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时滞 2-*D* 中立型离散状态模型正解的存在性*

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摘要: 在 Bandch 空间中利用不动点定理研究了时滞 2-*D* 二阶中立型离散时间状态模型正解的存在性.

关键词: 时滞 2-*D* 中立型离散状态模型; 不动点定理; 正解

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Existence of positive solution of delay 2-*D* neutral discrete state model

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Abstract: This paper is concerned with a class of delay 2-*D* neutral discrete state model. The conditions of the existence of eventually positive solutions are established by fixed point theorem in Bandch space.

Key words: delay 2-*D* neutral discrete state model; fixed point theorem; positive solution

1 引言(Introduction)

时滞 2-*D* 离散时间状态模型在控制理论[1],分子轨道[2],粒子物理[3],偏微分方程数值计算[4]等许多实际和理论领域都有广泛的应用,对于该系统的研究才刚刚开始[5~16],而在其定性理论的研究中,一直困扰人们的问题是正解的存在性,本文在 Banach 空间中利用不动点定理研究了二阶中立型时滞 2-*D* 离散时间状态模型正确的存在性.

考虑时滞 2-*D* 二阶中立型离散时间状态模型

$$T^2(\Delta_1,\Delta_2)(x_{mn}-cx_{m-k,n-l})+p_{mn}x_{m-\sigma,n-\tau}=0, \tag{1.1}$$

其中

$$T^2(\Delta_1,\Delta_2)=T(\Delta_1,\Delta_2)\cdot T(\Delta_1,\Delta_2),$$

$$T(\Delta_1,\Delta_2)(x_{mn})=\Delta_1 x_{mn}+\Delta_2 x_{mn}+Ix_{mn},$$

而 $\Delta_1 x_{mn}=x_{m+1,n}-x_{mn}$, $\Delta_2 x_{mn}=x_{m,n+1}-x_{mn}$, $Ix_{mn}=x_{mn}$, $p_{mn}\in(-\infty,\infty)$,即 p_{mn} 是振动系数,令 $N_0=\{0,1,2,\cdots\}$, 但对于任意的 $m,n\in N_0$ 有 $p_{mn}\neq 0$, 并且 $c\geqslant 0$, $k,l,\sigma,\tau\in N_0$, $(m,n)\in N_0^2=N_0\times N_0$, 又式(1.1)对应的连续型时滞偏微分方程为

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(F(x,y)-cF(x-k,y-l))+p(x,y)F(x-\sigma,y-\tau)=0, \tag{1.2}$$

从而我们可以利用离散型的相应结果来逼近连续型的一些相关性质.而对于系统(1.1)的初始值;解的存在唯一性等问题看文[15,16],关于系统(1.1)的正解通常由下列给出:

定义　对于双指标状态序列 $\{x_{mn}\}$ 满足系统(1.1),则称 $\{x_{mn}\}$ 是系统(1.1)的一个解,设 M,N 是正整数,若 $m\geqslant M$, $n\geqslant N$ 有 $x_{mn}>0$,则称 $\{x_{mn}\}$ 是系统(1.1)的一个正解序列.

2 系统(1.1)正解的存在性(Existence of positive solution of system (1.1))

对于任意的 $(m,n)\in N_0^2$,取 $s,t\in N_0$, 并且满足

$$s\geqslant m,\ t\geqslant n. \tag{2.1}$$

令

$$R_{st}(x)=\sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[i-(s+k-1)]p_{i,n+l}x_{i-\sigma,n+l-\tau}+\sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[j-(t+l-1)]p_{m+k,j}x_{m+k-\sigma,j-\tau}. \tag{2.2}$$

从式(2.2)我们首先得到一个有用的引理:

引理　$T^2(\Delta_1,\Delta_2)(R_{st}(x))\mid_{\substack{s=m\\t=n}}=$

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$p_{m+k,n+l}\cdot x_{m+k-\sigma,n+l-\tau}$.

证 首先利用附录A中的(A1),(A2),我们得到

$$T(\Delta_1,\Delta_2)(R_{st}(x)) = R_{s+1,t}(x)+R_{s,t+1}(x)-R_{st}(x) = -A_{st}(x).$$

从而

$$T^2(\Delta_1,\Delta_2)(R_{sk}(x)) = T(\Delta_1,\Delta_2)(-A_{st}(x)) = -(A_{s+1,t}(x)+A_{s,k+1}(x)-A_{sk}(x)). \quad (2.3)$$

对式(2.3)再利用附录A中的(A3)我们有

$$T^2(\Delta_1,\Delta_2)(R_{sk}(x))\big|_{\substack{s=m\\k=n}} = -(A_{s+1,t}(x)+A_{s,t+1}(x)-A_{st}(x))\big|_{\substack{s=m\\t=n}} = p_{m+k,n+l}x_{m+k-\sigma,n+l-\tau}.$$

为了行文方便,记 $\bar{M}=(M,N),\omega=(m,n)$,并且若 $m\geqslant M,n\geqslant N$,记为 $\omega\geqslant\bar{M}$.

定理 对于充分大的正整数 M,N 和 $s,t,m,n\in N_0$,设式(2.1)成立并且

i) $c>\frac{5}{4},k,l,\sigma,\tau\geqslant 1$;

ii) $\sum\limits_{i=M}^{\infty}i\,|\,p_{in}\,|+\sum\limits_{j=N}^{\infty}j\,|\,p_{mj}\,|<\infty$.

其中 $\omega\geqslant\bar{M}$,而且当 $\bar{M}\to\infty$ 时,$\sum\limits_{i=M}^{\infty}i\,|\,p_{in}\,|+\sum\limits_{j=N}^{\infty}j\,|\,p_{mj}\,|$ 关于 $\omega\geqslant\bar{M}$ 是一致趋于零的.则式(1.1)存在有界的正解

证 由于 $c>\frac{5}{4}$,利用ii),则对充分大的 M,N,我们有

$$\sum_{i=M}^{\infty}i\,|\,p_{in}\,|+\sum_{j=N}^{\infty}j\,|\,p_{mj}\,|<\frac{4c-5}{4}.$$

以下考虑Banach空间

$$l_\infty^{\bar{M}}=\{y=y_{st}\mid\omega\geqslant\bar{M},\ y_{st}\in\mathbb{R}=(-\infty,\infty)\},$$

而在 $l_\infty^{\bar{M}}$ 中定义子集合

$$D=\{y_{st}\in l_\infty^{\bar{M}}\mid\frac{c}{2}\leqslant y_{st}\leqslant 2c,\omega\geqslant\bar{M}\},$$

并且定义

$$Z_k=\{(i,j):i\geqslant 0,j\geqslant 0,i+j=k\},\quad k=0,1,2,\cdots,$$

则利用文[5]中的序关系,则可以把 N_0^2 分为不同的类,即有

$$Z_0=\{(0,0)\},\ Z_1=\{(1,0),(0,1)\},\ Z_2=\{(2,0),(0,2),(1,1)\},\cdots,$$

设 $M_0=(m_0,n_0)$ 为 N_0^2 中的初始值,对于充分大的 k 记

$$\Psi_k=Z_k\cup Z_{k+1}\cup Z_{k+2}\cup\cdots,$$

又在 D 上定义映射 T 为:

$$T(\{x_{st}\})=\begin{cases}\frac{1}{c}x_{s+k,t+l}+\frac{1}{c}\Big(\sum\limits_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[i-(s+k-1)]p_{i,n+l}x_{i-\sigma,n+l-\tau}+\\ \sum\limits_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[j-(t+l-1)]\cdot p_{m+k,j}x_{m+k-\sigma,j-\tau}\Big):\omega\in\Psi_k,\\ 2c:\omega\in N_0^2\setminus\Psi_k.\end{cases}$$

易见有 $TD\subset D$,令 $x,y\in D$,则有

$$\begin{aligned}&|\,T(x_{st})-T(y_{st})\,|\leqslant\\&\frac{1}{c}\,|\,x_{s+k,t+l}-y_{s+k,t+l}\,|+\\&\frac{1}{c}\Big(\sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[i-(s+k-1)]\,|\,p_{i,n+l}\,|\,|\,x_{i-\sigma,n+l-\tau}-y_{i-\sigma,n+l-\tau}\,|+\\&\sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[j-(t+l-1)]\,|\,p_{m+k,j}\,|\,|\,x_{m+k-\sigma,j-\tau}-y_{m+k-\sigma,j-\tau}\,|\Big)\leqslant\\&\frac{1}{c}\left(1+\frac{4c-5}{4}\right)\|x-y\|=\frac{4c-1}{4c}\|x-y\|.\end{aligned}$$

由于 $0<\frac{4c-1}{4c}<1$,从而 T 是 D 上的一个压缩映象,则存在一个元素 $\{x_{mn}\}\in D$,使得

$$\begin{aligned}x_{st}=&\frac{1}{c}x_{s+k,t+l}+\frac{1}{c}\Big(\sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[i-(s+k-1)]p_{i,n+l}x_{i-\sigma,n+l-\tau}+\\&\sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}p_{m+k,j}[j-(t+l-1)]x_{m+k-\sigma,j-\tau}\Big).\end{aligned}$$

令 $y_{st}=\frac{1}{c}x_{st}$,得

$$\begin{aligned}cy_{st}=&y_{s+k,t+l}+\\&\Big(\sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[i-(s+k-1)]p_{i,n+l}y_{i-\sigma,n+l-\tau}+\\&\sum_{j=t+1}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[j-(t+l-1)]p_{m+k,j}y_{m+k-\sigma,j-\tau}\Big).\end{aligned}$$

从而有

$$\begin{aligned}y_{s+k,t+l}-cy_{st}=&-\Big(\sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n+1}[i-(s+k+1)]p_{i,n+l}y_{i-\sigma,n+l-\tau}+\\&\sum_{j=t+1}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[j-(t+l-1)]p_{m+k,j}y_{m+k-\sigma,j-\tau}\Big)\\&\xlongequal{\text{由}(2.2)}-R_{st}(y).\end{aligned}\quad (2.4)$$

即亦，$y_{s+k,t+l}-cy_{st}=-R_{st}(y)$，对式(2.4)两边运行 $T^2(\Delta_1,\Delta_2)$，即

$$T^2(\Delta_1,\Delta_2)(y_{s+k,t+l}-cy_{st})=$$
$$T^2(\Delta_1,\Delta_2)(-R_{st}(y))=$$
$$-T(\Delta_1,\Delta_2)(T(\Delta_1,\Delta_2)(R_{st}(y)))\xlongequal{\text{由(A2),(A1)}}$$
$$-T(\Delta_1,\Delta_2)(-A_{st}(y))=T(\Delta_1,\Delta_2)(A_{st}(y)).$$

即

$$T^2(\Delta_1,\Delta_2)(y_{s+k,t+l}-cy_{st})=T(\Delta_1,\Delta_2)(A_{st}(y)). \tag{2.5}$$

对式(2.5)取 $s=m,t=n$，即

$$T^2(\Delta_1,\Delta_2)(y_{s+k,t+l}-cy_{st})\mid_{\substack{s=m\\t=n}}=$$
$$T(\Delta_1,\Delta_2)(A_{st}(y))\ \ _{\substack{s=m\\t=n}}\xlongequal{\text{由附录中的(A3)}}$$
$$-p_{m+k,n+l}y_{m+k-\sigma,n+l-\tau},$$

从而有

$$T^2(\Delta_1,\Delta_2)(y_{m+k,n+l}-cy_{mn})+$$
$$p_{m+k,n+l}y_{m+k-\sigma,n+l-\tau})=0. \tag{2.6}$$

而式(2.6)意味着由 $\left\{y_{st}=\frac{1}{c}x_{st}\right\}$当$s=m,t=n$ 时是系统(1.1)的正解，并且满足$\frac{1}{2}\leqslant y_{mn}\leqslant 2$，即解是有界的.

最后作为应用看一个例子：取

$$p_{mn}=\Big[2\Big(\frac{1}{((m+1)(n-1))^3}+\frac{1}{((m+1)n)^3}+$$
$$\frac{1}{((m-1)(n-1))^3}+\frac{1}{((m-1)(n+1))^3}+$$
$$\frac{1}{(m(n+1))^3}+\frac{5}{2(mn)^3}\Big)-$$
$$4\Big(\frac{1}{((m-1)n)^3}+\frac{1}{(m(n-1))^3}\Big)-\Big(\frac{1}{(m(n+2)^3}\Big)+$$
$$\frac{1}{((m+2)n)^3}+\frac{2}{((m+1)(n+1))^3}\Big)\Big]\cdot$$
$$((m-2)(n-1))^3.$$

考虑时滞 2-D 离散时间状态模型

$$T^2(\Delta_1,\Delta_2)(X_{mn}-2x_{m-1,n-1})+$$
$$p_{mn}x_{m-2,n-1}=0,\ m>2,\ n>1, \tag{2.7}$$

则系统(2.7)满足定理 1 的全部条件，从而系统(2.7)存在正的解，事实上 $\left\{x_{mn}=\frac{1}{(mn)^3}\right\}$即是这样的一个解.

以上我们在时滞 2-D 中立型离散状态模型中利用 Banach 空间中的压缩映象原理得到了正确序列的存在性，从而为进一步研究时滞 2-D 中立型离散状态模型的反馈控制、变结构控制、参数控制奠定了基础.

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附录 A(Appendix A)

关于引理中(A1),(A2),(A3)式的证明:

进一步整理(2.2)有

$$
\begin{aligned}
R_{st}(x) = & \sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[i-(s+k)]p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& 2\sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n-1}[j-(t+l)]p_{m+k,j}x_{m+k-\sigma,j-\tau} + \\
& 2\sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{m+k,j}x_{m+k-\sigma,j-\tau} = \\
& 2\sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}[i-(s+k)]p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& 2\sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& 2\sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}[j-(t+l)]p_{m+k,j}x_{m+k-\sigma,j-\tau} + \\
& 2\sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{m+k,j}x_{m+k-\sigma,j-\tau}.
\end{aligned}
$$

同理算出

$$
\begin{aligned}
R_{s+1,t}(x) = & \sum_{i=s+k+1}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}[i-(s+k)]p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}[j-(t+l)]p_{m+k,j}x_{m+k-\sigma,j-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{m+k,j}x_{m+k-\sigma,j-\tau} = \\
& \sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}[i-(s+k)]p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}[j-(t+l)]p_{m+k,j}x_{m+k-\sigma,j-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{m+k,j}x_{m+k-\sigma,j-\tau},
\end{aligned}
$$

$$
\begin{aligned}
R_{s,t+1}(x) = & \sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}[i-(s+k)]p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}[j-(t+l)]p_{m+k,j}x_{m+k-\sigma,j-\tau} + \\
& \sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{i,n+l}x_{i-\sigma,n+l-\tau}.
\end{aligned}
$$

从而有

$$
\begin{aligned}
& R_{s+1,t}(x) + R_{s,t+1}(x) + R_{st}(x) = \\
& -\left(\sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{i,n+l}x_{i-\sigma,n+l-\tau} + \right. \\
& \left.\sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{m+k,j}x_{m+k-\sigma,j-\tau}\right).
\end{aligned} \tag{A1}
$$

又令

$$
\begin{aligned}
A_{st} = & \sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n}p_{m+k,j}x_{m+k-\sigma,j-\tau}.
\end{aligned} \tag{A2}
$$

从而有

$$
\begin{aligned}
A_{s+1,t}(x) = & \sum_{i=s+k+1}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n+1}p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n+1}p_{m+k,j}x_{m+k-\sigma,j-\tau} = \\
& \sum_{i=s+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n+1}p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n+1}p_{m+k,j}x_{m+k-\sigma,j-\tau} - \\
& \left(\frac{1}{2}\right)^{s+t-m-n+1}p_{s+k,n+l}x_{s+k-\sigma,n+l-\tau}.
\end{aligned}
$$

同理有

$$
\begin{aligned}
A_{s,t+1}(x) = & \sum_{i=s'+k}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n+1}p_{i,n+l}x_{i-\sigma,n+l-\tau} + \\
& \sum_{j=t+l}^{\infty}\left(\frac{1}{2}\right)^{s+t-m-n+1}p_{m+k,j}x_{m+k-\sigma,j-l} - \\
& \left(\frac{1}{2}\right)^{s+t-m-n+1}p_{m+k,t+l}x_{m+k-\sigma,t+l-\tau}.
\end{aligned}
$$

因此得到

$$
\begin{aligned}
& A_{s+1,t}(x) + A_{s,t+1}(x) - A_{st}(x) = \\
& -\left(\frac{1}{2}\right)^{s+t-m-n+1}(p_{s+k,n+l}x_{s+k-\sigma,n+l-\tau} + p_{m+k,t+l}x_{m+k-\sigma,t+l-\tau}).
\end{aligned}
$$

从而有

$$
\begin{aligned}
& [A_{s+1,t}(x) + A_{s,t+1}(x) - A_{st}(x)]\,|_{\substack{s=m\\t=n}} = \\
& -\left(\frac{1}{2}\right)^{s+t-m-n+1}(p_{s+k,n+l}x_{s+k-\sigma,n+l-\tau} + \\
& p_{m+k,t+l}x_{m+k-\sigma,t+l-\tau})\,|_{\substack{s=m\\t=n}} = \\
& -p_{m+k,n+l}x_{m+k-\sigma,n+l-\tau}.
\end{aligned} \tag{A3}
$$

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