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Research on special LRP based on two-phase improved genetic algorithm

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Abstract: A two-phase improved genetic algorithm with control switch system architecture was proposed to solve location routing problems (LRP). Random switch was constructed to control mutation calculation and to improve the population diversity in the method. This improved GA (IGA) architecture made it possible to search the solution space efficiently without local optimization to some extent. A case study using computer simulation shows that the IGA with switch control system achieves significant improvement compared to a recent LRP heuristic.

Key words: location routing problems (LRP); improved genetic algorithm (IGA); logistic optimization; random switch

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基于两阶段改进遗传算法的一类特殊 LRP 问题研究

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摘要: 提出了一种带有随机开关的两阶段改进遗传算法并应用于集成化物流中的定位-运输路线安排(LRP)优化问题.该方法采用随机开关控制遗传算法中的变异运算,实现了空间的有效搜索,并且在一定程度上,避免了"局部最优现象"的发生.通过计算机仿真实验,证明了该算法求解 LRP 问题的有效性和准确性.

关键词: 定位-运输路线安排(LRP)问题; 改进遗传算法(IGA); 物流系统优化; 随机开关

1 Introduction

The conceptual foundation of LRP studies date back to Von Boventer, Maranzana, Webb, Lawrence and Pengilly, Christofids and Eilon and Higgins[1]. Although these earlier studies are far from capturing the total complexity of LRP, they first recognized the close interface between location and transportation decisions. Cooper generalized the transportation-location problem that aimed to find the optimal-location of supply sources and to minimize the transportation cost from sources to destinations[2]. Watson-Gandy and Dohrn may be some of the first authors credited to consider the multiple-drop nature of the vehicle routes within the location-transportation framework[3]. But LRP belongs to NP-hard, it is the model and solution of the algorithm that is the core for LRP. Two distinct types of methods were used to solve LRP. These are exact algorithms and heuristics.

Much research is made on solving scheduling problem with genetic algorithm. An improved GA is proposed to solve LRP in this paper. The solution is hard to find. In this paper GA was investigated as a heuristic technique for obtaining optimal or nearly optimal solutions to the vehicle routing problems with capacity and time constraints. This paper describes the difficulties for the solution of a special LRP. And the goal of transportation costs and facilities cost are considered, which satisfy minimum routing costs and costs of establishment and operation of the facilities. In this paper, cost is objective function, which makes it possible to minimize the total costs. The two-phase IGA method with random switch is proposed to solve LRP. First, location-allocation is solved, and then, vehicle routing problem is obtained by IGA. The population diversify is improved without the local optimization by IGA. The method constructs ran-

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dom switch to control mutation operator and to improve search precision. The method overcomes local optimization of traditional GA in some sense. At the end of the paper the authors also provide the future trends in this area and some possible paths of further logistics systematic optimization research.

2 Descriptions of LRP

Location-Routing Problem (LRP) is one of the problems in integrated logistics optimizations. It can be defined as follows. A feasible set of potential facility sites and location and expected demands of each customer are given. Each customer is to be assigned to one of the facilities, which will meet its demand. The shipments of customer demand are carried out by vehicles, which are dispatched from the facilities and operated on routes that include multiple customers. There is a fixed cost associated with opening a facility at each potential site. A distribution cost associated with any routing of vehicles includes the cost of acquiring the vehicles used in the routing, and the cost of delivery operations. The cost of delivery operations is linear in the total distance traveled by the vehicles. The LRP is used to determine the location of the facilities and the vehicle routes from the facilities to customers to minimize the sum of the location and distribution costs such that the vehicle capacities are not exceeded.

A constraint-based model is presented for the location routing problem. The hypotheses are as follows: ① The transportation is just in time. ② The facility is both starting point and destination of circular vehicle routing; each facility serves more than two customers. ③ Nature of demand/supply is deterministic. ④ There are multiple facilities. ⑤ Size of vehicle fleets is a single vehicle. ⑥ Vehicle capacities are determined. The total amount of goods is limited in every route by each vehicle's capability. ⑦ Facility capacities are undetermined; not all facilities have been chosen in every decision. ⑧ Each customer is served by one and only one vehicle. Considering the complexity of supply/ demand markets, it is assumed that they are retail markets. ⑨ Each facility is considered as a separate entity, not linked to the other facilities. ⑩ The objective is to minimize total costs.

3 Mathematical model of LRP

Decision variables considered are as follows.

$$X_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ goes from customer } i \text{ to customer } j, \\ & i \in S,\ j \in S,\ k \in V,\ i \neq j, \\ 0, & \text{otherwise}, \end{cases}$$

$$Z_r = \begin{cases} 1, & \text{if a facility is established at site } r, r \in G, \\ 0, & \text{otherwise}. \end{cases}$$

Model parameters are given as follows.

$G = \{r \mid r = 1,\cdots,m\}$ — the set of m feasible sites of candidate facility, $H = \{i \mid i = m+1,\cdots,m+N\}$ — the set of N customers to be served, $S = \{G\} \cup \{H\}$ — the set of all feasible sites and customers (it is also referred to nodes), $V = \{v_k \mid k = 1,\cdots,K\}$ — the set of K vehicles available for routing from the facilities, C_{ij} — average annual cost per distance traveling from node i to node j, $i \in S, j \in S$, C_k — the annual cost of acquiring vehicle $k(k = 1,\cdots,K)$, F_r — annual cost of establishing and operating a facility at site $r(r = 1,\cdots,m)$, q_j — average number of units demands by customer $j(j \in H)$, Q_k — capacity of vehicle $k(k = 1,\cdots,K)$, d_{ij} — distance from node i to node j, X_{rmk}, X_{rjk} — traveling by vehicle k from node r to customer m or to customer j, respectively.

Model of a special LRP is defined as follows.

The objective function of LRP is

$$f(x) = \min \sum_{i\in S}\sum_{j\in S}\sum_{k\in V} C_{ij}X_{ijk} + \sum_{k\in V}\Big(C_k \sum_{r\in G}\sum_{j\in H} X_{rjk}\Big) + \sum_{r\in G} F_r Z_r \tag{1}$$

subject to

$$\sum_{k\in V}\sum_{i\in S} X_{ijk} = 1,\ \forall j \in H, \tag{2}$$

$$\sum_{i\in H}\sum_{j\in S} q_j X_{ijk} \leqslant Q_k,\ \forall k \in V, \tag{3}$$

$$\sum_{i\in S} X_{ipk} - \sum_{j\in S} X_{pjk} = 0,\ \forall k \in V,\ p \in S, \tag{4}$$

$$\sum_{r\in G}\sum_{j\in H} X_{rjk} \leqslant 1,\ \forall k \in V, \tag{5}$$

$$\sum_{k\in V} X_{rjk} + Z_r + Z_j \leqslant 2,\ \forall m = 1,\cdots,R,\ r \in G, \tag{6}$$

$$\sum_{k\in V}\sum_{i\in H} X_{rik} - Z_r \geqslant 0,\ \forall r \in G, \tag{7}$$

$$\sum_{i\in H} X_{rik} - Z_r \leqslant 0,\ \forall k \in V,\ r \in G, \tag{8}$$

$$X_{ijk} = 0 \text{ or } 1,\ \forall i,j \in S,\ k \in V, \quad (9)$$

$$Z_r = 0 \text{ or } 1,\ \forall r \in G. \quad (10)$$

In this model, the objective function minimizes the total cost of routing, establishing and operating the facilities. Constraint (2) ensures that each customer is served by one and only one vehicle. Constraint (3) ensures that the vehicle capacity constraints are not exceeded for any of the vehicles used in routing, while (4) is the route continuity constraint, which implies that the vehicle should leave every point entered by the same vehicle. Constraint (5) guarantees that each vehicle is routed from one depot. Constraint (6) guarantees that there is no link between any two depots. Constraints (7) and (8) require that a vehicle only be from a depot if that depot is opened. The last two of constraints are the integer constraints.

4 Solution to special LRP based on two-phase IGA

Bruno proposed two-phase tabular search architecture for the solution of the LRP. First introduced in his paper, the two-phase approach offers a computationally efficient strategy that integrates facility location and routing decisions[4]. Our domestic scholars proposed some improvement of evolutionary algorithm (EA). A chaotic model, to control mutation operation in EA, constructs a random switching[5]. In order to solve an equipment replacement problem a new method, the genetic algorithm, is proposed. In the paper the random switching controlled crossover, the global (or near global) optimal solutions can be found[6].

4.1 IGA-based solution of LRP

A new method was proposed based on two-phase improved genetic algorithm with switch to solve a special LRP[7]. There are two steps in the suggested method. First, GA decides the location of potential facilities. Second, using improved GA solves vehicle routing problem. IGA was used to search the optimal route in the second phase, which introduces control switch into GA's selection to keep the population diversion without local optimization. The optimal route is sought to be minimized total cost, which ensures more economical benefits in distribution corporations.

Algorithm code of first phase It takes on natural number code to choose the locations of potential facilities.

Algorithm code of second phase The representation of a solution here is an integer string of length where N is the number of customers in determined route. Each gene in the string or chromosome is the node number assigned to customer originally. And the sequence of the genes in the chromosome is the order of visiting these customers.

Selection operation of second phase is according to individual K of genetic code, calculating fitness

$$f_k = 1/\sum_{i\in S}\sum_{j\in S}\sum_{k\in V} C_{ij}X_{ijk} + \sum_{k\in V}(C_k\sum_{r\in G}\sum_{j\in H}X_{rjk}) + \sum_{r\in G}F_rZ_r. \quad (11)$$

Selection probability

$$P_k = f_k/\sum_{i=1}^{L} f_i, \quad (12)$$

where L is the numbers of individual. The individual is chosen when its fitness is large.

The individual of first-phase genetic code is chosen only if its second-phase one exists. Only mutation operator is given in the first phase, that is to say, it produces natural number $k \in (1, N)$. Gene a_k is taken by a random number $[1, J_k]$. The crossover and mutation operators of the second phase are limited in string. LOX operator designs the crossover. Mutation operator is constructed with random switch by typical cell-network model, i. e. mutation operator is controlled by switch values. The mutation happens when switch values surpass β, (β depends on special problem, usually equal to 0.5). Otherwise when the values exceed β, the mutation individual is produced as follows: .

$$a_j = P_{ik} + \frac{f}{f\Sigma}N(0,1),\ k = 0 \sim n, \quad (13)$$

where f is fitness of individual i, $f\Sigma$ is the total fitness of population; P_{ik} is the weight k of the individual i.

This method constructs random switch to control mutation calculation in order to improve the population efficiency. The suggested method avoids the local optimization. The efficiency and accuracy of the method are proved by computational simulation, which is better than traditional GA.

4.2 Steps of two-phase IGA

Step 1 Produce the individual number L_1 in the first phase by the good reaction to cost. The total number is L_1.

Step 2 Produce the individual number L_2 in the second phase by the good reaction to cost. The total number is $L_1 L_2$,

Step 3 Evaluate the fitness of the individual in the second phase

$$f_k = 1/\sum_{i\in S}\sum_{j\in S}\sum_{k\in V} C_{ij}X_{ijk} + \sum_{k\in V}(C_k\sum_{r\in G}\sum_{j\in H}X_{rjk}) + \sum_{r\in G}F_rZ_r.$$

Step 4 Calculate selection probabilities in the second phase

$$P_k = f_k/\sum_{i=1}^{L} f_i.$$

Step 5 Cross and mutate to the individual in the second phase, which is limited in the children stings; mutation is used in the same vehicle routes by random switch.

Step 6 Update the old population with the newly generated population.

Step 7 If the certain number of generation in the second phase reaches G_2, go to Step 8, or else go to Step 3.

Step 8 If the certain number of generation in the first phase reaches G_1, stop.

Note Selections, crossover, mutation to the individual of the first phase, and the specific operating is in the context.

5 Analysis of computational results

In the following examples, six potential facilities, and thirty customs with different demand were analyzed. Each was served with the three different types of distribution vehicle that includes big truck, medium truck, and small truck. Their carrying capacity is as follows: 10 ton, 6 ton, and 2 ton. The annual cost establishing and operating a facility F_r is equal to 160 RMB/each, the transportation cost C_{ij} is equal to 2 RMB per ton, per km. The two-phase IGA was discussed to search the optimal routes based on the data in Fig. 1. The results of this method are given in Table 1. The genetic operators are as follows: generation population is 200; crossover rate is 0.90; mutation rate is 0.001; termination generation is 200. From the simulation results, the number of choosing potential facilities depends on special conditions. The objective value of five facility chosen is less than that of six ones. The reason is that the total cost increases by one more facility chosen.

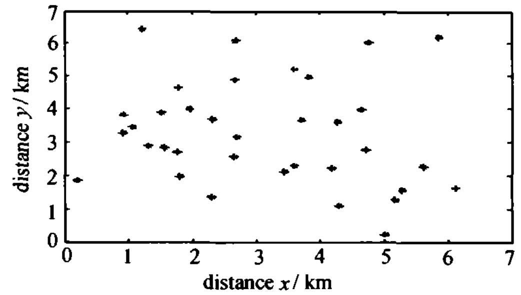

Fig. 1 Coordinate positions of six potential facilities and thirty customers

Table 1 Simulation results

problem scale	optimal routes for chosen facilities	function value
6/30	① PF2-C27-C9-C7-C19-C6 ② PF3-C10 ③ PF4-C18-C22-C26-C25-C21 ④ PF5-C5-C8-C17-C11-C15-C16-C14 ⑤ PF6-C3-C1-C23-C2-C12-C13-C30-C24-C2-C28-C29-C4	5265.67

6 Conclusion

In the present paper, the two-phase improved genetic algorithm for choosing facility is proposed. It combines the customer's demand into minimum wrap. Then IGA with switch control is used to search these optimal routes. The two-phase approach with control switch for the LRP is proposed that ensures the search efficiently and produces good solutions while avoiding local optimization. The proposed method can solve the special LRP, which is different in custom positions, demands and vehicle routes with self-adaptation. Thus the emphasis is directed towards developing a method, which will provide reliable and practical solution. The method leaves some room as well as a way for further research.

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Appendix B

In this appendix, we use Young's inequality[8]

$$xy \leqslant \frac{\varepsilon^p}{p} | x |^p + \frac{1}{q\varepsilon^q} | y |^q,$$

where $\varepsilon > 0$, the constants $p > 1$ and $q > 1$ satisfy $(p-1)(q-1) = 1$, and $(x, y) \in \mathbb{R}^2$.

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