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Neural Network-Based Adaptive Tracking Control for a Class of Nonlinear Systems *

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Abstract: A neural network-based adaptive tracking control scheme is proposed for a class of nonlinear systems. Two RBF neural networks are used to approximate the unknown nonlinear system, and a sliding model control term is used to eliminate the effects of the network inherent approximation errors and external disturbance. This control scheme can ensure the global stability of closed loop system and the asymptotical convergence of output tracking error.

Key words: neural networks; nonlinear systems; output tracking; approximate error; adaptive control
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基于神经网络的一类非线性系统自适应跟踪控制

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摘要: 提出一种非线性系统的自适应神经跟踪控制方案.通过利用 RBF 神经网络对未知非线性系统建模,并用一个滑模控制项消除网络建模误差和外部干扰的影响,从而能够保证闭环系统的全局稳定性和输出跟踪误差渐近收敛于零.

关键词: 神经网络; 非线性系统; 输出跟踪; 逼近误差; 自适应控制

1 Introduction

In recent years, the neural network-based control of highly uncertain nonlinear system has been intensively studied. The major potential is that neural networks can approximate arbitrary nonlinear function through learning. For the adatpive control systems based on neural networks, the network weights need to be updated using the network's output error, and the adaptive control law is synthesized based on the output of networks. Hence, the central research topics in the fields of neural control include the convergence of the weight training algorithms and the stability of the closed loop control systems. However, it is difficult that the stability, error convergence, and robustness are fully proved for these control systems based on off-line trained neural network beause of the highly nonlinear of the neural networks[1]. The recent developments using adaptive neural networks for direct adaptive control, as in [2 ~ 5], have made a great progress in view to solve the above problems. For example, Chen[2], Mario[3], Man[4], and Fabri et al [5] have proposed adaptive neural control schemes based on Lyapunov synthesis approach so that the stability of closed loop system and asymptotic error convergence can be guaranteed. Jin et al [6] proposed an adaptive neural tracking controller for nonlinear system, but the control scheme didn't consider the effect of the network approximation error that is inherent due to the use of a finite number of units in neural networks, in reason of implementation constraint.

In this paper, we propose a new neural network-based direct adaptive feedback control scheme for a class of nonlinear systems. Two RBF neural networks are used to model the dynamics of nonlinear systems. And then, a sliding mode control term is used to eliminate the effect of the inherent network approximation errors and external disturbance such that the proposed control scheme can ensure the asymptotic convergence of the output tracking error and the global stability of closed-loop system.

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2 Problem statement

Considering nonlinear SISO affine system

$$y^{(n)} = f(x) + g(x)u + d, \qquad (1)$$

where $x \in \mathbb{R}^n$ is the state vectors, u is the control input, y is the system output, $f(x)$ and $g(x)$ are unknown nonlinear smooth functions on $\mathbb{R}^n$, d is the external bounded disturbance, i.e. $|d| \leqslant d_0$, with d_0 known positive constant. The following assumptions are required.

A1) $g(x)$ is bounded away from zero, i.e. $|g(x)| \geqslant g_0 > 0$, with g_0 a known lower bound. The assumption A1) implies that $g(x)$ is strictly either positive or negative for all x. From now on, without losing generality, we shall assume that $g(x)$ is strictly positive.

A2) The desired output $y_d(t)$ is a continuously differentiable function, and its first n derivatives $y_d^{(1)}, \cdots, y_d^{(n)}$ are uniformly bounded.

The control objective is to find a control $u(t)$ that will force the output $y(t)$ to track asymptotically the desired output $y_d(t)$.

Since $g(x)$ in system (1) is bounded away from zero, its inverse is well defined. Thus, when $f(x)$, $g(x)$ are known and the external disturbance d does not exist, there exists the feedback linearization control law $u(t) = \dfrac{-f(x) + v(t)}{g(x)}$ so that the resulted input-output dynamics of system (1) is $y^{(n)} = v(t)$, where $v(t)$ is an auxiliary input. Choose

$$v(t) = y_d^{(n)} - \alpha_{r-1}e^{(n-1)} - \cdots - \alpha_0 e, \qquad (2)$$

where $e = y - y_d$ is the output tracking error. Then, the error dynamics of system (1) can be obtained as follows

$$e^{(n)} + \alpha_{n-1}e^{(n-1)} + \cdots + \alpha_0 e = 0. \qquad (3)$$

If $\alpha_{n-1}, \cdots, \alpha_0$ are chosen such that the polynomial

$$\Gamma(s) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_0$$

is Hurwitz, the error equation (3) is asymptotically stable. Hence, the output tracking error e of system (1) asymptotically converges to zero.

Define the error vector $S = (e, e^{(1)}, \cdots, e^{(n-1)})^{\mathrm{T}}$. Then, the error equation (3) can be rewritten as

$$\dot{S} = AS, \qquad (4)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & \cdots & \vdots \\ 0 & \cdots & \ddots & \ddots & 0 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n-1} \end{bmatrix}$$

is a Hurwitz matrix. Hence, there is a positive matrix P_0 satifying the following Lyapunov equation

$$P_0 A + A^{\mathrm{T}} P_0 = -I. \qquad (5)$$

Now we consider the case that the nonlinear continue functions $f(x)$ and $g(x)$ are completely unknown and there exists an external disturbance d. Since it is proven that neural networks can approximate a wide range of nonlinear functions to any desired degree of acccuracy, we can get the estimation $\hat{f}(x)$ and $\hat{g}(x)$ of unknown nonlinear function $f(\cdot)$ and $g(\cdot)$ using neural networks. And then, in a relatively straightforward manner, we will get the following modified control law

$$u_{al} = \frac{-\hat{f}(x) + v(t)}{\hat{g}(x)}. \qquad (6)$$

Substituting (6) into system (1), we obtain the error dynamics as

$$y^{(n)} = v(t) + (f(x) - \hat{f}(x)) + (g(x) - \hat{g}(x))u_{al}(t) + d. \qquad (7)$$

Due to the use of a finite number of units in neural networks, in reason of implementation constraint, the network approximation error is inevitable. That means

$$f(x) - \hat{f}(x) \neq 0, \; g(x) - \hat{g}(x) \neq 0.$$

Thus, the stability of error equation (7) can not be guaranteed.

To eliminate the effects of the network approximation error and the external disturbance d, we will redesign controller (6) by augmenting a compensation term u_{sl}, such that the control input $u(t)$ becomes

$$u(t) = u_{al}(t) + u_{sl}(t). \qquad (8)$$

3 Adaptive neural controller

3.1 Neural approximation

Now, we will employ two RBF networks to approximate the unknown nonlinear functions $f(\cdot)$ and $g(\cdot)$, respectively, and get the estimations $\hat{f}(x)$ and $\hat{g}(x)$ as

$$\hat{f}(x) = \hat{\theta}_1^{\mathrm{T}}\phi(x), \; \hat{g}(x) = \hat{\theta}_2^{\mathrm{T}}\varphi(x), \qquad (9)$$

where $\hat{\theta}_1$ and $\hat{\theta}_2$ are estimations of the network weight vectors θ_1 and θ_2, respectively. Later we will give the adaptive updating algorithms of network weights to provide the suitable performance of the nets. The vectors $\phi(x)$ and $\varphi(x)$ are Guassian type of functions whose ith element, respectively, are defined as

$$\phi_i(x) = \exp(-\|x - c_{1i}\|^2/\sigma_{1i}^2),$$

$$\varphi_i(x) = \exp(-\|x - c_{2i}\|^2/\sigma_{2i}^2),$$

with c_{ji} and $\sigma_{ji}(j = 1,2)$ representing the center and the spread of the ith basis function, respectively. In actural application, c_{ji} and σ_{ji} are predetermined by using the local training technique[7].

Further, the following assumptions are made:

A3) Given arbitrarily small positive constant w_f and w_g, we could always find (nonunique) optimal weight vector θ_1^* and θ_2^*, such that the network approximation errors ϵ_f, ϵ_g satisfy

$$\begin{aligned} |\epsilon_f| &= |\theta_1^{*\mathrm{T}}\phi(x) - f(x)| < w_f, \\ |\epsilon_g| &= |\theta_2^{*\mathrm{T}}\varphi(x) - g(x)| < w_g. \end{aligned} \tag{10}$$

A4) The optimal weights θ_1^* and θ_2^* are bounded.

Then, from expression (10), we have

$$\begin{aligned} f(x) &= \theta_1^{*\mathrm{T}}\phi(x) + \epsilon_f, \\ g(x) &= \theta_2^{*\mathrm{T}}\varphi(x) + \epsilon_g. \end{aligned} \tag{11}$$

Remark 1 Assumption A3) reflects the approximation capability of neural networks. It is proven that the network approximation error can become arbitrarily small when the number of weights is large enough[8].

Remark 2 It is known from assumptions A1) and A3) that g_0 can be chosen such that $g^*(x) \geqslant g_0$, where $g^*(x) = \theta_2^{*\mathrm{T}}\varphi(x)$.

3.2 The design of neural controller

To eliminate the effects of the network approximation error and external disturbance, we synthesize the robust control term u_{sl} in controller (8) as follows

$$u_{sl} = -k_{sl}\,\mathrm{sgn}(e_1) \tag{12}$$

with e_1 being the filtered error, $e_1 = P_{nn}S$, where P_{nn} is the nth row vector of matrix P_0. k_{sl} is the control gain given by the following expression

$$k_{sl} = \frac{w_f + |u_{al}|\, w_g + d_0}{g_0}. \tag{13}$$

Substituting the control law (8) into (1), we get the error dynamics of system (1) as

$$\dot{S} = AS + B, \tag{14}$$

where $B = (0,0,\cdots,b)^{\mathrm{T}}$, with

$$\begin{aligned} b = {} & -\tilde{\theta}_1^{\mathrm{T}}\phi(x) - \tilde{\theta}_2^{\mathrm{T}}\varphi(x)u_{al} + \\ & g(x)u_{sl} + \epsilon_f + \epsilon_g u_{al} + d. \end{aligned} \tag{15}$$

The weight updating algorithms are chosen as

$$\dot{\hat{\theta}} = \eta_1\phi(x)e_1,\ \dot{\hat{\theta}} = \eta_2\varphi(x)e_1u_{sl} \tag{16}$$

with η_1 and η_2 being positive adaptive rates.

Besides, because the control law (8) is not well-defined when $\hat{g}(x) = 0$. Therefore, to guarantee the boundedness of the control signal, we take the following parameter-resetting mechanism[5].

When $\hat{g}(x) < g_0$, letting

$$\hat{\theta}_2(t^+) = \hat{\theta}_2(t) + (g_0 - \hat{g}(x))\,\|\varphi(x)\|^{-2}\varphi(x), \tag{17}$$

where t denotes the time when $\hat{g}(x) < g_0$, and t^+ denotes the time just after the resetting mechanism (17) is activated.

It is easy to learn that the parameter-resetting mechanism (17) can ensure $\hat{g}(x) \geqslant g_0$ after a weight reset.

In the following, we use the Lyapunov approach to prove the stability of the closed loop system.

Theorem Considering system (1) with assumption A1) to A4). If the controller is designed by expressions (6), (8), (12) and (13), and the weight vectors are adjusted by adaptive mechanism (16) and (17), then the output tracking error of system (1) asymptotically converges to zero.

Proof Defining a candidate Lyapunov function as

$$V = \frac{1}{2}S^{\mathrm{T}}P_0S + \frac{1}{2}\mathrm{tr}(\tilde{\theta}_1^{\mathrm{T}}\eta_1^{-1}\tilde{\theta}_1) + \frac{1}{2}\mathrm{tr}(\tilde{\theta}_2^{\mathrm{T}}\eta_2^{-1}\tilde{\theta}_2), \tag{18}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i^*\ (i = 1,2)$ is the estimation errors, and $\dot{\tilde{\theta}}_i = \dot{\hat{\theta}}_i$.

Differentiating V with respect to time along the state trajectories of the error equation (14), and using expression (5), we can get

$$\begin{aligned} \dot{V} = {} & -\frac{1}{2}S^{\mathrm{T}}S - \tilde{\theta}_1^{\mathrm{T}}\phi(x)e_1 - \tilde{\theta}_2^{\mathrm{T}}\varphi(x)e_1u_{sl} + \\ & (g(x)u_{sl} + \epsilon_f + \epsilon_g u_{al} + d)e_1 + \\ & \mathrm{tr}(\tilde{\theta}_1^{\mathrm{T}}\eta_1^{-1}\dot{\tilde{\theta}}_1) + \mathrm{tr}(\tilde{\theta}_2^{\mathrm{T}}\eta_2^{-1}\dot{\tilde{\theta}}_2). \end{aligned}$$

Further, using the adaptive updating rules (16), the above expression becomes

$$\dot{V} = -\frac{1}{2}S^{\mathrm{T}}S + (g(x)u_{sl} + \epsilon_f + \epsilon_g u_{al} + d)e_1.$$

Using expressions (12) and (13), and considering Assumptions A1) and A3), and then, the following inequality holds

$$(g(x)u_{sl} + \epsilon_f + \epsilon_g u_{al} + d)e_1 \leqslant 0.$$

Hence, we get $\dot{V} \leqslant -\frac{1}{2}S^{\mathrm{T}}S \leqslant 0$.

Now, considering the change of function V in the case of $\hat{\theta}_2$ updated by the parameter-resetting mechanism (17), from remark 2, it can be easily deduced that

$$\Delta V = \frac{1}{2\eta_2}[\hat{g}(x)-g_0+2(g_0-g^*(x)](g_0-\hat{g}(x))\|\varphi(x)\|^{-2} \leqslant -\frac{1}{2\eta_2}(g_0-\hat{g}(x))^2\|\varphi(x)\|^{-2} \leqslant 0.$$

So far, it has been shown that V monotonically decreases. Then, it can be concluded that the close loop system is globally stable and $S, \tilde{\theta}_1, \tilde{\theta}_2$ are uniformly bounded.

Furthermore, it can be easily obtained that $v(t), \hat{\theta}_1$ and $\hat{\theta}_2$ are uniformly bounded. Hence, using boundedness of $\phi(x)$ and $\varphi(x)$, we conclude that $\hat{f}(x)$ and $\hat{g}(x)$ are bounded. Because the parameter-resetting mechanism (17) can ensure that $\hat{g}(x) \geqslant g_0 > 0$, we can get, from expression (8), the uniform boundedness of u_{a1}, and then, the same for u_{al} in (12). Thus, b is uniformly bounded since all terms on the right of (15) are bounded. Hence, (14) implies that $\dot{S}$ is uniformly bounded, so that S is uniformly continuous.

Let $V_1(t) = V(t) - \int_0^t(\dot{V} + \frac{1}{2}S^TS)d\tau$. Since $\dot{V} \leqslant -\frac{1}{2}S^TS$, then $V_1(t) \geqslant 0$, so that $V_1(t)$ is bounded below. Further, $\dot{V}_1(t) = -\frac{1}{2}S^TS$ imples that $\dot{V}_1(t) \leqslant 0$, so it is semi-negative-definite. Finally, $\dot{V}_1(t)$ is uniformly continuous since S is uniformly continuous. Hence, using Barbalat's lemma, we can deduce that $\lim_{t\to\infty}\dot{V}_1(t) = 0$, so that $S \to 0$ as $t \to \infty$. This implies that the tracking error e and its derivatives $e^{(k)}(k = 1, 2, \cdots, n)$ asymptotically converge to zero.

4 Simulation

Considering a second-order SISO nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -x_1 + (x_1^2 - 1)x_2 + u + d(t), \\ y = x_1. \end{cases}$$

Assuming $f(x) = -x_1 + (x_1^2 - 1)x_2$ is unknown. $d(t)$ is external disturbance, $d(t) = 0.2\sin t$. A $2 \times 20 \times 1$ RBF neural network is used to approximate the unknown nonlinear function $f(x)$. Take initial weight vector $\hat{\theta}(0) = 0$, the spread $\sigma_i = 0.25$. The center c_i is randomly chosen in $(-0.2, 0.2)$. Fig. 1 shows the simulation result. It can be seen that the closed-loop system with this control algorithm has excellent control performance.

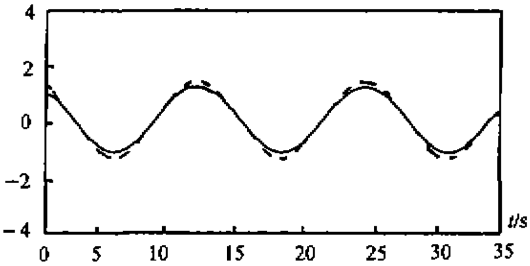

Fig. 1 Tracking trajectory of closed-loop system

5 Conclusion

We proposed in this paper an adaptive neural control scheme for a class of nonlinear system. The weight updating algorithm are derived based on Lyapunov method. The sliding mode technique is used to synthesize a robustifying term that can effectively eliminate the effect of the network reconstruction error and external disturbance.

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bination operator and gene leap operator continually produce new schemata, while selection operator, on one hand, maintains the excellent schemata with high fitness, but on the other hand, falls into disuse bad schemata with low fitness. Similarly with TGA, through genetic operator processing schemata, the individuals in the population continually move towards the optimal individual in PGA, finally the optimal solution can be gained.

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