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Robust Control for Nonlinear Systems with Bounded Perturbation *

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Abstract: This paper discusses the robust stabilization via state feedback for uncertain nonlinear systems. The uncertainty is described by gain bounded perturbation function on the state variables. The main result shows that a feedback control law can be obtained by constructing a positive definite Lyapunov-like function, if the considered system is of robust minimum-phase and its nominal system has relative degree one.

Key words: nonlinear systems; robust stabilization; uncertainty; state feedback
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具有有界摄动的非线性系统鲁棒控制

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摘要: 针对不确定性由增益有界的摄动函数表示的非线性系统,讨论了状态反馈鲁棒镇定问题.结果表明,如果所讨论的系统具有鲁棒最小相位特性,且其标称系统相对阶为 1,则反馈控制律可以通过构造满足本文引理条件的正定函数得到.

关键词: 非线性系统; 鲁棒稳定; 不确定性; 状态反馈

1 Introduction

As the differential geometry was introduced to the control theory, the essential structure of nonlinear systems was elucidated using the differential geometrical tool. In the past decade, a lot of approaches based on the system structure have been proposed to design nonlinear control systems in a general sense. For example, it has been shown that a desired Lyapunov function for robust stability can be constructed by recursive way, if the system has the triangular structure. The case of parametric uncertainty was considered in [1], and more broader class of uncertainty, which is described by gain bounded unknown function on the state variables, has been addressed by [2] and [3]. Robust stabilization and robust L_2 design problem have been investigated in the literature. A common characteristic of these approaches is that the robustness of the system is guaranteed by a fixed Lyapunov function of nominal system.

In this paper, we consider the robust stabilization problem for nonlinear systems with gain bounded uncertainty. The uncertainty is described by a perturbed function in the state space model of the systems. We will show that a feedback controller can be obtained by constructing a positive definite Lyapunov-like function, which ensures the robust convergence of the system at the equilibrium, if the considered system is of robust minimum-phase and the nominal system has relative degree one. It should be noted that though this paper only

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discusses the case of relative degree one, the proposed method can be also extended to the case where the relative degree is larger than one.

2 Problem present

Consider the nonlinear system with the uncertainty

$$\begin{cases} \dot{x} = f(x)(1 + \Delta_f(x)) + g(x)(1 + \Delta_g(x))u, \\ y = h(x), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^m, f(x), g(x)$ and $h(x)(f(0) = 0, h(0) = 0)$ are known smooth mappings. $\Delta_f(x)$ and $\Delta_g(x)$ are unknown scalar functions which describe the unmodeled error.

Assume that the system satisfies the following assumptions.

A1 The gain of $\Delta_f(x)$ is bounded, i.e. there is a known function $n(x)$ such that

$$|\Delta_f(x)| \leqslant |n(x)|, \ \forall x.$$

A2 $\Delta_g(x)$ is uniformly bounded, i.e. there is a sufficient small positive definite function $m(x)$ such that

$$|\Delta_g(x)| \leqslant 1 - m(x), \ \forall x.$$

A3 The model of system (1) has the relative degree $r = 1$, i.e.

$$L_g h(x) \neq 0, \forall x \in \mathbb{R}^n.$$

Under the geometrical condition, $G = \operatorname{span}\{g_1(x), g_2(x), \cdots g_m(x)\}$ is involutive distribution and the vector field $\{\tilde{g}_1(x), \tilde{g}_2(x), \cdots, \tilde{g}_m(x)\} = g(x)[L_g h(x)]^{-1}$ complete[4]. It is possible to find a suitable function $z = T(x)(z \in \mathbb{R}^{n-r})$ such that system (1) is transformed into the following form.

$$\begin{cases} \dot{z} = \tilde{f}(z, y)(1 + \tilde{\Delta}_f(z, y)), \\ \dot{y} = a(z,y)(1 + \tilde{\Delta}_f(z,y)) + b(z,y)(1 + \tilde{\Delta}_g(z,y))u, \end{cases} \tag{2}$$

where

$$\begin{bmatrix} z \\ y \end{bmatrix} = \phi(x) = \begin{bmatrix} T(x) \\ h(x) \end{bmatrix}, \tag{3}$$

and

$$\tilde{f}(z, y) = L_f T(\phi^{-1}(z, y)),$$
$$\tilde{\Delta}_f(z, y) = \Delta_f[\phi^{-1}(z, y)],$$
$$\tilde{\Delta}_g(z, y) = \Delta_g[\phi^{-1}(z, y)],$$
$$a(z, y) = L_f h(\phi^{-1}(z, y)),$$
$$b(z, y) = L_g h(\phi^{-1}(z, y)),$$
$$|\tilde{\Delta}_f| \leqslant |\tilde{n}(z, y)|,$$
$$|\tilde{\Delta}_g| \leqslant 1 - \tilde{m}(z, y),$$

It is clear that the zero dynamics of the system can be described as

$$\dot{z} = f_0(z)(1 + \tilde{\Delta}_f(z, 0)), \ f_0(z) = \tilde{f}(z, 0). \tag{4}$$

Furthermore, we assume that the system is of robust minimum-phase[2].

A4 There exists a positive definite function $W(z)$ such that its derivative along any trajectory of (4) satisfies $\dot{W}(z)|_{(4)} < -\varepsilon W(z), \forall \tilde{\Delta}_f$, where $\varepsilon > 0$.

The purpose of this paper is to seek a smooth feedback control $u = \alpha(x)$ such that the trajectory $x(t; 0, x_0)$ of the closed-loop system (1) satisfies $\lim\limits_{t \to \infty} x(t; 0, x_0) = 0$ for any $\Delta_f(x), \Delta_g(x)$ satisfying Conditions A1 and A2 and any initial state $x_0 \in \mathbb{R}^n$.

For simplicity of terminology, we say the system to be globally robust stable if system (1) has the above performance.

The following lemmas will be used to prove our main result.

Lemma 1[5] Consider the nonlinear system $\dot{x} = f(x)$. If there exist a positive definite function $V(x)$ and $\varphi(t)$ satisfying $\begin{cases} 0 < \varphi(t) < \infty, \forall t \\ \int_0^\infty \varphi(s) \mathrm{d}s < \infty \end{cases}$ such that the inequality $\dot{V}(x) \leqslant -\varepsilon V(x) + \varphi(t), \forall t$ holds when $\varepsilon > 0$, then for any initial value $x(0)$, $\lim\limits_{t \to \infty} x(t) = 0$.

Lemma 2[2] There exists a smooth and positive real function $\lambda_1(z) > 0$ such that the inequality

$$L_{f_0} W(z) + \frac{\lambda_1(z)}{2} \| L_{f_0} W(z) \|^2 + \frac{1}{2\lambda_1(z)} \| \tilde{n}(z, 0) \|^2 \leqslant - \varepsilon W(z), \ \forall z \neq 0 \tag{5}$$

holds if and only if the uncertain system (1) satisfies the Assumption A4.

3 Robust feedback controller design

3.1 Case of $\Delta_f(x) \equiv 0$

Because the smooth vector function $\tilde{f}(z, y)$ can be decomposed $\tilde{f}(z, y) = f_0(z) + f_1(z, y)y$, system (2) can be described as

$$\begin{cases} \dot{z} = f_0(z) + f_1(z, y)y, \\ \dot{y} = a(z, y) + b(z, y)(1 + \tilde{\Delta}_g)u. \end{cases} \tag{6}$$

Theorem 1 Assume that system (1) satisfies Conditions A2 ~ A4. Then a desired feedback control law ensuring the robust stability of closed-loop system globally is given by

$$u(z,y)=b^{-1}(z,y)\{c(z,y)-a_M(z,y)\}, \quad (7)$$

where

$$\begin{cases} a_M(z,y) = a(z,y) + [L_{f_1}W(z)]^{\mathrm{T}} + \frac{1}{2}\varepsilon y, \\ c(z,y) = -\frac{a_M a_M^{\mathrm{T}} y}{\tilde{m}(z,y)\{|y^{\mathrm{T}}a_M| + \gamma e^{-\beta t}\}}, \\ \quad (\gamma > 0, \beta > 0). \end{cases} \quad (8)$$

Proof Construct a positive definite function using $W(z)$ in Condition A4 as follows:

$$V(z,y) = W(z) + \frac{1}{2}y^{\mathrm{T}}y. \quad (9)$$

Calculating the derivative of V along the closed-loop system consisting of (6) and (7) yields

$$\begin{aligned} &\dot{V}(z,y)|_{(6)_{cl}} = \\ &L_{f_0}W(z) - \frac{1}{2}\varepsilon y^{\mathrm{T}}y + y^{\mathrm{T}}c(z,y) + \\ &y^{\mathrm{T}}\tilde{\Delta}_g c(z,y) - y^{\mathrm{T}}\tilde{\Delta}_g a_M(z,y) \leqslant \\ &-\varepsilon V + y^{\mathrm{T}}c(z,y) + y^{\mathrm{T}}\tilde{\Delta}_g c(z,y) + |y^{\mathrm{T}}a_M|. \end{aligned} \quad (10)$$

Using (8) and Condition A2: $\frac{1-|\Delta_g(\phi^{-1})|}{m(\phi^{-1})} \geqslant 1$, $\forall (z,y)$, the following inequality can be obtained for any $\Delta_g(x)$.

$$\begin{aligned} &\dot{V}(z,y)|_{(6)_{cl}} \leqslant \\ &-\varepsilon V(z,y) - \frac{|y^{\mathrm{T}}a_M|^2}{|y^{\mathrm{T}}a_M| + \gamma e^{-\beta t}} + |y^{\mathrm{T}}a_M| \leqslant \\ &-\varepsilon V(z,y) + \gamma e^{-\beta t}, \ \forall t \geqslant 0. \end{aligned} \quad (11)$$

Choosing $\varphi(t) = \gamma e^{-\beta t}$, then we have

$$\dot{V}(z,y)|_{(6)_{cl}} \leqslant -\varepsilon V(z,y) + \varphi(t), \ \forall t \geqslant 0.$$

Therefore, Theorem 1 is followed by Lemma 1.

3.2 Case of $\Delta_f(x) \neq 0, \Delta_g(x) \neq 0$

Theorem 2 Assume that system (1) satisfies Conditions A1 ~ A4. Then a desired feedback control law ensuring the globally robust stability of closed-loop system is given by

$$u(z,y) = b^{-1}(z,y)\{c_0(z,y) - a_0(z,y)\}, \quad (12)$$

where

$$\begin{aligned} c_0(z,y) &= \frac{a_0 a_0^{\mathrm{T}} y}{\tilde{m}(z,y)\{|y^{\mathrm{T}}a_0| + \gamma e^{-\beta t}\}}, \\ a_0(z,y) &= S^{\mathrm{T}}(z,y) + \frac{\lambda(z)}{2}c_1(z,y) + \\ &\frac{1}{2\lambda(z)}c_2(z,y) + \frac{1}{2}\varepsilon y, \end{aligned} \quad (13)$$

$$\begin{aligned} S^{\mathrm{T}}(z,y) &= (L_{f_1}W)^{\mathrm{T}} + a(z,y), \\ c_1(z,y) &= S^{\mathrm{T}}(2L_{f_0}W + Sy), \\ c_2(z,y) &= M^{\mathrm{T}}(2\tilde{n}(z,0) + My), \end{aligned} \quad (14)$$

and $\lambda(z)$ is a positive real function satisfying (5), $M(z,y)$ is the row vector function satisfying the decomposition $\tilde{n}(z,y) = \tilde{n}(z,0) + M(z,y)y$.

Proof The closed-loop system consisting of (3) and (12) is given by

$$\begin{bmatrix} \dot{z} \\ \dot{y} \end{bmatrix} = F(z,y) + E(z,y)\tilde{\Delta}_f + G(z,y,\tilde{\Delta}_g), \quad (15)$$

where

$$F(z,y) = \begin{bmatrix} f_0(z) + f_1(z,y)y \\ a(z,y) - a_0(z,y) \end{bmatrix},$$

$$E(z,y) = \begin{bmatrix} f_0(z) + f_1(z,y)y \\ a(z,y) \end{bmatrix},$$

$$G(z,y,\tilde{\Delta}_g) = \begin{bmatrix} 0 \\ c_0(z,y) - \tilde{\Delta}_g a_0(z,y) + \tilde{\Delta}_g c_0(z,y) \end{bmatrix}.$$

With Assumption A1 in mind, calculating the derivative of V along (15) yields

$$\begin{aligned} &\dot{V}(z,y)|_{(15)} = \\ &L_F V(z,y) + L_E V(z,y)\tilde{\Delta}_f + L_G V(z,y) \leqslant \\ &L_F V(z,y) + \frac{\lambda(z)}{2}\|L_E V(z,y)\|^2 + \\ &\frac{1}{2\lambda(z)}\|\tilde{n}(z,y)\|^2 + L_G V(z,y). \end{aligned} \quad (16)$$

Furthermore, it is easy to show that:

$$\begin{aligned} &L_F V(z,y) + \frac{\lambda(z)}{2}\|L_E V(z,y)\|^2 + \frac{1}{2\lambda(z)}\|\tilde{n}(z,y)\|^2 = \\ &L_{f_0}W + \frac{\lambda(z)}{2}\|L_{f_0}W\|^2 + \frac{1}{2\lambda(z)}\|\tilde{n}(z,0)\|^2 + \\ &y^{\mathrm{T}}\{(L_{f_1}W)^{\mathrm{T}} + a - a_0\} + \lambda(z)y^{\mathrm{T}}[(L_{f_1}W)^{\mathrm{T}} + a]L_{f_0}W + \\ &\frac{\lambda(z)}{2}y^{\mathrm{T}}[(L_{f_1}W)^{\mathrm{T}} + a][(L_{f_1}W)^{\mathrm{T}} + a]^{\mathrm{T}}y + \\ &\frac{1}{\lambda(z)}y^{\mathrm{T}}M^{\mathrm{T}}\tilde{n}(z,0) + \frac{1}{2\lambda(z)}y^{\mathrm{T}}M^{\mathrm{T}}My, \ \forall t \end{aligned} \quad (17)$$

holds for all z and y. Hence, by substituting (13) and (14) into (17), we have

$$\begin{aligned} &L_{f_0}W(z,y) + \frac{\lambda_1(z)}{2}\|L_{f_0}W(z,y)\|^2 + \\ &\frac{1}{2\lambda_1(z)}\|\tilde{n}(z,0)\|^2 - \frac{1}{2}\varepsilon y^{\mathrm{T}}y \leqslant \end{aligned}$$

$$- \varepsilon W(z,y) - \frac{1}{2}\varepsilon y^{\mathrm{T}} y =$$
$$- \varepsilon V(z,y). \tag{18}$$

Similar to the proof of Theorem 1, for any $\Delta_f(x)$ and $\Delta_g(x)$,

$$\dot{V}(z,y)\mid_{(15)} \leqslant$$
$$- \varepsilon V(z,y) + y^{\mathrm{T}} c_0(z,y) +$$
$$y^{\mathrm{T}} \tilde{\Delta}_g c_0(z,y) + | y^{\mathrm{T}} a_0(z,y) | \leqslant$$
$$- \varepsilon V(z,y) + \varphi(t),\ \forall t \geqslant 0. \tag{19}$$

Therefore, the proof of Theorem 2 is completed.

4 Simulation example

Consider a nonlinear system (1), where

$$x = (x_1, x_2)^{\mathrm{T}} \in \mathbb{R}^2,$$
$$f(x) = \begin{bmatrix} x_1 x_2 + x_2 \\ 3x_1 \end{bmatrix},\ g(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$h(x) = x_1 + x_2,$$
$$| \Delta_f(x) | \leqslant | n(x) |,\ | \Delta_g(x) | \leqslant 1 - m(x),$$
$$n(x) = \frac{1}{1 + \mathrm{e}^{-2x_2}} - \frac{1}{2},$$
$$m(x) = p(1 + \sin^2 x_2)(0 < p \leqslant 0.5).$$

Obviously, $L_g h(x) = 1$, i.e. the system has relative degree one. Hence, the system can be transformed into the normal form (3) by the following coordinate transformation.

$$\begin{bmatrix} z \\ y \end{bmatrix} = \phi(x) = \begin{bmatrix} T(x) \\ h(x) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{20}$$

where

$$\tilde{f}(z,y) = -3z + 3y = f_0(z) + f_1(z,y)y,$$
$$a(z,y) = z + (z+3)(y-z),\ b(z,y) = 1,$$
$$\tilde{\Delta}_f(z,y) = \Delta_f(\phi^{-1}(\begin{bmatrix} z \\ y \end{bmatrix})),\ \tilde{\Delta}_g(z,y) = \Delta_g(\phi^{-1}(\begin{bmatrix} z \\ y \end{bmatrix})),$$
$$\tilde{n}(z,y) = \tilde{n}(z,0) + M(z,y)y = \frac{1}{1 + e^{-2z}} - \frac{1}{2},$$
$$M(z,y) = 0,$$
$$\tilde{n}(z,0) = \frac{1}{1 + e^{-2z}} - \frac{1}{2},\ \tilde{m}(z,y) = p(1 + \sin^2 z).$$

Since the zero dynamics of the system is $\dot{z} = -3z(1 + \tilde{\Delta}_f(z,0))$, choosing the positive definite function $W(z) = \frac{1}{2}z^2$ and $\lambda(z) = \frac{1}{9}z^{-2}$, we get

$$L_{f_0} W(z) + \frac{\lambda}{2} | L_{f_0} W(z) |^2 + \frac{1}{2\lambda} | \tilde{n}(z,0) |^2 \leqslant$$
$$-3z^2 + \frac{\lambda}{2} \cdot 9z^4 + \frac{1}{2\lambda} \cdot \frac{1}{4} = -\frac{11}{8} z^2.$$

Thus, the inequality

$$L_{f_0} W(z) + \frac{\lambda}{2} | L_{f_0} W(z) |^2 + \frac{1}{2\lambda} | \tilde{n}(z,0) |^2 \leqslant$$
$$-\frac{11}{8} z^2 \leqslant -\frac{1}{2}\varepsilon z^2 = -\varepsilon W(z)$$

holds for sufficiently small ε ($\varepsilon \leqslant 2.75$).

According to Theorem 2, the following feedback control law is constructed for ensuring globally robust stability.

$$u(z,y) = c_0(z,y) - a_0(z,y),$$

where

$$c_0(z,y) = -\frac{a_0 a_0^{\mathrm{T}} y}{p(1 + \sin^2 z)\{| y^{\mathrm{T}} a_0 | + \gamma e^{-\beta t}\}},$$
$$a_0(z,y) = S^{\mathrm{T}}(z,y) + \frac{\lambda(z)}{2} c_1(z,y) + \frac{1}{2\lambda(z)} c_2(z,y),$$
$$S^{\mathrm{T}}(z,y) = 4z + (z+3)(y-z),$$
$$c_1(z,y) = s^{\mathrm{T}}(2L_{f_0} W + Sy),$$
$$L_{f_0} W = -3z^2,$$
$$c_2(z,y) = M^{\mathrm{T}}(2\tilde{n} + My) = 0,$$

and $p, \varepsilon, \gamma, \beta$ are positive numbers.

From the simulation, it can be seen that when ε, β are larger and p, γ smaller, the control input is very large, but the response time is short. The responses of u and y are shown in Fig.1 and Fig.2 respectively, when $p = 0.3, \varepsilon = 2, \gamma = 100, \beta = 20$ and the uncertainty

$$\Delta_f(x) = \frac{1}{1 + \mathrm{e}^{-2x_2}} - \frac{1}{2},\ \Delta_g(x) = p(1 + \sin^2 x_2).$$

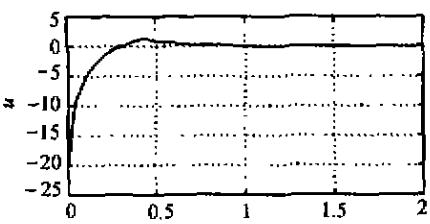

Fig. 1 The control law of system

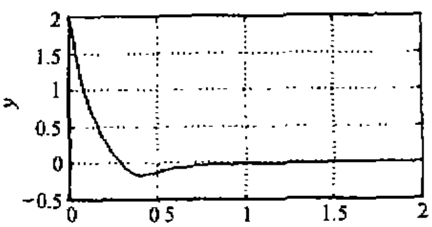

Fig. 2 Output curve of system

5 Conclusion

The robust stabilization of the minimum-phase nonlinear systems is discussed where the uncertainty is described by a perturbed function in the state space model. The main result shows that a smooth state feedback can be formed for ensuring globally robust stability if the system is of robust minimum-phase and the nominal system has relative degree one. The design method proposed in this paper can be extended to the case of the nonlinear system with the relative degree $r > 1$ under certain geometrical conditions.

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龚化淑 见本刊2001年第6期第826页.

申铁龙 见本刊2001年第1期第6页.

Main Contents of the Next Issue

A Hybrid Control for Autonomous Systems of Electric Power Supply ······ SUN Kai, ZHAO Qianchuan and ZHENG Dazhong

Optimal Control for Hybrid Systems Based on Dynamical Programming ······ YIN Zengshan, GAO Chunhua and LI Ping

Entrainment and Migration Control of Permanent-Magnet Synchronous Motor System ······ LI Zhong, ZHANG Bo and MAO Zongyuan

Generalized ANN Inverse Control Method ······ HE Dan, DAI Xianzhong and WANG Qin

Adaptive Selection of Crossover and Mutation Probability of Genetic Algorithm and Its Mechanism ······ CHEN Changzheng and WANG Nan

Analysis Method for Nash Strategy of Linear Time Variant Quadratic Differential Game via Wavelets (I) ······ ZHANG Chengke and WANG Xingyu

A Reachability and Properties of Discount Asset Optimization under Transaction Costs ······ XU Shimeng and ZHANG Yuzhong

Structure Analysis of Typical Fuzzy Controllers with Nonlinear Rules and Unevenly Distributed Membership Functions for Input and Output Variables ······ FAN Xingzhe and ZHANG Naiyao

A Spline Method for Computing a Class of Minimum-Energy Control for Multivariable Linear Systems ······ ZHANG Xinjian and LU Shirong

Adaptive Predictive Control of a Class of Nonlinear System ······ GUO Jian, CHEN Qingwei, ZHU Ruijun and HU Weili

Exponential Bidirectional Associative Memory Model with Intraconnection ······ CHEN Songcan and LIU Zheng

The Iterative Learning Control and Convergence Analysis for Nonlinear Industrial Process Control Systems ······ RUAN Xiao'e, WAN Baiwu and GAO Hongxia

Research of the Particle Size Neural Network Soft Sensor for Concentration Process ······ ZHANG Xiaodong, WANG Wei and WANG Xiaogang

Study on the Stability of the Volterra Series Model Based Adaptive Control Systems ······ DANG Yingnong and HAN Chongzhao

The Research of the Controllers for Machine Tool Linear Motor Direct Feed Drives ······ CHEN Zhihua, LI Shengyi, YANG Shunzhou and CUI Hongjuan

An Investigation on the Partially Control of the Torsional Vibration of the Turbogenerator Shaft System ··· HAO Zhiyong and FU Luhua