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A Family of Reliable Nonlinear H_∞ State-Feedback Controllers

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Abstract: The paper is concerned with the reliable controller parameterization problem. A procedure for designing a family of reliable nonlinear H_∞-state feedback controllers is presented. These controllers are obtained by interconnecting the 'central controller' with an asymptotically stable free system that satisfies one additional cascade condition. The resulting closed-loop nonlinear system is reliable in the sense that they provide guaranteed local internal stability and H_∞ performance not only when all actuators are operational but also when some of actuators experience outages. The results of this paper provide a deeper insight into the synthesis of the reliable nonlinear H_∞ state feedback.

Key words: nonlinear system; H_∞ theory; reliable control; Hamilton-Jacobi inequality

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一簇非线性 H_∞ 状态反馈可靠控制器

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摘要: 研究了非线性 H_∞ 状态反馈可靠控制器的参数化问题.用耗散理论,基于 Hamilton-Jacobi 不等式,构造出了一簇控制器,使得当有执行机构失效时,闭环系统仍渐近稳定且 L_2 增益有限.本文的结果为非线性 H_∞ 状态反馈可靠控制的综合提供了更深的视角.

关键词: 非线性系统; H_∞ 理论; 可靠控制; Hamilton-Jacobi 不等式

1 Introduction

In recent years, considerable attention has been paid to the design problems of reliable linear control systems achieving various reliability goals, and some design methods have been given by several authors (see[1~3] and the references therein). In Particular, Veilette et al[1] present a methodology for the design of reliable linear control systems by means of the algebraic Riccati equation approach from linear H_∞-control theory, such that the resulting designs provide guaranteed closed-loop stability and H_∞ performance not only when all control components are operating, but also in the case of some admissible control component outages.

The above results for linear systems have been extended to nonlinear systems by Yang et al[4,5], Liu et al[6] and by means of the Hamilton-Jacobi equation approach in nonlinear H_∞ control theory.

Frequently, in designing a control system there are other design objectives in addition to the underlying constraints of internal stability and disturbance attenuation. One way of solving these more complex control problems is to search over the set of controllers which solves the underlying H_∞-control problem for a controller which satisfies the additional design objectives. Therefore, an important problem is the parameterization of controllers which solve the nonlinear H_∞-control problem. Parameterization procedures of controllers solving locally the H_∞-control problem were given in [7~9]. The purpose of the present paper is to extend the technique developed in [9] to give a family of reliable nonlinear H_∞ state-feedback controllers. The resulting closed-loop nonlinear system is reliable in the sense that they provide guaranteed local internal stability and H_∞ performance not only when all actuators are operational but also when some of actuators experience outages.

2 Problem formulation and preliminaries

Consider a nonlinear system described by equations of the form

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$$\begin{cases} \dot{x} = f(x) + g_1(x)\omega + \sum_{j=1}^{m} g_{2j}(x)u_j, \\ z = [h^T(x) \quad u_1 \quad \cdots \quad u_m]^T, \\ y = x, \end{cases} \tag{1}$$

where x is the state defined in a bounded neighborhood x of $0 \in \mathbb{R}^n$, $\omega \in \mathbb{R}^q$, is the plant disturbance, $u = [u_1 \; u_2 \cdots u_m]^T \in \mathbb{R}^m$ denotes the control input, $z \in \mathbb{R}^{p+m}$ is the output to be regulated, and the measured output y considered in this paper is assumed to be equal to the state x. $f(x)$, $g_1(x)$, $g_{2j}(x)$ $(j = 1, \cdots, m)$ and $h(x)$ are all known smooth mappings defined in the neighborhood x with $f(0) = 0$ and $h(0) = 0$. To be more compact, we denote

$$g_2(x) = [g_{21}(x) \quad g_{22}(x) \quad \cdots \quad g_{2m}(x)].$$

Let $\Omega \subset \{1, 2, \cdots, m\}$ correspond to a selected subset to actuators susceptible to outages. Then, the problem considered in this paper is as follows.

Reliable control (RC) problem: Given system (1) and a positive constant γ, find a controller, such that for actuator outages corresponding to any $\sigma \subset \Omega$, the resulting closed-loop system is locally asymptotically stable and has a local L_2-gain less than or equal to γ.

For $\sigma \subset \Omega$, introduce the decomposition

$$g_2(x) = g_{2\sigma}(x) + g_{2\bar{\sigma}}(x),$$

where

$$g_{2\sigma}(x) = [\delta_\sigma(1)g_{21}(x) \; \delta_\sigma(2)g_{21}(x) \cdots \delta_\sigma(m)g_{2m}(x)]$$

with δ_σ defined as follows:

$$\delta_\sigma(i) = \begin{cases} 1, & \text{if } i \in \sigma, \\ 0, & \text{if } i \notin \sigma. \end{cases}$$

When actuator outages corresponding to $\sigma \subset \Omega$ occur, the resulting system can be described by

$$\begin{cases} \dot{x} = f(x) + g_{2\bar{\sigma}}(x)u_{\bar{\sigma}} + g_1(x)\omega, \\ z_{\bar{\sigma}} = [h(x)^T \quad u_{\bar{\sigma}}^T]^T. \end{cases} \tag{2}$$

The following inequality is obvious and will be used in the sequel:

$$g_{2\sigma}(x)g_{2\sigma}(x)^T \leqslant g_{2\Omega}(x)g_{2\Omega}(x)^T, \text{ for } \sigma \subset \Omega.$$

Definition 1[11] Suppose that $f(0) = 0$ and $h(0) = 0$. The pair $\{f, h\}$ is said to be locally detectable if there exists a neighborhood U of the point $x = 0$ such that, if $x(t)$ is any integral curve of $\dot{x} = f(x)$ with $x(0) \in U$, then $h(x(t)) = 0$ is defined for all $t \geqslant 0$ and $h(x(t)) = 0$ for all $t \geqslant 0$ implies $\lim\limits_{t \to \infty} x(t) = 0$.

3 Main results

Theorem 1 Suppose that $\{f, h\}$ is locally detectable. If the Hamilton-Jacobi inequality

$$H_1(x) =$$
$$V_x(x)f(x) + \frac{1}{4}V_x(\frac{1}{\gamma^2}g_1(x)g_1(x)^T -$$
$$(g_2(x)g_2(x)^T - 2g_{2\Omega}(x)g_{2\Omega}(x)^T))V_x^T +$$
$$h(x)^T h(x) \leqslant 0 \tag{3}$$

has positive definite smooth solution $V(x) > 0$ with $V(x(0)) = 0$), where

$$\omega_*(x) = \frac{1}{2\gamma^2}g_1(x)^T V_x(x)^T,$$

$$u_*(x) = -\frac{1}{2}g_2(x)^T V_x(x)^T.$$

Then, the controller

$$u = u_*(x) \tag{4}$$

solves the RC problem.

Proof Along the trajectory of system (2), we have

$$\frac{dV}{dt} =$$
$$V_x(x)f(x) + V_x(x)g_1(x)\omega +$$
$$V_x(x)g_2(x)u - V_x(x)g_{2\sigma}(x)u_\sigma \leqslant$$
$$V_x f + V_x g_1\omega + V_x g_2 u + \frac{1}{4}V_x g_{2\sigma}g_{2\sigma}^T V_x^T + u_\sigma^T u_\sigma \leqslant$$
$$V_x f + V_x g_1\omega + V_x g_2 u + \frac{1}{4}V_x g_{2\Omega}g_{2\Omega}^T V_x^T + u_\sigma^T u_\sigma =$$
$$V_x f + V_x g_1\omega + \| u + \frac{1}{2}g_2^T V_x^T \|^2 - u_{\bar{\sigma}}^T u_{\bar{\sigma}} - \frac{1}{4}g_{2\bar{\Omega}}g_{2\bar{\Omega}}^T V_x^T.$$

If $\omega = 0$ and $u = u_*(x)$, from the condition in Theorem 1 and $u_{*j}^2 \leqslant u_*^T u_*$, we have

$$\frac{dV}{dt} \leqslant -\gamma^2\omega_*(x)^T\omega_*(x) -$$
$$h(x)^T h(x) - u_{*\bar{\sigma}}(x)^T u_{*\bar{\sigma}}(x) \leqslant 0.$$

Observe that any trajectory satisfying $\dot{V}(x) = 0$ for all $t \geqslant 0$ is necessarily a trajectory of

$$\dot{x} = f(x) + g_{2\bar{\sigma}}(x)u_{*\bar{\sigma}}(x),$$

such that $x(t)$ is bounded and $h(x(t)) = 0$ for all $t \geqslant 0$. Since $\{f, h\}$ are locally detectable, $\lim\limits_{t \to \infty} x(t) = 0$. Thus, the closed-loop systems (2) and (4) are locally asymptotically stable by LaSalle' invariance principle.

On the other hand, we have

$$\frac{dV}{dt} + \| z_{\bar{\sigma}} \|^2 - \gamma^2 \| \omega \|^2 =$$

$$\frac{dV}{dt} + h^T h + u_{\bar\sigma}^T u_{\bar\sigma} - \gamma^2 \|\omega\|^2 \leqslant$$
$$V_x f + V_x g_1 \omega + V_x g_2 u +$$
$$\frac{1}{4} V_x g_{2\Omega} g_{2\Omega}^T V_x^T + \|u\|^2 + h^T h - \gamma^2 \|\omega\|^2 =$$
$$V_x f + \frac{1}{4\gamma^2} V_x g_1 g_1^T V_x^T - \frac{1}{4} V_x g_{2\bar\Omega} g_{2\bar\Omega}^T V_x^T + h^T h +$$
$$\|u + \frac{1}{2} g_2^T V_x^T\|^2 - \|\gamma\omega - \frac{1}{2\gamma} g_1^T V_x^T\|^2.$$

If $u = u_*(x)$, from the condition in Theorem 1, we have

$$\frac{dV}{dt} + \|z_{\bar\sigma}\|^2 - \gamma^2 \|\omega\|^2 \leqslant 0. \tag{5}$$

For any given $T > 0$, the integration of (5) from 0 to T yields

$$V(x(T)) - V(x(0)) + \int_0^T (\|z_{\bar\sigma}\|^2 - \gamma^2 \|\omega\|^2) dt \leqslant 0. \tag{6}$$

Therefore, $\int_0^T \|z_{\bar\sigma}\|^2 dt \leqslant \int_0^T \gamma^2 \|\omega\|^2 dt$ holds.

In the next section, a family of controllers solving the RC problem will be proposed. The family of controllers to be considered is described by dynamic equations of the form[9]

$$\begin{cases} \dot\xi = f(\xi) + g_1(\xi)\omega_*(\xi) + g_2(\xi)[u_*(\xi) + c(\eta)], \\ \dot\eta = a(\eta) + b(\eta)(x - \xi), \\ u = u_*(x) + c(\eta), \end{cases} \tag{7}$$

where $\xi \in \mathbb{R}^n$ and $\eta \in \mathbb{R}^s$ are defined on some neighborhoods of the origin. $a(\eta), b(\eta)$ and $c(\eta)$ are all smooth functions with $a(0) = 0$ and $c(0) = 0$. We show how $a(\eta), b(\eta)$ and $c(\eta)$ should be chosen to solve the RC problem.

The closed-loop system (1) and (7) can be described by

$$\begin{cases} \dot x_e = f_e(x_e) + g_e(x_e)[\omega - \omega_*(x)], \\ z = [h(x)^T\ u^T]^T = [h(x)^T\ (u_*(x) + c(\eta))^T]^T, \end{cases} \tag{8}$$

where

$$x_e = [x^T \quad \xi^T \quad \eta^T]^T,$$
$$f_e(x_e) =$$
$$\begin{bmatrix} f(x) + g_1(x)\omega_*(x) + g_2(x)u_*(x) + g_2(x)c(\eta) \\ f(\xi) + g_1(\xi)\omega_*(\xi) + g_2(\xi)u_*(\xi) + g_2(\xi)c(\eta) \\ a(\eta) + b(\eta)(x - \xi) \end{bmatrix},$$
$$g_e(x_e) = \begin{bmatrix} g_1(x) \\ 0 \\ 0 \end{bmatrix}.$$

Theorem 2 Consider system (8). Suppose that the condition in Theorem 1 is satisfied. If the Hamilton-Jacobi inequality

$$H_2(x_e) = W_{x_e} f_e(x_e) + c(\eta)^T c(\eta) +$$
$$\frac{1}{4\gamma^2} W_{x_e} g_e(x_e) g_e(x_e)^T W_{x_e}^T +$$
$$\frac{1}{2} W_{x_e} \begin{bmatrix} g_{2\Omega}(x) g_{2\Omega}(x)^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} W_{x_e}^T \leqslant 0 \tag{9}$$

has a positive definite smooth solution $W(x_e)$ with respect to $(x - \xi, \eta)$ (that is, $W(x_e) = 0$ for $x_e = [x^T \quad x^T \quad 0]^T$ and $W(x_e) > 0$ elsewhere) such that $H_2(x_e)$ is negative-definite with respect to $(x - \xi, \eta)$ ($H_2(x_e) = 0$ for $x_e = [x^T \quad x^T \quad 0]^T$ and $H_2(x_e) < 0$ elsewhere).

Then, the family of controllers (7) solve the RC problem.

Proof Suppose that there are actuator outages corresponding to $\sigma \subset \Omega$. Then the resulting closed-loop system can be described by

$$\begin{cases} \dot x_e = f_{e\bar\sigma}(x) + g_{e\bar\sigma}[\omega - \omega_*(x)], \\ z_{\bar\sigma} = [h(x)^T \quad u_{\bar\sigma}^T]^T, \end{cases} \tag{10}$$

where

$$x_e = [x^T \quad \xi^T \quad \eta^T]^T.$$
$$f_{e\bar\sigma}(x_e) =$$
$$\begin{bmatrix} f(x) + g_1(x)\omega_*(x) + g_{2\bar\sigma}(x)u_{*\bar\sigma}(x) + g_{2\bar\sigma}(x)c_{\bar\sigma}(\eta) \\ f(\xi) + g_1(\xi)\omega_*(\xi) + g_2(\xi)u_*(\xi) + g_2(\xi)c(\eta) \\ a(\eta) + b(\eta)(x - \xi) \end{bmatrix},$$
$$g_{e\bar\sigma}(x_e) = \begin{bmatrix} g_1(x) \\ 0 \\ 0 \end{bmatrix}.$$

We have

$$W_{x_e} f_{e\bar\sigma}(x_e) =$$
$$W_{x_e} f_e(x_e) - W_{x_e} \begin{bmatrix} g_{2\sigma}(x)(u_{*\sigma}(x) + c_\sigma(\eta)) \\ 0 \\ 0 \end{bmatrix} \leqslant$$
$$W_{x_e} f_e(x_e) + \frac{1}{2} W_{x_e} \begin{bmatrix} g_{2\Omega}(x) g_{2\Omega}(x)^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} W_{x_e}^T +$$
$$\frac{1}{2}[u_{*\sigma}(x) + c_\sigma(\eta)]^T [u_{*\sigma}(x) + c_\sigma(\eta)].$$

Note that

$$W_{x_e}(f_{e\bar{\sigma}}(x_e)+g_e(x_e)[\omega-\omega_*(x)])+\|c(\eta)\|^2-\gamma^2\|\omega-\omega_*(x)\|^2\leqslant$$
$$W_{x_e}(f_e(x_e)+g_e(x_e)[\omega-\omega_*(x)]+\|c(\eta)\|^2-\gamma^2\|\omega-\omega_*(x)\|^2+$$
$$\frac{1}{2}W_{x_e}\begin{bmatrix} g_{2\Omega}(x)g_{2\Omega}(x)^{\mathrm{T}} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}W_{x_e}^{\mathrm{T}}+$$
$$\frac{1}{2}[u_{*\sigma}(x)+c_\sigma(\eta)]^{\mathrm{T}}[u_{*\sigma}(x)+c_\sigma(\eta)]=$$
$$H_2(x_e)+W_{x_e}g_e(x_e)[\omega-\omega_*(x)]-\gamma^2\|\omega-\omega_*(x)\|^2-$$
$$\frac{1}{4\gamma^2}W_{x_e}g_eg_e^{\mathrm{T}}W_{x_e}^{\mathrm{T}}+\frac{1}{2}\|u_{*\sigma}(x)+c_\sigma(\eta)\|^2.$$

This shows that, along the trajectories of the closed-loop system (10),

$$\frac{\mathrm{d}W}{\mathrm{d}t}\leqslant$$
$$H_2(x_e)+W_{x_e}g_e(x_e)[\omega-\omega_*(x)]-$$
$$\gamma^2\|\omega-\omega_*(x)\|^2-\frac{1}{4\gamma^2}W_{x_e}g_e\,g_e^{\mathrm{T}}W_{x_e}^{\mathrm{T}}+$$
$$\frac{1}{2}\|u_{*\sigma}(x)+c_\sigma(\eta)\|^2-\|c_\sigma(\eta)\|^2+\gamma^2\|\omega-\omega_*(x)\|^2=$$
$$H_2(x_e)-\gamma^2\|\omega-\omega_*(x)-\frac{1}{2\gamma^2}g_e(x_e)^{\mathrm{T}}W_{x_e}^{\mathrm{T}}\|^2-$$
$$\|c_\sigma(\eta)\|^2+\gamma^2\|\omega-\omega_*(x)\|^2+\frac{1}{2}\|u_{*\sigma}(x)+c_\sigma(\eta)\|^2.$$

Similar arguments show that

$$\frac{\mathrm{d}V}{\mathrm{d}t}\leqslant$$
$$H_1(x)+V_xg_1\omega+V_xg_2u+\frac{1}{2}V_xg_{2\Omega}g_{2\Omega}^{\mathrm{T}}V_x^{\mathrm{T}}+$$
$$\frac{1}{2}u_\sigma^{\mathrm{T}}u_\sigma-h^{\mathrm{T}}h-\gamma^2\|\omega_*\|^2+\|u_*\|^2-2\|u_{*\Omega}\|^2=$$
$$H_1(x)-\|z\|^2-\gamma^2\|\omega\|^2-\gamma^2\|\omega-\omega_*(x)\|^2+$$
$$\|c_\sigma(\eta)\|^2+\frac{1}{2}u_\sigma^{\mathrm{T}}u_\sigma.$$

Consider the candidate Lyapunov function

$$U(x_e)=V(x)+W(x_e)>0.$$

Along the trajectories of the closed-loop system (10), we have

$$\frac{\mathrm{d}U}{\mathrm{d}t}\leqslant$$
$$H_1(x)+H_2(x_e)-\|z_{\bar{\sigma}}\|^2+\gamma^2\|\omega\|^2-$$
$$\gamma^2\|\omega-\omega_*(x)-\frac{1}{2\gamma^2}g_e^{\mathrm{T}}W_{x_e}^{\mathrm{T}}\|^2. \tag{11}$$

Setting $\omega=0$ in the above equality yields

$$\frac{\mathrm{d}U}{\mathrm{d}t}\leqslant H_1(x)+H_2(x_e)-\|z_{\bar{\sigma}}\|^2-$$
$$\gamma^2\|\omega_*(x)+\frac{1}{2\gamma^2}g_e^{\mathrm{T}}W_{x_e}^{\mathrm{T}}\|^2, \tag{12}$$

which is negative-semidefinite near $x_e=0$ by hypothesis. This proves that the equilibrium $x_e=0$ of the closed-loop system is stable. To prove asymptotic stability, observe that any trajectory satisfying $U(x_e)=0$ for all $t\geqslant 0$ is necessarily a trajectory of

$$\dot{x}=f(x)+g_{2\bar{\sigma}}(x)(u_{*\bar{\sigma}}(x)+c_{\bar{\sigma}}(\eta)),$$

such that $x(t)$ is bounded and $h(x(t))=0$ for all $t\geqslant 0$. In addition, the negative-definiteness of $H_2(x_e)$ with respect to $(x-\xi,\eta)$ implies that $x(t)=\xi(t)$ and $\eta(t)=0$ for all $t\geqslant 0$. Since $\{f,h\}$ is locally detectable, $\lim\limits_{t\to\infty}x(t)=0$, and thus $\lim\limits_{t\to\infty}\xi(t)=0$ all $t\geqslant 0$. Together with $\eta(t)=0$ for all $t\geqslant 0$, we can conclude asymptotic stability by LaSalle's invariance principle.

Furthermore, since $H_1(x)\leqslant 0$ and $H_2(x)\leqslant 0$ by hypothesis, (11) implies that

$$\frac{\mathrm{d}U}{\mathrm{d}t}+\|z_{\bar{\sigma}}\|^2-\gamma^2\|\omega\|^2\leqslant 0. \tag{13}$$

For any given $T>0$, integration of (13) from 0 to T yields

$$U(x(T))-U(x(0))+\int_0^T(\|z_{\bar{\sigma}}\|^2-\gamma^2\|\omega\|^2)\mathrm{d}t\leqslant 0.$$

Therefore, $\int_0^T\|z_{\bar{\sigma}}\|\,\mathrm{d}t\leqslant\int_0^T\gamma^2\|\omega\|^2\mathrm{d}t$ holds.

Theorem 2 has suggested a framework to solve the RC problem. More precisely, it has provided a general solution to the problem in question, in the sense that the free system parameters $a(\eta)$, $b(\eta)$ and $c(\eta)$ are not specified, except that the condition in Theorem 2 should be satisfied. These parameters provide a degree of freedom to achieve additional desired control performance.

Although a general solution has been given by Theorem 2, the function $H_2(x_e)$ thus found has $2n+s$ independent variables and actually involves the free system parameters. The following theorem, which is the main result of this paper, shows how the condition in Theorem 2 can be met by reducing the number of independent variables by n while imposing an additional cascade condition on the free system parameters.

Theorem 3 Suppose the condition in Theorem 2 is satisfied. When

$$A = f(\Psi) + g_1(\Psi)w_*(\Psi) + g_2(\Psi)u_*(\Psi),$$

let

$$\begin{cases} \begin{bmatrix} \dot{\Psi} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} A \\ a(\eta) + b(\eta)\Psi \end{bmatrix} + \begin{bmatrix} g_1(\Psi) \\ 0 \end{bmatrix} r, \\ \phi = c(\eta), \end{cases} \tag{14}$$

be any smooth nonlinear system with $a(0) = 0, c(0) = 0$ which satisfies the following condition:

The Hamilton-Jacobi inequality

$$H_3(\Psi,\eta) = [Q_\Psi \quad Q_\eta]\begin{bmatrix} A \\ a(\eta) + b(\eta)\Psi \end{bmatrix} + \frac{1}{4\gamma^2}Q_\Psi(g_1(\Psi)g_1(\Psi)^{\mathrm{T}} + 2\gamma^2 g_{2\Omega}(\Psi)g_{2\Omega}(\Psi)^{\mathrm{T}})Q_\Psi^{\mathrm{T}} + c(\eta)^{\mathrm{T}}c(\eta) \leqslant 0 \tag{15}$$

has a positive definite smooth solution $Q(\Psi,\eta):\mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}$ such that H_3 is negative definite near $(\Psi,\eta) = (0,0)$ and its Hessian matrix is nonsingular at $(\Psi,\eta) = (0,0)$.

Then, the family of controllers (7) solve the RC problem.

Proof We claim that $w(x_e) = Q(x - \xi, \eta)$ satisfies the condition in Theorem 2. Clearly $W(x_e) = Q(x - \xi, \eta)$ is positive definite with respect to $(x - \xi, \eta)$. It remains to prove that $H_2(x_e)$ is negative definite with respect to $(x - \xi, \eta)$. To this end, set

$$e = x - \xi,\ \Pi(e,\xi,\eta) = [H_2(x_e)]_{x=\xi+e}.$$

Then a simple calculation shows that

$$\Pi(0,\xi,0) = 0, \left[\frac{\partial \Pi(e,\xi,\eta)}{\partial e}\right]_{e=0,\eta=0} = 0,$$

$$\left[\frac{\partial \Pi(e,\xi,\eta)}{\partial e}\right]_{e=0,\eta=0} = 0.$$

Thus, similarly to the arguments in [12], $\Pi(e,\xi,\eta)$ can be expressed as

$$\Pi(e,\xi,\eta) = [e^{\mathrm{T}} \quad \eta^{\mathrm{T}}]\Pi_0(e,\xi,\eta)[e^{\mathrm{T}} \quad \eta^{\mathrm{T}}]^{\mathrm{T}},$$

for some continuous matrix $\Pi_0(e,\xi,\eta)$. Moreover, it is easy to verify that

$$\Pi_0(0,0,0) = \begin{bmatrix} \dfrac{\partial^2 H_3(\Psi,\eta)}{\partial \Psi^2} & \dfrac{\partial^2 H_3(\Psi,\eta)}{\partial \eta \partial \Psi} \\ \dfrac{\partial^2 H_3(\Psi,\eta)}{\partial \eta \partial \Psi} & \dfrac{\partial^2 H_3(\Psi,\eta)}{\partial \eta^2} \end{bmatrix}_{\Psi=0,\eta=0}$$

Since the latter matrix is negative definite from the condition in Theorem 3, $H_2(x_e)$ is negative definite with respect to $(x - \xi, \eta)$. Then, by Theorem 2 it is concluded that the family of controllers (7) solve the RC problem.

The condition in Theorem 3 is of conceptual, rather than computational, significance. It implies that cascade (14) has a locally asymptotically stable equilibrium $(\Psi,\eta) = (0,0)$ and has L_2-gain $\leqslant \gamma$. For $a(\eta)$, $b(\eta)$ and $c(\eta)$ satisfying the condition in Theorem 3, family (7) solve the RC problem. In other words, Theorem 3 provided a family of controllers solving the RC problem. These controllers are obtained by interconnecting the 'central controller' $u = u_*(x)$, with an asymptotically stable free system that satisfies the cascade condition.

4 Conclusion

The paper is concerned with the reliable controller parameterization problem. A procedure for designing a family of reliable nonlinear H$_\infty$-state feedback controllers is presented. These controllers are obtained by interconnecting the 'central controller' with an asymptotically stable free system that satisfies one additional cascade condition. The resulting closed-loop nonlinear system is reliable in the sense that they provide guaranteed local internal stability and H$_\infty$ performance not only when all actuators are operational but also when some of actuators experience outages. The results of this paper provide a deeper insight into the synthesis of the reliable nonlinear H$_\infty$ state feedback.

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更 正

本刊2001年第18卷第1期第147页,右栏第一行即“目前,我们所开展的‘无模型控制器’的研究就是企图用”移至左栏第二行,即:

控制理论的研究应该与计算机技术结合起来.

目前,我们所开展的“无模型控制器”的研究就是企图用一种新的思维方式来进行的.我们企图冲破现代控制理论的思维框架,……

论,Kalman滤波到非线性控制,无穷维系统,随机系统,适应控制,系统辨识等,出现大量的新结果,许多新刊物,新著作,使高技术中出现的众多问题得以解决,但一个客观事实是:……

特此更正并致歉意.

本刊编辑部