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Passivity of interconnected control systems with time-delays based on decentralized control

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Abstract: Passivity is not only an important property of a system, but also an important approach to control a system. Based on decentralized control method, the passivity of interconnected systems with time-delays is studied. Employing Lyapunov-Krasovskii functional method and Hamiltonian function method, the decentralized and passive controllers are presented explicitly by solving some linear matrix inequalities (LMIs). The new passivity criteria and the parameters of the passive controllers presented here are delay-dependent. By using the LMI toolbox in software Matlab, the conditions in the new results are easy to be verified and the parameters of the passive controllers are easy to be well set too, when the interconnected delays are known. A numerical example illustrates the availability of the theorems of passivity and the design procedure of the passive controllers.

Key words: decentralized control; passivity; interconnected systems; time-delay; linear matrix inequality

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时滞关联大系统基于分散控制的无源性

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摘要: 无源性不仅是系统的一个重要性质,而且是控制一个系统的重要途径.本文基于分散控制方法研究了关联大系统的无源性.利用 Lyapunov-Krasovskii 泛函方法和 Hamilton 函数方法,通过求解线性矩阵不等式(LMIs)得到了分散无源控制器的显式表达.所得到的新的无源性判据和无源控制器的参数是与时滞相关的.当关联时滞已知时,这些新结果的条件易于用 Matlab 中的 LMI 工具箱进行验证,因而控制器的设计也容易实现.最后用数值例子说明了所得到定理的有效性和无源控制器的设计过程.

关键词: 分散控制; 无源性; 关联系统; 时滞; 线性矩阵不等式

1 Introduction

Passivity is one of the important properties of networks and systems. Its application to the feedback stabilization of control systems has recently gained renewed attention[1~3]. In particular, Byrnes et al[1] find out when a finite-dimensional system can be rendered to be passive via state feedback. This important result highlights the passivity-based method for various control problems[4,5].

Over the years, the decentralized control of interconnected large-scale systems has received considerable attention and has been addressed numerously[6~9]. The decentralized control is the main and effective approach to the stabilization of interconnected control systems. Also, linear matrix inequality (LMI) method has become a powerful tool to solve the problems of large-scale systems (see, for example[2,10]). Passivity-based control should be an important tool to solve the stabilization problem for interconnected control systems. To implement the passivity-based control, first of all, the systems should be passive. However, as we know, few results were concerned with the passivity of interconnected control systems with interconnected delays.

The purpose of this paper is to study the passivity of interconnected control systems with interconnected time-delays. The Lyapunov-Krasovskii functional, Hamiltonian function, decentralized control and LMI approaches are combined to solve the problem. The decentralized controllers are delay-dependent and presented by solving some LMIs.

2 Problem description

Consider an interconnected system S with time-delays composed of N coupled subsystems S_i

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$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + E_i \omega_i(t) + \sum_{j=1}^{N} A_{ij} x_j(t-\tau_{ij}), \\ z_i(t) = C_i x_i(t) + D_i \omega_i(t), \\ x_i(t) = \varphi_i(t), \\ t \in [-\tau_i, 0],\ 0 < \tau_{ij} < \infty, \\ \tau_i = \max_{j \in \{1,2,\cdots,N\}} \{\tau_{ij}\}. \end{cases} \tag{1}$$

where $i \in \{1,2,\cdots,N\}$; $x_i(t) \in \mathbb{R}^{n_i}$ denotes the state vector of S_i; $u_i(t) \in \mathbb{R}^{m_i}$ denotes the controller vector of S_i; $\omega_i(t) \in \mathbb{R}^{q_i}$ denotes the exogenous input of S_i and $z_i(t) \in \mathbb{R}^{q_i}$ denotes the output vector of the subsystem S_i, respectively. We assume that $\omega_i(t) \in L_2(0,\infty)$. The matrices A_i, B_i and E_i, with appropriate dimensions, indicate the nominal subsystem. For $i, j \in \{1,2,\cdots,N\}$, A_{ij} denote the interconnection matrices between the subsystems S_i and S_j. Constants $\tau_{ij} \geqslant 0$ are interconnected delays. The initial value $\varphi_i(t) \in \mathbb{R}^{n_i}$ of the system is a continuous function defined on $t \in [-\tau_i, 0]$. Suppose that (A_i, B_i) is completely controllable and the state of the subsystem can be observed directly.

If we design a local state feedback controller for each subsystem S_i

$$u_i(t) = K_i x_i(t), \tag{2}$$

where $K_i \in \mathbb{R}^{m_i \times n_i}$ is the feedback gain matrix, the interconnected closed-loop system is

$$\begin{cases} \dot{x}_i(t) = (A_i + B_i K_i) x_i(t) + E_i \omega_i(t) + \sum_{j=1}^{N} A_{ij} x_j(t-\tau_{ij}), \\ z_i(t) = C_i x_i(t) + D_i \omega_i(t). \end{cases} \tag{3}$$

where $i = 1,2,\cdots,N$.

The target is to design such local state feedback controllers (2), guaranteeing internal stability and strict passivity of the closed-loop system (3).

Denote Ξ and Z the sets of admissible trajectories for the exogenous input and the output, respectively. The admissible trajectory can be $z(s) = H(s)\omega(s)$.

Definition 1[5] An operator $H: \Xi \mapsto Z$ is said to be strictly passive if there exist $\varepsilon_1 \geqslant 0, \varepsilon_2 \geqslant 0, \varepsilon_1 + \varepsilon_2 > 0$, such that

$$\int_0^T \omega^T(s) z(s) ds \geqslant -\beta^2 + \varepsilon_1 \int_0^T \omega^T(s)\omega(s) ds + \varepsilon_2 \int_0^T z^T(s) z(s) ds, \tag{4}$$

for all T, where $\beta \in \mathbb{R}$ is some constant, $(\omega(s), z(s)) \in \Xi \times Z$ denotes an admissible trajectory. The operator is input-strictly passive if $\varepsilon_1 > 0$ and output-strictly passive if $\varepsilon_2 > 0$.

The closed-loop systems (3) can be viewed as an input to output system, where $\omega_i(t)$ is the input, and $z_i(t)$ is the output. Thus, we have the following definition.

Definition 2 The large-scale closed-loop system (3) is said to be strictly passive if it is internally stable and the input ω and output z satisfy the inequality (4) for all solutions with null initial conditions. The decentralized controller vector $u = (u_1, \cdots, u_N)^T$ of the large-scale control system (1) is said to be a passive controller, if the closed-loop system (3) is strictly passive. Each component of $u = (u_1, \cdots, u_N)^T$ is defined by equation (2).

For a matrix E, we define a two-valued function $\delta(\cdot)$ as

$$\delta(E) = \begin{cases} 0, & E = 0, \\ 1, & E \neq 0. \end{cases}$$

Lemma 1 Suppose x and y are vectors or matrices with suitable dimensions, then for any positive number $\alpha > 0$ and positive definite matrix Q, we have

$$2x^T y \leqslant \alpha x^T Q^{-1} x + \alpha^{-1} y^T Q y.$$

3 Conditions of passivity

We always assume that the interconnected delays are known or can be measured. So, we consider the delay-dependent conditions of passivity.

To establish our main theorem, a Lyapunov-Krasovskii functional is introduced:

$$V_d(x_t) = \sum_{i=1}^{N} \{ x_i^T(t) P_i x_i(t) + \sum_{j=1}^{N} \int_{-\tau_{ij}}^{0} [\int_{t+\theta}^{t} r_1 x_j^T(s)(A_j + B_j K_j)^T (A_j + B_j K_j) x_j(s) ds + \sum_{k=1}^{N} \int_{t+\theta-\tau_{jk}}^{t} r_2 x_k^T(s) A_{jk}^T A_{jk} x_k(s) ds + \int_{t+\theta}^{t} r_3 \omega_j^T(s) E_j^T E_j \omega_j(s) ds] d\theta \}. \tag{5}$$

where $0 < P_i = P_i^T \in \mathbb{R}^{n_i \times n_i}$, and $r_1 > 0, r_2 > 0, r_3 > 0$ are weighting factors. Now from the system (3) we obtain

$$x_j(t-\tau_{ij}) = x_j(t) - \int_{-\tau_{ij}}^{0} \dot{x}_j(t+\theta) d\theta = x_j(t) - \int_{-\tau_{ij}}^{0} (A_j + B_j K_j) x_j(t+\theta) d\theta - \int_{-\tau_{ij}}^{0} E_j \omega_j(t+\theta) d\theta - \int_{-\tau_{ij}}^{0} \sum_{k=1}^{N} A_{jk} x_k(t+\theta-\tau_{jk}) d\theta. \tag{6}$$

Hence, the closed-loop system (3) becomes

$$\begin{cases}\dot{x}_i(t)=(A_i+B_iK_i)x_i(t)+E_i\omega_i(t)+\sum\limits_{j=1}^{N}A_{ij}x_j(t)-\\ \qquad \sum\limits_{j=1}^{N}A_{ij}\int_{-\tau_{ij}}^{0}(A_j+B_jK_j)x_j(t+\theta)\mathrm{d}\theta-\\ \qquad \sum\limits_{j=1}^{N}A_{ij}\int_{-\tau_{ij}}^{0}E_j\omega_j(t+\theta)\mathrm{d}\theta-\\ \qquad \sum\limits_{j=1}^{N}A_{ij}\int_{-\tau_{ij}}^{0}\sum\limits_{k=1}^{N}A_{jk}x_k(t+\theta-\tau_{jk})\mathrm{d}\theta,\\ z_i(t)=C_ix_i(t)+D_i\omega_i(t),\ i=1,2,\cdots,N.\end{cases}\tag{7}$$

Theorem 1 For the large-scale control system (1), suppose that there exist matrix $K_i\in\mathbb{R}^{m_i\times m_i}$, positive definite matrix $P_i\in\mathbb{R}^{n_i\times n_i}$ and constants $r_1>0, r_2>0, r_3>0, \alpha>0$ satisfying $D_i+D_i^{\mathrm{T}}-r_3(\sum\limits_{j=1}^{N}\tau_{ji})E_i^{\mathrm{T}}E_i>0$ and the following matrix inequalities:

$$\Pi_i(P_i)=\begin{bmatrix}\hat{A}_i & A_{d_0}^{\mathrm{T}} & -(C_i^{\mathrm{T}}-P_iE_i)\\ A_{d_0} & -J_{d_0} & O\\ -(C_i-E_i^{\mathrm{T}}P_i) & O & -(D_i+D_i^{\mathrm{T}}-r_3(\sum\limits_{j=1}^{N}\tau_{ji})E_i^{\mathrm{T}}E_i)\end{bmatrix}\leqslant 0,$$

$$i=1,2,\cdots,N,\tag{8}$$

where

$$\hat{A}_i=P_i(A_i+B_iK_i)+(A_i+B_iK_i)^{\mathrm{T}}P_i+\sum_{j=1}^{N}\tau_{ij}(r_1^{-1}+Nr_2^{-1}+r_3^{-1})P_iA_{ij}A_{ij}^{\mathrm{T}}P_i+\sum_{j=1}^{N}(\alpha^{-1}P_jP_j+\alpha A_{ji}^{\mathrm{T}}A_{ji}),$$

$$A_{d_0}=\Big[(\sum_{k=1}^{N}\tau_{k1})A_{1i}\ \cdots\ (\sum_{k=1}^{N}\tau_{kN})A_{Ni}\ \ (\sum_{k=1}^{N}\tau_{ki})(A_i+B_iK_i)\Big]^{\mathrm{T}}$$

$$J_{d_0}=\mathrm{diag}\Big\{(\sum_{k=1}^{N}\tau_{k1})r_2^{-1}I\ \cdots\ (\sum_{k=1}^{N}\tau_{kN})r_2^{-1}I\ \ (\sum_{k=1}^{N}\tau_{ki})r_1^{-1}I\Big\},$$

meanwhile, there exists at least one strict inequality among the N inequalities (8). Then $u=(u_1,\cdots,u_N)^{\mathrm{T}}$ is the passive controller of the system (1).

Proof Differentiating $V_d(x_t)$ along with the state trajectory of the system (7), by using Lemma 1, we obtain

$$\begin{aligned}\dot{V}_d(x_t)\leqslant&\sum_{i=1}^{N}\{x_i^{\mathrm{T}}(t)(A_i^{\mathrm{T}}P_i+P_iA_i+\\&K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i+P_iB_iK_i)x_i(t)+2x_i^{\mathrm{T}}(t)P_iE_i\omega_i(t)\}+\\&\sum_{j=1}^{N}2x_i^{\mathrm{T}}(t)P_iA_{ij}x_j(t)+\\&\sum_{j=1}^{N}r_1\tau_{ij}x_j^{\mathrm{T}}(t)(A_j+B_jK_j)^{\mathrm{T}}(A_j+B_jK_j)x_j(t)+\\&\sum_{j=1}^{N}r_2\tau_{ij}\sum_{k=1}^{N}x_k^{\mathrm{T}}(t)A_{jk}^{\mathrm{T}}A_{jk}x_k(t)+\\&\sum_{j=1}^{N}r_3\tau_{ij}\omega_j^{\mathrm{T}}(t)E_j^{\mathrm{T}}E_j\omega_j(t)+\\&\sum_{j=1}^{N}r_1^{-1}\tau_{ij}x_i^{\mathrm{T}}(t)P_iA_{ij}A_{ij}^{\mathrm{T}}P_ix_i(t)+\\&\sum_{j=1}^{N}r_2^{-1}\tau_{ij}\sum_{k=1}^{N}x_i^{\mathrm{T}}(t)P_iA_{ij}A_{ij}^{\mathrm{T}}P_ix_i(t)+\\&\sum_{j=1}^{N}r_3^{-1}\tau_{ij}x_i^{\mathrm{T}}(t)P_iA_{ij}A_{ij}^{\mathrm{T}}P_ix_i(t)\leqslant\\&\sum_{i=1}^{N}\{x_i^{\mathrm{T}}(t)(A_i^{\mathrm{T}}P_i+P_iA_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i+P_iB_iK_i+\\&\sum_{j=1}^{N}\tau_{ij}(r_1^{-1}+Nr_2^{-1}+r_3^{-1})P_jA_{ij}A_{ij}^{\mathrm{T}}P_i+\\&\sum_{j=1}^{N}(\alpha^{-1}P_jP_j+\alpha A_{ji}^{\mathrm{T}}A_{ji})+\\&r_1(\sum_{j=1}^{N}\tau_{ji})(A_i+B_iK_i)^{\mathrm{T}}(A_i+B_iK_i)+\\&r_2\sum_{j=1}^{N}(\sum_{k=1}^{N}\tau_{kj})A_{ji}^{\mathrm{T}}A_{ji})x_i(t)+\\&2x_i^{\mathrm{T}}(t)P_iE_i\omega_i(t)+\sum_{j=1}^{N}r_3\tau_{ij}\omega_j^{\mathrm{T}}(t)E_j^{\mathrm{T}}E_j\omega_j(t)\}.\end{aligned}$$

When $\omega_i(t)=0$, from the inequality (8) and Schur complement, we have $\dot{V}_d(x_t)<0$. Therefore, the system (7) is internal stable. The Hamiltonian $H(x,t)$ is defined and computed as

$$H(x,t)=-\dot{V}_d(x_t)+2z^{\mathrm{T}}(t)\omega(t)\geqslant-\sum_{i=1}^{N}Y_i^{\mathrm{T}}(t)\Pi(P_i)Y_i(t),\tag{9}$$

where $Y_i(t)=[x_i^{\mathrm{T}}(t)\ \ \omega_i^{\mathrm{T}}(t)]$.

From the condition (8), it follows $-\dot{V}_d(x_t)+2z^{\mathrm{T}}(t)\omega(t)>0$. Then we obtain

$$\int_0^T z^{\mathrm{T}}(t)\omega(t)\mathrm{d}t\geqslant\frac{1}{2}V_d(x(T))\geqslant 0$$

for all the solutions with the null initial value.

On the other hand, under the conditions of the Theorem 1, there is a sufficient small constant $\alpha>0$ such that all the conditions are still satisfied if D_i is replaced by $D_i-\alpha I_i$. By similar arguments, we have

$$\int_0^T[z(t)-\alpha\omega(t)]^{\mathrm{T}}\omega(t)\mathrm{d}t\geqslant 0,$$

that is

$$\int_0^T z^{\mathrm{T}}(t)\omega(t)\mathrm{d}t\geqslant\alpha\int_0^T\omega^{\mathrm{T}}(t)\omega(t)\mathrm{d}t.$$

It concludes that the closed-loop system (7) is strictly passive. The proof is completed.

Since the matrix inequalities (8) are not linear, we can not use the LMI toolbox in Matlab directly. Next, we consider an equivalent version of the theorem by using LMI expression.

Theorem 2 For the large-scale control system (1), suppose that there exist positive definite matrices $X_i, Y_i \in \mathbb{R}^{n_i \times n_i}$, and the constants $r_1 > 0, r_2 > 0, r_3 > 0, \alpha > 0$ such that $D_i + D_i^T - r_3(\sum_{j=1}^{N} \tau_{ji}) E_i^T E_i > 0$ and satisfying the following LMIs with respect to X_i, Y_i:

$$\begin{bmatrix} \hat{A}_i & X_i A_{di}^T & Y_i^T B_i^T & -(X_i C_i^T - E_i) \\ A_{di} X_i & -I_i & 0 & 0 \\ B_i Y_i & 0 & -J_i & 0 \\ -(C_i X_i - E_i^T) & 0 & 0 & -[D_i + D_i^T - r_3(\sum_{j=1}^{N} \tau_{ji}) E_i^T E_i \end{bmatrix} \leqslant 0. \tag{10}$$

where $i = 1, 2, \cdots, N$,

$$\hat{A}_i = A_i X_i + X_i A_i^T + B_i Y_i + Y_i B_i^T + \sum_{j=1}^{N} \tau_{ij} (r_1^{-1} + N r_2^{-1} + r_3^{-1}) A_{ij} A_{ij}^T + \alpha^{-1} N I_i,$$

$$J_i = r_1^{-1} \Big(\sum_{k=1}^{N} \tau_{ki} \Big)^{-1} I_i,$$

and matrix A_{di} is a Cholesky decomposition of the non-negative-definite matrix:

$$A_{di}^T A_{di} = \alpha \sum_{j=1}^{N} A_{ji}^T A_{ji} + r_1 \Big(\sum_{k=1}^{N} \tau_{ki} \Big) A_i^T A_i + r_3 \sum_{j=1}^{N} \Big(\sum_{k=1}^{N} \tau_{kj} \Big) A_i^T A_i \geqslant 0,$$

and meanwhile, there exists at least one strict inequality among the N inequalities (10). Then $u = (u_1, \cdots, u_N)$ is the passive controller of system (1), where $K_i = Y_i X_i^{-1}$.

Proof Using Lemma 1 and let $X_i = P_i^{-1}$ and $Y_i = K_i P_i^{-1}$ to the left-hand side of the inequality (10), we obtain

$$A_i X_i + B_i Y_i + A_i^T X_i + Y_i^T B_i^T + \sum_{j=1}^{N} \tau_{ij} (r_1^{-1} + N r_2^{-1} + r_3^{-1}) A_{ij} A_{ij}^T + \sum_{j=1}^{N} (\alpha^{-1} I_i + \alpha X_i A_{ji}^T A_{ji} X_i) + r_1 \Big(\sum_{k=1}^{N} \tau_{ki} \Big) (X_i A_i^T A_i X_i + Y_i^T B_i^T B_i Y_i) + r_2 \sum_{j=1}^{N} \Big(\sum_{k=1}^{N} \tau_{kj} \Big) X_i^T A_{ji}^T A_{ji} X_i +$$

$$(E_i - X_i C_i^T)(D_i + D_i^T - r_3 (\sum_{j=1}^{N} \tau_{ji}) E_i^T E_i)^{-1} (E_i^T - C_i X_i). \tag{11}$$

From Schur complement and the condition (10) it follows that each matrix (11) is non-positive or negative definite as in the proof of Theorem 1. It gives the passive controller and every feedback gain is $K_i = Y_i X_i^{-1}$. The proof is completed.

4 An illustrative example

An example is given to show the availability of our theorems and the design process of the decentralized controller vector.

In system (1), the coefficients and delays are assumed as

$$A = \begin{pmatrix} -1 & 0.35 \\ 0.56 & -1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} -0.51 & 0 \\ 0 & -0.51 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} -1.21 & 0 \\ 0 & -1.21 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix},$$

$$E_1 = \begin{pmatrix} 0.001 & 0 \\ 0 & 0.03 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -1.7 & 0 & -0.05 \\ 0.1 & -1 & -0.021 \\ 0.016 & 0.001 & -3.8 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} -1.42 & 0 & 0 \\ 0 & -1.42 & 0 \\ 0 & 0 & -1.42 \end{pmatrix},$$

$$C_2 = \begin{pmatrix} -1.072 & 0 & 0 \\ 0 & -1.072 & 0 \\ 0 & 0 & -1.072 \end{pmatrix},$$

$$D_2 = \begin{pmatrix} 4.2 & 0 & 0 \\ 0 & 4.2 & 0 \\ 0 & 0 & 4.2 \end{pmatrix},$$

$$E_2 = \begin{pmatrix} 0.001 & -0.01 & 0 \\ -0.1 & 0.03 & 0.01 \\ 0 & 0.021 & 0.01 \end{pmatrix}.$$

The interconnection of the two subsystems are described by the interconnected matrices

$$A_{11} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 0.25 & 0.71 & 0.32 \\ -0.61 & 0.13 & 0.441 \end{pmatrix},$$

$$A_{21} = \begin{pmatrix} 0.115 & 0.231 \\ 0.22 & 0.12 \\ -0.654 & 0.0253 \end{pmatrix},$$

$$A_{22} = \begin{pmatrix} 0.135 & 0 & 0 \\ 0 & 0.135 & 0 \\ 0 & 0 & 0.135 \end{pmatrix}.$$

$$\tau_{11} = 0.001, \ \tau_{12} = 0.1462,$$

$$\tau_{21} = 0.15, \ \tau_{22} = 0.1322.$$

It is an interconnected control system composed of a 2-dimensional subsystem and a 3-dimensional subsystem.

Select the parameters $r_1 = 2.5, r_2 = 2, r_3 = 0.43$ and $\alpha = 50$. By using LMI toolbox in Matlab and in light of the Theorem 2, we obtain the solutions (positive-definite matrices)

$$X_1 = \begin{pmatrix} 0.0879 & -0.0355 \\ -0.0355 & 0.4600 \end{pmatrix},\ Y_1 = \begin{pmatrix} 1.1620 & 0.0378 \\ 0.0378 & 1.1013 \end{pmatrix},$$

$$X_2 = \begin{pmatrix} 0.1435 & -0.0588 & 0.0735 \\ -0.0588 & 0.1380 & -0.0929 \\ 0.0735 & -0.0929 & 0.1903 \end{pmatrix},$$

$$Y_2 = \begin{pmatrix} 0.9353 & 0.0151 & -0.0375 \\ 0.0151 & 0.9525 & 0.0253 \\ -0.0375 & 0.0253 & 0.8711 \end{pmatrix}.$$

Then the feedback gain matrices are

$$K_1 = \begin{pmatrix} 13.6814 & 1.1377 \\ 1.4418 & 2.5053 \end{pmatrix}$$

and

$$K_2 = \begin{pmatrix} 8.6755 & 2.1053 & -2.5223 \\ 2.2244 & 10.9626 & 4.6273 \\ -2.4046 & 4.2719 & 7.5944 \end{pmatrix}.$$

From the gain matrices, the decentralized and passive controller vector $\boldsymbol{u}$ can be obtained.

5 Conclusion

In this paper, by using decentralized control method, Lyapunov-Krasovskii functional, Hamiltonian function and LMI method, passivity property is studied for interconnected control systems with time-delays. Some sufficient conditions are proposed in terms of some appropriate LMIs to guarantee the existence of decentralized and passive controllers. Our results are easy to be applied by using the LMI toolbox in the software Matlab. Therefore, they are easy to be implemented in practice. Also, the method can be extended to the case of time variant delays or the systems with uncertain parameters.

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