第 20 卷第 5 期
2003 年 10 月

控制理论与应用
Control Theory & Applications

Vol.20 No.5
Oct. 2003

Article ID: 1000 - 8152(2003)05 - 0673 - 05

Feedback stabilization of nonuniform Timoshenko beam with dynamical boundary

YAN Qing-xu[1], FENG De-xing[2]
(1. Information Engineering School, China University of Geosciences, Beijing 100083, China;
2. Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100080, China)

Abstract: The boundary feedback control problem for a nonuniform Timoshenko beam with a load at one end was studied. First, a boundary feedback control scheme was proposed, and the well-posedness of the corresponding closed loop system was established. Then by using the multiplier method, it was proved that the closed loop system was exponentially stable if two boundary feedback controls were applied simultaneously to the beam's tip where the load was carried.

Key words: boundary feedback control; Timoshenko beam; C_0 semigroups; exponential stability; multiplier method
CLC number: O175.24, O175.26 **Document code**: A

带动态边界的非均匀 Timoshenko 梁的反馈镇定

阎庆旭[1], 冯德兴[2]
(1. 中国地质大学 信息工程学院, 北京 100083; 2. 中国科学院 数学与系统科学研究院, 北京 100080)

摘要: 对于一端具负载的非均质 Timoshenko 梁,研究了其边界反馈镇定问题.首先提出了一种边界反馈控制方案,建立了相应的闭环系统的适定性.然后利用乘子法证明了,当两个边界反馈控制同时作用于梁的负载端时,闭环系统是指数稳定的.

关键词: 边界反馈控制; Timoshenko 梁; C_0 半群; 指数稳定性; 乘子法

1 Introduction

Consider the boundary feedback stabilization problem of the system governed by a nonuniform Timoshenko beam with dynamical boundary conditions. The system to be investigated in this paper is described as follows (see [1 ~ 5]):

$$\begin{cases} \rho\ddot{w} + (K(\varphi - w'))' = 0, & x \in (0,l),\quad t > 0, \\ I_\rho\ddot{\varphi} - (EI\varphi')' + K(\varphi - w') = 0, & x \in (0,l),\quad t > 0, \\ M\ddot{w}(l,t) = K(l)(\varphi(l,t) - w'(l,t)) + u_1(t), & t > 0, \\ J\ddot{\varphi}(l,t) = -EI(l)\varphi'(l,t) + u_2(t), & t > 0, \\ w(0,t) = \varphi(0,t) = 0, & t > 0, \end{cases} \tag{1.1}$$

where a nonuniform beam of length l moves in $w - x$ plane, ρ is the mass per unit length, $w(x,t)$ is the deflection of the beam from its equilibrium, and $\varphi(x,t)$ is the total rotatory angle of the beam at x, I_ρ and EI are the mass moment of inertia and rigidity coefficient of the cross section, respectively, and K is the shear modulus of elasticity. Here the boundary conditions at $x = 0$ in (1.1) means that the beam is clamped at $x = 0$, and at $x = l$, the beam is of a load of mass M and rotatory inertia J. Here and after the prime and the dot stand for the derivatives with respect to space variable x and time variable t, respectively. $u_1(t)$ and $u_2(t)$ represent the boundary feedback controls applied to the beam's right end $x = l$.

For the system (1.1), we apply the following linear boundary feedback controls:

$$\begin{cases} u_1(t) = -\alpha_1\dot{w}(l,t) + \alpha_2(\dot{\varphi}(l,t) - \dot{w}'(l,t)), & t > 0, \\ u_3(t) = -\beta_1\dot{\varphi}(l,t) - \beta_2\dot{\varphi}'(l,t), & t > 0 \end{cases} \tag{1.2}$$

with nonnegative constants $\alpha_1, \alpha_2, \beta_1, \beta_2$. Moreover, in this paper, we always assume that

Received date: 2002 - 09 - 29; Revised date: 2003 - 05 - 20.
Foundation item: supported by the National Natural Science Foundation of China (60174008).

Assumption S $\rho(x), I_\rho(x), K(x), EI(x) > 0$, $\forall x \in [0,l]$, and $\rho, I_\rho, K, EI \in C^1[0,l]$.

In recent years, the feedback stabilization problem on the beam has attracted a great deal of attention and a lot of results on various feedback stabilization problems of flexible beam equations have been turned out (see [1~7]). In paper [1], it is proved that with both force and moment feedback controls applied to just one end of a Timoshenko beam without end mass, the energy corresponding to the closed loop system decays exponentially. In paper [4], the feedback stabilization problem of a uniform Timoshenko beam system (1.1) with a tip mass and dissipative boundary feedback (1.2) was considered and under the condition that $\alpha_1, \alpha_2, \beta_1, \beta_2 > 0$, some interesting results for the stability of the closed loop system are obtained. In [6], the uniform boundary stabilization of a nonuniform Euler-Bernoulli beam is obtained by using the frequency domain multiplier method. In this paper, we consider the stabilization of a nonuniform Timoshenko beam with dynamical boundary and some boundary feedback controls. In a certain comparatively weak condition, the asymptotic stability of the closed loop system is derived, while in some comparatively strong conditions, the exponential stability of the closed loop system (1.1) and (1.2) are obtained based on some of the results obtained in [4] and [6] and some of the skills used there.

2 Well-posedness of closed loop system

Set

$$\begin{cases} \xi(t) = M\dot{w}(l,t) - \alpha_2(\varphi(l,t) - w'(l,t)), & t > 0, \\ \eta(t) = J\dot{\varphi}(l,t) + \beta_2\varphi'(l,t), & t > 0, \end{cases} \tag{2.1}$$

then we have

$$\begin{cases} \dot{\xi}(t) = K(l)(\varphi(l,t) - w'(l,t)) - \alpha_1\dot{w}(l,t), & t > 0, \\ \dot{\eta}(t) = -EI(l)\varphi'(l,t) - \beta_1\dot{\varphi}(l,t), & t > 0. \end{cases} \tag{2.2}$$

Now the closed loop system (1.1) and (1.2) becomes

$$\begin{cases} \rho\ddot{w} + (K(\varphi - w'))' = 0, \quad x \in (0,l), & t > 0, \\ I_\rho\ddot{\varphi} - (EI\varphi')' + K(\varphi - w') = 0, \quad x \in (0,l), & t > 0, \\ \dot{\xi}(t) = K(l)(\varphi(l,t) - w'(l,t)) - \alpha_1\dot{w}(l,t), & t > 0, \\ \dot{\eta}(t) = -EI(l)\varphi'(l,t) - \beta_1\dot{\varphi}(l,t), & t > 0, \\ w(0,t) = \varphi(0,t) = 0, & t > 0. \end{cases} \tag{2.3}$$

To incorporate the above closed loop system into a certain function space, we define a product Hilbert space $\mathscr{H}$ by

$$\mathscr{H} = V_0^1 \times L_\rho^2(0,l) \times V_0^1 \times L_{I_\rho}^2(0,l) \times \mathbb{C}^2,$$

where $V_0^k = \{\varphi \in H^k(0,l) \mid \varphi(0) = 0\}$ for $k = 1,2$, and $H^k(0,l)$ is the usual Sobolev space of order k. The inner product in $\mathscr{H}$ is defined as follows:

$$(Y_1, Y_2)_{\mathscr{H}} = \int_0^l K(\varphi_1 - w_1')(\bar{\varphi}_2 - \bar{w}_2')\mathrm{d}x + \int_0^l EI\varphi_1'\bar{\varphi}_2\mathrm{d}x + \int_0^l \rho z_1\bar{z}_2\mathrm{d}x + \int_0^l I_\rho\psi_1\bar{\psi}_2\mathrm{d}x + \gamma_1\xi_1\bar{\xi}_2 + \gamma_2\eta_1\bar{\eta}_2,$$

where $Y_k = [w_k, z_k, \varphi_k, \psi_k, \xi_k, \eta_k]^{\mathrm{T}} \in \mathscr{H}$ for $k = 1, 2$, and

$$\gamma_1 \triangleq \frac{K(l)}{K(l)M + \alpha_1\alpha_2}, \quad \gamma_2 \triangleq \frac{EI(l)}{EI(l)J + \beta_1\beta_2}.$$

We define a linear operator $\mathscr{A}$ in $\mathscr{H}$ by

$$\mathscr{A}[w \quad z \quad \varphi \quad \psi \quad \xi \quad \eta]^{\mathrm{T}} = \begin{bmatrix} z \\ -\rho^{-1}(K(\varphi - w'))' \\ \psi \\ I_\rho^{-1}(EI\varphi')' - K I_\rho^{-1}(\varphi - w') \\ K(l)(\varphi(l) - w'(l)) - \alpha_1 z(l) \\ -EI(l)\varphi'(l) - \beta_1\psi(l) \end{bmatrix},$$

$$[w \quad z \quad \varphi \quad \psi \quad \xi \quad \eta]^{\mathrm{T}} \in \mathscr{D}(\mathscr{A}),$$

$$\mathscr{D}(\mathscr{A}) = \{[w, z, \varphi, \psi, \xi, \eta]^{\mathrm{T}} \in \mathscr{H} \mid w, \varphi \in V_0^2, z, \psi \in V_0^1, \eta = J\psi(l) + \beta_2\varphi'(l), \xi = Mz(l) - \alpha_2(\varphi(l) - w'(l))\}.$$

Then we can write the closed loop system (2.3) as the following linear evolution equation in $\mathscr{H}$:

$$\dot{Y}(t) = \mathscr{A}Y(t), \tag{2.4}$$

where $Y(t) = [w(\cdot,t), \dot{w}(\cdot,t), \varphi(\cdot,t), \dot{\varphi}(\cdot,t), \xi(t), \eta(t)]^{\mathrm{T}}$.

Theorem 1 Let $\mathscr{A}$ be defined as above, then $\mathscr{A}$ generates a C_0 semigroup $T(t)$ of contraction in $\mathscr{H}$. Moreover, $\mathscr{A}$ has compact resolvent and $0 \in \rho(\mathscr{A})$.

Proof For any $Y = [w, z, \varphi, \psi, \xi, \eta]^{\mathrm{T}} \in D(\mathscr{A})$, integrating by parts and referring to the boundary conditions of $Y \in D(\mathscr{A})$, we have

$$\mathrm{Re}\,(\mathscr{A}Y, Y)_{\mathscr{H}} = -K(l)\gamma_1\alpha_2 \mid \varphi(l) - w'(l) \mid^2 - \alpha_1\gamma_1 M \mid z(l) \mid^2 -$$

$\gamma_2\beta_2 EI(l) \mid \varphi'(l) \mid^2 - \beta_1\gamma_2 J \mid \varphi(l) \mid^2$,

which implies the dissipativity of $\mathscr{A}$.

For the maximal dissipativity of $\mathscr{A}$ and for the fact that $0 \in \rho(\mathscr{A})$, it is sufficient to show that $\forall\, \tilde{Y} = [\tilde{w}, \tilde{z}, \tilde{\varphi}, \tilde{\psi}, \tilde{\xi}, \tilde{\eta}]^T \in \mathscr{H}$, there exists a unique $Y = [w, z, \varphi, \psi, \xi, \eta]^T \in D(\mathscr{A})$, such that $\mathscr{A}\, Y = \tilde{Y}$, i.e., $z = \tilde{w}, \psi = \tilde{\varphi}$, and

$$\begin{cases} -(K(\varphi - w'))' = \rho\tilde{z},\ x \in (0, l), \\ (EI\varphi')' - K(\varphi - w') = I_\rho\tilde{\psi},\ x \in (0, l), \\ K(l)(\varphi(l) - w'(l)) = \alpha_1\tilde{w}(l) + \tilde{\xi}, \\ -EI(l)\varphi'(l) = \beta_1\tilde{\varphi}(l) + \tilde{\eta}, \\ w(0) = \varphi(0) = 0. \end{cases} \tag{2.5}$$

But it can be easily shown by the general theory of ordinary differential equations.

The compactness of the resolvent of $\mathscr{A}$ is easily derived by using the Sobolev embedding theorem. With Lummer-Phillips theorem, the proof is then complete.

Thus according to the semigroup theory[8], we obtain:

Theorem 2 For any $Y_0 \in \mathscr{H}$, (2.4), and hence the closed loop system (1.1) and (1.2), has a unique weak solution $Y(t) = T(t)Y_0$, where $T(t)$ is the linear semigroup of contraction generated by $\mathscr{A}$. Moreover, if $Y_0 \in \mathscr{D}(\mathscr{A})$, then $Y(t) = T(t)Y_0$ is a strong solution to (2.4).

3 Asymptotic decay of closed loop system

We now discuss the asymptotic stability of the closed loop system (2.4) under Assumption S and $\alpha_1 + \alpha_2 > 0, \beta_1 + \beta_2 > 0$. The energy corresponding to the solution of the closed loop system (2.4) is

$$E(t) = \frac{1}{2}\Big[\int_0^l (EI \mid \varphi' \mid^2 + K \mid \varphi - w' \mid^2 + \rho \mid z \mid^2 + I_\rho \mid \varphi \mid^2)\mathrm{d}x + \gamma_1 \mid \xi \mid^2 + \gamma_2 \mid \eta \mid^2\Big],$$

where $Y(t) = [w(\cdot,t), \dot{w}(\cdot,t), \varphi(\cdot,t), z(\cdot,t), \xi(t), \eta(t)]^T$ is the solution to (2.4). It is easy to check that under Assumption S,

$$\dot{E}(t) = -K(l)\gamma_1\alpha_2 \mid \varphi(l) - w'(l) \mid^2 - \alpha_1\gamma_1 M \mid z(l) \mid^2 - \gamma_2\beta_2 EI(l) \mid \varphi'(l) \mid^2 - \beta_1\gamma_2 J \mid \varphi(l) \mid^2, \tag{3.1}$$

Let $i\mathbb{R}$ denote the imaginary axis.

Lemma 1 Assume that $\alpha_1 + \alpha_2 > 0, \beta_1 + \beta_2 > 0$ and that Condition S holds, then $i\mathbb{R} \subset \rho(\mathscr{A})$, the resolvent set of $\mathscr{A}$.

Proof We prove only the case of $\alpha_1 = \beta_1 = 0$, $\alpha_2 > 0, \beta_2 > 0$, as for the other cases, the proof is similar. Since $\mathscr{A}$ has compact resolvent, it is sufficient to prove $i\mathbb{R} \cap \sigma_p(A) = \emptyset$. If not, then there exists a nonzero eigenvalue $i\lambda \in i\mathbb{R}$ of $\mathscr{A}$. Let $\Psi = [w, z, \varphi, \psi, \xi, \eta]^T \in D(\mathscr{A})$ be an eigenfunction of $\mathscr{A}$ corresponding to $i\lambda$. From Assumption S and the fact that

$$0 = \mathrm{Re}\,((\mathscr{A} - i\lambda)\Psi, \Psi)_{\mathscr{H}} = -\frac{\alpha_2 K(l)}{M} \mid \varphi(l) - w'(l) \mid^2 - \frac{EI(l)\beta_2}{J} \mid \varphi'(l) \mid^2, \tag{3.2}$$

we have $\varphi(l) = w'(l)$ and $\varphi'(l) = 0$. So

$$0 = i\lambda\xi - K(l)(\varphi(l) - w'(l)) = i\lambda\xi = i\lambda(Mz(l) - \alpha_2(\varphi(l) - w'(l))) = i\lambda Mz(l) = (i\lambda)^2 Mw(l),$$

and hence $w(l) = 0$. Similarly, we can derive that $\varphi(l) = 0$. Then it follows that w and φ satisfy

$$\begin{cases} (K(w' - \varphi))' + \lambda^2\rho w = 0, \\ (EI\varphi')' - K(\varphi - w') + \lambda^2 I_\rho\varphi = 0, \\ w(0) = \varphi(0) = w(l) = \varphi(l) = w'(l) = \varphi'(l) = 0. \end{cases}$$

Thus, it is trivial to deduce that $w(x) = \varphi(x) = 0$, $\forall\, x \in [0, l]$, and therefore $\Psi = 0$, a contradiction. The proof is then complete.

Based on the criterion of asymptotic stability in [9], we get the main result in this section:

Theorem 3 Suppose that $\alpha_1 + \alpha_2 > 0, \beta_1 + \beta_2 > 0$ and that Assumption S holds. Then for any initial state $Y_0 \in \mathscr{H}$, the energy $E(t)$ corresponding to the solution of the closed loop system (2.4) decays asymptotically, i.e.,

$$\lim_{t\to+\infty} E(t) = 0.$$

4 Exponential decay of closed loop system

In this section, we derive that under the condition of $\alpha_1, \beta_1 = 0, \alpha_2, \beta_2 > 0$, the closed loop system (2.4) is of exponential decay. The main result in this section is the following:

Theorem 4 Suppose that $\alpha_1, \beta_1 = 0, \alpha_2, \beta_2 > 0$. Then the energy corresponding to the closed loop system (2.4) decays exponentially, i.e., for every $Y_0 \in \mathscr{H}$, there exist positive constants C, ω, independent of Y_0, such that

$$E(t) \leqslant Ce^{-\omega t} \| Y_0 \|^2.$$

Proof According to [10], it follows from Theorem 1 and Lemma 1 that we need only to prove that

$$\sup_{\lambda \in i\mathbb{R}} \| (\lambda - \mathscr{A})^{-1} \| < +\infty.$$

Assuming the contrary, then by virtue of the continuity of $(i\lambda - \mathscr{A})^{-1}$, there must be $\lambda_n \in i\mathbb{R}$ and $Z_n = [w_n, z_n, \varphi_n, \psi_n, \xi_n, \eta_n]^{\mathrm{T}} \in \mathscr{D}(\mathscr{A}), n = 1,2,\cdots$ such that

$$\| Z_n \|_{\mathscr{H}} = 1, \ |\lambda_n| \to +\infty (\text{as } n \to \infty), \tag{4.1}$$

and

$$\| (\lambda_n - \mathscr{A}) Z_n \|_{\mathscr{H}} \triangleq \| \tilde{Z}_n \|_{\mathscr{H}} = o(1), \tag{4.2}$$

where and afterwards, $\tilde{Z}_n = (\lambda_n - \mathscr{A}) Z_n = [\tilde{w}_n, \tilde{z}_n, \tilde{\varphi}_n, \tilde{\psi}_n, \tilde{\xi}_n, \tilde{\eta}_n]^{\mathrm{T}}$, $a_n = o(1)$ means that $a_n \to 0, (n \to \infty)$ for every $\{a_n\}$.

From (4.1) and (4.2),

$$(\tilde{Z}_n, Z_n)_{\mathscr{H}} = \frac{\alpha_2 K(l)}{M} | \varphi_n(l) - w_n'(l) |^2 + \frac{EI(l)\beta_2}{J} | \varphi_n'(l) |^2 + \lambda_n \| Z_n \|_{\mathscr{H}}^2 + \int_0^l K[(\varphi_n - w_n')(\bar{\psi}_n - \bar{z}_n') - (\bar{\varphi}_n - \bar{w}'_n)(\psi_n - z_n)]\mathrm{d}x + \int_0^l EI[\varphi_n'\bar{\psi}_n' - \bar{\varphi}_n'\psi_n']\mathrm{d}x = o(1). \tag{4.3}$$

Hence, we obtain

$$\mathrm{Re}\,(\tilde{Z}_n, Z_n)_{\mathscr{H}} = \frac{\alpha_2 K(l)}{M} | \varphi_n(l) - w_n'(l) |^2 + \frac{EI(l)\beta_2}{J} | \varphi_n'(l) |^2 = o(1). \tag{4.4}$$

Referring to the facts that $\lambda_n w_n - z_n = \tilde{w}_n$ and $\lambda_n \varphi_n - \varphi_n = \tilde{\varphi}_n$, we get

$$-i\mathrm{Im}\{(\tilde{Z}_n, Z_n)_{\mathscr{H}}\} = \lambda_n\Big[\int_0^l EI | \varphi_n' |^2\mathrm{d}x + \int_0^l K | \varphi_n - w_n' |^2\mathrm{d}x - \int_0^l \rho | z_n |^2\mathrm{d}x - \int_0^l I_\rho | \psi_n |^2\mathrm{d}x - \frac{1}{M} | \xi_n |^2 - \frac{1}{J} | \eta_n |^2\Big] = o(1). \tag{4.5}$$

From (4.2) ~ (4.5) and the fact that $\lambda_n\xi_n - K(l)(\varphi_n(l) - w_n'(l)) = \lambda_n\xi_n + o(1) = \tilde{\xi}_n = o(1)$, and $\lambda_n\eta_n + EI(l)\varphi_n'(l) = \lambda_n\eta_n + o(1) = \tilde{\eta}_n = o(1)$, we have

$$Mz_n(l) - \alpha_2(\varphi_n(l) - w_n'(l)) = Mz_n(l) + o(1) = o(1),$$

$$J\varphi_n(l) + \beta_2\varphi_n'(l) = J\psi_n(l) + o(1) = o(1).$$

Thus it follows from (4.1) ~ (4.5) that

$$\begin{cases} \varphi_n(l) - w_n'(l) = o(1), \ \varphi_n'(l) = o(1), \\ z_n(l) = o(1), \ \psi_n(l) = o(1), \\ \lambda_n w_n(l) = o(1), \ \lambda_n\varphi_n(l) = o(1), \ \lambda_n\xi_n = o(1), \\ \xi_n = o(1), \ \lambda_n\eta_n = o(1), \ \eta_n = o(1), \\ \int_0^l \rho | z_n |^2\mathrm{d}x + \int_0^l I_\rho | \varphi_n |^2 - \frac{1}{2} = o(1), \\ \int_0^l K | \varphi_n - w_n' |^2\mathrm{d}x + \int_0^l EI | \varphi_n' |^2\mathrm{d}x - \frac{1}{2} = o(1), \\ \int_0^l \rho | \lambda_n w_n |^2\mathrm{d}x + \int_0^l I_\rho | \lambda_n\varphi_n |^2\mathrm{d}x - \frac{1}{2} = o(1). \end{cases} \tag{4.6}$$

Based on the fact that $\lambda_n Z_n - \mathscr{A} Z_n = \tilde{Z}_n$, we have

$$\rho\lambda_n^2 w_n - (K(w_n' - \varphi_n))' = \rho(\tilde{z}_n + \lambda_n\tilde{w}_n), \tag{4.7}$$

$$I_\rho\lambda_n^2\varphi_n - [(EI\varphi_n')' - K(\varphi_n - w_n')] = I_\rho(\tilde{\psi}_n + \lambda_n\tilde{\varphi}_n). \tag{4.8}$$

Multiplying both sides of (4.7) by $(\mathrm{e}^{M_1 x} - 1)\bar{w}_n'$, integrating from 0 to l, referring to (4.2) and (4.6), and integrating by parts if necessary, we can arrive at

$$\int_0^l (M_1\mathrm{e}^{M_1 x}\rho + (\mathrm{e}^{M_1 x} - 1)\rho') | \lambda_n w_n |^2\mathrm{d}x + \int_0^l (M_1\mathrm{e}^{M_1 x}K - (\mathrm{e}^{M_1 x} - 1)K') | w_n' |^2\mathrm{d}x + 2\mathrm{Re}\int_0^l K(\mathrm{e}^{M_1 x} - 1)\bar{w}_n'\varphi_n'\mathrm{d}x = o(1), \tag{4.9}$$

where M_1 is a positive constant to be determined later on.

Similarly, multiplying both sides of (4.8) by $(\mathrm{e}^{M_1 x} - 1)\bar{\varphi}_n'$, integrating from 0 to l, referring to (4.6), and integrating by parts if necessary, we get

$$\int_0^l (M_1\mathrm{e}^{M_1 x}I_\rho + (\mathrm{e}^{M_1 x} - 1)I_\rho') | \lambda_n\varphi_n |^2\mathrm{d}x + \int_0^l (M_1\mathrm{e}^{M_1 x}EI - (\mathrm{e}^{M_1 x} - 1)EI') | \varphi_n' |^2\mathrm{d}x - \int_0^l K(\mathrm{e}^{M_1 x} - 1)w_n'\bar{\varphi}_n'\mathrm{d}x = o(1). \tag{4.10}$$

Adding (4.9) and (4.10), we obtain

$$\int_0^l (M_1\mathrm{e}^{M_1 x}\rho + (\mathrm{e}^{M_1 x} - 1)\rho') | \lambda_n w_n |^2\mathrm{d}x + \int_0^l (M_1\mathrm{e}^{M_1 x}K - (\mathrm{e}^{M_1 x} - 1)K') | w_n' |^2\mathrm{d}x + \int_0^l (M_1\mathrm{e}^{M_1 x}I_\rho + (\mathrm{e}^{M_1 x} - 1)I_\rho' | \lambda_n\varphi_n |^2\mathrm{d}x +$$

$$\int_0^l (M_1 e^{M_1 x} EI) - (e^{M_1 x} - 1) EI') \mid \bar{\varphi}_n' \mid^2 dx = o(1), \tag{4.11}$$

which, when M_1 is large enough, is an obvious contradiction to (4.6), the proof is then complete.

References:

[1] KIM J U, RENARDY Y. Boundary control of the Timoshenko beam [J]. *SIAM J of Control & Optimization*, 1987,25(6):1417 - 1429.

[2] TIMOSHENKO S. *Vibration Problems in Engineering* [M]. New York: Van Nostand, 1955.

[3] MORGÜL O. Boundary control of a Timoshenko beam attached to a rigid body: planar motion [J]. *Int J Control*, 1991, 54(4): 763 - 761.

[4] SHI D H, HOU S H, FENG D X. Feedback stabilization of a Timoshenko beam with an endmass [J]. *Int J Control*, 1998,69(2):285 - 300.

[5] SHI D H, FENG D X. Exponential decay of Timoshenko beam with locally distributed feedback [J]. *IMA J of Mathematical Control and Information*, 2001,18:395 - 403.

[6] LIU K S, LIU Z Y. Boundary stabilization of a nonhomogeneous beam with rotatory inertia at the tip [J]. *J of Computational and Applied Mathematics*, 2000, 114(1): 1 - 10.

[7] RUSSELL D L. Mathematical models for the elastic beam and their control-theoretic implications [A]. Brezis H, Crandall H G, Kapell F. *Semigroups, Theory and Applications* [C]. Essex, England: Longman, 1986,2:177 - 217.

[8] PAZY A. *Semigroup of Linear Operators and Applications to Partial Differential Equations* [M]. New York: Springer-Verlag, 1983.

[9] HUANG F L. On the asymptotical stability of linear dynamical systems in general Banach spaces [J]. *Chinese Science Bulletin*, 1983, 28 (10): 584 - 586.

[10] HUANG F L. Characteristic condition for exponential stability of linear dynamical systems in Hilbert spaces [J]. *Chinese Annual of Differential Equations*, 1985,1:43 - 56.

作者简介:

阎庆旭 (1955 —),男,中国地质大学副教授,1985 在山东大学获理学硕士学位,2002 年在中国科学院数学与系统科学研究院获理学博士学位,主要研究兴趣是弹性振动系统的反馈镇定,以及非线性动态系统的渐近分析等;

冯德兴 (1940 —),男,中国科学院数学与系统科学研究院系统科学研究所研究员,1964 年毕业于中国科学技术大学应用数学系,研究方向为分布参数系统控制理论与应用,E-mail: dxfeng@control.iss.ac.cn