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Adaptive high order differential feedback control for inverted pendulum system

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Abstract: Making full use of high order differential information extracted, an adaptive high order differential feedback controller is proposed, which does not depend on the model of SISO nonlinear affine system to a certain extent. Stability and robustness of the closed-loop system were analyzed. Through considering the position acceleration in dynamic equation of angle of pendulum as control input, the inverted pendulum system was then converted into interactional double nonlinear SISO affine systems. The pendulum system was thus successfully stabilized and regulated by using double adaptive high order differential feedback controller in series. Numeric simulations showed that the controller reaches satisfied effect for the benchmark model, and importantly had strong robustness for nonlinear friction term, parameter variations including length and mass of pendulum and mass of carriage, and external disturbance as well.

Key words: inverted pendulum system; high order differentiator; adaptive high order differential feedback controller; model free controller; robustness

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倒立摆系统自适应高阶微分反馈控制

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摘要: 利用提取的系统高阶微分信息,提出了自适应高阶微分反馈控制器.某种程度上该控制器不依赖于单输入单输出(SISO)非线性仿射系统的模型.并且分析了闭环系统的稳定性和鲁棒性.通过将摆角方程的位移加速度看作是控制输入,将倒立摆系统转化成相互影响的两个SISO仿射系统,从而用两个串级高阶微分反馈控制器成功地实现了倒立摆系统的镇定与调节.数字仿真表明,控制器对摆的基准模型实现了较为满意的控制,而且该控制方法对非线性摩擦项,对摆长、摆质量、小车质量等参数变化以及外扰动具有强鲁棒性.

关键词: 倒立摆系统; 高阶微分器; 自适应高阶微分反馈控制器; 不依赖模型控制器; 鲁棒性

1 Introduction

As a typical unstable nonlinear plant, the problems of stabilizing and regulating an inverted pendulum system have been a benchmark example in demonstrating and motivating various control design techniques[1~4]. For example, based on the model Chung and John[1] presented nonlinear controller to regulate the swing energy of the pendulum using L_∞ small-gain approach and Lin et al.[2] provided linear state feedback controller that balances the pendulum. Kawatani et al.[3] linearized nonlinear mathematical model of a parallel-type double inverted pendulum system, and then designed a stable controller by state feedback gain vector and full state observer. Yao et al.[4] firstly identified dynamic linearizing model through using fuzzy method, then based on the model design poles assignment controller to stabilize the system. All the controllers of these literatures rely on the nonlinear benchmark model or linearized model of the inverted pendulum. Some design approaches consider robust control for friction term etc.[1,3], but the uncertainty is smaller than benchmark model. In fact, it is the important characteristic of modern control theory that the controller relies on model of controlled plant according to a criterion.

We find that the measurable information and their differentials up to nth-order are significant in the affine system. The differentials not only are the changing rates of the output of the system, but also are the inner states of the system. Han[5] proposed auto-disturbance-rejection controller using high order differentials. But there is no analysis of stability and convergence for the closed-loop system with the controller.

We designed high order differentiator (HOD) being independent of the controlled plant and only relying on

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the signal itself in literature[6]. The HOD is able to approximate the real signal and extract its differentials up to nth-order. Stability and convergence of the HOD are proved.

Making use of the most of the information of the extracted differentials, we design adaptive high order differential feedback controller (HODFC) that does not rely on the model of the system, but depend on the differentials up to nth-order. Theoretical analysis shows that the HODFC achieves stability and convergence for the closed-loop system.

If we consider the acceleration as control input in dynamic equation of angle of pendulum, and do it as control objective in dynamic equation of position of carriage, then the pendulum system is converted into double nonlinear SISO affine system.

Therefore, using two HODFC, we can stabilize and regulate the inverted pendulum. The angle of the pendulum converges to zero and the position of the cart reaches to given aim using the controller. Because the controller does not rely on the model of inverted pendulum in some extent, the HODFC is robust for the disturbance and all the parameters variations. Simulations demonstrate the validity of the proposed theory. Furthermore, the HODFC does not rely on the velocity of position and that of angle, but only relies on the position and angle of the pendulum. Therefore, the controller is adaptive.

This paper is organized as follows: Section 2 presents the adaptive HODFC for SISO affine system based on the HOD. Section 3 converts the inverted pendulum system into affine system, and stabilizes and regulates it using the adaptive HODFC. In section 4 numerical experiments are performed to show the validity of control for inverted pendulum system.

2 Adaptive high order differential feedback control

Consider SISO affine system with disturbance, its differential equation is depicted as

$$\Pi \quad y^{(n)} = f(\boldsymbol{x}, t) + d(t) + u. \tag{1}$$

where u is the control input, y is the measured output, $y^{(i)}$ denotes the ith differential of y, $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} = [y, y^{(1)}, \cdots, y^{(n-1)}]^{\mathrm{T}} \in \mathbb{R}^n$ denotes output differential vector, and is also state vector of the system. $f(\cdot)$ is unknown smooth nonlinear bounded time-varying function. $d(t)$ is unknown bounded smooth disturbance. Initial conditions $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$.

The given target trajectory y_r exists differentials up to nth-order, and $y_r^{(n)}$ is continuous. If y_r does not satisfy these conditions, we soften it to meet the conditions. Setting given input differential vector $\boldsymbol{x}_r = [y_r, y_r^{(1)}, \cdots, y_r^{(n-1)}]^{\mathrm{T}} \in \mathbb{R}^n$, and given input extended differential vector $\boldsymbol{r} = [y_r, y_r^{(1)}, \cdots, y_r^{(n)}]^{\mathrm{T}}, \boldsymbol{r} \in \mathbb{R}^{n+1}$, extended output differential vector $\boldsymbol{z} = [y, y^{(1)}, \cdots, y^{(n)}]^{\mathrm{T}} \in \mathbb{R}^{n+1}$, and error extended differential vector $\boldsymbol{e} = \boldsymbol{r} - \boldsymbol{z} = [e_1, e_2, \cdots, e_{n+1}]^{\mathrm{T}} = [e, e^{(1)}, \cdots, e^{(n)}]^{\mathrm{T}} \subset \mathbb{R}^{n+1}$, where $e = y_r - y$.

In general, the output y and the given input y_r are known, but the output extended differential vector $\boldsymbol{z}$ and the given input extended differential vector $\boldsymbol{r}$ are unknown. Literature [6] proposed a class of HOD which extracted differentials up to n-th order for any measurable signal $y(t)$ which possesses nth-order differential. Setting $\hat{\boldsymbol{z}} = [\hat{y}, \hat{y}^{(1)}, \cdots, \hat{y}^{(n)}]^{\mathrm{T}}$ to express estimating vector of the extended differential vector $\boldsymbol{z} = [y, y^{(1)}, \cdots, y^{(n)}]^{\mathrm{T}}$ (note that $\hat{y}^{(i)}$ denotes estimate of $y^{(i)}$, is not the ith differential of $\hat{y}$).

The presented HOD is represented as combined expression, i.e. connecting n_o order dynamic system (2) with $n+1$ order algebra equation (3).

$$\Sigma: \begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} + a_i(y - \hat{x}_1), \ 1 \leqslant i \leqslant n_o - 1, \\ \dot{\hat{x}}_{n_o} = a_{n_o}(y - \hat{x}_1). \end{cases} \tag{2}$$

$$\begin{cases} \hat{y} = \hat{x}_1, \\ \hat{y}^{(i)} = \hat{x}_{i+1} + a_i(y - \hat{x}_1), i = 2, \cdots, n, \end{cases} \tag{3}$$

where n_o is the order of the system Σ, generally, setting $n_o \geqslant n + 1$, $\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_{n_o}$ are the states of the system Σ, $a_i (i = 1, \cdots, n_o)$ are the parameters. The problem is how to obtain filtering signal $\hat{y}$ based on the measured signal $y(t)$, furthermore, to obtain estimating signal $\hat{y}^{(1)}, \cdots, \hat{y}^{(n)}$.

Obviously, the stability of the HOD is identical to the system Σ. Making Laplace transformation for Eq. (2), we easily obtain transfer function from y to $\hat{x}_1$,

$$\Phi(s) = \frac{a_1 s^{n_o - 1} + \cdots + a_{n_o - 1} s + a_{n_o}}{s^{n_o} + a_1 s^{n_o - 1} + \cdots + a_{n_o - 1} s + a_{n_o}}.$$

If the parameters $a_i (i = 1, \cdots, n_o)$ are not correctly given, all the extracted differentials by the HOD are possibly not ideal, even the system Σ is unstable. We analyzed the parameters design of the system Σ based on root-locus in literature [6]. The parameters are given by the following form:

$$\begin{cases} a_i = K C_{n_o - 1}^{i-1} a^{i-1}, K = n_o^{n_o} a / (n_o - 1)^{n_o - 1}, \\ a \in [2, 30], i = 1, 2, \cdots, n_o. \end{cases} \tag{4}$$

Note that the HOD has been simplified into two adjustable parameters n_0 and a.

We had the following remark for the HOD under the parameters formula (4) (see literature [6]):

1) The HOD does not rely on the model of the estimated system Π, and it is an additional system based on signal $y(t)$; 2) The HOD is an asymptotically stable systems; 3) The HOD holds higher convergence, and satisfies

$$\lim_{a\to\infty}\hat{y}^{(i)} = y^{(i)}, i = 0,\cdots,n. \tag{5}$$

where $\hat{y}^{(0)}$ denotes $\hat{y}$, where $a \to \infty$ is strictly in mathematics. In practice only taking $a \in [2,30]$, the HOD has higher precision.

In the following, we study the control problem based on the differentials up to n-th order.

Assumption 1 The extended differential vector of the output z and extended differential vector of reference input $\boldsymbol{r}$ are known, and $y^{(n)}$ is continuous.

Theorem 1 For the time-varying nonlinear system (1) with unknown model and with unknown disturbance, the HODFC is represented as

$$u = \boldsymbol{K}\boldsymbol{e} + \hat{u}, \tag{6}$$

where $\boldsymbol{K} = [k_n, k_{n-1}, \cdots, k_1, 1] \in \mathbb{R}^{1\times(n+1)}$, the polynomial $s^n + k_1 s^{n-1} + \cdots + k_n$ is a Hurwitz polynomial, and $\hat{u}$ is the filtering value of the control u, which satisfies

$$\dot{\hat{u}} = -\lambda\hat{u} + \lambda u, \tag{7}$$

where λ is a large positive number, $\hat{u}_0 = 0, u_0 = 0$. The HODFC makes the closed-loop system asymptotically stable, and has strong robustness for parameters and disturbance changing of the system, and meets convergence

$$\lim_{t\to\infty}\lim_{\lambda\to\infty}\boldsymbol{x} = \boldsymbol{x}_r. \tag{8}$$

Proof From Eq.(1) and definition of $\boldsymbol{e}$, we have

$$\begin{aligned}\dot{e}_n &= y_r^{(n)} - y^{(n)} = \\ &y_r^{(n)} - (f(\boldsymbol{x},t) + d(t) + u) = \\ &y_r^{(n)} - y^{(n)} + y^{(n)} - (f(\boldsymbol{x},t) + d(t) + u).\end{aligned}$$

Furthermore, the following form holds

$$\begin{cases}\dot{e}_1 = e_2,\\ \dot{e}_2 = e_3,\\ \quad\vdots\\ \dot{e}_n = y_r^{(n)} - y^{(n)} + y^{(n)} - (f(\boldsymbol{x},t) + d(t) + u).\end{cases} \tag{9}$$

Setting $\boldsymbol{\varepsilon} = \boldsymbol{x} - \boldsymbol{x}_r = [e_1, e_2, \cdots, e_n]^{\mathrm{T}} \in \mathbb{R}^n$,

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \cdots & 1\\ 0 & 0 & 0 & \cdots & 0\end{bmatrix} \in \mathbb{R}^{n\times n},\ b = \begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix} \in \mathbb{R}^n,$$

$$A_m = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \cdots & 1\\ -k_n & -k_{n-1} & -k_{n-2} & \cdots & -k_1\end{bmatrix} \in \mathbb{R}^{n\times n}$$

From Eq.(9), we have

$$\begin{aligned}&\dot{\boldsymbol{\varepsilon}} = A\boldsymbol{\varepsilon} + b(y_r^{(n)} - y^{(n)} + y^{(n)} - (f(\boldsymbol{x},t) + d(t) + u)) = \\ &A_m\boldsymbol{\varepsilon} + b(\boldsymbol{K}'\boldsymbol{\varepsilon} + y_r^{(n)} - y^{(n)} + y^{(n)} - (f(\boldsymbol{x},t) + d(t) + u)) = \\ &A_m\boldsymbol{\varepsilon} + b(\boldsymbol{K}\boldsymbol{e} + y^{(n)} - (f(\boldsymbol{x},t) + d(t) + u)).\end{aligned} \tag{10}$$

where $\boldsymbol{K}' = [k_n, k_{n-1}, \cdots, k_1] \in \mathbb{R}^{1\times(n)}$ makes A_m is a Hurwitz matrix, it means that there exists matrix $P = P^{\mathrm{T}} > 0$, for any positive matrix Q, satisfies

$$PA_m + A_m^{\mathrm{T}}P = -Q.$$

Identically, $\boldsymbol{K}$ makes $s^n + k_1 s^{n-1} + \cdots + k_n$ become a Hurwitz polynomial. In Eq.(10), Let

$$\boldsymbol{K}\boldsymbol{e} + y^{(n)} - (f(\boldsymbol{x},t) + d(t) + u) = 0, \tag{11}$$

We have the control law

$$u = \boldsymbol{K}\boldsymbol{e} + y^{(n)} - (f(\boldsymbol{x},t) + d(t)). \tag{12}$$

Because the sum term $f(\boldsymbol{x},t) + d(t)$ is unknown, the control law is unable to be realized. From the system (1), we have

$$y^{(n)} - (f(\boldsymbol{x},t) + d(t)) = u. \tag{13}$$

But u is the control law which required to be gained, so it is also unable to be realized. We consider the filtering value $\hat{u}$ of u is able to be realized because the filtering has lag property from formula (7). Use $\hat{u}$ to replace u, and it means

$$\hat{u} \approx y^{(n)} - (f(\boldsymbol{x},t) + d(t)). \tag{14}$$

Substitute (14) into (12), we gain controller (6). Substitute (6) into (10), we have the following important expression of the closed-loop system

$$\dot{\boldsymbol{\varepsilon}} = A_m\boldsymbol{\varepsilon} + b(u - \hat{u})). \tag{15}$$

In the following we prove stability, convergence and robustness for the closed-loop system (15). From (7), the equation is a filtering expression. The filtering $\hat{u}$ is realized via integrator, so the filtering $\hat{u}$ is necessarily continuous no matter whether u is continuous. It means that the continuity of the filtering $\hat{u}$ does not rely on that of u, as long as u is integrable. Furthermore, from the assumption 1 that $y^{(n)}$ and $y_r^{(n)}$ are continuous, so $y^{(i)}$ and $y_r^{(i)}(i = 0,\cdots,n-1)$ must be also continuous, because of $i < n$. Hence we obtain that $\boldsymbol{e}$ is continuous. Therefore we have the control law u which must be continuous from (6). From (7) again, we have

$$\hat{u} = \lambda u/(s + \lambda). \tag{16}$$

From (16) and continuity of u, we obtain

$$\lim_{\lambda\to\infty}\hat{u} = u. \tag{17}$$

Because A_m is a Hurwitz matrix, the closed-loop control system is asymptotically stable, and from (15) and (17) again, we have

$$\lim_{t\to\infty}\lim_{\lambda\to\infty}\boldsymbol{\varepsilon} = 0. \tag{18}$$

Hence we obtain convergent remark (8). Because the controller (6) does not rely on the model of the system, and only rely on signals of given input and output and their differentials and high order differentials, the controller is strong robust for the function $f(\cdot)$ and disturbance $d(t)$.

Explanation

1) Where $\lambda\to\infty$ is only rigorous for mathematics meanings. Because the control law u is continuous, taking $\lambda\in[5,100]$, filtering $\hat{u}$ can excessively approximate u. The filtering $\hat{u}$ can be completely gained by other filtering equation expect for Eq. (7).

2) The HODFC not only realizes that the output of the closed-loop system y track given input y_r, but also does that the differentials up to $(n-1)$ th of the output $y^{(i)}$ track the differentials up to $(n-1)$ th of given input $y_r^{(i)}$, which is different from the general objective that the output of the closed-loop system track given input.

3) The HODFC has distinct physical meanings. The control law has two terms, in which one term $\hat{u}$ overcomes or counteracts sum term $f(\boldsymbol{x},t)+d(t)$, another term $\boldsymbol{Ke}$ makes the closed-loop system asymptotically stable which can be seen from Eq. (11) to Eq. (18). We call the sum term $f(\boldsymbol{x},t)+d(t)$ as generalized disturbance.

4) From the Assumption 1, the HODFC is non-adaptive for $\boldsymbol{z}$, and $\boldsymbol{r}$. From (1), output y possesses differentials up to nth-order, which satisfy the condition of measurable input signal for HOD, therefore we estimate $\boldsymbol{z}$ and $\boldsymbol{r}$ via the HOD to obtain $\hat{\boldsymbol{z}}$ and $\hat{\boldsymbol{r}}$ based on the output y and given input y_r. Furthermore from (2), $n_0\geq n+1$, hence $\hat{y}^{(n)}$ is continuous. The controller (6) is converted into the following form:

$$u = \boldsymbol{K}\hat{\boldsymbol{e}} + \hat{u}, \tag{19}$$

where $\hat{\boldsymbol{e}} = \hat{\boldsymbol{r}} - \hat{\boldsymbol{z}}$, hence we yield adaptive HODFC.

Fig. 1 shows the realized diagram of the adaptive HODFC for nonlinear system based on HOD.

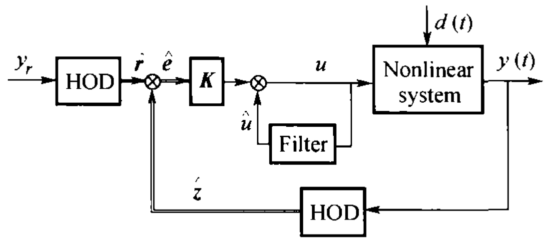

Fig. 1 Realized diagram of adaptive high order differential feedback control based on the HOD for nonlinear system

3 The inverted pendulum system analysis, stabilization and regulation

Consider the benchmark model of the inverted pendulum system with linear smooth friction[7]

$$(J+mL^2)\varphi^{(2)} = -(mL\cos\varphi)y^{(2)} + mLg\sin\varphi, \tag{20.1}$$

$$(M+m)y^{(2)} = -by^{(1)} - (mL\cos\varphi)\varphi^{(2)} + mL\sin\varphi(\varphi^{(1)})^2 + u, \tag{20.2}$$

where φ denotes the angle of the pendulum with respect to vertical line, y denotes the position of the carriage, J denotes inertia of the pendulum, L denotes distance between center of mass of pendulum and tip attached to the carriage, M and m respectively denotes mass of the carriage and the pendulum, g denotes the acceleration due to gravity, b denotes the linear coefficient of friction for the carriage, u denotes the external force applied to the carriage.

3.1 Stabilization problem of pendulum

Design the closed-loop feedback control to achieve the following double objectives[7]

$$\lim_{t\to\infty}\varphi(t) = \varphi_r = 0, \tag{21.1}$$

$$\lim_{t\to\infty}y(t) = y_r = 0. \tag{21.2}$$

This is synchronously stabilization problem for the angle of pendulum and the position of carriage. Furthermore, if we take $y_r\neq 0$, the stabilization problem is also the regulation problem. Therefore, we mostly study the stabilization problem.

From Eqs. (20.1) and (20.2), the pendulum system is a class of single input and two output system. Hence, we can't directly apply the adaptive HODFC to control it. But we can convert the system into two SISO affine systems, and use two controllers to control it.

In order to control a system, we should grasp its given objective, output, control force and controlled plant. From Eq. (20.1) of angle of pendulum, the output is angle φ, and the given objective is expression (21.1), and the drive force is $y^{(2)}$ whose second integral is also the output y of the carriage with pendulum. For Eq. (20.2) of the carriage, it is activated by external force u, and produces output acceleration $y^{(2)}$, the term $-(mL\cos\varphi)\varphi^{(2)} + mL\sin\varphi(\varphi^{(1)})^2$ is couple term which can be taken into as generalized disturbance, and the given objective is expression (21.2).

According to the above analyses, Eq. (20.1) and Eq. (20.2) can be considered into double SISO affine system. For Eq. (20.1), to achieve the objective Eq. (21.1), we can use one outer-loop adaptive HODFC to obtain control law $y_{1r}^{(2)}$. For the Eq. (20.2), we look the y_{1r} (i.e second integral of $y_{1r}^{(2)}$) as given objectives (its extended differential vector is $\boldsymbol{r}_1 = [y_{1r}, y_{1r}^{(1)}, y_{1r}^{(2)}]^{\mathrm{T}})$, and design one inner-loop adaptive HODFC to obtain control law u, then the control law u can achieve objective (21.1). The whole system makes up of a control one in series. The structure diagram of the closed-loop inverted pendulum system is shown in Fig.2.

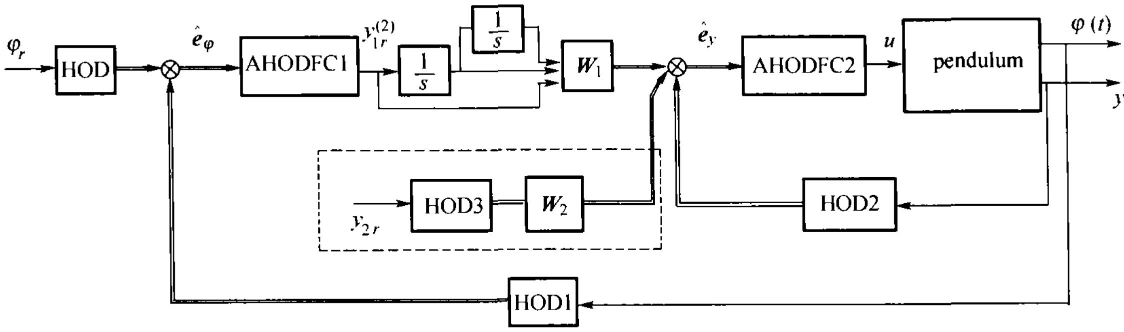

Fig. 2 Realized diagram of double adaptive HODFC based on the HOD for inverted pendulum system

Notice that the control law u possibly does not meet the objective (21.2). To achieve objective (21.2), we give another constrained objective $y_{2r} = 0$ for the inner-loop control HODFC, and its extended given input is $\boldsymbol{r}_2 = [y_{2r}, y_{2r}^{(1)}, y_{2r}^{(2)}]^{\mathrm{T}} = [0,0,0]^{\mathrm{T}}$ (the dotted line in the Fig.2). To synthesize (21.1) and (21.2), we define a weighted given input extended vector for inner-loop control HODFC

$$\boldsymbol{r} = \boldsymbol{W}_1\boldsymbol{r}_1 + \boldsymbol{W}_2\boldsymbol{r}_2. \tag{22}$$

where $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are weights. In the following, we give weighting scheme:

If control is only stabilization problem, i.e $y_{2r} = 0$, weighing is not any significant, because of $\boldsymbol{r}_2 = [0,0,0]^{\mathrm{T}}$, take $\boldsymbol{r} = \boldsymbol{r}_1$ or identically take $\boldsymbol{W}_1 = [1,1,1]$, $\boldsymbol{W}_2 = [0,0,0]$. If control is regulation problem i.e, $y_r \neq 0$, $\boldsymbol{r}_2 = [y_{2r}, y_{2r}^{(1)}, y_{2r}^{(2)}]^{\mathrm{T}} \neq [0,0,0]^{\mathrm{T}}$, take $\boldsymbol{W}_2 = [1,1,1]$. From the above analyses, we know that the most important objective fractional value is $y_{1r}^{(2)}$ in vector $\boldsymbol{r}_1$, because $y_{1r}^{(2)}$ is also control law in outer-loop HODFC, so take $\boldsymbol{W}_1 = [0,0,w]$, $w \in [0.7,2]$. In general, the pendulum system is disturbed more severe, and the parameters change bigger, w is taken bigger.

In order to unify the stabilization and regulation problem, and make the formula simpler, we consider stabilization problem as regulation problem. It means that we take $\boldsymbol{W}_1 = [0,0,w]$, $w \in [0.7,2]$, $\boldsymbol{W}_2 = [1,1,1]$ in both stabilization and regulation.

From (6) and (7), for the outer-loop HODFC1, the control law $y_{1r}^{(2)}$ is depicted as

$$y_{1r}^{(2)} = \boldsymbol{K}_\varphi \hat{\boldsymbol{e}}_\varphi + \hat{y}_{1r}^{(2)},\ \dot{\hat{y}}_{1r}^{(2)} = -\lambda_1 \hat{y}_{1r}^{(2)} + \lambda_1 y_{1r}^{(2)}. \tag{23}$$

The error differential vector is represented as

$$\hat{\boldsymbol{e}}_\varphi = \hat{\boldsymbol{z}}_{\varphi r} - \hat{\boldsymbol{z}}_\varphi = -[\hat{\varphi}, \hat{\varphi}^{(1)}, \hat{\varphi}^{(2)}]^{\mathrm{T}}. \tag{24}$$

where $\hat{\boldsymbol{z}}_{\varphi r} = [0,0,0]^{\mathrm{T}}$.

For inner-loop HODFC2, and the control law u is depicted as

$$u = -\boldsymbol{K}_y \hat{\boldsymbol{e}}_y + \hat{u},\ \dot{\hat{u}} = -\lambda_2\hat{u} + \lambda_2 u. \tag{25}$$

in which $\hat{\boldsymbol{e}}_y = \hat{\boldsymbol{r}} - \hat{\boldsymbol{z}}_y$, $\hat{\boldsymbol{r}} = \boldsymbol{W}_1\boldsymbol{r}_1 + \boldsymbol{W}_2\hat{\boldsymbol{r}}_2$, $\boldsymbol{r}_1 = [y_{1r}, y_{1r}^{(1)}, y_{1r}^{(2)}]^{\mathrm{T}}$, $\hat{\boldsymbol{r}}_2 = [0,0,0]^{\mathrm{T}}$, $\hat{\boldsymbol{z}}_y = [\hat{y}, \hat{y}^{(1)}, \hat{y}^{(2)}]^{\mathrm{T}}$. We consider coefficient of control term $y^{(2)}$ being negative (i.e reverse action) in Eq. (20.1), hence take $-\boldsymbol{K}_y$ (not $\boldsymbol{K}_y$) to counteract the reverse action in Eq.(25).

3.2 Regulation problem of the position of pendulum

During the time range $t \in [0, t_d]$, to design feedback controller which satisfies section 3.1 requirement of stabilization problem, and $t \in [t_d, T]$, where $T > t_d \geqslant 0$ is another given time, the system satisfies

$$\varphi(t) \to 0, \tag{26.1}$$

$$y(t) \to y_r \neq 0. \tag{26.2}$$

where y_r is a softened variable by given objective of position y_d (see literature [7]).

The regulation problem is the same as stabilization problem except for $y_r \neq 0$, hence the control scheme is the same as that of Section 3.1 except for

$$\boldsymbol{r}_2 = [y_{2r}, y_{2r}^{(1)}, y_{2r}^{(2)}]^{\mathrm{T}} \neq [0,0,0]^{\mathrm{T}}.$$

Notice 1 1) The controller HODFC1 and HODFC2 are all adaptive, because they are only based on output variables φ and y of system, not based on the states $\varphi^{(1)}$ and $y^{(1)}$. We can obtain estimation $\hat{\varphi}^{(1)}, \hat{\varphi}^{(2)}$ and $\hat{y}^{(1)}, \hat{y}^{(2)}$, based on φ and y using the HOD.

2) The double adaptive HODFC does not rely on the parameters and function relations of the models (20.1) and (20.2). Therefore, the controller has strong robustness for parameters and function relation changing.

4 Simulation and robustness verification for pendulum system

4.1 Stabilization problem

We take the benchmark parameters (see literature [7]) of (20.1) and (20.2), $M = 1.320$ kg, $L = 0.25$ m, $m = 0.109$ kg, $b = 0.1\ \mathrm{N/(m \cdot s^{-1})}$, $J = mL^2/3$, $g =$

$9.8\ \mathrm{m/s^2}$, and take initial values $\varphi(0) = \varphi^{(1)}(0) = y(0) = y^{(1)}(0) = 0$, and impulse force $40\delta(t)$ N. Design controller to make the closed-loop system satisfy the following requirement:

$$|\varphi(t)| \leqslant 0.02\ \mathrm{rad},\ t \geqslant 0.5\ \mathrm{s}, \quad (27.1)$$
$$|y(t)| \leqslant 0.01\ \mathrm{m},\ t \geqslant 1\ \mathrm{s}, \quad (27.2)$$
$$|u(t)| \leqslant 10\ \mathrm{V} \quad (27.3)$$

and give the cost $I = \int_0^6 \varphi^2(\tau)\,d\tau$.

Firstly, we research stabilization of the system. The controller is double adaptive HODFC, i.e Eqs. (23), (24) and (25). Take the same parameters for the HOD1, the HOD2, and the HOD3 as $n_0 = 5, a = 5$, and take the parameters of the HODFC1 as $K_\varphi = [16,8,1]^T, \lambda_1 = 10$ in Eq. (23), and that of the HODFC2 in Eq. (25) as $K_y = [25,10,1]^T, \lambda_2 = 6, W_1 = [0,0,0.75], W_2 = [1,1,1]$, the sample time $\tau = 0.005$. Fig.3 (a), (b), (c) show the angle of pendulum $\varphi(t)$, position of the carriage $y(t)$ and driving force $u(t)$ (includes impulse force) respectively. To observe clearly initial transient process, we plot $u(t)$ in $t \in [0,1]$. Obviously, $\varphi(t)$, $y(t)$ and $u(t)$ satisfy requirement (27.1), (27.2) and (27.3) respectively. In fact, the angle satisfies $|\varphi(t)| \leqslant 0.02$ rad, $t \geqslant 0.1$ s; the position satisfies $|y(t)| \leqslant 0.01$ m; $t \geqslant 0$ s; because initial impulse force is $40\delta(t) \gg 10\delta(t)$, initial transient process appears $|u(t)| \geqslant 10$ V, but only at $t \geqslant 0.01$ s, satisfies $|u(t)| < 10$ V. Calculate the cost $I = \int_0^6 \varphi^2(\tau)\,d\tau = 0.0149$.

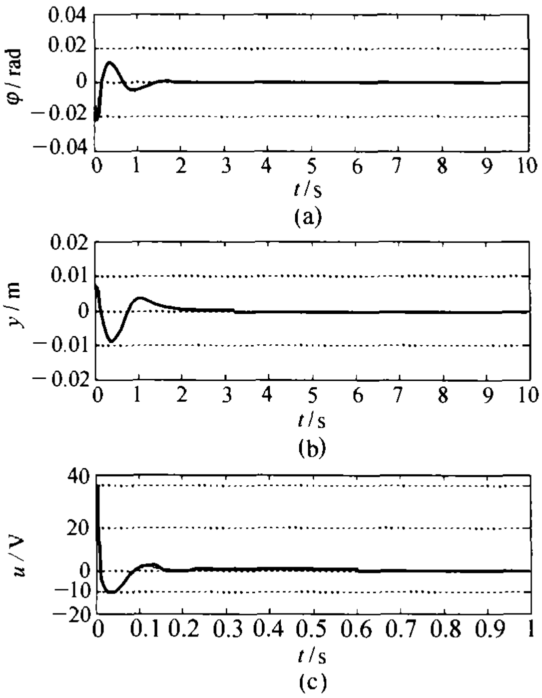

Fig. 3 Angle φ, position y and force u in stabilization

4.2 Regulation problem

The parameters of the controlled pendulum system are the same as that of section 4.1. Designing a controller satisfies the above requirement, and reaches another new requirement that given objective of position $y_d = 0.2$ m after $t_d = 5$ s satisfies $y(t) \to 0.2, \varphi(t) \to 0$, and does

$$|\varphi(t)| \leqslant 0.02\ \mathrm{rad}, t \geqslant 6\ \mathrm{s}, \quad (28.1)$$
$$|y(t) - y_d| \leqslant 0.01\ \mathrm{m}, t \geqslant 7\ \mathrm{s}, \quad (28.2)$$
$$|u(t)| \leqslant 10\ \mathrm{V}, \quad (28.3)$$

and the cost $I = \int_0^{10} \{\varphi^2(\tau) + (y(\tau) - y_d)^2\}\,d\tau$.

The controller is still the double adaptive HODFC, and its parameters are taken the same as that of the above controller in stabilization problem. The softened objective curve is represented as

$$y_{td} = \begin{cases} 0, & 0 \leqslant t < t_d, \\ 0.2, & t \geqslant t_d, \end{cases}$$

$$y_{2r} = \frac{16}{s^2 + 8s + 16} y_{td}.$$

Fig.4 (a), (b), and (c) show $\varphi(t)$, $y(t)$ and $u(t)$ respectively. The angle $\varphi(t)$ satisfies (28.1) at $t \geqslant 5.525$ s, the position $y(t)$ does (28.2) at $t \geqslant 6.9$ s, and driving force $u(t)$ does (28.3). The cost

$$I = \int_0^{10} \{\varphi^2(\tau) + (y(\tau) - y_d)^2\}\,d\tau = 46.2761.$$

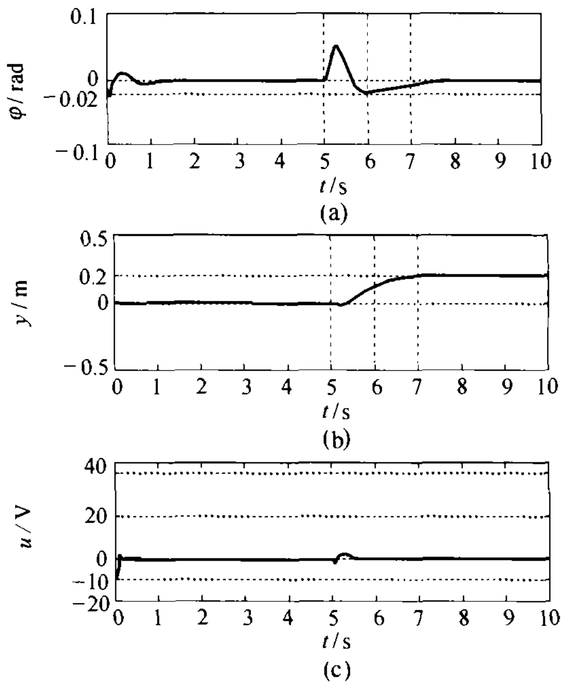

Fig. 4 Angle φ, position y and force u in regulation

The double adaptive HODFC in the above simulation does not rely on the parameters of system and some function relation.

4.3 Verification of robustness under simulation

We consider complicated model with nonlinear smooth friction or with disturbance, and parameters changing or time-varying.

4.3.1 Robust control for friction and parameter changing

Consider the inverted pendulum system with nonlinear smooth friction and that of node of pendulum[8], and parameters changing. The model is depicted as

$$(J + mL^2)\varphi^{(2)} = -(mL\cos\varphi)y^{(2)} - C\varphi^{(1)} + mLg\sin\varphi, \tag{29.1}$$

$$(M+m)y^{(2)} = -b\,|\,y^{(1)}\,| - (mL\cos\varphi)\varphi^{(2)} + mL\sin\varphi(\varphi^{(1)})^2 + u, \tag{29.2}$$

where $C = 0.12$ is coefficient of friction for node of pendulum. Increase the coefficient of smooth nonlinear friction $b = 0.95$, and magnify the value of m 4 times than that in benchmark model, i.e $m = 0.436$, and take $L = 0.625$. Notice that the values of the three parameters go beyond requirement in literature [8]. Other parameters and initial condition are the same as that of benchmark model (20.1) and (20.2).

a) We examine robust stabilization. The parameters of the controller are completely the same as that of the above controller except for the weight $W_1 = [0\ 0\ 1.15]$. The control results are shown in Fig. 5. Obviously, $\varphi(t)$, $y(t)$ and $u(t)$ respectively still satisfy requirement (27.1), (27.2) and (27.3). The cost $I = \int_0^6 \varphi^2(\tau)d\tau = 0.0151$.

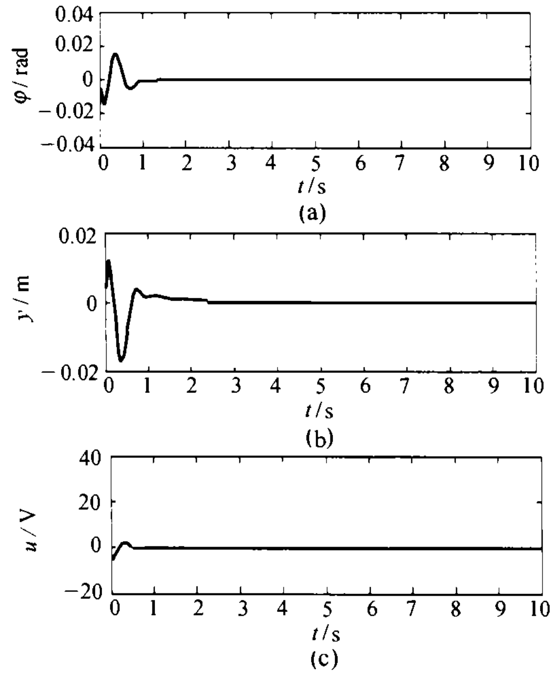

Fig. 5 Angle φ, position y and force u in robust stabilization

b) We examine regulation problem. The parameters of controller are completely the same as that of the above robust stabilization a). The control results are shown in the Fig.6. The angle $\varphi(t)$, position $y(t)$ and driving force $u(t)$ respectively still satisfy requirements (28.1), (28.2) and (28.3). The cost

$$I = \int_0^{10} \{\varphi^2(\tau) + (y(\tau) - y_d)^2\} d\tau = 47.6302.$$

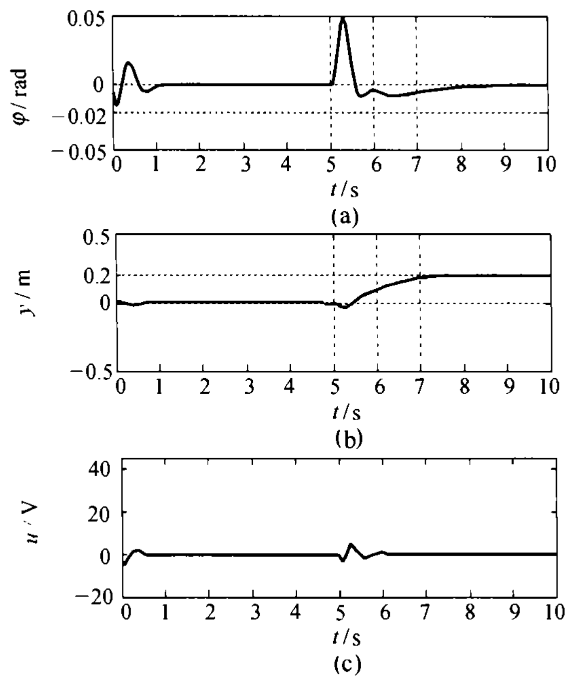

Fig. 6 Angle φ, position y and force u in robust regulation

4.3.2 Robust control for disturbed system and parameters M being time-varying

The Eq.(29.2) is converted into the following form:

$$(M(t) + m)y^{(2)} = -b\,|\,y^{(1)}\,| - (mL\cos\varphi)\varphi^{(2)} + mL\sin\varphi(\varphi^{(1)})^2 + u + d(t), \tag{30}$$

where

$$M(t) = \sin(t) + M_0, M_0 = 1.320, t \in [0,20], \tag{31}$$

$$d(t) = \begin{cases} 2, & 10 < t < 11, \\ 0, & \text{others}. \end{cases} \tag{32}$$

And other parameters are the same as that of system (29.1) and (29.2).

a) We examine stabilization problem. Take $W_1 = [0, 0, 1.25]$, other parameters still do not change. The control results are shown in Fig. 7. Obviously, $\varphi(t)$, $y(t)$ and $u(t)$ respectively still satisfy requirements (27.1), (27.2) and (27.3). $I = \int_0^{20} \varphi^2(\tau)d\tau = 0.0203$. From the figure we clearly see that the parameter time-varying has little effectiveness, and the disturbance activate at time $10 < t < 11$, but the influence promptly disappear, which demonstrate that the double adaptive has strong robustness for the disturbance and parameter changing.

b) We examine regulation problem. The controller is as the same as that of the above a) in section 4.3.2. The control results are shown in Fig. 8. Obviously, $\varphi(t)$, $y(t)$ and $u(t)$ respectively still satisfy requirements (28.1), (28.2) and (28.3). Calculate the cost

$$I = \int_0^{20} \{\varphi^2(\tau) + (y(\tau) - y_d)^2\} d\tau = 47.9324.$$

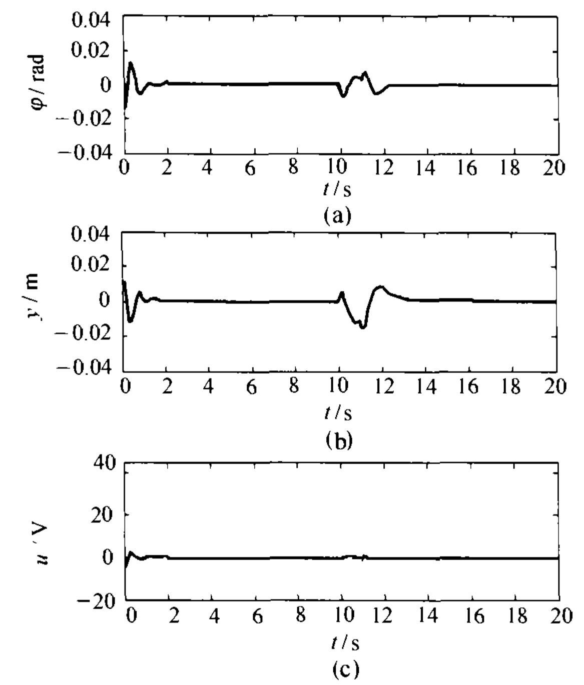

Fig. 7 Angle φ, position y and force u in robust stabilization (disturbed time-varying system)

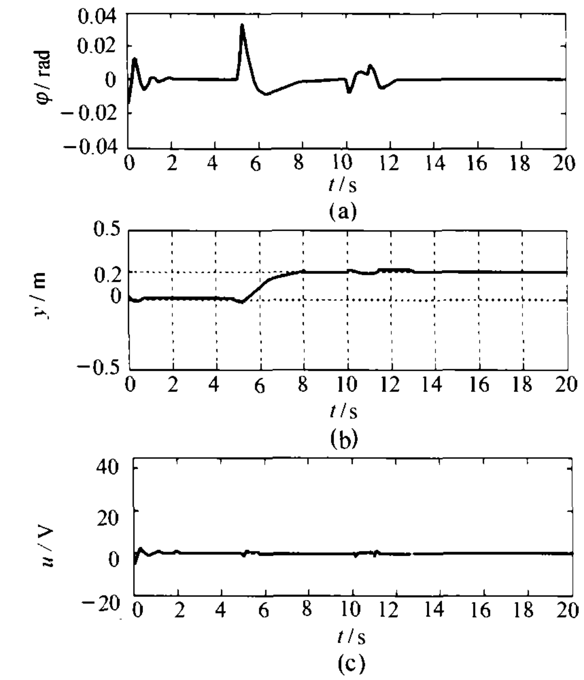

Fig. 8 Angle φ, position y and force u in robust stabilization (disturbed time-varying system)

Notice 2 The problem in section 4.3.2 goes beyond studied scope in literature [8], we aim at demonstrating the proposed adaptive HODFC that is valid and robust. In fact, the adaptive HODFC is still robust for other parameters changing and other bounded disturbance, and even for function relations changing (certainly, if change function relations, possibly the system is not a inverted pendulum system, but we can demonstrate that the controller is strong in robustness).

5 Conclusion

This paper presents the adaptive HODFC for SISO affine system, which does not rely on the model of the controlled plant, and analyzes stability and robustness of the closed-loop system. The stabilization and regulation synchronously are studied as the same problem in the inverted pendulum system. The system is successfully stabilized and regulated through converted it into double affine one and using the proposed adaptive HODFC.

In fact, in further work, we present adaptive MHODFC (multivariable HODFC), and are successfully applied in control and synchronization for chaos system, and in control for AC timing system.

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