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Robust H_∞ control of neutral delay systems with Markovian jumping parameters

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Abstract: The problem of H_∞ control of neutral delay systems with Markovian jumping parameters and polytopic uncertainty was addressed. By using a descriptor model transformation of the system and Moon's inequality for bounding cross terms, a new delay-dependent sufficient condition for the existence of a memoryless H_∞ state feedback controller was derived in terms of linear matrix inequalities.

Key words: Markovian jump linear systems; time-delay systems; H_∞ control; delay-dependent criteria

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参数服从马尔可夫切换中立型时滞系统鲁棒 H_∞ 控制

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摘要: 研究参数服从马尔可夫切换及具有仿射不确定性的中立型时滞系统 H_∞ 状态反馈控制器设计问题.通过将系统表示为等价的奇异系统,并利用 Moon 不等式放大向量积,基于线性矩阵不等式,得到了使系统存在无记忆 H_∞ 状态反馈控制器与时滞相关的新的充分条件.

关键词: 马尔可夫切换系统; 时滞系统; H_∞ 控制; 线性矩阵不等式;时滞相关准则

1 Introduction

Markovian jump linear systems (MJLS) are hybrid systems with two components in their state vectors: The first component refers to the mode, and the second, the state. The mode is described by a continuous Markovian process with a finite state space; the state in each mode is described by a system of differential equations.

For MJLS with time-delays, the problems of stability and H_∞ control have been extensively studied (the reader can refer to [1 ~ 3] and the references therein). Recently, delay-dependent conditions for stability and H_∞ control of MJLS with constant delay were presented in [4]. However, the work did not consider the time-delay in the controller output. In this note, we are concerned with the problem of H_∞ control for uncertain neutral delay systems with Markovian jumping parameters. The time-delays in this system are assumed to be mode-dependent. To the best of our knowledge, this problem has not been investigated prior to the present paper. Based on the application of the descriptor model transformation and Moon's inequality on inner product of two vectors, new delay-dependent solutions to the problems of stability and H_∞ control are derived in terms of linear matrix inequalities.

2 Model description

Let $\{r(t), t \geqslant 0\}$ be a time homogeneous Markovian process with right continuous trajectories taking values from a finite set $S = \{1, 2, \cdots, N\}$, with stationary transition probabilities:

$$P\{r(t+\Delta) = j \mid r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta), & i = j, \end{cases}$$

where $\Delta > 0, \lim\limits_{\Delta \to 0} o(\Delta)/\Delta = 0$, and $\pi_{ij} \geqslant 0$ for $i \neq j$,

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$\pi_{ii} < 0$ with $\sum_{j=1, j\neq i}^{N} \pi_{ij} = -\pi_{ii}$ for each $i \in S$. Consider a continuous-time linear hybrid neutral delay system with N models. Suppose that the system mode is governed by $\{r(t), t \geqslant 0\}$ and let the system dynamics be described as follows:

$$\begin{cases} \dot{x}(t) - F(r(t))\dot{x}(t-h) = \\ A(r(t))x(t) + A_d(r(t))x(t-\tau(r(t))) + \\ B(r(t))u(t) + B_w(r(t))w(t), \\ z(t) = C(r(t))x(t) + C_d(r(t))x(t-\tau(r(t))) + \\ \qquad D(r(t))u(t) + D_w(r(t))w(t), \\ x(t) = \varphi(t),\ r(t) = r_0,\ t \in [-\bar{h}, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^q$ is the exogenous disturbance input, $z(t) \in \mathbb{R}^p$ is the output to be controlled. $\tau(i) \geqslant 0$ is the constant delay time of the state in the system when the mode is in i. $\bar{\tau} = \max\{\tau(i), i \in S\}$, $\bar{h} = \max\{\bar{\tau}, h\}$. $r_0 \in S$ and $\varphi(t) \in C[\bar{h}, 0]$ are the initial conditions of the mode and the continuous state, respectively. To simplify the notation, $M(i)$ will be denoted by M_i.

The aim of this paper is to design a state feedback control law

$$u(t) = K(r(t)) \tag{2}$$

such that for a given real number $\gamma > 0$, the closed-loop system (1) with controller (2) and $w(t) = 0$ is stochastically stable and the following inequality holds

$$\mathrm{E}\int_0^\infty z^{\mathrm{T}}(t)z(t)\mathrm{d}t \leqslant \int_0^\infty w^{\mathrm{T}}(t)w(t)\mathrm{d}t$$

for all functions $w(t) \in L_2[0, \infty)$.

For the sake of convenience, we introduce the following notations.

$$\lambda = \max\{|\pi_{ii}|, i \in S\},\ \underline{\tau} = \min\{\tau_i, i \in S\},$$
$$\mu = 1 + \lambda(\bar{\tau} - \underline{\tau}),\ \rho_i = \tau_i + \lambda/2(\bar{\tau}^2 - \underline{\tau}^2).$$

3 Main results

In this section, we first consider the stability of system (1) with $u(t) = 0$, $w(t) = 0$.

Theorem 1 The jump linear system (1) with $u(t) = 0$, $w(t) = 0$ is stochastically stable if for each $i \in S$, there exist matrices $P_i > 0, P_j, M_{ij}, S_j > 0, W_{ij}, j = 1, 2, W_{i3}$, and $Q > 0$ satisfy the coupled matrix inequalities:

$$\begin{bmatrix} \Omega_{i1} & \Omega_{i2} & P_{i1}^{\mathrm{T}}A_{di} - M_{i1} & P_{i1}^{\mathrm{T}}F_i \\ * & \Omega_{i3} & P_{i2}^{\mathrm{T}}A_{di} - M_{i2} & P_{i2}^{\mathrm{T}}F_i \\ * & * & -S_1 & 0 \\ * & * & 0 & -S_2 \end{bmatrix} < 0,$$

$$\begin{bmatrix} W_{i1} & W_{i2} & M_{i1} \\ W_{i2}^{\mathrm{T}} & W_{i3} & M_{i2} \\ M_{i1}^{\mathrm{T}} & M_{i2}^{\mathrm{T}} & Q \end{bmatrix} \geqslant 0,$$

where

$$\Omega_{i1} = A_i^{\mathrm{T}}P_{i1} + P_{i1}^{\mathrm{T}}A_i + \tau_i W_{i1} + M_{i1} + M_{i1}^{\mathrm{T}} + \sum_{l=1}^{N}\pi_{il}P_l + \mu S_1,$$

$$\Omega_{i2} = P_i - P_{i1}^{\mathrm{T}} + A_i^{\mathrm{T}}P_{i2} + \tau_i W_{i2} + M_{i2}^{\mathrm{T}},$$

$$\Omega_{i3} = -P_{i2} - P_{i2}^{\mathrm{T}} + \tau_i W_{i3} + \rho_i Q + S_2.$$

The proof of Theorem 1 can be achieved as follows. First represent (1) with $u(t) = 0$ and $w(t) = 0$ in the equivalent descriptor form:

$$\dot{x}(t) = y(t),$$
$$0 = -y(t) + F(r(t))y(t-h) + [A(r(t)) + A_d(r(t))]x(t) - A_d(r))\int_{t-\tau(r(t))}^{t} y(s)\mathrm{d}s.$$

Take stochastic Lyapunov functional $V(t)$ to be

$$V(t) = x^{\mathrm{T}}(t)P(r(t))x(t) + \int_{t-\tau(r(t))}^{t} x^{\mathrm{T}}(s)S_1x(s)\mathrm{d}s + \int_{-\tau(r(t))}^{0}\mathrm{d}\theta\int_{t+\theta}^{t} y^{\mathrm{T}}(s)Qy(s)\mathrm{d}s + \lambda\int_{-\bar{\tau}}^{-\underline{\tau}}\mathrm{d}\theta\int_{t+\theta}^{t}[y^{\mathrm{T}}(s)Qy(s)(s-t-\theta) + x^{\mathrm{T}}(s)S_1x(s)] + \int_{t-h}^{t} y^{\mathrm{T}}(s)S_2y(s)\mathrm{d}s.$$

Then, using Moon's inequality[5] and the techniques in [6], we can obtain Theorem 1.

Now we apply Theorem 1 to the H$_\infty$ control problem of system (1), we get the following theorem.

Theorem 2 If for each $i \in S$, and for some prescribed scalar ε_i, there exist matrices $X_i > 0, Y_i, Z_i, \bar{K}_i, U_j > 0, \bar{W}_{ij}, j = 1, 2, \bar{W}_{i3}, R > 0$, satisfying the following coupled matrix inequalities:

$$\begin{bmatrix} \bar{\Omega}_{i1} & \bar{\Omega}_{i2} & 0 & 0 & H_{i1} & L_{i21} & L_{i1} \\ * & \bar{\Omega}_{i3} & L_{i3} & B_{wi} & 0 & L_{i22} & 0 \\ 0 & * & -\Phi_3 & 0 & H_{i2} & 0 & 0 \\ 0 & * & 0 & -\gamma^2 I & D_{wi}^{\mathrm{T}} & 0 & 0 \\ * & 0 & * & * & -I & 0 & 0 \\ * & * & 0 & 0 & 0 & -\Phi_{i2} & 0 \\ * & 0 & 0 & 0 & 0 & 0 & -\Phi_{i1} \end{bmatrix} < 0, \tag{3}$$

and

$$\begin{bmatrix} \bar{W}_{i1} & \bar{W}_{i2} & 0 \\ \bar{W}_{i2}^{\mathrm{T}} & \bar{W}_{i3} & \varepsilon_i A_{di} R \\ 0 & \varepsilon_i R A_{di}^{\mathrm{T}} & R \end{bmatrix} \geqslant 0, \tag{4}$$

where

$\bar{\Omega}_{i1} = Z_i + Z_i^{\mathrm{T}} + \pi_{ii} X_i + \tau_i \bar{W}_{i1}$,

$\bar{\Omega}_{i2} = X_i (A_i^{\mathrm{T}} + \varepsilon_i A_{di}^{\mathrm{T}}) + \bar{K}_i^{\mathrm{T}} B_i^{\mathrm{T}} - Z_i^{\mathrm{T}} + Y_i + \tau_i \bar{W}_{i2}$,

$\bar{\Omega}_{i3} = -Y_i - Y_i^{\mathrm{T}} + \tau_i \bar{W}_{i3}$,

$\Phi_{i1} = \mathrm{diag}\,[X_1, \cdots, X_{i-1}, X_{i+1}, \cdots, X_N]$;

$L_{i1} = [\sqrt{\pi_{i1}} X_i, \cdots, \sqrt{\pi_{ii-1}} X_i, \sqrt{\pi_{ii+1}} X_i, \cdots, \sqrt{\pi_{iN}} X_i]$,

$\Phi_{i2} = \mathrm{diag}\,[\mu U_1, U_2, \rho_i R]$,

$L_{i21} = [\mu X_i \quad Z_i^{\mathrm{T}} \quad \rho_i Z_i^{\mathrm{T}}]$, $L_{i22} = [0 \quad Y_i^{\mathrm{T}} \quad \rho_i Y_i^{\mathrm{T}}]$,

$\Phi_3 = \mathrm{diag}\,[U_1, U_2]$, $L_{i3} = [(1-\varepsilon_i) A_{di} U_1 \quad F_i U_2]$,

$H_{i1} = X C_i^{\mathrm{T}} + \bar{K}_i^{\mathrm{T}} D_i^{\mathrm{T}}$, $H_{i2}^{\mathrm{T}} = [C_{di} U_1 \quad 0]$.

Then the controller (2) with $K_i = \bar{K}_i X_i^{-1}$ stabilizes system (1) and the closed loop system verifies noise attenuation level γ.

Theorem 2 is derived for the system (1) where the system matrices are all known. However, since the LMIs of (3) and (4) are affine in the system matrices, Theorem 2 can be extended to the case where the system matrices are not exactly known and they reside within a given polytope. For fixed mode $i \in S$, denoting $M_i = [F_i\, A_i\, A_{di}\, B_i\, B_{wi}\, C_i\, C_{di}\, D_i\, D_{wi}]$, we assume that $M_i \in \mathrm{cov}\,\{M_i^j, j = 1, \cdots, v\}$, here $M_i^j = [F_i^j \quad A_i^j \quad A_{di}^j \quad B_i^j \quad B_{wi}^j \quad C_i^j \quad C_{di}^j \quad D_i^j \quad D_{wi}^j]$.

Theorem 3 Consider the system (1), where the system matrices reside within the polytope M_i for each $i \in S$. If for each $i \in S$, and for some prescribed scalar ε_i, there exist matrices $X_i > 0, Y_i^j, Z_i^j, \bar{K}_i, U_l > 0, \bar{W}_{il}^j, l = 1,2, \bar{W}_{i3}^j, R > 0$, satisfying (3) and (4), where $F_i, A_i, A_{di}, B_i, B_{wi}, C_i, C_{di}, D_i, D_{wi}, Y_i, Z_i, \bar{W}_{il}, l = 1,2,3$, are taken with the upper index j for all $j = 1, \cdots, v$, Then the controller (2) with $K_i = \bar{K}_i X_i^{-1}$ stabilizes system (1) and the closed loop system verifies noise attenuation level γ.

4 Conclusion

In this note, we have studied the problem of H_∞-control for neutral delay systems with Markovian jumping parameters. Based linear matrix inequalities, efficient delay-dependent sufficient solutions have been obtained.

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