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# Controller design for a class of controlled Petri nets

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**Abstract**: A novel controller synthesis method for solving forbidden states avoidance problem was presented by exploiting the structural properties of Petri nets. The method could be used to design state feedback controllers for a special class of controlled Petri nets in which all precedence path subnets were state machines. The synthesized controller is maximally permissive under no concurrency assumption.

**Key words**: discrete event systems; Petri nets; controller synthesis

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# 一类受控 Petri 网的控制器设计

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**摘要**: 通过挖掘 Petri 网的内在的结构特性, 获得了一种新的解决禁止状态避免问题的控制器设计方法. 这种设计方法适用于一类具有特殊结构的受控 Petri 网(即所有前向路径子网是状态机)的状态反馈控制器设计. 在非并发的假设条件下, 所综合的控制器是最大允许.

**关键词**: 离散事件动态系统; Petri 网; 控制器综合

## 1 Introduction

The controller synthesis problem is to design a controller that restricts the behavior of controlled Petri net (CtlPN) to some control specifications[1]. The algorithms proposed by Holloway et al[2], Krogh et al[3] and Boel et al[4] are typically path-based methods. The main drawback of the path-based methods is that they are only applicable to a small class of CtlPN's[5]. The approach proposed by Li and Wonham[6] is a kind of typical linear integer programming approach. It can be applied for CtlPN's in which the uncontrollable subnets are loop-free. The S-decreases method proposed by Chen is based on dual LIP approach. The method can be used for CtlPN's in which all influentially uncontrolled subnets are forward and backward conflict-free[5].

Through exploiting the structural properties of Petri nets, for a special class of CtlPN's in which all precedence path subnets are state machines, the control policy can be obtained via determining whether or not a marking satisfies a collection of inequalities in this paper. We claim that the CtlPN consider here cannot be addressed by the reported methods[2~6]. Furthermore, the designed controller is maximally permissive under no concurrency assumption.

## 2 Controlled Petri nets

### 2.1 Ordinary Petri nets

An ordinary Petri net is a triple $G := (P, T, E)$ with the set of places $P$, the set of transitions $T$, the set of directed arcs $E \subseteq (P \times T) \cup (T \times P)$, and the incidence matrix $E: P \times T \rightarrow \{0, 1, -1\}$ defined as [7]:

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$$E(p,t) = \begin{cases} 1, & \text{if } (t,p) \in E, \\ -1, & \text{if } (p,t) \in E, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

The set of all input and output places of a transition $t \in T$ is defined as $^{\cdot}t = \{p \in P \mid (p,t) \in E\}$ and $t^{\cdot} = \{p \in P \mid (t,p) \in E\}$, respectively. Similarly, the set of all input and output transitions of a place $p \in P$ is defined as $^{\cdot}p = \{t \in T \mid (t,p) \in E\}$ and $p^{\cdot} = \{t \in T \mid (p,t) \in E\}$, respectively. A state machine (SM) is an ordinary PN such that $|^{\cdot}t| = |t^{\cdot}| = 1$ for all $t \in T$.

A marking of a PN is a function $m: P \to Z_+$. The set of all possible markings is denoted by $M$. A transition $t$ is state-enabled by a marking $m \in M$ if for each $p \in {}^{\cdot}t$ such that $m(p) \geqslant 1$. For a given marking $m$, a state-enabled transition $t$ may be fired, and will result in a new marking $m'$ defined by the following equation:

$$m'(p) = m(p) - |p^{\cdot} \cap \{t\}| + |^{\cdot}p \cap \{t\}|, \ \forall p \in P. \quad (2)$$

The evolution of a PN from a marking $m$ to a marking $m'$ after firing transition $t$ is denoted by $m[t\rangle m'$, where $t$ is state-enabled by $m$, and $m'$ is defined by the equation (2). A firing sequence from $m_0$ is a sequence of transition $\sigma = (t_0, t_1, \cdots, t_{k-1})$ such that $m_0[t_0\rangle m_1 [t_1\rangle \cdots [t_{k-1}\rangle m_k$, denoted as $m_0[\sigma\rangle m_k$. After executing a firing sequence $\sigma$, the new marking can be decided by the following state transition equation:

$$m' = m + E \cdot \bar{\sigma}, \quad (3)$$

where $\bar{\sigma}(t) := \sum_{i=0}^{k-1} |\{t\} \cap \{t_i\}|$ is a firing count vector.

A marking $m'$ is reachable from $m$ in $G$ if there exists a firing sequence $\sigma$ such that $m[\sigma\rangle m'$. The set of all firing sequences in $G$ is denoted by $\Sigma(G)$. The set of transitions occurring in the firing sequence $\sigma$ is denoted by $T(\sigma)$. The set of all reachable markings in $G$ is denoted by $R_\infty(m)$. For $M' \subseteq M$, the set of all reachable markings from $M'$ is denoted by $R_\infty(M') := \bigcup_{m \in M'} R_\infty(m)$.

### 2.2 Controlled Petri nets

Formally, a CtlPN is a tuple $G^c := (G, T^c)$, where $G = (P, T, E)$ is an ordinary Petri net and $T^c \subseteq T$ ($T^u = T \setminus T^c$) is the set of controllable (uncontrollable) transitions. The state of a CtlPN $G^c$ is determined by its marking, which is the distribution of tokens in the state places. A CtlPN is shown in Fig. 1, where circles representing state places, bars representing uncontrollable transitions, and bars with the letter $c$ representing controllable transitions.

A control of a CtlPN is a function $u: T \to \{0,1\}$ with $\forall t \in T^u, u(t) \equiv 1$. The set of all controls is denoted by $U$. $u_0 \in U$ is called the zero control, if $\forall t \in T^c$, $u_0(t) = 0$. A transition $t$ is control-enabled if $u(t) = 1$. A transition $t$ is enabled under the given marking $m$ and control $u$ if $t$ is both state-enabled and control-enabled. For a given marking $m$ and control $u$, an enabled transitions $t$ can be fired, and will result in a new marking $m'$ defined by the equation (2).

Given a CtlPN $G^c$ and a control $u$, if there exists a firing sequence $\sigma$ such that $m[\sigma\rangle m'$ and $t_i$ is control-enabled, then the marking $m'$ can be reached from $m$ under $u$ in $G^c$. The marking set $R_k(u, m)$ denotes all markings that can be reached from $m$ within $k$ steps under the control $u$. The set of all reachable markings under the control $u$ is defined by $R_\infty(u, m) := \lim_{k \to \infty} R_k(u, m)$.

### 2.3 Precedence path subnet

A path $\pi = (p_1 t_1 p_2 t_2 \cdots t_{k-1} p_k)$ is a string of nodes such that both of the beginning and end nodes are places, $t_i \neq t_j$ and $p_i \neq p_j$ with $i \neq j$, and $t_i \in p_i^{\cdot} \cap {}^{\cdot}p_{i+1}$ for $\forall i \in \{i, \cdots, k-1\}$. The set composed of all places occurring in $\pi$ is denoted by $\pi_p$.

**Definition 1** Given a place $p \in P$ in $G^c$, a precedence path $\pi(p)$ of $p$ is a path $(p_1 t_1 p_2 \cdots t_{k-1} p_k)$ such that: 1) $p = p_k$; 2) $\forall i \in \{1, \cdots, k-1\}, t_i \in T^u$; The set of all precedence path of $p$ is denoted by $\Pi(p)$. Define place set $\Pi_p(p) := \bigcup_{\pi \in \Pi(p)} \pi_p(p)$.

**Definition 2** Given a place $p \in P$ in $G^c$, a precedence path subnet $PPS(p)$ is a subnet $\hat{P}, \hat{T}, \hat{E})$, where $\hat{P} = \Pi_p(p), \hat{T} = {}^{\cdot}\hat{P} \cap T^u$ and $\hat{E} = (\hat{P} \times \hat{T} \cup \hat{T} \times \hat{P}) \cap E$.

**Example 1** Consider the CtlPN shown in Fig. 1. The precedence path subnets of $p_1$ and $p_5$ are illustrated in Fig.2 and Fig.3 respectively.

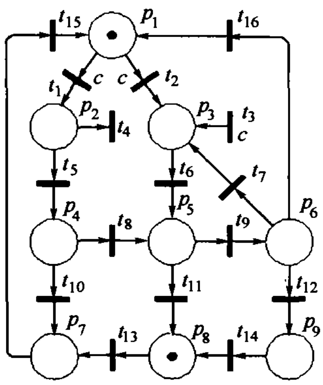


Fig. 1 Controlled Petri net

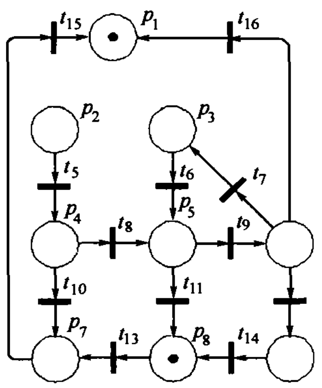


Fig. 2 Precedence path subnet of $p_1$

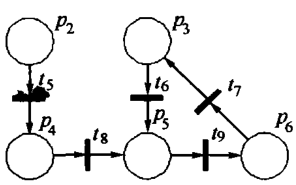


Fig. 3 Precedence path subnet of $p_5$

## 3 Admissible markings

We consider a legal set defined as follows:

$$M_{F,k,v} = \{m \in M \mid \sum_{p \in F} v(p) \cdot m(p) \leqslant k\}, \tag{4}$$

where $F \subseteq P$ is said to be constraint place set, $k \in \mathbb{Z}_+$ is a non-negative constant scalar, and $v: F \to \mathbb{R}_+$ is a non-negative constant $n$-vector. It can also be expressed as the following form:

$$M_{F,k,v} = \{m \in M \mid v^{\mathrm{T}} \cdot m \leqslant k\}, \tag{5}$$

where $v(p) = 0$ for $\forall p \notin F$. The admissible set of $M_{F,k,v}$ is defined as follows:

$$A_{F,k,v} = \{m \in M \mid R_\infty(u_0, m) \subseteq M_{F,k,v}\}. \tag{6}$$

According to (4), $A_{F,k,v}$ can also be described as

$$A_{F,k,v} = \{m \in M \mid \forall m' \in R_\infty(u_0, m), \sum_{p \in F} v(p) \cdot m'(p) \leqslant k\}. \tag{7}$$

From (3), (5) and (7), we have the equation

$$A_{F,k,v} = \{m \in M \mid v^{\mathrm{T}} \cdot m + v^{\mathrm{T}} \cdot E \cdot \bar{\sigma}^* \leqslant k\}, \tag{8}$$

where $\bar{\sigma}^*$ is the solution to the following optimization problem[6]:

$$\max_{\sigma \in \Sigma(G_u)} v^{\mathrm{T}} \cdot E \cdot \bar{\sigma}, \tag{9}$$

Usually, the optimization problem is a nonlinear integer program[6]. However, if all precedence path subnets of the constraint places are state machines, then the problem can be converted into determining whether or not a marking satisfies a collection of inequalities.

**Definition 3** Given a set $F' \subseteq F$ such that $\forall p_1, p_2 \in F', v(p_1) = v(p_2) = v'$ and $\forall p_3 \in F \setminus F'$, $v(p_3) \neq v'$ is called equivalent constraint place set. A set composed of all equivalent constraint place sets of $F$ is denoted by $\mathscr{F}$, and it can be indexed by the subscript set $I = \{1, \cdots, s\}$ where $s$ is some integer, i.e., $\mathscr{F} = \bigcup_{i \in I} \{F_i\}$.

By Definition 3, $\forall F_i \in F$ and $\forall p \in F_i, v(p)$ is fixed and is denoted by $v_i$. Henceforth, we can rewrite (7) as follows:

$$A_{F,k,v} = \{m \in M \mid \sum_{m' \in R_\infty(u_0, m)} (v_i \cdot \sum_{p \in F_i} m'(p)) \leqslant k\}. \tag{10}$$

**Definition 4** For $F_i \in \mathscr{F}(i \in I)$, we define the following place sets:

1) $(F_i, \prec) := \{F_j \in \mathscr{F} \mid j \in I, v_i > v_j\}$;

2) $^\circ F_i := \bigcup_{p \in F_i} \Pi_p(p)$;

3) $^\circ(F_i, \prec) := \bigcup_{\hat{F} \in (F_i, \prec)} {}^\circ\hat{F}$;

4) $^* F_i := {}^\circ F_i \setminus {}^\circ(F_i, \prec)$.

**Definition 5** For $\mathscr{F} = \{F_i \subseteq F \mid i \in I\}$, we define the following transition sets:

1) $\forall i \in I, T_i := {}^\cdot({}^* F_i) \cap ({}^* F_i)^\cdot \cap T^u$;

2) $T_b = \bigcup_{i \in I}(({}^* F_i)^\cdot \setminus {}^\cdot({}^* F_i)) \cap T^u$;

3) In special, $T_0 := T^u \setminus (\bigcup_{i \in I} T_i \cup T_b)$.

**Remark 1** For $\forall p \in F$, if $PPS(p)$ is a state machine, then ${}^\cdot T_i \cap {}^\cdot T_j = \Phi$, ${}^\cdot T_i \cap T_j^\cdot = \Phi$ and $T_i^\cdot \cap {}^\cdot T_j = \Phi$ for $\forall i, j \in \{0, 1, \cdots, s\}$ with $i \neq j$.

**Lemma 1** Let $\forall p \in F_i, PPS(p)$ be a state machine. Then there is a firing sequence $\sigma$ with $T(\sigma) \subseteq T_i$

such that $\sum_{p\in F_i} m'(p) = \sum_{p\in {}^*F_i} m(p)$ where $m[\sigma\rangle m'$.

**Proof** From Definition 4, $\forall p_l \in {}^*F_i, l \in \{1,\cdots,|{}^*F_i|\}$, there is a path $\pi = (p_{l0}t_{l1}p_{l1}t_{l2}\cdots p_{l(k_l-1)}t_{lk_l}p_{lk_l})$ such that for $j = 1,\cdots,k, p_{lj} \in {}^*F_i, t_{lj} \in T_i$ and $p_{lk_l} \in F_i$. Since $\forall p \in F_i, PPS(p)$ is a state machine, $\sigma'_l = (t_{l1}t_{l2}\cdots t_{lk_l}p_{lk_l})$ is a firing sequence and it can be executed by $m(p_l)$ times, i.e.,

$$\sigma_l = \underbrace{\sigma'_l:\sigma'_l:\cdots:\sigma'_l}_{m(p)}$$

is a firing sequence such that $T(\sigma_l) \subseteq T_i$. After firing $\sigma_l$, all tokens in $p_l$ move into $p_{lk}$ and the tokens in other places do never change. From the above description, $\sigma = \sigma_1:\cdots:\sigma_{|{}^*F_i|}$ is a firing sequence such that $T(\sigma_i) \subseteq T_i$. After firing $\sigma$, it means that all tokens in ${}^*F_i$ move into $F_i$ and the tokens in other places do never change. Henceforth, the Lemma is true.

**Remark 2** From the proof of Lemma 1, it follows that after firing any sequence $\sigma$ with $T(\sigma) \subseteq T_i$, we have that $\sum_{p\in F_i} m'(p) \leqslant \sum_{p\in {}^*F_i} m(p)$.

**Lemma 2** Let $\forall p \in F, PPS(p)$ be a state machine. Then there is a firing sequence $\sigma = \sigma_1:\cdots:\sigma_s$ with $T(\sigma_i) \subseteq T_i(i = 1,\cdots,s)$ satisfying that after the firing of $\sigma$, it holds that $\sum_{p\in F} v(p)\cdot m'(p) = \sum_{i\in I}(v_i \cdot \sum_{p\in {}^*F_i} m(p))$, where $m[\sigma\rangle m'$.

**Proof** We prove the lemma by induction on the cardinality of $\mathscr{F}$, i.e., $|\mathscr{F}|$.

a) Induction base: when $|\mathscr{F}| = 1$, from Lemma 1, it is true;

b) Induction hypothesis: when $|\mathscr{F}| = s-1$, it is true;

c) Induction procedure: when $|\mathscr{F}| = s$, it implies that $\mathscr{F} = \mathscr{F}' \cup \{F_s\}$ where $\mathscr{F}' = \{F_1,\cdots,F_{s-1}\}$. It is obvious that $|\mathscr{F}'| = s-1$. By induction hypothesis, there is a firing sequence $\sigma' = \sigma_1:\cdots:\sigma_{s-1}$ such that $T(\sigma_i) \subseteq T_i(i = 1,\cdots,s-1)$. After firing $\sigma'$, it holds that

i) $m_{s-1}(p)\,|_{p\in {}^*F_s} = m(p)\,|_{p\in {}^*F_s}$;

ii) $\sum_{p\in \bigcup_{i=1}^{s-1}F_i} (v(p)\cdot m_{s-1}(p)) = \sum_{i=1}^{s-1}(v_i\cdot \sum_{p\in {}^*F_i} m(p))$.

By Lemma 1, there is a firing sequence $\sigma_s$ with $T(\sigma_s) \subseteq T_i$. After firing $\sigma_s$, it holds that

iii) $m_s(p)\,|_{p\in \bigcup_{i=1}^{s-1}F_i} = m_{s-1}(p)\,|_{p\in \bigcup_{i=1}^{s-1}F_i}$;

iv) $\sum_{p\in F_s} m_s(p) = \sum_{p\in {}^*F_s} m_{s-1}(p)$.

From i), ii), iii) and iv), we have that

$$\sum_{p\in F}(v(p)\cdot m_s(p)) =$$
$$\sum_{p\in \bigcup_{i=1}^{s-1}F_i}(v(p)\cdot m_s(p)) + \sum_{p\in F_s}(v_s\cdot m_s(p)) =$$
$$\sum_{p\in \bigcup_{i=1}^{s-1}F_i}(v(p)\cdot m_{s-1}(p)) + \sum_{p\in {}^*F_s}(v_s\cdot m_{s-1}(p)) =$$
$$\sum_{i=1}^{s-1}(v_i\cdot \sum_{p\in {}^*F_i} m(p)) + \sum_{p\in {}^*F_s}(v_s\cdot m(p)) =$$
$$\sum_{i=1}^{s}(v_i\cdot \sum_{p\in {}^*F_i} m(p)).$$

Hence, when $|\mathscr{F}| = s$, the lemma is true.

**Remark 3** From Remark 2 and Lemma 2, it implies that after firing any sequence $\sigma = \sigma_1:\cdots:\sigma_s$ with $T(\sigma_i) \subseteq T_i(i = 1,\cdots,s)$, we have that:

$$\sum_{p\in F} v(p)\cdot m'(p) \leqslant \sum_{i\in I}(v_i\cdot \sum_{p\in {}^*F_i} m(p)).$$

**Lemma 3** Given an ordinary Petri net $G$ and two transitions $t_1, t_2 \in T, {}^\cdot t_1 \cap {}^\cdot t_2 = \Phi$ and ${}^\cdot t_1 \cap t_2^\cdot = \Phi$. If $(t_2t_1)$ is a firing sequence, then $(t_1t_2)$ is also a firing sequence.

**Proof** Lemma 3 comes from Lemma 5 in [7].

**Lemma 4** Given an ordinary Petri net $G$ and two sets of transitions $T_1, T_2 \in T, {}^\cdot T_1 \cap {}^\cdot T_2 = \Phi$ and ${}^\cdot T_1 \cap T_2^\cdot = \Phi$. If $m[\sigma\rangle \hat{m}$ with $T(\sigma) \subseteq T_1 \cup T_2$, then there exists $\tilde{\sigma} = \sigma_1:\sigma_2$ satisfying $T(\sigma_i) \subseteq T_i(i = 1,2)$ and $m[\tilde{\sigma}\rangle \hat{m}$.

**Proof** We prove the lemma by induction on occurring counts of the elements of $T_1$ in $\sigma$ which denoted by cout $(\sigma, T_1)$.

a) Induction base: When cout $(\sigma, T_1) = 1$, suppose $\sigma = (t_1\cdots t_i\cdots t_k)$ such that $t_1,\cdots,t_{i-1},t_{i+1},\cdots,t_k \in T_2$ and $t_i \in T_1$. By applying Lemma 3 multiple times in succession, $\tilde{\sigma} = (t_it_1\cdots t_{i-1}t_{i+1}\cdots t_k)$ is a firing sequence. So, the lemma is true;

b) Induction hypothesis: When cout $(\sigma, T_1) = l-1$, the Lemma is true;

c) Induction procedure: When cout $(\sigma, T_1) = l$, suppose $\sigma = \sigma^1 : t : \sigma^2$ such that $t \in T_1$ and $T(\sigma^2) \subseteq T_2$. It implies that cout $(\sigma^1, T_1) = l - 1$. From the induction hypothesis, $\sigma_1^1 : \sigma_2^1 : t : \sigma^2$ is a firing sequence such that $m[\sigma_1^1 : \sigma_2^1 : t : \sigma^2\rangle \hat{m}$, where $T(\sigma_1^1) \subseteq T_1$ and $T(\sigma_2^1) \subseteq T_2$. By applying Lemma 3 multiple times in succession, $\sigma_1^1 : t : \sigma_2^1 : \sigma^2$ is a firing sequence such that $m[\sigma_1^1 : t : \sigma_2^1 : \sigma^2\rangle \hat{m}$. Let $\sigma_1 = \sigma_1^1 : t$ and $\sigma_2 = \sigma_2^1 : \sigma^2$, then $\tilde{\sigma} = \sigma_1 : \sigma_2$ is a firing sequence. Henceforth, the lemma is true.

**Lemma 5** Let $\forall p \in F, PPS(p)$ be a state machine. If there is a firing sequence $\sigma$ such that $T(\sigma) \subseteq \bigcup_{i=0}^{s} T_i$ and $m[\sigma\rangle \hat{m}$, then there must exist $\tilde{\sigma} = \sigma_0 : \sigma_1 : \cdots : \sigma_n$ with $T(\sigma_i) \subseteq T_i (i = 0, 1, \cdots, s)$ such that $m[\tilde{\sigma}\rangle \hat{m}$.

**Proof** By Remark 1, we can obtain the lemma by applying Lemma 4 multiple times in succession.

**Lemma 6** Let $\forall p \in F, PPS(p)$ be a state machine. If there is a firing sequence $\sigma$ such that $T(\sigma) \subseteq T^u$, $T(\sigma) \cap T_b \neq \Phi$, and $m[\sigma\rangle \hat{m}$ then

$$\sum_{p \in F} (v(p) \cdot \hat{m}(p)) < \max_{m' \in R_\infty(u_0, m)} \left(\sum_{p \in F} v(p) \cdot m'(p)\right).$$

**Proof** It suffices to show that if $\sigma = \sigma_1 : t : \sigma_2$ where $t \in T_b$ and $T(\sigma_1), T(\sigma_2) \subseteq \bigcup_{i \in I} T_i$, then $\sum_{p \in F} (v(p) \cdot \hat{m}(p)) < \max_{m' \in R_\infty(u_0, m)} (\sum_{p \in F} v(p) \cdot m'(p))$. Considering $m[\sigma_1\rangle m_1[t > m_2[\sigma_2\rangle \hat{m}$, there are three cases:

1) For ${}^{\cdot}t \subseteq {}^{*}F_i$ and $t^{\cdot} \subseteq {}^{*}F_j$ with $v_i > v_j$: If the transition $t$ is fired at $m_1$, then it is true that

$$m_2(p) \mid_{p \in {}^{\cdot}t} = m_1(p) \mid_{p \in {}^{\cdot}t} - 1$$
$$m_2(p) \mid_{p \in t^{\cdot}} = m_1(p) \mid_{p \in t^{\cdot}} + 1$$
$$m_2(p) \mid_{p \notin {}^{\cdot}t \cup t^{\cdot}} = m_1(p) \mid_{p \notin {}^{\cdot}t \cup t^{\cdot}}.$$

From Remark 3, it holds that

$$\sum_{p \in F} v(p) \cdot \hat{m}(p) \leqslant \sum_{i \in I} (v_i \cdot \sum_{p \in {}^{*}F_i} m_2(p)),$$

namely,

$$\sum_{p \in F} (v(p) \cdot \hat{m}(p)) \leqslant \sum_{i \in I} (v_i \cdot \sum_{p \in {}^{*}F_i} m_1(p)) + v_j - v_i.$$

By Lemma 5, we also have a firing sequence $\sigma' = \sigma_1 : \sigma_2'$ such that $T(\sigma_2') \subseteq \bigcup_{i \in I} T_i$ and $\sum_{p \in F} (v(p) \cdot \tilde{m}(p)) = \sum_{i \in I} (v_i \cdot \sum_{p \in {}^{*}F_i} m_1(p))$. Henceforth,

$$\sum_{p \in F} (v(p) \cdot \hat{m}(p)) < \sum_{p \in F} (v(p) \cdot \tilde{m}(p)) \leqslant \max_{m' \in R_\infty(u_0, m)} \left(\sum_{p \in F} v(p) \cdot m'(p)\right).$$

2) For ${}^{\cdot}t \subseteq {}^{*}F_i$ and $t^{\cdot} \subseteq {}^{*}F_j$ with $v_i < v_j$, this case does not exist;

3) For ${}^{\cdot}t \subseteq {}^{*}F_i$ and $t^{\cdot} \not\subseteq \bigcup_{j \in I} {}^{*}F_j$, the proof is similar to the one of cases 1).

**Lemma 7** Let $\forall p \in F, PPS(p)$ be a state machine. Then $\max_{\hat{m} \in R_\infty(u_0, m)} (\sum_{p \in F} v(p) \cdot \hat{m}(p)) = \sum_{i \in I} (v_i \cdot \sum_{p \in {}^{*}F_i} m(p))$.

**Proof** From Lemma 6, it suffices to show that for any $\sigma$ with $T(\sigma) \subseteq T^u$ and $T(\sigma) \cap T_b = \Phi$ such that $\sum_{p \in F} (v(p) \cdot \hat{m}(p)) \leqslant \sum_{i \in I} (v_i \cdot \sum_{p \in {}^{*}F_i} m(p))$. From Lemma 5, $\tilde{\sigma} = \sigma_0 : \sigma_1 : \cdots : \sigma_s$ is a firing sequence such that $m[\tilde{\sigma}\rangle \hat{m}$ and $T(\sigma_i) \subseteq T_i (i = 0, \cdots, s)$. From Remark 3, we have that $\sum_{p \in F} (v(p) \cdot \hat{m}(p)) \leqslant \sum_{i \in I} (v_i \cdot \sum_{p \in {}^{*}F_i} m(p))$. By Lemma 2, we know that the lemma is true.

From the above lemma, we can rewrite (10) as follows:

$$A_{F,k,v} = \{m \in M \mid \sum_{i \in I} (v_i \cdot \sum_{p \in {}^{*}F_i} m(p)) \leqslant k\} \tag{11}$$

and it is natural to obtain the following theorem:

**Theorem 1** Given a CtlPN $G^c$ and a legal set $M_{F,k,v}$. If $PPS(p)$ is a state machine, then $A_{F,k,v} = \{m \in M \mid c^{\mathrm{T}} \cdot m \leqslant k\}$ with some constant $n$-vector $c \in \mathbb{R}_+^n$.

**Example 2**(continued) Consider the CtlPN Fig. 1 with the legal set $M_{F,k,v} = (m(p_1) + 2m(p_5) \leqslant 3)$. We have that $F = \{p_1, p_5\}, F_1 = \{p_1\}, F_2 = \{p_5\}$, ${}^{*}F_1 = \{p_1, p_7, p_8, p_9\}$ and ${}^{*}F_2 = \{p_2, p_3, p_4, p_5, p_6\}$. By Lemma 7, it holds that $A_{F,k,v} = (c^{\mathrm{T}} \cdot m \leqslant k)$ where $c = (1,2,2,2,2,2,1,1,1)^{\mathrm{T}}$ and $k = 3$.

## 4 State feedback controllers

Define a state feedback controller as follows[6]:

$$u(t_i, m) = \begin{cases} 1, & \text{if } m[t_i\rangle m',\ m' \in A_{F,k,v}, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

From Theorem 1, we can directly obtain the following theorem:

**Theorem 2** Given a CtlPN $G^c$ and a legal set $M_{F,k,v}$. If $\forall p \in F, PPS(p)$ is a state machine, then the state feedback control policy is given by

$$\forall t_i \in T^c,\ u(t_i, m) = (c^T \cdot m \leqslant d_i) \quad (13)$$

where $d_i$ is some constant.

**Proof** By the state transition equation (2), we have $m' = m + E \cdot \bar{\sigma}$, where $\bar{\sigma}(t_i) = 1$. By Theorem 1, it holds that $c^T \cdot (m + E \cdot \bar{\sigma}) \leqslant k$. It means that there is constant $d_i = k - c^T \cdot E \cdot \bar{\sigma}$ such that $c^T \cdot m \leqslant d_i$. Henceforth, the theorem is true.

Under the assumption of no concurrency (NC), the controller defined as (12) is the unique maximally permissive control policy for a given $m \in M_{F,k,v}$[6]. So, the state feedback controller designed by (13) is also the unique maximally permissive.

**Example 3** (continued) Consider the CtlPN shown in Fig. 1 with the initial marking $m = (1,0,0,0,0,0,0,1,0)^T$ and the legal set $M_{F,k,v} = (m(p_1) + 2m(p_5) \leqslant 3)$. By Theorem 1, we have $A_{F,k,v} = (c^T \cdot m \leqslant k)$, where $c = (1,2,2,2,2,2,1,1,1)^T$ and $k = 3$. According to (13), we obtain that

$$u(t_1, m) = (c^T \cdot m \leqslant 2) = 1,$$
$$u(t_2, m) = (c^T \cdot m \leqslant 2) = 1,$$
$$u(t_3, m) = (c^T \cdot m \leqslant 1) = 0.$$

**Remark 4** If the control objective is given by an intersection of $M_{F_j,k_j,v_j}$ ($j \in J$, $J$ is an index set), then Theorem 2 can be directly extended as follows: If $\forall j \in J, \forall p \in F_j, PPS(p)$ is a state machine, then the state feedback control policy $u$ is given by

$$\forall t_i \in T^c,\ u(t_i, m) = \bigwedge_{j \in J} (c_j^T \cdot m \leqslant d_{i,j}), \quad (14)$$

where $c_j$ is some constant $n$-vector and $d_{i,j}$ is some constant.

**Example 4** (the cat and mouse problem) The problem is a popular example in the field of DES control and the model in Fig. 4 is taken from [9], where the transitions $t_7$ and $t_9$ are uncontrollable (For details, please refer to [9]). The control objective is given as follows:

$$M_{F_1,k_1,v_1} = (m(p_1) + m(p_6) \leqslant 1),$$
$$M_{F_2,k_2,v_2} = (m(p_2) + m(p_7) \leqslant 1),$$
$$M_{F_3,k_3,v_3} = (m(p_3) + m(p_8) \leqslant 1),$$
$$M_{F_4,k_4,v_4} = (m(p_2) + m(p_9) \leqslant 1),$$
$$M_{F_5,k_5,v_5} = (m(p_5) + m(p_{10}) \leqslant 1).$$

By Theorem 1, we have the following expressions:

$$A_{F_1,k_1,v_1} = (m(p_1) + m(p_6) \leqslant 1),$$
$$A_{F_2,k_2,v_2} = (m(p_2) + m(p_4) + m(p_7) \leqslant 1),$$
$$A_{F_3,k_3,v_3} = (m(p_3) + m(p_8) \leqslant 1),$$
$$A_{F_4,k_4,v_4} = (m(p_2) + m(p_4) + m(p_9) \leqslant 1),$$
$$A_{F_5,k_5,v_5} = (m(p_5) + m(p_{10}) \leqslant 1).$$

From Remark 4, the state feedback control policy is given as follows (For simplicity, we only show the control of $t_1$):

$$u(t_1, m) =$$
$$(m(p_1) + m(p_6) \leqslant 2) \wedge$$
$$(m(p_2) + m(p_4) + m(p_7) \leqslant 0) \wedge$$
$$(m(p_3) + m(p_8) \leqslant 2) \wedge$$
$$(m(p_2) + m(p_4) + m(p_9) \leqslant 0) \wedge$$
$$(m(p_5) + m(p_{10}) \leqslant 2).$$

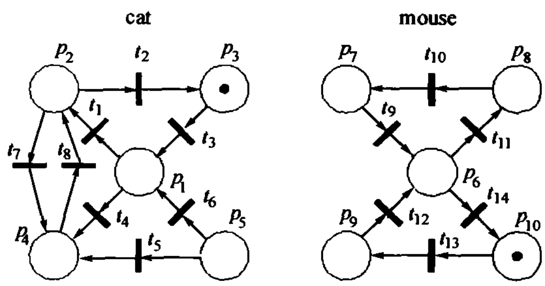


Fig. 4 Cat and mouse problem

## 5 Conclusions

In this paper, we have shown that if all precedence path subnets are state machines, then the computation of state feedback control policy becomes a matter of determining whether or not the current marking satisfies a collection of inequalities. Furthermore, the controllers designed by using Theorem 2 are maximally permissive under the assumption of no concurrency.

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