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Robust dissipativity and feedback stabilization for interval linear impulsive systems

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Abstract: How an interval linear impulsive system could be robustly dissipative with respect to the quadratic supply rate was illustrated. Moreover, the conditions under which a robustly dissipative interval linear impulsive system could be asymptotically stabilized by a state feedback control law. Finally, an example was given as the application.

Key words: interval impulsive dynamical system; robust dissipativity; supply rate; storage function; feedback stabilization

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区间线性脉冲系统的鲁棒耗散性及其反馈镇定

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摘要: 主要讨论了区间线性脉冲系统相对于二次供给率的鲁棒耗散性问题,得到了该系统是鲁棒耗散的充分条件,而且还给出了这样的鲁棒耗散系统能被一个状态反馈控制律所镇定的条件,最后,给出了一个例子.

关键词: 区间线性脉冲系统; 鲁棒耗散性; 供给率; 储存函数; 反馈镇定

1 Introduction

For decades, a lot of attention has been paid to dissipativity. In the literature of nonlinear control, dissipativity concept was initially introduced by Willems (1972) in his seminal two-part papers[1,2] in terms of an inequality involving the storage function and supply rate. The extension of the work of Willems to the case of affine nonlinear systems was carried out by Hill and Moylan (1976,1980)[3,4] and references therein. Byrnes and Isidori et.al started to research the dissipativity and stabilization of nonlinear continuous systems in terms of geometric nonlinear system theory in [5,6] and references therein. Recently, although [7 ~ 11] have extended the notions of classical dissipativity theory by using generalized storage functions and supplying rates for impulsive systems and left-continuous systems, the results obtained by them do not include the robust dissipativity and feedback stabilization of dissipative nonlinear impulsive systems. In this paper, by employing the methods of Lyapunov and matrix inequality, we have investigated the conditions under which an interval linear impulsive systems is robustly dissipative with respect to the quadratic supply rate. Moreover, by utilizing the stability results for general impulsive systems[12] instead of the LaSalle invariance principle, we have investigated the conditions under which a robust dissipative interval linear impulsive system can be asymptotically stabilized by linear state feedback. At last, we present an example as the application of the results obtained by us.

2 Preliminaries

An impulsive system has the form

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$$\begin{cases} \dot{x}(t) = f_c(x(t)) + g_c(x(t))u_c(t), & t \neq t_k, \\ \Delta x(t) = f_d(x(t)) + g_d(x(t))u_d(t), & t = t_k, \\ y_c(t) = h_c(x(t)), & t \neq t_k, \\ y_d(t) = h_d(x(t)), & t = t_k, \end{cases} \tag{1}$$

where $x(t_0) = x_0, t \geqslant 0, x(t) \in \mathbb{R}^n, \Delta x(t_k) = x(t_k^+) - x(t_k), u_c(t) \in U_c \subseteq \mathbb{R}^{m_c}, u_d \in U_d \subseteq \mathbb{R}^{m_d}$, $y_c(t) \in \mathbb{R}^{l_c}, y_d \in \mathbb{R}^{l_d}, f_c: \mathbb{R}^n \to \mathbb{R}^n$ is Lipschitz continuous and satisfies $f_c(0) = 0, g_c: \mathbb{R}^n \to \mathbb{R}^{n\times m_c}$, $f_d: \mathbb{R}^n \to \mathbb{R}^n$ is continuous and satisfies $f_d(0) = 0$, $g_d: \mathbb{R}^n \to \mathbb{R}^{n\times m_d}$, $h_c: \mathbb{R}^n \to \mathbb{R}^{l_c}$ and satisfies $h_c(0) = 0$, $h_d: \mathbb{R}^n \to \mathbb{R}^{l_d}$ and satisfies $h_d(0) = 0$. Here, we assume that $u_c(\cdot)$ and $u_d(\cdot)$ are restricted to the class of admissible inputs consisting of measurable functions $(u_c(t), u_d(t)) \in U = (U_c, U_d)$ for all $t \geqslant 0$, where the constraint set U is given with $(0,0) \in U$.

In this paper, we consider the quadratic supply rate (γ_c, γ_d)[7], which is given by

$$\gamma_c(u_c, y_c) = y_c^{\mathrm{T}} R_c y_c + 2y_c^{\mathrm{T}} S_c u_c + u_c^{\mathrm{T}} Q_c u_c, \tag{2}$$

$$\gamma_d(u_d, y_d) = y_d^{\mathrm{T}} R_d y_d + 2y_d^{\mathrm{T}} S_d u_d + u_d^{\mathrm{T}} Q_d u_d, \tag{3}$$

where symmetric matrices $R_c \in \mathbb{R}^{l_c\times l_c}, S_c \in \mathbb{R}^{l_c\times m_c}, Q_c \in \mathbb{R}^{m_c\times m_c}, R_d \in \mathbb{R}^{l_d\times l_d}, S_d \in \mathbb{R}^{l_d\times m_d}$, $Q_d \in \mathbb{R}^{m_d\times m_d}$, with $Q_c \geqslant 0, , Q_d \geqslant 0$.

Definition 1 An impulsive system of the form (1) is said to be dissipative with respect to supply rate (γ_c, γ_d) if there exists a $C^r(r \geqslant 0)$ nonnegative function $V: \mathbb{R}^n \to \mathbb{R}$ with $V(0) = 0$, called storage function, such that for all $(u_c, u_d) \in U$ the following dissipation inequality holds

$$V(x(t)) \leqslant V(x(t_0)) + \int_{t_0}^{t} \gamma_c(u_c(s), y_c(s))\mathrm{d}s + \sum_{k\in N[t,\hat{t})} \gamma_d(u_d(t_k), y_d(t_k)), \tag{4}$$

where $x(t)(t \geqslant t_0)$ is a solution to (1) with $(u_c, u_d) \in U$ and $x(t_0) = x_0$.

Remark 1 The special cases of dissipativity are the passivity and nonexpansivity with respect to the following supply rate (5) and (6), respectively (see [7]):

$$(\gamma_c, \gamma_d) = (2y_c^{\mathrm{T}} u_c,\ 2y_d^{\mathrm{T}} u_d), \tag{5}$$

$$(\gamma_c, \gamma_d) = (r_c^2 u_c^{\mathrm{T}} u_c - y_c^{\mathrm{T}} y_c, r_d^2 u_d^{\mathrm{T}} u_d - y_d^{\mathrm{T}} y_d), \tag{6}$$

where $r_c > 0, r_d > 0$ are constants.

The interval linear impulsive systems discussed in this paper are described by

$$\begin{cases} \dot{x}(t) = A_c x(t) + \Delta A_c x(t) + B_c u_c(t), & t \neq t_k, \\ \Delta x(t) = (A_d - I_n)x(t) + \Delta A_d x(t) + \\ \qquad B_d u_d(t), & t = t_k, \\ y_c(t) = C_c x(t), & t \neq t_k, \\ y_d(t) = C_d x(t), & t = t_k, \end{cases} \tag{7}$$

where $x(t_0) = x_0, A_c \in \mathbb{R}^{n\times n}, B_c \in \mathbb{R}^{n\times m_c}, C_c \in \mathbb{R}^{l_c\times n}, A_d \in \mathbb{R}^{n\times n}, B_d \in \mathbb{R}^{n\times m_d}, C_d \in \mathbb{R}^{l_d\times n}$, and $\Delta A_c \in N[Q_c, P_c] = \{\Delta A_c = (\Delta a_{ij}) \in \mathbb{R}^{n\times n}: q_{ij}^c \leqslant \Delta a_{ij} \leqslant p_{ij}^c\}, \Delta A_d \in N[Q_d, P_d]$ represent the interval matrices, where $Q_c = (q_{ij}^c) \in \mathbb{R}^{n\times n}, P_c = (p_{ij}^c) \in \mathbb{R}^{n\times n}$.

Definition 2 Interval linear impulsive system (7) is said to be robustly dissipative with respect to the supply rate (γ_c, γ_d) given by (2) and (3) if for any $\Delta A_c \in N[Q_c, P_c], \Delta A_d \in N[Q_d, P_d]$, the system is dissipative with respect to the supply rate (γ_c, γ_d).

Remark 2 Similarly, we can define the robust passivity and robust nonexpansivity for the interval linear impulsive system with respect to the supply rate (5) and (6), respectively. By [14], an interval matrix $\Delta A \in N[Q, P] = \{A = (a_{ij}) \in \mathbb{R}^{n\times n}: q_{ij} \leqslant a_{ij} \leqslant p_{ij}, i,j = 1,2,\cdots,n\}$ can be described as $\Delta A = A_0 + E\Sigma F$, where $A_0 = \frac{1}{2}(P+Q), H = (h_{ij})_{n\times n} = \frac{1}{2}(P-Q)$, $\Sigma \in \Sigma^* = \{\Sigma \in \mathbb{R}^{n^2\times n^2}: \Sigma = \mathrm{diag}\{\varepsilon_{11}, \cdots, \varepsilon_{nn}\}, |\varepsilon_{ij}| \leqslant 1, i,j = 1,2,\cdots,n\}, EE^{\mathrm{T}} = \mathrm{diag}\{\sum_{j=1}^{n} h_{1j}, \sum_{j=1}^{n} h_{2j}, \cdots, \sum_{j=1}^{n} h_{nj}\}, F^{\mathrm{T}}F = \mathrm{diag}\{\sum_{j=1}^{n} h_{j1}, \sum_{j=1}^{n} h_{j2}, \cdots, \sum_{j=1}^{n} h_{jn}\}$.

Hence, we can formulate the system (7) as the following interval linear impulsive system

$$\begin{cases} \dot{x}(t) = (A_c + A_{c0})x(t) + E_c\Sigma_c F_c x(t) + B_c u_c(t), & t \neq t_k, \\ \Delta x(t) = (A_d - I_n + A_{d0})x(t) + E_d\Sigma_d F_d x(t) + \\ \qquad B_d u_d(t), & t = t_k, \\ y_c(t) = C_c x(t), & t \neq t_k, \\ y_d(t) = C_d x(t), & t = t_k, \end{cases} \tag{8}$$

where $\Delta A_c = A_{c0} + E_c\Sigma_c F_c, \Delta A_d = A_{d0} + E_d\Sigma_d F_d$, and $\Sigma_c \in \Sigma^*, \Sigma_d \in \Sigma^*$.

Lemma 1[11] Let $\Sigma \in \Sigma^*$, then for any positive constant λ and any $\xi \in \mathbb{R}^{n^2}$, $\eta \in \mathbb{R}^{n^2}$ the following inequality holds

$$2\xi^{\mathrm{T}}\Sigma\eta \leqslant \lambda^{-1}\xi^{\mathrm{T}}\xi + \eta^{\mathrm{T}}\eta. \tag{9}$$

3 Robust dissipativity

Theorem 1 Assume that there exist matrices $X \in \mathbb{R}^{n\times n}$, $L_c \in \mathbb{R}^{p_c\times n}$, $L_d \in \mathbb{R}^{p_d\times n}$, $W_c \in \mathbb{R}^{p_c\times m_c}$, and $W_d \in \mathbb{R}^{p_d\times m_d}$, with X positive definite, and positive constants λ_1, λ_2 such that the following equations hold.

$$(A_c + A_{c0})^{\mathrm{T}}X + X(A_c + A_{c0}) + \lambda_1 XE_cE_c^{\mathrm{T}}X + \frac{1}{\lambda_1}F_c^{\mathrm{T}}F_c - C_c^{\mathrm{T}}R_cC_c + L_c^{\mathrm{T}}L_c = 0, \tag{10}$$

$$XB_c - C_c^{\mathrm{T}}S_c + L_c^{\mathrm{T}}W_c = 0, \tag{11}$$

$$Q_c - W_c^{\mathrm{T}}W_c = 0, \tag{12}$$

$$(A_d + A_{d0})^{\mathrm{T}}(X + \lambda_2 XE_dE_d^{\mathrm{T}}X)(A_d + A_{d0}) - (1-\mu)X + \lambda_2^{-1}F_d^{\mathrm{T}}F_d - C_d^{\mathrm{T}}R_dC_d + L_d^{\mathrm{T}}L_d = 0, \tag{13}$$

$$(A_d + A_{d0})^{\mathrm{T}}(X + \lambda_2 XE_dE_d^{\mathrm{T}}X)B_d - C_d^{\mathrm{T}}S_d + L_d^{\mathrm{T}}W_d = 0, \tag{14}$$

$$Q_d - B_d^{\mathrm{T}}(X + \lambda_2 XE_dE_d^{\mathrm{T}}X)B_d - W_d^{\mathrm{T}}W_d = 0, \tag{15}$$

where $\mu = \dfrac{\| E_d \|^2 \cdot \| F_d \|^2 \cdot \lambda_{\max}(X)}{\lambda_{\min}(X)}$, $\|\cdot\|$ stands for Euclid norm, $\lambda_{\max}(X)$ and $\lambda_{\min}(X)$ are the largest and smallest eigenvalues of matrix X, respectively. Then the interval linear impulsive system (8) is robustly dissipative with respect to the supply rate (γ_c, γ_d) given by (2) and (3).

Proof Let $V(x) = x^{\mathrm{T}}Xx$, then V is C^1 and positive definite function.

First, we shall show $\dot V(x) \leqslant \gamma_c(u_c, y_c)$, for all $t_k < t \leqslant t_{k+1}$, $k \in \mathbb{N}$.

From (10) ~ (12) and by Lemma 1, for $t_k < t \leqslant t_{k+1}$, $k \in \mathbb{N}$, we get

$$\begin{aligned}
&\dot V(x) = \\
&x^{\mathrm{T}}\{(A_c^{\mathrm{T}} + A_{c0}^{\mathrm{T}})X + X(A_c + A_{c0})\}x + 2x^{\mathrm{T}}XE_c\Sigma_cF_cx + 2x^{\mathrm{T}}XB_cu_c = \\
&x^{\mathrm{T}}\{(A_c^{\mathrm{T}} + A_{c0}^{\mathrm{T}})X + X(A_c + A_{c0})\}x + \lambda_1x^{\mathrm{T}}XE_cE_c^{\mathrm{T}}Xx + \frac{1}{\lambda_1}x^{\mathrm{T}}F_c^{\mathrm{T}}F_cx + 2x^{\mathrm{T}}XB_cu_c = \\
&x^{\mathrm{T}}\{(A_c^{\mathrm{T}} + A_{c0}^{\mathrm{T}})X + X(A_c + A_{c0}) + \lambda_1XE_cE_c^{\mathrm{T}}X + \frac{1}{\lambda_1}F_c^{\mathrm{T}}F_c\}x + 2x^{\mathrm{T}}XB_cu_c = \\
&x^{\mathrm{T}}C_c^{\mathrm{T}}R_cC_cx - x^{\mathrm{T}}L_c^{\mathrm{T}}L_cx + 2x^{\mathrm{T}}C_c^{\mathrm{T}}S_cu_c - 2x^{\mathrm{T}}L_c^{\mathrm{T}}W_cu_c = \\
&y_c^{\mathrm{T}}R_cy_c + 2y_c^{\mathrm{T}}S_cu_c + u_c^{\mathrm{T}}Q_cu_c - u_c^{\mathrm{T}}Q_cu_c - x^{\mathrm{T}}L_c^{\mathrm{T}}L_cx - 2x^{\mathrm{T}}L_c^{\mathrm{T}}W_cu_c = \\
&\gamma_c(u_c, y_c) - \| L_cx + W_cu_c \|^2 \leqslant \gamma_c(u_c, y_c).
\end{aligned} \tag{16}$$

Hence, by (16), $\dot V(x(t)) \leqslant \gamma_c(u_c(t), y_c(t))$ holds for all $t_k < t \leqslant t_{k+1}$, $k \in \mathbb{N}$.

Next, we shall show $\Delta V(x(t_k)) \leqslant \gamma_d(u_d(t_k), y_d(t_k))$, $k \in \mathbb{N}$.

By system (8), for $t = t_k$, we get

$$\begin{aligned}
&\Delta V(x_k) = \\
&\{(A_d + A_{d0})x_k + B_du_d\}^{\mathrm{T}}X\{(A_d + A_{d0})x_k + B_du_d\} + 2x_k^{\mathrm{T}}F_d^{\mathrm{T}}\Sigma_d^{\mathrm{T}}E_d^{\mathrm{T}}X\{(A_d + A_{d0})x_k + B_du_d\} + \\
&x_k^{\mathrm{T}}F_d^{\mathrm{T}}\Sigma_d^{\mathrm{T}}E_d^{\mathrm{T}}XE_d\Sigma_dF_dx_k - x_k^{\mathrm{T}}Xx_k,
\end{aligned} \tag{17}$$

where $x_k = x(t_k)$, $u_d = u_d(t_k)$.

Since $X > 0$, there exists a nonsingular matrix U such that $X = U^{\mathrm{T}}U$. Hence,

$$\begin{aligned}
&x_k^{\mathrm{T}}F_d^{\mathrm{T}}\Sigma_d^{\mathrm{T}}E_d^{\mathrm{T}}XE_d\Sigma_dF_dx_k = \\
&x_k^{\mathrm{T}}F_d^{\mathrm{T}}\Sigma_d^{\mathrm{T}}E_d^{\mathrm{T}}U^{\mathrm{T}}UE_d\Sigma_dF_dx_k = \\
&\| UE_d\Sigma_dF_dx_k \|^2 \leqslant \| U \|^2 \cdot \| E_d \|^2 \cdot \| F_d \|^2 \cdot x_k^{\mathrm{T}}x_k = \\
&\lambda_{\max}(X) \cdot \| E_d \|^2 \cdot \| F_d \|^2 \cdot x_k^{\mathrm{T}}x_k \leqslant \\
&\frac{\lambda_{\max}(X) \cdot \| E_d \|^2 \cdot \| F_d \|^2}{\lambda_{\min}(X)} \cdot x_k^{\mathrm{T}}Xx_k = \mu \cdot x_k^{\mathrm{T}}Xx_k,
\end{aligned} \tag{18}$$

where $\mu = \dfrac{\lambda_{\max}(X) \cdot \| E_d \|^2 \cdot \| F_d \|^2}{\lambda_{\min}(X)}$.

Hence, by using Lemma 1 and through (13) ~ (15), (17), (18), we get

$$\begin{aligned}
&\Delta V(x_k) \leqslant \\
&x_k^{\mathrm{T}}\{(A_d + A_{d0})^{\mathrm{T}}X(A_d + A_{d0}) - (1-\mu)X\}x_k + 2x_k^{\mathrm{T}}(A_d + A_{d0})^{\mathrm{T}}XB_du_d + 2x_k^{\mathrm{T}}F_d^{\mathrm{T}}\Sigma_d^{\mathrm{T}}E_d^{\mathrm{T}}X\{(A_d + A_{d0})x_k + B_du_d\} + u_d^{\mathrm{T}}B_d^{\mathrm{T}}XB_du_d \leqslant \\
&x_k^{\mathrm{T}}\{(A_d + A_{d0})^{\mathrm{T}}X(A_d + A_{d0}) - (1-\mu)X\}x_k + 2x_k^{\mathrm{T}}(A_d + A_{d0})^{\mathrm{T}}XB_du_d + \lambda_2^{-1}x_k^{\mathrm{T}}F_d^{\mathrm{T}}F_dx_k + \\
&\lambda_2 \cdot \{(A_d + A_{d0})x_k + B_du_k\}^{\mathrm{T}}XE_dE_d^{\mathrm{T}}X\{(A_d + A_{d0})x_k + B_du_k\} + u_d^{\mathrm{T}}B_d^{\mathrm{T}}XB_du_d = \\
&x_k^{\mathrm{T}}\{(A_d + A_{d0})^{\mathrm{T}}(X + \lambda_2XE_dE_d^{\mathrm{T}}X)(A_d + A_{d0}) + \lambda_2^{-1}F_d^{\mathrm{T}}F_d - (1-\mu)X\}x_k + 2x_k^{\mathrm{T}}(A_d + A_{d0})^{\mathrm{T}}(X + \lambda_2XE_dE_d^{\mathrm{T}}X)B_du_d + u_d^{\mathrm{T}}B_d^{\mathrm{T}}(X + \lambda_2XE_dE_d^{\mathrm{T}}X)B_du_d = \\
&x_k^{\mathrm{T}}C_d^{\mathrm{T}}R_dC_dx_k - x_k^{\mathrm{T}}L_d^{\mathrm{T}}L_dx_k + 2x_k^{\mathrm{T}}C_d^{\mathrm{T}}S_du_d - 2x_k^{\mathrm{T}}L_d^{\mathrm{T}}W_du_d + u_d^{\mathrm{T}}Q_du_d - u_d^{\mathrm{T}}W_d^{\mathrm{T}}W_du_d = \\
&\gamma_d(u_d, y_d) - \| L_dx_k + W_du_d \|^2 \leqslant \gamma_d(u_d, y_d),
\end{aligned}$$

where $x_k = x(t_k), u_d = u_d(t_k), y_d = y_d(t_k)$.

Hence, $\Delta V(x(t_k)) \leqslant \gamma_d(u_d(t_k), y_d(t_k))$ holds for all $k \in \mathbb{N}$. Therefore, by Theorem 2.1 in [7], the interval linear impulsive system given by (8) is robust dissipative with respect to the supply rate (γ_c, γ_d) given by (2) and (3). The proof is complete.

Corollary 1 Assume that there exist matrices $X \in \mathbb{R}^{n\times n}, L_c \in \mathbb{R}^{p_c\times n}, L_d \in \mathbb{R}^{p_d\times n}, W_c \in \mathbb{R}^{p_c\times m_c}$, and $W_d \in \mathbb{R}^{p_d\times m_d}$, with X positive definite, and positive constants λ_1, λ_2 such that the following equations hold.

$$(A_c + A_{c0})^T X + X(A_c + A_{c0}) + \lambda_1 XE_cE_c^T X + \frac{1}{\lambda_1}F_c^T F_c + C_c^T C_c + L_c^T L_c = 0, \tag{19}$$

$$XB_c + L_c^T W_c = 0, \tag{20}$$

$$r_c^2 I - W_c^T W_c = 0, \tag{21}$$

$$(A_d+A_{d0})^T(X+\lambda_2 XE_dE_d^T X)(A_d+A_{d0}) - (1-\mu)X + \lambda_2^{-1}F_d^T F_d + C_d^T C_d + L_d^T L_d = 0, \tag{22}$$

$$(A_d + A_{d0})^T(X + \lambda_2 XE_dE_d^T X)B_d + L_d^T W_d = 0, \tag{23}$$

$$r_d^2 I - B_d^T(X + \lambda_2 XE_dE_d^T X)B_d - W_d^T W_d = 0, \tag{24}$$

where $\mu = \dfrac{\| E_d \|^2 \cdot \| F_d \|^2 \cdot \lambda_{\max}(X)}{\lambda_{\min}(X).}$.

Then the interval linear impulsive system (8) is robustly nonexpansive with respect to the supply rate (γ_c, γ_d) given by (6).

Proof The result is a direct consequence of Theorem 1 with $Q_c = r_c^2 I, S_c = 0, R_c = -I, Q_d = r_d^2 I, S_d = 0$, and $R_d = -I$.

Remark 3 Theorem 1 can also be reduced to the robust KYP Lemma[13] when the impulses are eliminated. The details are omitted here.

4 Stabilization by state feedback

Theorem 2 Let X be positive definite matrix satisfying (10) ~ (15). If there exist $K_c \in \mathbb{R}^{m_c\times n}, K_d \in \mathbb{R}^{m_d\times n}$, and constants α_c, α_d with $\alpha_d > -1$ such that

Condition 1

$$K_c^T(S_cC_c - W_c^T L_c) + (C_c^T S_c - L_c^T W_c)K_c + C_c^T R_c C_c - L_c^T L_c - \alpha_c \cdot X \leqslant 0, \tag{25}$$

$$K_d^T(Q_d - W_d^T W_d)K_d + (C_d^T S_d - L_d^T W_d)K_d + K_d^T(S_dC_d - W_d^T L_d) + C_d^T R_d C_d - L_d^T L_d - \alpha_d \cdot X \leqslant 0. \tag{26}$$

Condition 2 Denote $\mu_k \triangleq \alpha_c(t_{k+1} - t_k) + \ln(1 + \alpha_d)$, then $\mu_k \leqslant 0$, for all $k \in \mathbb{N}$.

Then the state feedback control law

$$(u_c, u_d) = (K_c x, K_d x) \tag{27}$$

can stabilize the equilibrium $x = 0$ of dissipative interval linear impulsive system (8), and asymptotically stabilize the equilibrium $x = 0$ if $\sum_{k=1}^{\infty} \mu_k = -\infty$.

Proof By Theorem 1, the interval linear impulsive system (8) is robustly dissipative with respect to the quadratic supply rate (γ_c, γ_d). Let $V(x) = x^T Xx$, then V is C^1 and positive definite. We consider the closed-loop system to consist of (8) and (27).

First, we shall show $\dot V(x) \leqslant \alpha_c \cdot V(x)$, for all $t_k < t \leqslant t_{k+1}, k \in \mathbb{N}$.

From (10) ~ (12) and by Condition 1 for $t_k < t \leqslant t_{k+1}, k \in \mathbb{N}$, we get

$$\begin{aligned}
&\dot V(x) = \\
&x^T\{(A_c^T + A_{c0}^T)X + X(A_c + A_{c0})\}x + 2x^T XE_c\Sigma_c F_c x + 2x^T XB_cK_c x \leqslant \\
&x^T\{(A_c^T + A_{c0}^T)X + X(A_c + A_{c0})\}x + \lambda_1 x^T XE_cE_c^T Xx + \frac{1}{\lambda_1}x^T F_c^T F_c x + 2x^T XB_cK_c x = \\
&x^T\{(A_c^T + A_{c0}^T)X+X(A_c + A_{c0})+\lambda_1 XE_cE_c^T X + \frac{1}{\lambda_1}F_c^T F_c + K_c^T B_c^T X + XB_cK_c\}x = \\
&x^T\{C_c^T R_c C_c - L_c^T L_c + K_c^T(S_cC_c - W_c^T L_c) + (C_c^T S_c - L_c^T W_c)K_c\}x \leqslant \alpha_c \cdot x^T Xx.
\end{aligned} \tag{28}$$

Hence, by (28), $\dot V(x(t)) \leqslant \alpha_c \cdot V(x)$ holds for all $t_k < t \leqslant t_{k+1}, k \in \mathbb{N}$.

Next, we shall show $V(x_k^+) \leqslant (1 + \alpha_d) \cdot V(x_k)$, where $x_k^+ = x_k + \Delta x_k, k \in \mathbb{N}$.

Using Condition 1 and (13) ~ (15), we get

$$\begin{aligned}
&V(x_k^+) = \\
&\{(A_d + A_{d0})x_k + B_dK_dx_k\}^T X\{(A_d + A_{d0})x_k + B_dK_dx_k\} + 2x_k^T F_d^T \Sigma_d^T E_d^T X\{(A_d + A_{d0})x_k + B_dK_dx_x\} + x_k^T F_d^T \Sigma_d^T E_d^T XE_d\Sigma_d F_d x_k \leqslant \\
&x_k^T\{(A_d + A_{d0})^T X(A_d + A_{d0}) + \frac{\lambda_{\max}(X) \| E_d \|^2 \cdot \| F_d \|^2}{\lambda_{\min}(X)} \cdot X\}x_k + \\
&2x_k^T(A_d + A_{d0})^T XB_dK_dx_k + 2x_k^T F_d^T \Sigma_d^T E_d^T X\{(A_d + A_{d0})x_k + B_dK_dx_k\} + x_k^T K_d^T B_d^T XB_dK_dx_k \leqslant \\
&x_k^T\{(A_d + A_{d0})^T(A_d + A_{d0}) + \mu \cdot X + \lambda_2^{-1}F_d^T F_d +
\end{aligned}$$

$$(A_d + A_{d0})^T X B_d K_d + K_d^T B_d^T X (A_d + A_{d0}) + \lambda_2[(A_d + A_{d0}) + B_d K_d]^T X E_d E_d^T X[(A_d + A_{d0}) + B_d K_d] + K_d^T B_d^T X B_d K_d\} x_k = x_k^T\{(A_d + A_{d0})^T (X + \lambda_2 X E_d E_d^T X)(A_d + A_{d0}) + \mu \cdot X + \lambda_2^{-1} F_d^T F_d + K_d^T B_d^T (X + \lambda_2 X E_d E_d^T X)(A_d + A_{d0}) + (A_d + A_{d0})^T (X + \lambda_2 X E_d E_d^T X) B_d K_d + K_d^T B_d^T (X + \lambda_2 X E_d E_d^T X) B_d K_d\} x_k = x_k^T\{C_d^T R_d C_d - L_d^T L_d + X + K_d^T (Q_d - W_d^T W_d) K_d + (C_d^T S_d - L_d^T W_d) K_d + K_d^T (S_d C_d - W_d^T L_d)\} x_k \leqslant (1 + \alpha_d) x_k^T X x_k, \tag{29}$$

where $\mu = \dfrac{\| E_d \|^2 \cdot \| F_d \|^2 \cdot \lambda_{\max}(X)}{\lambda_{\min}(X)}$. Hence, by (29), $V(x_k^+) \leqslant (1 + \alpha_d) \cdot V(x_k)$ holds for all $k \in \mathbb{N}$. (28), (29) and Condition 2 imply that all the conditions of Theorem 3.1 in [12] hold. Hence, the closed-loop interval linear impulsive system given by (8) and (27) is stable, and asymptotically stable if in addition $\sum_{k=1}^{\infty} \mu_k = -\infty$. Therefore, the dissipative interval linear impulsive system (8) can be stabilized by the state feedback control law (27).

Corollary 2 Let X be positive definite matrix satisfying (10) ~ (15). If there exist $K_c \in \mathbb{R}^{m_c \times n}$, $K_d \in \mathbb{R}^{m_d \times n}$, and constants α_c, α_d with $\alpha_d > -1$ such that

1)

$$K_c^T B_c^T X + X B_c K_c - C_c^T C_c - L_c^T L_c - \alpha_c \cdot X \leqslant 0, \tag{30}$$

$$K_d^T (r_d^2 I - W_d^T W_d) K_d - L_d^T W_d K_d - K_d^T W_d^T L_d - C_d^T C_d - L_d^T L_d - \alpha_d \cdot X \leqslant 0. \tag{31}$$

2) Denote $\mu_k \triangleq \alpha_c(t_{k+1} - t_k) + \ln(1 + \alpha_d)$, then $\mu_k \leqslant 0$, for all $k \in \mathbb{N}$.

Then the state feedback control law

$$(u_c, u_d) = (K_c x, K_d x) \tag{32}$$

can stabilize the equilibrium $x = 0$ of the nonexpansive interval linear impulsive system (8), and asymptotically stabilize the equilibrium $x = 0$ if in addition $\sum_{k=1}^{\infty} \mu_k = -\infty$.

Proof The result is a direct consequence of Theorem 1 and Corollary 1 with $Q_c = r_c^2 I, S_c = 0, R_c = -I, Q_d = r_d^2 I, S_d = 0, R_d = -I$, and $X B_c = -L_c^T W_c$.

5 Example

As the application of the results obtained in Parts 3, 4, we give an example in this section. Here, the numerical calculation procedure is coded and executed by using the software MATLAB. We give the matrices of system (8) as follows

$$A_c = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & -1 \\ -2 & 0 & -1 \end{pmatrix}, \quad B_c = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix},$$

$$C_c = \begin{pmatrix} 1.000 & 0.1111 & 0.8522 \\ -1.4387 & 0.1000 & -0.2207 \\ -1.6971 & -0.7264 & 0.1000 \end{pmatrix},$$

$$A_d = \begin{pmatrix} 0.1 & 0 & 0.1 \\ 0 & 0.1 & 0 \\ 0.1 & -0.2 & 0.1 \end{pmatrix}, \quad B_d = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix},$$

$$C_d = \begin{pmatrix} 1 & -0.2155 & -0.0812 \\ 0.0385 & 0.1 & -0.9713 \\ 1.3851 & 0.8964 & 0.1 \end{pmatrix},$$

$A_{c0} = 0, A_{d0} = 0, D_c = 0, D_d = 0, E_d = F_d = 0$, and E_c, F_c satisfying

$$E_c E_c^T = \operatorname{diag}\{0.1, 0.1, 0.1\},$$
$$F_c^T F_c = \operatorname{diag}\{0.1, 0.1, 0.03\}.$$

The matrices in the quadratic supply rate (2) and (3) is given by $Q_c = 4, S_c = 0, R_c = -I_3; Q_d = 4, S_d = 0, R_d = -I_3$. Then, by solving equations (10) ~ (15), we derive one of the solutions as follows:

$$X = \begin{pmatrix} 3.0000 & 1.0000 & 0.0000 \\ 1.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix},$$

$$L_c = \begin{pmatrix} 2.0000 & 1.0000 & 1.0000 \\ 0.0000 & -1.0000 & 0.0000 \\ 0.0000 & -1.0000 & 0.0000 \end{pmatrix},$$

$$W_c = \begin{pmatrix} -2.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix},$$

$$L_d = \operatorname{diag}\{0.2000, 0.3000, 0.000\},$$
$$W_d = (1.000, -1.000, 1.000)^T,$$

$\lambda_1 = 2$, and for any $\lambda_2 > 0$.

Hence, the system obtained is robust dissipative with respect to the quadratic supply rate by Corollary 1. Moreover, we can design the state feedback controller by employing Conditions 1 and 2 of Corollary 2. Let μ_k

$= 0.1, \Delta_k = t_{k+1} - t_k = 0.1, \alpha_c = 1, \alpha_d = -0.18127$ be fixed, then we can get many (K_c, K_d) satisfying Corollary 2. For example, $K_c = (0.5 \quad 0 \quad 0.5)$, $K_d = (0.2 \quad 0.1 \quad 0.1)$, where the state feedback controller $(u_c, u_d) = (K_c x, K_d x)$ asymptotically stabilizes the dissipative interval linear impulsive system.

6 Conclusions

By employing the methods of Lyapunov and matrix inequality, we have investigated the conditions under which an interval linear impulsive system is robustly dissipative with respect to the quadratic supply rate. Dissipativity theory is always linked with the feedback stabilization theory. By utilizing the stability results for general impulsive systems, we have derived the sufficient conditions under which a robustly dissipative interval linear impulsive system can be asymptotically stabilized by a feedback control law. As for the stabilization for dissipative uncertain nonlinear impulsive systems, we will discuss it in our future papers.

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