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Adaptive iterative feedback control for nonlinear system with unknown control gain

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Abstract: An adaptive iterative feedback control approach was proposed for a class of single-input, single-output uncertain nonlinear systems with completely unknown control gain. Unlike the ordinary iterative learning control which required some preconditions of stability on the learning gain, the adaptive iterative feedback control achieved the convergence through an unknown feedback gain in a Nussbaum-type function. It was proved that the repetitive tracking error sequence was asymptotic to the interval $[-\delta, \delta]$ for arbitrary prescribed $\delta > 0$ when iterations go to infinity. Simulation is carried out to show the validity of the proposed method.

Key words: adaptive iterative feedback control; unknown control gain; repetitive tracking
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基于未知控制增益的非线性系统自适应迭代反馈控制

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摘要: 针对一类单输入单输出不确定非线性重复跟踪系统,提出一种基于完全未知控制增益的自适应迭代反馈控制.与普通迭代学习控制需要学习增益稳定性前提条件不同,所提自适应迭代反馈控制律通过不断修改 Nussbaum 形式的反馈增益达到收敛.证明当迭代次数 $i \to \delta$ 时,重复跟踪误差可一致收敛到任意小界 δ. 仿真显示了所提控制方法的有效性.

关键词: 自适应迭代反馈控制; 未知控制增益; 重复跟踪

1 Introduction

Iterative learning controls are designed based on the discrete Lyapunov method and the control output is updated in an affine fashion such as P type or D type learning (see [1,2]). They require some preconditions of stability on the learning gains. For example, given a linear dynamical system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \in \mathbb{R}^n, \ x(0) = x_0 \in \mathbb{R}^n, \\ y(t) = Cx(t) \in \mathbb{R}^m, \ u(t) \in \mathbb{R}^l, \end{cases} \tag{1}$$

the ordinary iterative learning control requires the precondition of $\| I - CBL \| < 1$ for the stability on the learning gain, where L is the learning gain and $\| \cdot \|$ denotes a function norm. [3] proposes an adaptive iterative learning controller for a class of single-input, single-output linear time-invariant system that does not require the above precondition, but needs to know the sign of the first Markov parameter i.e. the sign of CB.

Nussbaum-type gain was originally proposed by [4], and has been adopted in the adaptive control of first-order nonlinear systems and nonlinearly perturbed high-order linear systems (see [5 ~ 10]). [11] proposes an asymptotic regulating control scheme for a class of time-varying uncertain nonlinear systems with both additive nonlinear uncertainties and unknown multiplicative terms with Nussbaum-type gain in adaptive feedback control.

This paper proposes a new approach — adaptive iterative feedback control — for a class of single-input, single-output unknown nonlinear systems. Without any preconditions of stability on the feedback gain, the adaptive

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iterative feedback control obtains the convergence through an unknown feedback gain in a Nussbaum-type function. Through rigorous proof, this paper shows the repetitive tracking error sequence is asymptotic to the interval $[-\delta,\delta]$ for arbitrary prescribed $\delta>0$ when iterations go to infinity.

2 Problem formulation

In this paper, we consider the single-input, single-output uncertain nonlinear system

$$\begin{cases}\dot{x}_i(t)=f(x_i(t),t)+b(x_i(t),t)u_i(t)\in\mathbb{R}^n,\\ x_i(0)=x_0\in\mathbb{R}^n,\\ y_i(t)=Cx_i(t)\in\mathbb{R},\ u_i(t)\in\mathbb{R},\end{cases}\tag{2}$$

where i denotes the iteration sequence, $x_i(t)\in\mathbb{R}^n$ is the physically measurable state vector, $u_i(t)\in\mathbb{R}$ is the control input, and $y_i(t)\in\mathbb{R}$ is the system output.

Given a finite initial state $x_i(0)$ and a finite time interval $[0,T_f]$, the control objective is to design an iterative feedback control such that, as $i\to\infty$, the system output $y_i(t)$ of the nonlinear uncertain system (2) tracks the desired trajectory $y_d(t)\in\mathbb{R}$ which satisfies the following dynamical system over $[0,T_f]$

$$\begin{cases}\dot{x}_d(t)=f(x_d(t),t)+b(x_d(t),t)u_d(t)\in\mathbb{R}^n,\\ x_d(0)=x_0\in\mathbb{R}^n,\\ y_d(t)=Cx_d(t)\in\mathbb{R},\ u_d(t)\in\mathbb{R},\end{cases}\tag{3}$$

i.e. as $i\to\infty$, the repetitive tracking error sequence $e_i(t)=y_d(t)-y_i(t)\to 0$. As part of the repeatability, the initial state $x_i(0)=x_d(0)=x_0$ is available for all trials.

For notational convenience, we define

$$f_i(t):=f(x_i(t),t),\ b_i(t):=b(x_i(t),t),$$
$$f_d(t):=f(x_d(t),t),\ b_d(t):=b(x_d(t),t).$$

For this system we make the following assumptions:

A1) $Cf_i(t),Cf_d(t),Cb_i(t),Cb_d(t)$ and $u_d(t)$ are unknown but have both upper bound and lower bound.

A2) $0<\theta_{\min}\leqslant|Cb_i(t)|\leqslant\theta_{\max}$, where $|\cdot|$ denotes the absolute value, $\theta_{\min}$ and $\theta_{\max}$ are positive constants, and the sign of $Cb_i(t)$ is unknown but unchanged during the trials.

3 Design of control law

Throughout this paper, we choose a Nussbaum-type function

$$v(\xi):=\cos\left(\frac{\pi}{2}\xi\right)\exp(\xi^2),\tag{4}$$

where $\xi\in\mathbb{R}$.

Define

$$\gamma:=\begin{cases}\gamma_a,\ \text{if } v(\xi)\geqslant 0,\\ \gamma_b,\ \text{if } v(\xi)<0,\end{cases}$$

where $\gamma_a,\gamma_b\in\mathbb{R}$ and $\gamma_a,\gamma_b>0$, then for every $s_0\in\mathbb{R}$, the Nussbaum-type function $v(\cdot)$ has the properties[4]

$$\lim_{s\to\infty}\sup\frac{1}{s}\int_{s_0}^{s}\gamma v(\xi)\mathrm{d}\xi=+\infty,\tag{5}$$

$$\lim_{s\to\infty}\inf\frac{1}{s}\int_{s_0}^{s}\gamma v(\xi)\mathrm{d}\xi=-\infty,\tag{6}$$

where $s\geqslant s_0$ and $s\neq 0$.

The proposed control law is given below

$$u_i(t)=v(k_i(t))e_i(t),\tag{7}$$

$$\dot{k}_i(t)=\begin{cases}|e_i(t)|\cdot(|e_i(t)|-\delta),\ \text{if } |e_i(t)|>\delta,\\ 0,\qquad \text{if } |e_i(t)|\leqslant\delta,\end{cases}\tag{8}$$

$$k_i(0)=k_{i-1}(T_f),\tag{9}$$

where i denotes the iterative sequence, $u_i(t)$ is the control input, and $k_i(t)$ is the parameter of the Nussbaum-type function.

Theorem Under the control laws (7) ~ (9), if $x_i(0)=x_d(0)$, then $k_i(t)$ and $e_i(t)$ are bounded and $e_i(t)$ is asymptotic to the interval $[-\delta,\delta]$ for arbitrary prescribed $\delta>0$ when iterations go to infinity, i.e. $|e_i(t)|\leqslant\delta$ as $i\to\infty$.

Proof Define the function

$$V(\beta):=\begin{cases}\frac{1}{2}\beta^2,\ \text{if } \beta>0,\\ 0,\quad \text{if } \beta\leqslant 0,\end{cases}$$

then

$$V(|e_i(t)|-\delta)=$$
$$\begin{cases}\frac{1}{2}(|e_i(t)|-\delta)^2,\ \text{if } |e_i(t)>\delta,\\ 0,\qquad \text{if } |e_i(t)|\leqslant\delta,\end{cases}\quad t\in[0,T_f].$$

If $e_i(t)>\delta$, then

$$\frac{\mathrm{d}}{\mathrm{d}t}V(|e_i(t)|-\delta)=(e_i(t)-\delta)\dot{e}_i(t)=$$
$$(e_i(t)-\delta)[C(f_d(t)-f_i(t))+Cb_d(t)u_d(t)-$$
$$Cb_i(t)v(k_i(t))e_i(t)].$$

Similarly if $e_i(t)<-\delta$, then

$$\frac{\mathrm{d}}{\mathrm{d}t}V(|e_i(t)|-\delta) =$$
$$(e_i(t)+\delta)[C(f_d(t)-f_i(t))+Cb_d(t)u_d(t)-$$
$$Cb_i(t)v(k_i(t))e_i(t)].$$

Suppose $|e_i(t)| \leqslant \delta$ as $i \to \infty$ does not hold, i.e. $|e_i(t)| > \delta$. Then from (8) and (9) we know $k_i(t)$ is unbounded i.e. $k_i(t) \to \infty$ as $i \to \infty$. From A1) we know $C(f_d(t)-f_i(t)) + Cb_d(t)u_d(t)$ is bounded, so we can obtain $-c\,|e_i(t)| \leqslant C(f_d(t)-f_i(t)) + Cb_d(t)u_d(t) \leqslant c\,|e_i(t)|$ when $|e_i(t)| > \delta$, where c is a finite positive constant.

From (8) we know

$$\dot{k}_i(t) = \begin{cases} e_i(t)\cdot(e_i(t)-\delta), & \text{if } e_i(t) > \delta, \\ 0, & \text{if } |e_i(t)| \leqslant \delta, \\ e_i(t)\cdot(e_i(t)+\delta), & \text{if } e_i(t) < -\delta. \end{cases}$$

If $Cb_i(t) > 0$, i.e. $0 < \theta_{\min} \leqslant Cb_i(t) \leqslant \theta_{\max}$, define

$$\theta := \begin{cases} \theta_{\min}, & \text{if } v(k_i(t)) \geqslant 0, \\ \theta_{\max}, & \text{if } v(k_i(t)) < 0, \end{cases}$$

then $-Cb_i(t)v(k_i(t)) \leqslant -\theta v(k_i(t))$.

So when $e_i(t) > \delta$,

$$\frac{\mathrm{d}}{\mathrm{d}t}V(|e_i(t)|-\delta) \leqslant$$
$$(e_i(t)-\delta)[c\,|e_i(t)| - \theta v(k_i(t))e_i(t)] =$$
$$(c-\theta v(k_i(t)))\dot{k}_i(t). \tag{10}$$

Similarly when $e_i(t) < -\delta$,

$$\frac{\mathrm{d}}{\mathrm{d}t}V(|e_i(t)|-\delta) \leqslant (c-\theta v(k_i(t)))\dot{k}_i(t). \tag{11}$$

Integrating (10) and (11) separately, for i-th iteration, in-th interval $[t_{in0}, t_{inf}]$ in which $|e_i(t)| > \delta$, $0 \leqslant t_{in0} < t_{inf} \leqslant T_f$, we obtain

$$0 \leqslant V(e_i(t)) =$$
$$\int_{in0}^{t}(c-\theta v(k_i(t)))\dot{k}_i(t)\mathrm{d}t =$$
$$\int_{k_i(t_{in0})}^{k_i(t)}(c-\theta v(k_i(t)))\mathrm{d}k_i(t),\ t \in [t_{in0}, t_{inf}].$$

Supposing there are im intervals in which $|e_i(t)| > \delta$ in every i-th iteration, and from (8) and (9), we can obtain

$$0 \leqslant \sum_{j=1}^{i-1}\sum_{jn=1}^{jm}V(e_j(t_{jnf})) + \sum_{jn=1}^{in-1}V(e_i(t_{jnf})) + V(e_i(t)) =$$
$$\sum_{j=1}^{i-1}\int_{k_j(0)}^{k_j(T_f)}(c-\theta v(k))\mathrm{d}k + \int_{k_i(0)}^{k_i(t)}(c-\theta v(k))\mathrm{d}k =$$
$$\int_{k_1(0)}^{k_i(t)}(c-\theta v(k))\mathrm{d}k =$$
$$c(k_i(t)-k_1(0)) - \int_{k_1(0)}^{k_i(t)}\theta v(k)\mathrm{d}k, \tag{12}$$

here $t \in [t_{in0}, t_{inf}]$ and $|e_i(t)| > \delta$.

When $k_i(t) > 0$, divided by $k_i(t)$ in (11), we obtain

$$0 \leqslant c\left(1-\frac{k_1(0)}{k_i(t)}\right) - \frac{1}{k_i(t)}\int_{k_1(0)}^{k_i(t)}\theta v(k)\mathrm{d}k. \tag{13}$$

From the previous assumption we know the monotone function $k_i(t)$ is unbounded as $i \to \infty$. Then when $k_i(t) \to \infty$, (13) contradicts one or the other of properties (5) and (6). So $k_i(t)$ is bounded as $i \to \infty$ and the previous assumption does not hold. And from (8) and (9) we know $e_i(t)$ is bounded and $|e_i(t)| \leqslant \delta$ as $i \to \infty$.

If $Cb_i(t) < 0$, i.e. $-\theta_{\max} \leqslant Cb_i(t) \leqslant -\theta_{\min} < 0$, then $0 < \theta_{\min} \leqslant -C_{b_i}(t) \leqslant \theta_{\max}$, similarly we know the theorem holds.

4 Example

Define $x(t) = [\xi_1(t), \xi_2(t)]^{\mathrm{T}}$ and consider the two order single-input, single-output nonlinear systems Σ_1:

$$\dot{\xi}_1(t) = -\sin(\xi_1(t)) + 0.5\cos(\xi_2(t)),$$
$$\dot{\xi}_2(t) = 0.5\cos(\xi_1(t)) + (2.5+0.15\sin(\xi_1(t)) - 0.1\cos(\xi_2(t)))u,$$
$$y(t) = 3\xi_1(t) + 2\xi_2(t),$$

and Σ_2:

$$\dot{\xi}_1(t) = -\sin(\xi_1(t)) + 0.5\cos(\xi_2(t)),$$
$$\dot{\xi}_2(t) = 0.5\cos(\xi_1(t)) - (2.5+0.15\sin(\xi_1(t)) - 0.1\cos(\xi_2(t)))u,$$
$$y(t) = 3\xi_1(t) + 2\xi_2(t).$$

The parameters are unknown to the controller. The only difference between Σ_1 and Σ_2 is that $b(t)_{\Sigma_1} = -b(t)_{\Sigma_2}$, i.e. $(Cb(t))_{\Sigma_1} = -(Cb(t))_{\Sigma_2}$. Obviously Σ_1 and Σ_2 both satisfy the assumptions A1) and A2). By way of illustration, the control tasks of Σ_1 and Σ_2 are both to track the desired trajectory $y_d = 8t^2 - 5t^3$, $t \in [0,1]$.

The continuous feedback strategies (7) ~ (9) guarantee the tracking errors of Σ_1 and Σ_2, which both converge asymptotically to an arbitrary prescribed small bound δ when the iterative sequence $i \to \infty$.

Figures 1 ~ 3 depict the iterative trackings of the system Σ_1 and Σ_2 with the initial data $x_i(0) = [0,0]^{\mathrm{T}}$, $k_1(0) = 0$ and $\delta = 0.1$. From Figs. 1 ~ 3 we can see that the adaptive iterative feedback control can obtain the error convergence without knowing the sign of $Cb_i(t)$. And from Fig.3 we can see with the increase of the iteration sequence i, the feedback gain $v(k_i(t))$ is bounded and its sign is changed to be the same as the sign of $Cb_i(t)$.

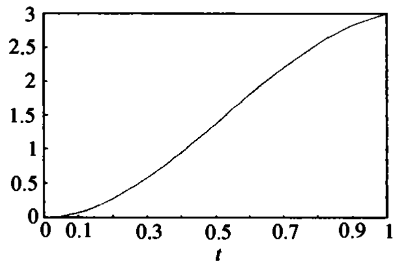
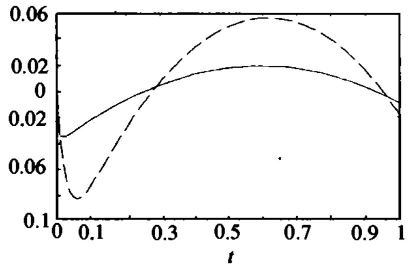
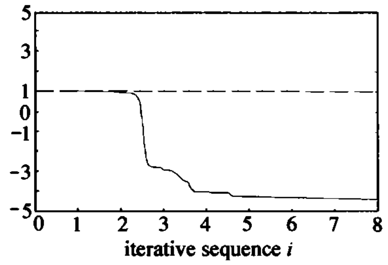


Fig. 1 Desired trajectory $y_d(t)$



Σ_1:------ Σ_2:——

Fig. 2 8th tracking error $e(t)$



Σ_1:------ Σ_2:——

Fig. 3 Feedback gain $v(k_i(t))$ for 1st to 8th tracking

5 Conclusions

Based on the idea of adaptive feedback control, this paper proposes that an adaptive iterative feedback control obtains the error convergence through an unknown feedback gain in a Nussbaum-type function in repetitive tracking control.

Without any preconditions about feedback gain and through rigorous proof we have concluded that the unknown feedback gain is bounded. This as a result guarantees the validity of the adaptive iterative feedback control.

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