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VSS-based coordinated control of Ice-skater Robot

SONG Li-bo[1,2], Lü Tian-sheng[1], ZHANG Pei-yan[1], ZHANG Jia-liang[2]
(1. School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200030, China
(2. School of Mechanical Engineering, Dong Hua University, Shanghai 200051, China;)

Abstract: A new type of leg-wheeled hybrid mobile robot, Ice-skater Robot, was developed based on the rolling and skating principle. Then, such coordinated control methods were disccussed as the model reference control method, the algorithm control method and the finite state control method briefly. And the motions in the normal direction and the tangent direction of the wheels were analyzed and the kinematic equation, the path-based nonholonomic kinematic state space used in the acceleration control were also obtained. At the same time, the switching functions satisfying the sliding mode reachable condition to realize the coordination of the wheels, the legs and between the wheels and the legs were designed. The experiment verifies the practicability of the coordinated control method based on the nonholonomic kinematic state space and the variable structure control with sliding mode method.

Key words: Ice-skater Robot; state space; variable structure control with sliding mode (VSS); coordinated control
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基于滑模变结构方法的溜冰机器人协调控制器设计

宋立博[1,2]，吕恬生[1]，张培艳[1]，张家梁[2]
(1.上海交通大学 机械工程学院，上海 200030;2.东华大学 机械工程学院，上海 200051)

摘要：根据轮滑原理设计了腿轮混合结构从动轮式移动机器人——溜冰机器人.在简要介绍模型基准控制法、算法控制法和有限状态控制法的基础上,研究了滚轮的法向和切向运动,得出基于运动路径的、无奇异性的非完整运动学状态空间.同时,在考虑控制系统时间延迟的基础上,建立了符合滑模可达性条件的机械腿、滚轮和机械腿与滚轮相互协调的切换函数.实验证实了这种基于非完整运动学状态空间和滑模变结构方法的协调控制策略的可行性.

关键词：溜冰机器人；状态空间；滑模变结构控制；协调控制

1 Introduction

Legged, wheeled and tracked mobile robots are widely used in such different circumstances as manufacturing, energy, service, space-flight and military affairs. But these mobile robots may not be applied to the star exploration, the rescuing, some high temperature sites and unpredictable terrains due to their single constructions. And some leg-wheeled hybrid construction mobile robots were developed in Japan and Germany to gain high terrain adaptability, stabilization, operation ability, high energy efficiency and fast motion velocity at the same time, and their gait planning methods and kinematics were also discussed[1~4]. Ref. [5] researched the circular turning gaits and its control of a combined wheel-leg vehicle developed by China and Germany.

But there are hardly any papers dealing with the principle and the control problems of the passive wheel mobile robots in China. This paper will discuss the design of the control system of the Ice-skater Robot based on the variable structure control with sliding mode (abbr. VSS) method.

2 Principle and construction

This new leg-wheeled mobile robot equipped with wheels at the ends of the legs is based on the rolling skating principle. If the wheels are inclined to the motion direction of the robot with some angles, the passive wheels will slide on the ground because the normal friction forces are greater than the tangent friction forces

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of the wheels, the result force will drive the robot because the lateral forces may be cancelled if the related legs and wheels maintain symmetrical synchronically[1]. The robot can be quadruped or hexapod to maintain balance, but it also can be biped. According to this principle, the Ice-skater Robot prototype are developed, as shown in Fig. 1.

The Ice-skater Robot is a new quadruped leg-wheeled hybrid mobile robot with four passive wheels installed at the end of the legs without any direct driving equipment, steering equipment and braking equipment. To make the wheels vertical to the ground, the parallel mechanism is used as legs. Its construction is very simple, and the driving force is stable because the wheels can keep perpendicular to the ground at any time.

To simplify the control system, it is not necessary for all the legs and the wheels to be used as the propulsion unit. The two front legs can rest in their inner limited positions, and the two rear wheels and legs must be controlled synchronically and symmetrically to generate the driving force. Then, the Ice-skater Robot can skate on the surface along the desired curve defined by the two front orientation wheels. This is a basic gait, the top board can keep balance and four wheels can stay at the ground simultaneously.

Fig. 1 Prototype of Ice-skater Robot

3 Some methods used in coordinated control

It was pointed out in Ref. [6] that the coordinated control of mobile robots can be converted into the position control and the on-off control of joint motors through servomechanisms to simplify the control algorithm. The coordinated control is conducted with three methods.

1) Model reference control method.

The on-off control signals or the joint motor commands are generated using some dynamic equations, such as the Lagrange equation and the Hamilton equation, or the kinematics equations. It is sometimes hard to use because the high – order differential motion equations must be solved to get joint trajectories which may be non-continuous functions of the time. And if there is some slippage between the wheels and the ground, it is impossible to get the precise dynamic equations and the kinematics equations.

2) Algorithm control method.

The algorithm control method is another version of the model reference control method. This method is simpler than the first one, and however, the computing intensity will be increased due to the complexity of the motion equations. When there are no sensory feedbacks, the order of the differential equations can not be lowered easily by imposing some constraints on the system.

3) Finite state control method.

This method is based on the observation that the simple modes of locomotion can be divided into a finite number of discretely related states, and the joints' rotation can be accomplished by some on-off motor signals. Under this circumstance, the on-off signals are determined not by the solution of motion differential equations or kinematics equations, but by these finite motion states after transformed. Though these discrete states can keep balance, it is also not possible to ensure the balance of the robot during transformation.

The variable structure control with sliding mode is a kind of nonlinearly robust control method. It is the on-off control method with desired sliding mode[7]. And the precise dynamic model or the kinematics model of the Ice-skater Robot is not needed, but it is easy to get the state space based on its principle. The state space of the Ice-skater Robot and the on-off switching of the motors make use of the VSS method possible.

4 Setup of nonholonomic kinematic state space

On the basis of the rigid-body and the nonholonomic constraint assumptions, the kinematics and inverse kinematics of the Ice-skater Robot are discussed fully and accurately in Ref. [8]. It has proven that only the orientation angles of the wheels satisfy the following equation

$$\forall (i,j,k)\ i \neq j \text{ and } j \neq k,\quad \sin(\boldsymbol{\beta}_j - \boldsymbol{\beta}_i)(x_k\cos\boldsymbol{\beta}_k + y_k\sin\boldsymbol{\beta}_k) = 0. \tag{1}$$

Then there exist the instantaneous center of rotation (ICR) which the Ice-skater Robot moves around. It was pointed out in Ref. [8] that if the orientation angles of the wheels in singular posture are selected as the local coordinates, the robot will be in the singular posture and the state space will be singular. To eliminate the singularity, the motion path based state space can be set up.

If the output is $\boldsymbol{\xi}$, the inputs are $\boldsymbol{u}_1, \boldsymbol{u}_2$ and $\boldsymbol{u}_3$, and the state variable is $\boldsymbol{\zeta}$, the following state space can represent the kinematics characteristics of Ice-skater Robot clearly

$$\begin{cases} \dot{\boldsymbol{\xi}} = \boldsymbol{\zeta}, \\ \dot{\boldsymbol{\zeta}} = \boldsymbol{u}_1, \\ \dot{\boldsymbol{\beta}} = \mathrm{diag}\ (k_1,k_2,k_3,k_4)(A(\boldsymbol{\xi},\boldsymbol{\zeta}) \cdot \boldsymbol{\zeta} + \\ \qquad B(\boldsymbol{\xi},\boldsymbol{\zeta})\cdot\boldsymbol{u}_1) + (I_4 - \mathrm{diag}\ (k_1,k_2,k_3,k_4))\cdot\boldsymbol{u}_2, \\ \dot{\boldsymbol{\alpha}} = \mathrm{diag}\ (k_1,k_2,k_3,k_4)(A(\boldsymbol{\xi},\boldsymbol{\zeta}) \cdot \boldsymbol{\zeta} + \\ \qquad B(\boldsymbol{\xi},\boldsymbol{\zeta}) \cdot \boldsymbol{u}_1) + (I_2 - \mathrm{diag}\ (k_5,k_6)) \cdot \boldsymbol{u}_3, \\ \dot{\boldsymbol{\theta}} = J_r^{-1} \cdot J_t(\boldsymbol{\Psi} + \boldsymbol{\beta}) \cdot \boldsymbol{\zeta}. \end{cases} \tag{2}$$

Where, $A(\boldsymbol{\xi},\boldsymbol{\zeta}), B(\boldsymbol{\xi},\boldsymbol{\zeta})$ are coefficients related to the motion path, $k_i (i = 1,2,\cdots,6)$ is equal to 0 or 1. When $k_1 = k_2 = k_5 = k_6$, the wheels and the legs will move simultaneously, this gait can be called "simultaneous mode". But if $(k_1 = k_2) \neq (k_5 = k_6)$ during different times, it will be named "independent mode".

It can be seen that it is an acceleration control system without singularity. And it can be called nonholonomic kinematic state space because it involves the dynamic nonholonomic constraints and the kinematic constraints. At the same time, this state space also shows the principle and motion sequence of the legs and the wheels clearly.

5 Algorithm of coordinated control

The coordinated control of the Ice-skater Robot can be realized by two steps. First, the rear-left leg and the rear-left wheel must be coordinated to produce the friction force, and so must the rear-right leg and the rear-right wheel. Then, the coordinated control of these four wheels and legs could be realized to produce the propulsion force.

Taken into on-off switching motions due to the adjustment of the orientation angles of the wheels and posture of the legs account, the switching functions of the wheel and the leg can be designed with the feedback of the desired position of the leg and the wheel respectively. That is to say, only if the wheels and the legs are in desired positions, can the legs and the wheels adjust the orientation angles and the postures. To identify the inner and the outer desired positions, the integer function int can be used in the switching functions. And the signal function sgn is also integrated into the switching functions to change the signs of these switching functions when the legs and the wheels are in their desired positions.

For example, the normalized switching function of the rear-left leg $s_1(\boldsymbol{\alpha}_1, \boldsymbol{\beta}_1)$ and that of the rear-right function $s_2(\boldsymbol{\alpha}_2, \boldsymbol{\beta}_2)$ can be defined with the orientation angles of the rear-left wheel $\boldsymbol{\beta}_1$ and that of the rear-right wheel $\boldsymbol{\beta}_2$:

$$s_1(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1) = \mathrm{int}\left(\frac{2\boldsymbol{\beta}_1}{(1 + \mathrm{sgn}\ (\boldsymbol{\beta}_1))\boldsymbol{\beta}_{1\max} + (1 - \mathrm{sgn}\ (\boldsymbol{\beta}_1))\boldsymbol{\beta}_{1\min}}\right) \cdot \delta(\tau - t) \cdot \varepsilon_1, \tag{3}$$

$$s_2(\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2) = \mathrm{int}\left(\frac{2\boldsymbol{\beta}_2}{(1 + \mathrm{sgn}\ (\boldsymbol{\beta}_2))\boldsymbol{\beta}_{2\max} + (1 - \mathrm{sgn}\ (\boldsymbol{\beta}_2))\boldsymbol{\beta}_{2\min}}\right) \cdot \delta(\tau - t) \cdot \varepsilon_2, \tag{4}$$

Where, $\boldsymbol{\beta}_{1\max}$ and $\boldsymbol{\beta}_{1\min}$ are the desired maximum and the minimum positions of the rear-left wheel, $\boldsymbol{\beta}_{2\max}$ and $\boldsymbol{\beta}_{2\min}$ are the desired maximum and the minimum positions of the rear-right wheel. And $\tau(s)$ is delay time of the control system, t is the time timed when the switching motion begins, and $\delta(\tau - t)$ is the step function.

And the switching functions of the rear-left wheel and the rear-right wheel can also be defined according to the postures of the rear-left leg $\boldsymbol{\alpha}_1$ and of the rear-right leg $\boldsymbol{\alpha}_2$:

$$s_3(\boldsymbol{\beta}_1,\boldsymbol{\alpha}_1) = \mathrm{int}\left(\frac{2\boldsymbol{\alpha}_1}{(1 + \mathrm{sgn}\ (\boldsymbol{\alpha}_1))\boldsymbol{\alpha}_{1\max} + (1 - \mathrm{sgn}\ (\boldsymbol{\alpha}_1))\boldsymbol{\alpha}_{1\min}}\right) \cdot \delta(\tau - t) \cdot \varepsilon_3, \tag{5}$$

$$s_4(\boldsymbol{\beta}_2,\boldsymbol{\alpha}_2) = \mathrm{int}\left(\frac{2\boldsymbol{\alpha}_2}{(1 + \mathrm{sgn}\ (\boldsymbol{\alpha}_2))\boldsymbol{\alpha}_{2\max} + (1 - \mathrm{sgn}\ (\boldsymbol{\alpha}_2))\boldsymbol{\alpha}_{2\min}}\right) \cdot \delta(\tau - t) \cdot \varepsilon_4. \tag{6}$$

Where, α_{1max} and α_{1min} are the desired maximum and the minimum positions of the rear-left leg, α_{2max} and α_{2min} are the desired maximum and the minimum positions of the rear-right leg, and $\varepsilon_i (i = 1,2,3,4)$ are arbitrary positive numbers.

It can be seen that when $s_i > 0 (i = 1,2,3,4), \dot{s} \leqslant 0$; and when $s_i < 0, \dot{s} \geqslant 0$ due to the delay time of the control system. Namely, the switching functions (3) ~ (6) satisfy the generalized reachable condition $s_i \cdot \dot{s} \leqslant 0$ of the sliding mode control.

Secondly, these switching functions (3) ~ (6) can be "and" together as switching functions to control the Ice-skater Robot to generate the driving force:

$$\begin{cases} \boldsymbol{\beta}_1 = \boldsymbol{\beta}_{1max} - \dot{\boldsymbol{\beta}}_1 t_1, & s_1 > 0 \cap s_3 > 0 \cap s_2 < 0 \cap s_4 < 0, \\ \boldsymbol{\beta}_1 = \boldsymbol{\beta}_{1min} + \dot{\boldsymbol{\beta}}_1 t_1, & s_1 < 0 \cap s_3 < 0 \cap s_2 > 0 \cap s_4 > 0, \\ \boldsymbol{\beta}_2 = \boldsymbol{\beta}_{2max} - \dot{\boldsymbol{\beta}}_2 t_2, & s_2 > 0 \cap s_4 > 0 \cap s_1 < 0 \cap s_3 < 0, \\ \boldsymbol{\beta}_2 = \boldsymbol{\beta}_{2min} + \dot{\boldsymbol{\beta}}_2 t_2, & s_2 < 0 \cap s_4 < 0 \cap s_1 > 0 \cap s_3 > 0, \end{cases} \tag{7}$$

$$\begin{cases} \boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_{1max} - \dot{\boldsymbol{\alpha}}_1 t_3, & s_1 < 0 \cap s_3 > 0 \cap s_2 > 0 \cap s_4 < 0, \\ \boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_{1min} + \dot{\boldsymbol{\alpha}}_1 t_3, & s_1 > 0 \cap s_3 < 0 \cap s_2 < 0 \cap s_4 > 0, \\ \boldsymbol{\alpha}_2 = \boldsymbol{\beta}_{2max} - \dot{\boldsymbol{\alpha}}_2 t_4, & s_2 < 0 \cap s_4 > 0 \cap s_1 > 0 \cap s_3 < 0, \\ \boldsymbol{\alpha}_2 = \boldsymbol{\alpha}_{2min} + \dot{\boldsymbol{\alpha}}_2 t_4, & s_2 > 0 \cap s_4 < 0 \cap s_1 < 0 \cap s_3 > 0. \end{cases} \tag{8}$$

When the Ice-skater Robot turns leftward or rightward, the orientation angles of the front-left wheel $\boldsymbol{\beta}_3$ and that of the front-right wheel $\boldsymbol{\beta}_4$ determining the motion direction can be calculated by the inverse kinematics[8]:

$$\begin{cases} \boldsymbol{\beta}_3 = \cos^{-1}\left(\dfrac{\rho \mp l_w}{\rho_3}\right), \\ \boldsymbol{\beta}_4 = \cos^{-1}\left(\dfrac{\rho \pm l_w}{\rho_4}\right). \end{cases} \tag{9}$$

Where, ρ, ρ_3 and ρ_4 are the distances between the ICR and the longitudinal symmetry of the robot, the contact points between the front-left wheel and the front-right wheel and the ground respectively.

The Eqs. (7) ~ (9) constitute the coordinated control system of the Ice-skater Robot.

6 Experiment

The desired positions can be obtained with the winding resistors and other parts if some A/D modules are available. As to the PLC without the A/D module, the desired positions would be replaced by the limited positions defined by some jiggling switches that are installed at the inner positions and the outer positions of legs and wheels easily. Eqs. (7) and (8) can be realized by the if-then law, the sketch map and the real PLC control system are shown in Fig.2 and Fig.3.

The DC regulator ① is the YJ83/2 dual stable voltage and current power supply, the Battery ② is BT—12M7—OAT12V7.0Ah/20HR rechargeable cell. The MELSEC FX2N—64MR ③ with FXWIN2.0 programming language is the central control unit and the OMRON MK2P—I 250VAC/28VDC/7A OMRON relays and JQX—13F 280VAC/28VDC/10A relays ④ are used to drive legs and wheels. And others are the power lines from relays to the robot and the feedback signal lines to the PLC. The TD3023G—24H—6K100 DC motors and TE 35QG—24 DC motors are used to drive legs and wheels respectively. Meanwhile, the "Hongqi" 3A 250VAC jiggling switches are used to feedback positions of legs and wheels.

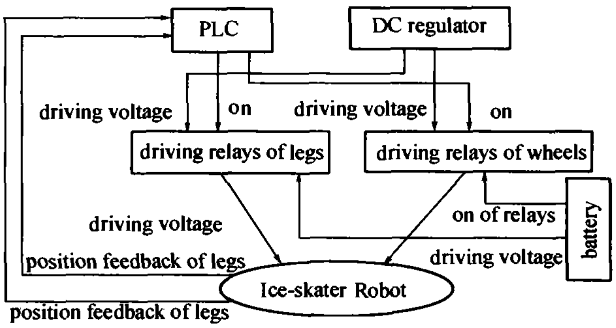

Fig. 2 Control construction of Ice-skater Robot

Fig. 3 Real control system of Ice-skater Robot

When $\alpha_{1max} = -\alpha_{2min} = 30°$, $\alpha_{1min} = -\alpha_{2max} = -15°$, $\boldsymbol{\beta}_{1max} = -\boldsymbol{\beta}_{2min} = 45°$, $\boldsymbol{\beta}_{1min} = -\boldsymbol{\beta}_{2max} = -45°$, $t_1 = t_2 = 0.25$ s, $t_3 = t_4 = 1.0$ s and $\tau = 0.1$ s, experiments were conducted in the independent mode. Due to the DC regulator, DC 12 V is supplied to wheel motors. Though there is slight slippage of two rear wheels, the Ice-skater Robot finished straight skating motion in average velocity of 0.2 m/s when $\boldsymbol{\beta}_3 = \boldsymbol{\beta}_4 = 0°$, and right-turning in 0.8 r/s when $\rho = 50$ cm, $\boldsymbol{\beta}_3 = -35.2°$ and $\boldsymbol{\beta}_4 = -46.0°$ defined in Eq.(9). More details can be referred in Ref. [8].

7 Conclusion

This paper mainly concerns the nonholonomic kinematic state space and the coordinated control problems of the passive wheel Ice-skater Robot. Taken the balance of the top board into account, the two front wheels can be used to define the motion direction, two rear wheels and two rear legs can be used to generate the driving force when two front legs rest in their inner limited positions. And the motion of legs and wheels can be determined by the desired positions of the wheels and the legs respectively. When the switching functions of legs and legs are "and" together with the orientation angles of two front wheels defined by the inverse kinematics, the coordinated control system will be developed. If the desired positions are replaced by the limited positions of legs and wheels, the coordinated control would be logic if-then law.

References:

[1] ENDO G, HIROSE S. Leg-wheel hybrid walking vehicle [J]. *J of Robotics Society of Japan*, 2000, 18(2): 112 - 119.

[2] MÜLLER J, HILLER M. Design of an energy optimal hydraulic concept for the large-scale combined legged and wheeled vehicle ALDURO [C/OL]//the Tenth Congress on the Theory of Machine and Mechanisms, Oulu, Finland [2001 - 06 - 15]. http://www.mechatronik.uni - duisburg.de/ robotics/alduro/publications.html.

[3] WANG Qimin, ZHU Jinkang. *Robotic Engineering and Its Application* [M]. Beijing: National Defense Industry Press, 1989: 39 - 45.
(王岐民,朱进康.机器人工程与应用[M].北京:国防工业出版社,1989:39 - 45.)

[4] EIJI N, SEI N. Leg-wheel robot: a futuristic mobile platform for forestry industry [C/OL][2001 - 12]. http:// www.robotics.is.tohoku.ac.jp/lab/robot/charoit - 2/papers.

[5] LIU Hongyi, WEN Bangchun, SORG H. Circular turning gaits of a combined wheel-leg vehicle [J]. *J of Northeastern University*, 1995, 15(3): 258 - 262.
(柳洪义,闻邦椿,斯欧格 H.一种腿轮结合式步行机的圆路径步态[J].东北大学学报,1995,15(3):258 - 262.)

[6] WOON L C, GE S S, CHEN X Q, et al. Adaptive neural network control of coordinated manipulators [J]. *J of Robotic Systems*, 1999, 16(4): 195 - 211.

[7] WANG Fengyao. *Variable structure control system with sliding mode* [M]. Beijing: Mechanical Industry Press, 1995.
(王丰尧.滑模变结构控制[M].北京:机械工业出版社,1995.)

[8] SONG Libo. *Study on kinematics and dynamics of leg-wheeled passive wheel type mobile robot-Ice-skater Robot* [D]. Shanghai: Shanghai Jiao Tong University, 2002.
(宋立博.从动轮式溜冰机器人运动学与动力学研究[D].上海:上海交通大学,2002.)

作者简介:

宋立博 (1973 —),男,1999 年于南京理工大学获工学硕士学位,2002 年于上海交通大学获工学博士学位,现在东华大学作博士后研究工作,主要研究领域:特种机器人,机器人智能控制技术,机电一体化,现场总线技术以及先进制造与自动化技术等,E-mail: lbsong@mail.dhu.edu.cn;

吕恬生 (1945 —),男,1982 年于上海交通大学获工学硕士学位,现为上海交通大学机械工程学院教授,博士生导师,上海交通大学工程训练中心主任,主要研究领域: 特种机器人,多移动机器人系统及其智能控制等,主持 863 项目多项,已发表论文 70 余篇;

张培艳 (1976 —),女,2001 年于吉林工业大学获工学硕士学位,现为上海交通大学机械电子工程专业博士研究生,主要研究领域:机器人智能控制技术,机电一体化技术等;

张家梁 (1964 —),男,1986 年于合肥工业大学获工学硕士学位,2002 年于上海交通大学获工学博士学位,现为东华大学机械工程学院高工、副教授,主要研究领域:特种机器人、机电一体化、先进制造与自动化技术等.