a point around-7610. The third one still lies at the far left of these two. The system's behaviour is dominated by the low frequency root. Comparing the root-locus at different K_d, it is shown that the Buck converter by the integrating function control law (4) can be stabilized when the gain K_d is proper.

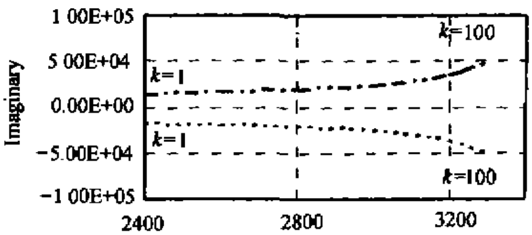

Fig. 5 Root locus when K_d=0

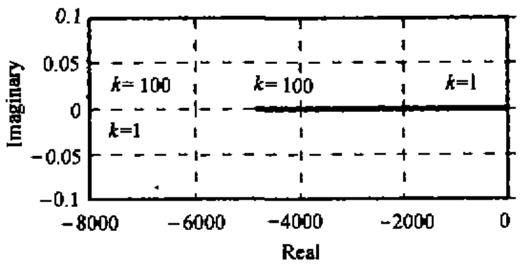

Fig. 6 Root locus when K_d=0.02

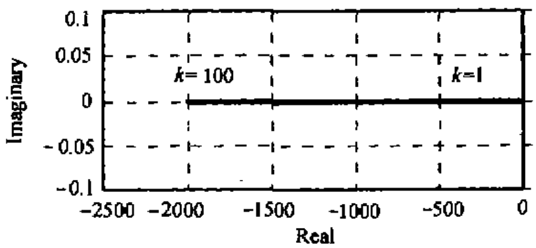

Fig. 7 Root locus when K_d=0.05

4 Conclusion

The above analysis shows that with the integrating function control law, (4), the root locus of the closed-loop system can be put in the left half plane to ensure stable operation. It should also be noted that in the above analysis, no small-signal assumption is made. With the nonlinear integrating function control law, the nonlinear Buck converter is changed into a linear system. Therefore, the dynamic behaviour of the closed-loop system and the results are valid for both the small-signal and large-signal variation.

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