

线性不确定多时滞系统的 α -鲁棒控制

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摘要: 通过分析时滞系统超越型的特征方程的根的分布, 并结合线性矩阵不等式(LMI)技术研究了含多个不确定常时滞的线性不确定时滞系统的可 α -鲁棒镇定及其控制器设计问题, 得到了相应的 α -鲁棒无记忆反馈控制律. 不同于一般的结果, 本方法得到的控制器不但使得系统可鲁棒镇定, 而且闭环系统特征方程的根的实部均小于等于某个指定的负数. 结果表示为 LMI 形式, 易于进行数值处理. 最后以一个数值例子显示了所得结果的有效性及其应用方法.

关键词: α -鲁棒镇定; 时滞不确定系统; 频域法; 线性矩阵不等式

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α -robust stabilization for linear uncertain systems with multiple delays

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Abstract: The α -robust stabilization and α -robust controller design for linear uncertain systems with multiple unknown constant delays were studied by analyzing the distribution of the roots of their transcendental characteristic equations and using LMI technology, and corresponding α -robust memoryless feedback control law was obtained. Being different from common results, the obtained controller not only ensured the system robust stabilization, but also let the real part of all roots of the closed-loop system's characteristic equation less than or equal to a certain negative. The result was presented in the form of Linear Matrix Inequality(LMI) problem so that it was easy to be calculated. A computation example was given to illustrate the proposed method.

Key words: α -robust stabilization; uncertain systems with time-delays; frequency-domain techniques; LMI approach

1 引言(Introduction)

频域方法能为线性时滞系统的稳定性分析与综合提供充分必要条件. 然而, 为获得充分必要结果, 必须直接求解超越特征方程, 这通常是非常复杂和困难的工作(见文献[1~5]及其参考文献). 本文采用频域方法研究了含多个常时滞的线性不确定系统的时滞依赖的可 α -鲁棒镇定的条件, 所得结果为指数稳定型并表示为 LMI 形式, 且由此得到的结果可保证系统的特征方程的根在复平面的 $\text{Re } s \leq -\alpha < 0$ 的区域范围内. 最后以一个数值例子显示其具体应用.

2 系统描述与准备工作(System statement and preliminaries)

考虑下面的含多个未知常时滞的线性不确定系统

$$\dot{x}(t) = (A_0 + \Delta A_0)x(t) + \sum_{k=1}^m (A_k + \Delta A_k)x(t - \tau_k) + (B_0 + \Delta B_0)u(t) + (B_1 + \Delta B_1)u(t - h). \quad (1)$$

其中: $x(t) \in \mathbb{R}^n$ 为状态向量, $u(t) \in \mathbb{R}^l$ 为控制向

量; $B_0, B_1, A_0, A_k, k = 1, 2, \dots, m$ 是具有适当维数的常矩阵; $h \in \mathbb{R}_+$; $\tau_k \in \mathbb{R}_+, k = 1, 2, \dots, m$ 为未知常时滞; $\Delta A_k = D_k F_k E_k, k = 0, 1, 2, \dots, m, \Delta B_i = D_{B_i} F_{B_i} E_{B_i}, i = 0, 1$ 是不确定项; $F_j F_j^T \leq I, j = 1, 2, \dots, m; F_{B_0} F_{B_0}^T \leq I, F_{B_1} F_{B_1}^T \leq I$.

本文旨在研究在无记忆反馈控制律 $u(t) = Kx(t)$ 作用下, 系统(1)的 α -鲁棒镇定条件. 为此, 将 $u(t) = Kx(t)$ 代入系统(1), 得

$$\dot{x}(t) = (A_0 + \Delta A_0 + (B_0 + \Delta B_0)K)x(t) + \sum_{k=1}^m (A_k + \Delta A_k)x_k(t - \tau_k) + (B_1 + \Delta B_1)Kx(t - h) = \bar{A}_0 x(t) + \sum_{i=1}^{m+1} \bar{A}_i x_k(t - \tau_k). \quad (2)$$

其中

$$\begin{aligned} \bar{A}_0 &= A_0 + \Delta A_0 + (B_0 + \Delta B_0)K, \\ \bar{A}_i &= A_i + \Delta A_i, i = 1, \dots, m, \\ \bar{A}_{m+1} &= (B_1 + \Delta B_1)K, \tau_{m+1} = h. \end{aligned}$$

定义 1 对于系统(1),若存在正数 $\tau_{kM} > 0$,使得在 $\tau_k \in [0, \tau_{kM}]$, $k = 1, 2, \dots, m+1$ 情形下,其式(2)的特征方程^[6]

$$\det [sI_n - (\bar{A}_0 + \sum_{k=1}^{m+1} e^{-\tau_k s} \bar{A}_k)] = 0 \quad (3)$$

所有的根均位于复平面的 $\operatorname{Re} s \leq -\alpha < 0$ 的区域内,则称系统(1)是可 α -镇定的,其控制律 $u(t) = Kx(t)$ 为 α -镇定控制律。

引理 1 D, E, F 为具有适当维数的实矩阵,且 $F^T F \leq I$,则对于任意标量 $\varepsilon > 0$,下面的不等式成立^[7]:

$$DFE + E^T F^T D^T \leq \varepsilon DD^T + \varepsilon^{-1} E^T E. \quad (4)$$

3 主要结果(Main result)

定理 1 对于 $h_M = \tau_{(m+1)M} > 0, \tau_{kM} > 0, k = 1, 2, \dots, m$,若存在标量 $\alpha > 0$ 及对称正定矩阵 $P \in \mathbb{R}^{n \times n}$ 和 $S_k \in \mathbb{R}^{n \times n}, k = 1, 2, \dots, m+1$,使得

$$\begin{bmatrix} \bar{P}\bar{A}_0 + \bar{A}_0^T P + 2\alpha P + \sum_{i=1}^m e^{a\tau_i} S_i + e^{a h_M} S_{m+1} & \bar{P}\bar{A}_1 & \cdots & \bar{P}\bar{A}_m & P(B_1 + \Delta B_1)K \\ \bar{A}_1^T P & -e^{a\tau_1} S_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \bar{A}_m^T P & 0 & \cdots & -e^{a\tau_m} S_m & 0 \\ K^T(B_1 + \Delta B_1)^T P & 0 & \cdots & 0 & -e^{a h_M} S_{m+1} \end{bmatrix} \leq 0. \quad (5)$$

其中

$$\bar{A}_0 = (A_0 + \Delta A_0 + (B_0 + \Delta B_0)K),$$

$$\bar{A}_i = A_i + \Delta A_i, i = 1, \dots, m,$$

则系统(1)是可 α -镇定的,相应的 α -镇定控制器为 $u(t) = Kx(t)$ 。

证 令 $\tilde{s} = \tilde{\sigma} + j\tilde{\omega}$ 是特征方程(3)的根,则

$$\det [\tilde{s}I - (\bar{A}_0 + \sum_{k=1}^{m+1} e^{-\tau_k \tilde{s}} \bar{A}_k)] = 0.$$

记

$$\begin{aligned} W(\tilde{\sigma}, \tilde{\omega}, \tau_k) &= \bar{A}_0 + \sum_{k=1}^{m+1} e^{-\tau_k \tilde{s}} \bar{A}_k = \\ &\bar{A}_0 + \sum_{k=1}^{m+1} \cos(-\tau_k \tilde{\omega}) e^{-\tau_k \tilde{\sigma}} \bar{A}_k + j \sum_{k=1}^{m+1} \sin(-\tau_k \tilde{\omega}) e^{-\tau_k \tilde{\sigma}} \bar{A}_k. \end{aligned} \quad (6)$$

其中 $\tau_k \in [0, \tau_{kM}]$. 则必存在非零向量 $\tilde{y}: = \tilde{y}(\tilde{\sigma}, \tilde{\omega}, \tau_k) \in \mathbb{C}^n$, 使得

$$(\tilde{s}I - W(\tilde{\sigma}, \tilde{\omega}, \tau_k))\tilde{y} = 0,$$

即 $W(\tilde{\sigma}, \tilde{\omega}, \tau_k)\tilde{y} = (\tilde{\sigma} + j\tilde{\omega})\tilde{y}$,

则

$$\tilde{\sigma} =$$

$$\begin{aligned} &(\tilde{y}^* (PW(\tilde{\sigma}, \tilde{\omega}, \tau_k) + W^*(\tilde{\sigma}, \tilde{\omega}, \tau_k)P)\tilde{y}) / (2\tilde{y}^* P\tilde{y}) = \\ &(\tilde{y}^* T_0 \tilde{y} + \sum_{k=1}^{m+1} e^{-\tau_k \tilde{\sigma}} \tilde{y}^* (\bar{P}\bar{A}_k e^{-j\tau_k \tilde{\omega}} + e^{-j\tau_k \tilde{\omega}} \bar{A}_k^T P)\tilde{y}) / (2\tilde{y}^* P\tilde{y}). \end{aligned} \quad (7)$$

其中 $T_0 = \bar{P}\bar{A}_0 + \bar{A}_0^T P$.

设 $L_k \in \mathbb{R}^{n \times n}, k = 1, 2, \dots, m+1$ 为对称可逆矩阵,根据 Cauchy-Schwarz 不等式:对任意的 $x, y \in \mathbb{C}^n$,不等式 $|x^* y| \leq \sqrt{x^* x y^* y}$ 成立;根据常用不等式:对任意 $a, b \in \mathbb{R}$,有 $2ab \leq a^2 + b^2$, 得到

$$\tilde{\sigma} =$$

$$\begin{aligned} &(\tilde{y}^* T_0 \tilde{y} + \sum_{k=1}^{m+1} e^{-\tau_k \tilde{\sigma}} \tilde{y}^* (\bar{P}\bar{A}_k e^{-j\tau_k \tilde{\omega}} + e^{j\tau_k \tilde{\omega}} \bar{A}_k^T P)\tilde{y}) / (2\tilde{y}^* P\tilde{y}) = \\ &(\tilde{y}^* T_0 \tilde{y} + \sum_{k=1}^{m+1} e^{-\tau_k \tilde{\sigma}} (\tilde{y}^* \bar{P}\bar{A}_k L_k^{-1} L_k e^{-j\tau_k \tilde{\omega}} \tilde{y} + \tilde{y}^* e^{j\tau_k \tilde{\omega}} L_k L_k^{-1} \bar{A}_k^T P \tilde{y})) / (2\tilde{y}^* P\tilde{y}) \leq \\ &(\tilde{y}^* T_0 \tilde{y} + \sum_{k=1}^{m+1} 2e^{-\tau_k \tilde{\sigma}} (\tilde{y}^* \bar{P}\bar{A}_k L_k^{-1} L_k^{-1} \cdot \bar{A}_k^T P \tilde{y} \tilde{y}^* L_k L_k^*)^{\frac{1}{2}}) / (2\tilde{y}^* P\tilde{y}) \leq \\ &(\tilde{y}^* T_0 \tilde{y} + \sum_{k=1}^{m+1} e^{-\tau_k \tilde{\sigma}} \tilde{y}^* (\bar{P}\bar{A}_k S_k^{-1} \bar{A}_k^T P + S_k)\tilde{y}) / (2\tilde{y}^* P\tilde{y}). \end{aligned} \quad (8)$$

其中 $S_k = L_k L_k$.

下面,用反证法来证明这个定理.注意到,由 Schur 定理,条件(5)等价于

$$T_0 + \sum_{k=1}^{m+1} e^{a\tau_{kM}} (\bar{P}\bar{A}_k S_k^{-1} \bar{A}_k^T P + S_k) \leq -2\alpha P, \quad (9)$$

令条件(5)成立.假设式(3)有一个根 $\tilde{s} = \tilde{\sigma} + j\tilde{\omega} \in \mathbb{C}$,使得 $\tilde{\sigma} > -\alpha$,则由式(8)和式(9),得到

$$-\alpha < \tilde{\sigma} \leq$$

$$\begin{aligned} &(\tilde{y}^* T_0 \tilde{y} + \sum_{k=1}^{m+1} e^{-\tau_k \tilde{\sigma}} \tilde{y}^* (\bar{P}\bar{A}_k S_k^{-1} \bar{A}_k^T P + S_k)\tilde{y}) / (2\tilde{y}^* P\tilde{y}) \leq \\ &(\tilde{y}^* T_0 \tilde{y} + \sum_{k=1}^{m+1} e^{a\tau_{kM}} \tilde{y}^* (\bar{P}\bar{A}_k S_k^{-1} \bar{A}_k^T P + S_k)\tilde{y}) / (2\tilde{y}^* P\tilde{y}) \leq \\ &-\alpha. \end{aligned} \quad (10)$$

上式是一个矛盾,所以假设不成立. 证毕.

定理 2 对于 $h_M = \tau_{(m+1)M} > 0, \tau_{kM} > 0, k = 1, 2, \dots, m$,若存在适当维数的矩阵 Y ,标量 $\alpha > 0$, $\varepsilon_{B_0}, \varepsilon_{B_1}, \varepsilon_k > 0, k = 1, 2, \dots, m$ 及对称正定矩阵 $X \in$

$\Xi^{n \times n}$ 及 $V_k \in \Xi^{n \times n}, k = 1, 2, \dots, m+1$, 使得下式 成立:

$$\begin{bmatrix} W & A_1 X & \cdots & A_m X & B_1 Y & X E_0^T & Y^T E_{B_0}^T & 0 & \cdots & 0 & 0 \\ X A_1^T & -e^{-\alpha \tau_1} V_1 & \cdots & 0 & 0 & 0 & 0 & X E_1^T & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ X A_m^T & 0 & \cdots & -e^{-\alpha \tau_m} V_m & 0 & 0 & 0 & 0 & \cdots & X E_m^T & 0 \\ Y^T B_1^T & 0 & \cdots & 0 & -e^{-\alpha h} V_{m+1} & 0 & 0 & 0 & \cdots & 0 & Y^T E_{B_1}^T \\ E_0 X & 0 & \cdots & 0 & 0 & -\epsilon_0 I & 0 & 0 & \cdots & 0 & 0 \\ E_{B_0} Y & 0 & \cdots & 0 & 0 & 0 & -\epsilon_{B_0} I & 0 & \cdots & 0 & 0 \\ 0 & E_1 X & \cdots & 0 & 0 & 0 & 0 & -\epsilon_1 I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & E_m X & 0 & 0 & 0 & 0 & \cdots & -\epsilon_m I & 0 \\ 0 & 0 & \cdots & 0 & E_{B_1} Y & 0 & 0 & 0 & \cdots & 0 & -\epsilon_{B_1} I \end{bmatrix} \leq 0. \quad (11)$$

其中

$$\begin{aligned} W = & A_0 X + B_0 Y + (A_0 X + B_0 Y)^T + 2\alpha X + \\ & \epsilon_{B_0} D_{B_0} D_{B_0}^T + \epsilon_{B_1} D_{B_1} D_{B_1}^T + \epsilon_0 D_0 D_0^T + \\ & \sum_{k=1}^m [e^{-\alpha \tau_k} V_k + \epsilon_k D_k D_k^T] + e^{-\alpha h} V_{m+1}, \end{aligned} \quad (12)$$

则系统(1)可 α -鲁棒镇定, 相应的 α -鲁棒控制器为 $u(t) = YX^{-1}x(t)$.

证 由定理1可知, 使系统(1)可 α -镇定的充分条件是下式成立:

$$G + U + U^T \leq 0. \quad (13)$$

其中:

$$G = \begin{bmatrix} P(A_0 + B_0 K) + (A_0 + B_0 K)^T P + 2\alpha P + \sum_{i=1}^m e^{\alpha \tau_i} S_i + e^{\alpha h} S_{m+1} & P A_1 & \cdots & P A_m & P B_1 K \\ & A_1^T P & & & -e^{-\alpha \tau_1} S_1 & \cdots & 0 & 0 \\ & \vdots & & & \vdots & \ddots & \vdots & \vdots \\ & A_m^T P & & & 0 & \cdots & -e^{-\alpha \tau_m} S_m & 0 \\ & K^T B_1^T P & & & 0 & \cdots & 0 & -e^{-\alpha h} S_{m+1} \end{bmatrix}, \quad (14)$$

$$\begin{aligned} U = & \begin{bmatrix} P D_{B_0} \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_{B_0} [E_{B_0} K \quad 0 \quad \cdots \quad 0] + \begin{bmatrix} P D_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_0 [E_0 \quad 0 \quad \cdots \quad 0] + \cdots + \\ & \begin{bmatrix} P D_m \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_m [0 \quad \cdots \quad 0 \quad E_m] + \begin{bmatrix} P D_{B_1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_{B_1} [E_{B_0} K \quad 0 \quad \cdots \quad 0]. \end{aligned} \quad (15)$$

根据引理1, 有

$$U + U^T \leq \begin{bmatrix} \sum_{i=0}^m \epsilon_i P D_i D_i^T P & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{\epsilon_0} E_0^T E_0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} +$$

$$\begin{bmatrix} K^T E_{B_0}^T & 0 & \cdots & 0 & 0 \\ 0 & E_1^T & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & E_m^T & 0 \\ 0 & 0 & \cdots & 0 & K^T E_{B_1}^T \end{bmatrix} \begin{bmatrix} \frac{1}{\epsilon_{B_0}} I & 0 & \cdots & 0 & 0 \\ 0 & \frac{1}{\epsilon_1} I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\epsilon_m} I & 0 \\ 0 & 0 & \cdots & 0 & \frac{1}{\epsilon_{B_1}} I \end{bmatrix} \begin{bmatrix} E_{B_0} K & 0 & \cdots & 0 & 0 \\ 0 & E_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & E_m & 0 \\ 0 & 0 & \cdots & 0 & E_{B_1} K \end{bmatrix}. \quad (16)$$

将式(16)代入式(13),同时左右同乘 $\text{diag}[P^{-1} \cdots P^{-1}]$,并记 $X = P^{-1}$, $Y = KX$, $V_i = XS_i X$, $i = 1, 2, \dots, m+1$,然后应用Schur补技术,可得式(11),即式(11)成立保证式(13)成立,定理得证.

4 数值例子(Numerical example)

考虑系统(1),其中:

$$A_0 = \begin{bmatrix} -2 & 0.7 \\ 0.5 & -5 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & -0.1 \\ 3 & -0.3 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0.2 \end{bmatrix},$$

$$\tau_1 \leq \tau_{1M} = 0.5, h \leq h_M = 0.3;$$

不确定项

$$\Delta A_k = D_k F_k E_k, k = 0, 1,$$

$$\Delta B_i = D_{B_i} F_{B_i} E_{B_i}, i = 0, 1.$$

其中

$$D_0 = D_1 = D_{B_0} = D_{B_1} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$E_0 = E_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, E_{B_0} = E_{B_1} = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

$$F_k^T F_k \leq I, k = 0, 1, F_{B_i}^T F_{B_i} \leq I, i = 0, 1.$$

要求设计的控制器使得系统特征方程的根满足 $\text{Re } s \leq -\alpha = -0.285$,应用MATLAB LMI工具箱,求解式(12)得到

$$X = \begin{bmatrix} 0.3257 & -0.0708 \\ -0.0708 & 1.1257 \end{bmatrix}, V_1 = \begin{bmatrix} 0.4960 & -0.2640 \\ -0.2640 & 1.8922 \end{bmatrix},$$

$$V_2 = \begin{bmatrix} 0.2437 & -0.2521 \\ -0.2521 & 0.9781 \end{bmatrix}, Y = \begin{bmatrix} 0.0430 & -0.0742 \\ -0.0742 & -0.1714 \end{bmatrix},$$

$$\epsilon_0 = 1.5856, \epsilon_1 = 1.6439, \epsilon_{B_0} = 0.7289, \epsilon_{B_1} = 0.7002.$$

反馈控制器 $u(t) = Kx(t)$ 的增益矩阵 $K = YX^{-1}$

$$= \begin{bmatrix} 0.0866 & -0.2090 \\ -0.3592 & -0.6043 \end{bmatrix}, \text{经验算以上参数使式(11)}$$

成立,根据定理2, $u(t) = Kx(t)$ 是 α -鲁棒控制器.

5 小结(Conclusion)

本文针对一类具有范数有界不确定系数和未知常时滞的系统,利用频域的方法,研究了系统的 α -鲁棒镇定问题,所得结果为时滞依赖的LMI形式,最后通过一个例子说明了结论的有效性及其应用方法.

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