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带非线性扰动的不确定多时变时滞系统H_∞鲁棒稳定性

马新军[1,3]，向少华[2]，胥布工[2]，黄德先[3]

(1. 广东省科学院 自动化工程研制中心, 广东 广州 510070; 2. 华南理工大学 自动化科学与工程学院, 广东 广州 510640;

3. 清华大学 自动化系, 北京 100084)

摘要: 研究了带非线性扰动的不确定多时变时滞系统的H_∞鲁棒稳定性. 其中不确定项是范数有界的, 非线性扰动满足线性约束. 基于Lyapunov-Krasovskii泛函, 根据矩阵不等式技术和非线性处理方法, 提出了两个新的时滞稳定性的结果. 最后给出了两个示例, 表明本文提出的方法与存在的文献结果相比, 具有较少的保守性和良好的有效性.

关键词: 鲁棒稳定性; 时滞; 不确定; 非线性扰动

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H-infinity robust stability for uncertain systems with multiple time-varying delays and nonlinear perturbations

MA Xin-jun[1,3], XIANG Shao-hua[2], XU Bu-gong[2], HUANG De-xian[3]

(1.Automation Engineering R & M Center, Guangdong Academy of Sciences, Guangzhou Guangdong 510070, China;
2.College of Automation Science & Technology, South China University of Technology,
Guangzhou Guangdong 510640, China;
3.Department of automation, Tsinghua University, Beijing 100080, China)

Abstract: The robust H-infinity stability is studied for uncertain systems with multiple time-varying delays and nonlinear perturbations. The uncertainties are norm-bounded, and the nonlinear perturbations meet linear constraints. Based on Lyapunov-Krasovskii functional, two new delay-dependent results are presented in terms of matrix inequalities technique and nonlinear dealing method. In the end, two examples are given to illustrate that the presented method is less conservative and more effective than the existing ones.

Key words: robust stability; time delay; uncertain; nonlinear perturbations

1　引言(Introduction)

对时滞不确定系统的研究已有较多的结果, 这些研究结果主要是关于稳定性和镇定的两个方面: 一方面是时滞相关的结果, 如Li[1~3], 关[4], Su[5], Yan[6], Park[7], Lee[8]；另一方面是时滞无关的结果, 如Kim[9]和Li[10,11]. 近年来时滞相关的稳定性研究吸引了许多研究者的注意, 并且得到了一些积极的结果. 由于时滞相关的稳定性结果提供了使得系统保持稳定的时滞的大小, 因而比时滞无关的稳定性结果具有较少的保守性. 由于不确定在理论和实践上有着许多不同的表现形式, 带干扰的不确定系统引起了广泛的关注, 如Guan[4]和Su[10]. Guan[4]获得了带一项时滞和非线性扰动的不确定系统的时滞相关的稳定性结果, Li[10]提出了一个多时滞不确定非线性系统的时滞无关的稳定性结果. 本文研究带非线性扰动的多时变时滞不确定系统的时滞相关鲁棒稳定性.

基于Lyapunov-Krasovskii泛函和矩阵不等式技术, 本文提出了一个非线性扰动的多时变时滞不确定系统的时滞相关H_∞鲁棒稳定性结果. 最后通过两个数值示例说明了本文的方法得到的结果的有效性和较少的保守性.

2　准备及系统描述(System description and preliminary)

考虑以下带非线性扰动的多时变时滞不确定系统

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$$\begin{cases} \dot{x}(t) = (A_0 + \Delta A_0(t))x(t) + Hw(t) + \\ \qquad \sum_{i=1}^{k} (A_i + \Delta A_i(t))x(t - \tau_i(t)) + \\ \qquad f(t, x(t), x(t-\tau_1(t)), \cdots, x(t-\tau_k(t))), \\ x(t) = \phi(t),\ \ t \in [-\max\limits_{i=1,2,\cdots,k} \bar{\tau}_i, 0], \\ y(t) = Cx(t). \end{cases} \tag{1}$$

其中: $x(t) \in \mathbb{R}^n$ 是状态向量, $u(t) \in \mathbb{R}^m$是控制向量, $w(t)$是属于$L_2[0,\infty)$的扰动输入, $y(t) \in \mathbb{R}^m$是受控输出, $\phi(t)$是初始条件; $\tau_i(t)$, $i = 0, 1, \cdots, k$, 表示时变时滞, 且满足$0 \leqslant \tau_i(t) \leqslant \bar{\tau}_i$, $\dot{\tau}_i(t) \leqslant \eta_i < 1$; A_i, C,和H, $i = 0, 1, \cdots, k$是具有适当维数的实矩阵; ΔA_i, $i = 0, 1, \cdots, k$表示时变参数不确定项, 并可表示为

$$\begin{aligned} &\Delta A_i = DF(t)E_i, i = 0, 1, \cdots, k, \\ &F(t) = \mathrm{diag}(F_0(t), F_1(t), \cdots, F_k(t)) \end{aligned} \tag{2}$$

其中: D 和E_i, $i = 0, 1, \cdots, k$ 是具有适当维数的实矩阵, 且$F_i(t) \in \mathbb{R}^{n_i \times n_i}$, $i = 0, 1, \cdots, k$是Lebesgue可测的未知实时变矩阵, 且满足

$$F_i^{\mathrm{T}}(t)F_i(t) \leqslant I. \tag{3}$$

简单起见, 设

$$f(t) := f(t, x(t), x(t-\tau_1(t)), \cdots, x(t-\tau_k(t))).$$

非线性扰动是时变的并满足

$$\begin{aligned} &f^{\mathrm{T}}(t)f(t) \leqslant \\ &a_0^2 x^{\mathrm{T}}(t)G_0^{\mathrm{T}}G_0 x(t) + \\ &\sum_{i=1}^{k} a_i^2 x^{\mathrm{T}}(t-\tau_i(t))G_i^{\mathrm{T}}G_i x(t-\tau_i(t)). \end{aligned} \tag{4}$$

其中: $a_i (i = 0, 1, \cdots, k)$是标量常数, $G_i (i = 0, 1, \cdots, k)$是具有适当维数的常实矩阵.

以下的引理是得到主要的结果必须使用的.

引理 1[8] 设$a(\cdot) \in \mathbb{R}^{n_a}, b(\cdot) \in \mathbb{R}^{n_b}$和$N(\cdot) \in \mathbb{R}^{n_a \times n_b}$定义在区间$\Omega$, 则对任意矩阵$X \in \mathbb{R}^{n_a \times n_a}, Y \in \mathbb{R}^{n_a \times n_b}$和$Z \in \mathbb{R}^{n_b \times n_b}$, 以下不等式成立:

$$\begin{aligned} &-2\int_{\Omega} a^{\mathrm{T}}(s)Nb(s) \leqslant \\ &\int_{\Omega} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y-N \\ Y^{\mathrm{T}}-N^{\mathrm{T}} & Z \end{bmatrix} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix} \mathrm{d}s. \end{aligned} \tag{5}$$

其中

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geqslant 0. \tag{6}$$

引理 2[11] 设D, F, E 和A是具有适当维数的实矩阵, 且有$FF^{\mathrm{T}} \leqslant I$, 则对任意标量$\varepsilon > 0$, 以下不等式成立:

$$DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} \leqslant \varepsilon DD^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E. \tag{7}$$

引理 3[12] 设$\Omega_0(x)$, $\Omega_1(x)$是$\mathbb{R}^n$上的任意二次型函数, 如果存在任意标量$\rho > 0$使得对任意的非零$x \in \mathbb{R}^n$有$\Omega_0(x) - \rho\Omega_1(x) < 0$成立, 则对任意满足$\Omega_1(x) \leqslant 0$的$x \in \mathbb{R}^n - \{0\}$有$\Omega_0(x) \leqslant 0$成立.

定义 对给定正常数$\gamma > 0$, 如果$w(t) = 0$时, 系统(1)是稳定的; 且在零初始条件下及$w(t) \in L_2[0,\infty) \neq 0$情况下, 有$\|y(t)\|_2 \leqslant \gamma\|w(t)\|_2$, 则称系统(1)是具有$\mathrm{H}_\infty$范数界$\gamma > 0$鲁棒稳定的.

3 主要结果(Main Results)

本节首先研究当$\Delta A_i = 0$, $i = 1, \cdots, k$时, 系统(1)的稳定性, 并得到一个时滞相关H_∞鲁棒稳定的充分条件.

定理 1 系统(1)当$\Delta A_i = 0, i = 0, 1, \cdots, k$时对任意满足$0 \leqslant \tau_i(t) \leqslant \bar{\tau}_i$, $\dot{\tau}_i(t) \leqslant \eta_i < 1$的时变时滞是具有$\mathrm{H}_\infty$范数界$\gamma > 0$鲁棒稳定的, 如果存在有$P > 0, Q_i > 0, \rho > 0, X_i, Y_i$和$Z_i$使得以下不等式成立:

$$\begin{bmatrix} \Omega & L_1 & \cdots & L_k & PH & P & A_0^{\mathrm{T}}\bar{Z} \\ L_1^{\mathrm{T}} & L_{11} & 0 & 0 & 0 & 0 & A_1^{\mathrm{T}}\bar{Z} \\ \vdots & 0 & \vdots & 0 & 0 & 0 & \vdots \\ L_k^{\mathrm{T}} & 0 & 0 & L_{kk} & 0 & 0 & A_k^{\mathrm{T}}\bar{Z} \\ H^{\mathrm{T}}P & 0 & 0 & 0 & -\gamma^2 I & 0 & H^{\mathrm{T}}\bar{Z} \\ P & 0 & 0 & 0 & 0 & -\rho & \bar{Z} \\ \bar{Z}A_0 & \bar{Z}A_1 & \cdots & \bar{Z}A_k & \bar{Z}H & \bar{Z} & -\bar{Z} \end{bmatrix} < 0, \tag{8}$$

$$\begin{bmatrix} X_i & Y_i \\ Y_i^{\mathrm{T}} & Z_i \end{bmatrix} \geqslant 0. \tag{9}$$

其中

$$\begin{aligned} &\Omega = PA_0 + A_0^{\mathrm{T}}P + C^{\mathrm{T}}C + \rho a_0^2 G_0^{\mathrm{T}}G_0 + \\ &\qquad \sum_{i=1}^{k} \left(\bar{\tau}_i X_i + Y_i + Y_i^{\mathrm{T}} + Q_i\right), \\ &L_{ii} = -(1-\eta_i)Q_i + \rho a_i^2 G_i^{\mathrm{T}}G_i, L_i = PA_i - Y_i, \\ &\bar{Z} = \sum_{i=1}^{k} \frac{\bar{\tau}_i}{1-\eta_i} Z_i,\ \ i = 1, \cdots, k. \end{aligned}$$

证 由于

$$x(t-\tau_i(t)) = x(t) - \int_{t-\tau_i(t)}^{t} \dot{x}(s)\,\mathrm{d}s, \tag{10}$$

故有

$$\begin{aligned} \dot{x}(t) = (A_0 + \sum_{i=1}^{k} A_i)x(t) + Hw(t) + \\ f(t) - \sum_{i=1}^{k} A_i \int_{t-\tau_i(t)}^{t} \dot{x}(s)\,\mathrm{d}s. \end{aligned} \tag{11}$$

选取Lyapunov泛函

$$V = V_1 + V_2 + V_3. \tag{12}$$

其中

$$V_1 = x^{\mathrm{T}}(t)\,Px(t)\,,$$

$$V_2 = \sum_{i=1}^{k}\frac{1}{1-\eta_i}\int_{-\tau_i(t)}^{0}\int_{t+r}^{t}\dot{x}^{\mathrm{T}}(s)\,Z_i\dot{x}(s)\,\mathrm{d}s\mathrm{d}r,$$

$$V_3 = \sum_{i=1}^{k}\int_{t-\tau_i(t)}^{t}x^{\mathrm{T}}(s)\,Q_ix(s)\,\mathrm{d}s.$$

因此, 沿着(11)的解可得

$$\dot{V}_1 = 2x^{\mathrm{T}}(t)P\sum_{i=1}^{k}A_ix(t)+2x^{\mathrm{T}}(t)PHw(t)\,2x^{\mathrm{T}}(t)Pf(t)- 2\sum_{i=1}^{k}x^{\mathrm{T}}(t)\,PA_i\int_{t-\tau_i(t)}^{t}\dot{x}(s)\,\mathrm{d}s. \tag{13}$$

对任意$t-\tau_i(t)\leqslant s\leqslant t$, 定义式(5)中的$a(\cdot)$, $b(\cdot)$和N为$a(s)=x(t)$, $b(s)=\dot{x}(s)$和$N_i=PA_i$, 利用引理1可得

$$-2x^{\mathrm{T}}(t)PA_i\int_{t-\tau_i(t)}^{t}\dot{x}(s)\mathrm{d}s\leqslant x^{\mathrm{T}}(t)(\bar{\tau}_iX_i+Y_i-PA_i+Y_i^{\mathrm{T}}-A_i^{\mathrm{T}}P+Q_i)x(t)+2x^{\mathrm{T}}(t)(PA_i-Y_i)x(t-\tau_i(t))+\int_{t-\tau_i(t)}^{t}\dot{x}^{\mathrm{T}}(s)Z_i\dot{x}(s)\mathrm{d}s.$$

同样有

$$\dot{V}_2\leqslant \sum_{i=1}^{k}\bar{\tau}_i/(1-\eta_i)\,\dot{x}^{\mathrm{T}}(t)\,Z_i\dot{x}(t)-\sum_{i=1}^{k}\int_{t-\tau_i(t)}^{t}\dot{x}^{\mathrm{T}}(s)Z_i\dot{x}(s)\mathrm{d}s = (A_0x(t)+\sum_{i=1}^{k}A_ix(t-\tau_i(t))+f(t)+Hw(t))^{\mathrm{T}}\times \bar{Z}(A_0x(t)+\sum_{i=1}^{k}A_ix(t-\tau_i(t))+f(t)+Hw(t))-\sum_{i=1}^{k}\int_{t-\tau_i(t)}^{t}\dot{x}^{\mathrm{T}}(s)\,Z_i\dot{x}(s)\,\mathrm{d}s, \tag{14}$$

$$\dot{V}_3\leqslant \sum_{i=1}^{k}x^{\mathrm{T}}(t)\,Q_ix(t)-\sum_{i=1}^{k}(1-\eta_i)\,x^{\mathrm{T}}(t-\tau_i(t))\,Q_ix(t-\tau_i(t))\,. \tag{15}$$

故当$w(t)=0$, 可得

$$\dot{V}=\dot{V}_1+\dot{V}_2+\dot{V}_3\leqslant\Xi_0(t)=z^{\mathrm{T}}(t)\,\Xi_0z(t)\,.$$

其中

$$z^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t)\ \ q^{\mathrm{T}}(t)\ \ f^{\mathrm{T}}(t)],$$

$$q^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t-\tau_1(t))\ \cdots\ x^{\mathrm{T}}(t-\tau_k(t))],$$

$$\Xi_0=\begin{bmatrix}G_{11} & G_{12} & G_{13}\\ G_{12}^{\mathrm{T}} & G_{22} & G_{23}\\ G_{13}^{\mathrm{T}} & G_{23}^{\mathrm{T}} & G_{33}\end{bmatrix}, \tag{16}$$

$$G_{11}=PA_0+A_0^{\mathrm{T}}P+\sum_{i=1}^{k}(\bar{\tau}_iX_i+Y_i+Y_i^{\mathrm{T}}+Q_i)+A_0^{\mathrm{T}}\bar{Z}A_0,$$

$$G_{12}=\left[PA_1-Y_1+A_0^{\mathrm{T}}\bar{Z}A_1\ \cdots\ PA_k-Y_k+A_0^{\mathrm{T}}\bar{Z}A_k\right],$$

$$G_{13}=P+A_0^{\mathrm{T}}\bar{Z},\ G_{33}=\bar{Z},\ G_{23}=\left[A_1^{\mathrm{T}}\bar{Z}_1\ \cdots\ A_k^{\mathrm{T}}\bar{Z}\right],$$

$$G_{22}=\begin{bmatrix}\bar{L}_1+A_1^{\mathrm{T}}\bar{Z}A_1 & A_1^{\mathrm{T}}\bar{Z}A_2 & \cdots & A_1^{\mathrm{T}}\bar{Z}A_k\\ A_2^{\mathrm{T}}\bar{Z}A_1 & \bar{L}_2+A_2^{\mathrm{T}}\bar{Z}A_2 & \cdots & A_1^{\mathrm{T}}\bar{Z}A_k\\ \vdots & \vdots & \vdots & \vdots\\ A_k^{\mathrm{T}}\bar{Z}A_1 & A_k^{\mathrm{T}}\bar{Z}A_2 & \cdots & \bar{L}_k+A_k^{\mathrm{T}}\bar{Z}A_k\end{bmatrix},$$

$$\bar{L}_i=-(1-\eta_i)\,Q_i,\ \ i=1,2,\cdots,k.$$

此外, 不等式(4)可以写为

$$\Xi_1(t)=z^{\mathrm{T}}(t)\cdot\mathrm{diag}(-a_0^2G_0^{\mathrm{T}}G_0,-a_1^2G_1^{\mathrm{T}}G_1,\cdots,-a_k^2G_k^{\mathrm{T}}G_k,I)\cdot z(t)\leqslant 0. \tag{17}$$

由引理3, 如果存在有$\rho>0$使得

$$\Xi_0(t)-\rho\Xi_1(t)=z^{\mathrm{T}}(t)\,\Xi z(t)<0,$$

则意味着下式成立:

$$\Xi=\begin{bmatrix}\bar{G}_{11} & G_{12} & G_{13}\\ G_{12}^{\mathrm{T}} & \bar{G}_{22} & G_{23}\\ G_{13}^{\mathrm{T}} & G_{23}^{\mathrm{T}} & \bar{G}_{33}\end{bmatrix}<0, \tag{18}$$

$$\begin{bmatrix}X_i & Y_i\\ Y_i^{\mathrm{T}} & Z_i\end{bmatrix}\geqslant 0,\ \ i=1,2,\cdots,k. \tag{19}$$

其中

$$\bar{G}_{11}=G_{11}+\rho a_0^2G_0^{\mathrm{T}}G_0,\ \ \bar{G}_{33}=\bar{Z}-\rho I,$$

$$\bar{G}_{22}=\begin{bmatrix}\bar{L}_1+A_1^{\mathrm{T}}\bar{Z}A_1 & A_1^{\mathrm{T}}\bar{Z}A_2 & \cdots & A_1^{\mathrm{T}}\bar{Z}A_k\\ A_2^{\mathrm{T}}\bar{Z}A_1 & \bar{L}_2+A_2^{\mathrm{T}}\bar{Z}A_2 & \cdots & A_1^{\mathrm{T}}\bar{Z}A_k\\ \vdots & \vdots & \vdots & \vdots\\ A_k^{\mathrm{T}}\bar{Z}A_1 & A_k^{\mathrm{T}}\bar{Z}A_2 & \cdots & \bar{L}_k+A_k^{\mathrm{T}}\bar{Z}A_k\end{bmatrix},$$

$$\bar{L}_i=-(1-\eta_i)\,Q_i+\rho a_i^2G_i^{\mathrm{T}}G_i,\ i=1,2,\cdots,k.$$

故有对任意满足式(17)的$z\in\mathbb{R}^{3n}-0$,

$$\dot{V}\leqslant\Xi_0(t)<0$$

成立. 根据Lyapunov-Krasovskii稳定性定理, 式(18)和(19)意味着系统(1)当$\Delta A_i=0$ $(i=01,\cdots,k)$和$w(t)=0$时, 是渐进稳定的.

为证明$\|y(t)\|_2\leqslant\gamma\|w(t)\|_2$, 对任意非零$w\in L_2[0,\infty)$, 假定零初始条件和$V(t,x(t))\,|_{t=0}=0$,

$$\begin{aligned}J_{yw}=&\int_0^{\infty}\left(y^{\mathrm T}(t)\,y(t)-\gamma^2w^{\mathrm T}(t)\,w(t)\right)\mathrm dt\leqslant\\&\int_0^{\infty}\left(y^{\mathrm T}(t)\,y(t)-\gamma^2w^{\mathrm T}(t)\,w(t)+\dot V\right)\mathrm dt+\\&V(t,x(t))\,|_{t=0}-V(t,x(t))\,|_{t=\infty}=\\&\varPi_0(t)=z^{\mathrm T}(t)\,\varPi_0z(t)\,.\end{aligned}\tag{20}$$

其中

$$z^{\mathrm T}(t)=[x^{\mathrm T}(t)\ \ q^{\mathrm T}(t)\ \ f^{\mathrm T}(t)\ \ w^{\mathrm T}(t)]$$
$$q^{\mathrm T}(t)=[x^{\mathrm T}(t-\tau_1(t))\ \ \cdots\ \ x^{\mathrm T}(t-\tau_k(t))]$$

和

$$\varPi_0=\begin{pmatrix}U_{11}&G_{12}&G_{13}&G_{14}\\G_{12}^{\mathrm T}&G_{22}&G_{23}&G_{24}\\G_{13}^{\mathrm T}&G_{23}^{\mathrm T}&G_{33}&G_{34}\\G_{14}^{\mathrm T}&G_{24}^{\mathrm T}&G_{34}^{\mathrm T}&G_{44}\end{pmatrix},$$
$$U_{11}=C^{\mathrm T}C+G_{11},\ \ G_{14}=A_0^{\mathrm T}\bar ZH,$$
$$G_{24}=\left[H^{\mathrm T}\bar ZA_1\ \cdots\ H^{\mathrm T}\bar ZA_k\right]^{\mathrm T},$$
$$G_{34}=H^{\mathrm T}\bar Z,\ \ G_{44}=H^{\mathrm T}\bar ZH-\gamma^2I.$$

不等式(4)可写为

$$\begin{aligned}\varPi_1(t)=z^{\mathrm T}(t)\mathrm{diag}(&-a_0^2G_0^{\mathrm T}G_0,-a_1^2G_1^{\mathrm T}G_1,\cdots,\\&-a_k^2G_k^{\mathrm T}G_k,I,0)z(t)\leqslant 0.\end{aligned}\tag{21}$$

由引理3, 若存在有实数$\rho>0$使得$\varPi_0(t)-\rho\varPi_1(t)=z^{\mathrm T}(t)\,\varPi z(t)<0$, 则意味着下式成立:

$$\varPi=\begin{pmatrix}\bar U_{11}&G_{12}&G_{13}&G_{14}\\G_{12}^{\mathrm T}&\bar G_{22}&G_{23}&G_{24}\\G_{13}^{\mathrm T}&G_{23}^{\mathrm T}&\bar G_{33}&G_{34}\\G_{14}^{\mathrm T}&G_{24}^{\mathrm T}&G_{34}^{\mathrm T}&G_{44}\end{pmatrix}<0,\tag{22}$$

$$\begin{bmatrix}X_i&Y_i\\Y_i^{\mathrm T}&Z_i\end{bmatrix}\geqslant 0,\ \ i=1,2,\cdots,k.\tag{23}$$

其中 $\bar U_{11}=U_{11}+\rho a_0^2G_0^{\mathrm T}G_0$,
则对任意满足式(21)的$z\in\mathbb R^{4n}-\{0\}$有$J_{yw}=\varPi_0(t)<0$成立. 利用Schur补[13], 可得如果式(8)和(9)成立, 系统(1)当$\Delta A_i=0\ (i=0,1,\cdots,k)$时是具有$\mathrm H_\infty$范数界$\gamma>0$鲁棒稳定的.

以下研究具有不确定性的系统(1)的鲁棒稳定性, 并得到一个时滞相关具有范数界$\gamma>0$的$\mathrm H_\infty$鲁棒稳定的判据.

定理 2 系统(1)对任意满足$0\leqslant\tau_i(t)\leqslant\bar\tau_i$, $\dot\tau_i(t)\leqslant\eta_i<1$的时变时滞是具有范数界$\gamma>0$的$\mathrm H_\infty$鲁棒稳定的, 如果存在有$\Delta=\mathrm{diag}(\varepsilon_0I_{n0},\varepsilon_1I_{n1},\cdots,\varepsilon_kI_{nk})>0$及$P>0,Q_i>0,\rho>0,X_i,Y_i$和$Z_i$使得以下不等式成立:

$$\begin{bmatrix}\varOmega&L_1&\cdots&L_k&PH&P&A_0^{\mathrm T}\bar Z&PD&E_0^{\mathrm T}\Delta\\L_1^{\mathrm T}&L_{11}&0&0&0&0&A_1^{\mathrm T}\bar Z&0&E_1^{\mathrm T}\Delta\\\vdots&0&\ddots&0&0&0&\vdots&0&\vdots\\L_k^{\mathrm T}&0&0&L_{kk}&0&0&A_k^{\mathrm T}\bar Z&0&E_k^{\mathrm T}\Delta\\H^{\mathrm T}P&0&0&0&-\gamma^2I&0&H^{\mathrm T}\bar Z&0&0\\P&0&0&0&0&-\rho&\bar Z&0&0\\\bar ZA_0&\bar ZA_1&\cdots&\bar ZA_k&\bar ZH&\bar Z&-\bar Z&\bar ZD&0\\D^{\mathrm T}P&0&0&0&0&0&D^{\mathrm T}\bar Z&-\Delta&0\\\Delta E_0&\Delta E_1&\cdots&\Delta E_k&0&0&0&0&-\Delta\end{bmatrix}<0,\tag{24}$$

$$\begin{bmatrix}X_i&Y_i\\Y_i^{\mathrm T}&Z_i\end{bmatrix}\geqslant 0.\tag{25}$$

其中

$$\varOmega=PA_0+A_0^{\mathrm T}P+\rho a_0^2G_0^{\mathrm T}G_0+C^{\mathrm T}C+\sum_{i=1}^{k}\left(\bar\tau_iX_i+Y_i+Y_i^{\mathrm T}+Q_i\right),$$
$$L_{ii}=-(1-\eta_i)\,Q_i+\rho a_i^2G_i^{\mathrm T}G_i,L_i=PA_i-Y_i,$$
$$\bar Z=\sum_{i=1}^{k}\frac{\bar\tau_i}{1-\eta_i}Z_i,\ i=1,\cdots,k.$$

证 用J表示式(8)左侧的矩阵, 并用A_i+DFE_i取代式(8)$A_i,i=0,1,\cdots,k$, 则式(8)可写为

$$J+\bar DF\bar E+\bar E^{\mathrm T}F^{\mathrm T}\bar D^{\mathrm T}<0.\tag{26}$$

其中

$$\bar D=\left[D^{\mathrm T}P\ 0\ \cdots\ 0\ \ D^{\mathrm T}\bar Z\right]^{\mathrm T},$$
$$\bar E=\left[E_0\ E_1\ \cdots\ E_k\ \ 0\ 0\ 0\right].$$

根据引理2和式(3), 若存在有矩阵

$$\varDelta=\mathrm{diag}(\varepsilon_0I_{n0},\varepsilon_1I_{n1},\cdots,\varepsilon_kI_{nk})>0\tag{27}$$

满足

$$J+\bar D\varDelta^{-1}\bar D^{\mathrm T}+\bar E^{\mathrm T}\varDelta\bar E<0,\tag{28}$$

对式(28)使用Schur补[13], 立刻可得式(24).

4 示例及仿真(Examples and simulation)

以下给出了带非线性项和不带非线性项的不确定时变时滞系统的示例.

例 1 考虑以下带非线性扰动的不确定时变时滞系统

$$\begin{cases}A_0=\begin{bmatrix}0&0\\0&-1\end{bmatrix},A_1=\begin{bmatrix}-2&-0.5\\0&-1\end{bmatrix},H=\begin{bmatrix}1\\1\end{bmatrix},\\G_0=\begin{bmatrix}1&0\\0&1\end{bmatrix},G_1=\begin{bmatrix}1&0\\0&1\end{bmatrix},E_1=\begin{bmatrix}0&0&1&0\\0&0&0&1\end{bmatrix}^{\mathrm T},\\D=\begin{bmatrix}0.2&0&0.2&0\\0&0.2&0&0.2\end{bmatrix},E_0=\begin{bmatrix}1&0&0&0\\0&1&0&0\end{bmatrix}^{\mathrm T},\\C=\begin{bmatrix}0&1\end{bmatrix},\ a_0=0.2,\ a_1=0.1.\end{cases}\tag{29}$$

应用定理2, 可知当$\gamma = 1.95$时, 对任意时变时滞$0 \leqslant \tau_1(t) \leqslant \bar{\tau}_1 = 2.4999, \dot{\tau}_1(t) = \eta_1 = 0$, 系统(29)是渐进稳定的; 当$\gamma = 1.95$时, 对任意时变时滞$0 \leqslant \tau_1(t) \leqslant \bar{\tau}_1 = 1.7498, \dot{\tau}_1(t) \leqslant \eta_1 = 0.3$, 系统(29)是渐进稳定的.图1为系统(29)在$\bar{\tau} = 2.45, x_0 = [1.4, -1.8]$时的状态输出.

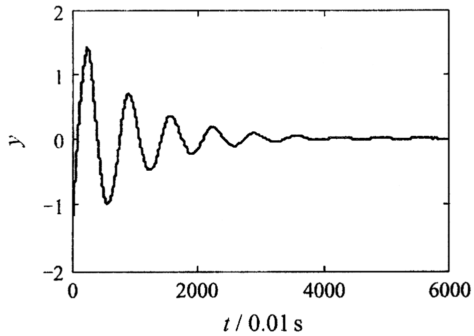

图 1 系统输出y

Fig. 1 System output y

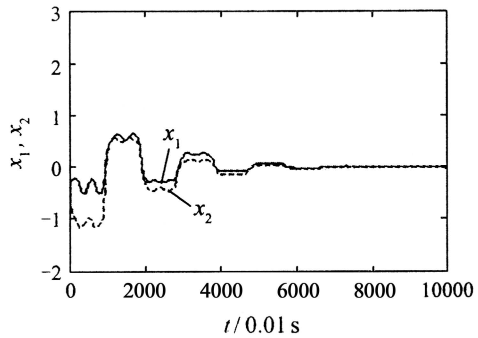

图 2 系统状态

Fig. 2 System state

例 2 当$f(\cdot) = 0$时, 考虑以下不确定时滞系统

$$\begin{cases} A_0 = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 0 \\ -0.8 & -1 \end{bmatrix}, \\ F = \begin{bmatrix} 0.2 & 0 & 0.2 & 0 \\ 0 & 0.2 & 0 & 0.2 \end{bmatrix}, \\ E_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, E_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \|\Delta A_0\| \leqslant 0.2, \ \|\Delta A_1\| \leqslant 0.2, \ \forall t. \end{cases} \tag{30}$$

文献中存在的时滞相关的判据提供了保证系统鲁棒稳定的最大的时滞界$\bar{\tau}$, 如$\bar{\tau} = 3.0369$[5]和$\bar{\tau} = 0.4428$[1]. 而利用本文定理2得到的最大时滞界为$\bar{\tau} = \infty$. 这意味着本文得稳定性判据表明系统(30)实际上是时滞无关稳定的. 图2为系统(30)在$\bar{\tau} = 14, x_0 = [2.4 \ \ -1.7]$时的状态轨迹.

注 本文采用了非线性处理技巧和PARK P[7]不等式的变化形式, 即LEE Y S等[8]中的不等式引理.而应用文献[7]中PARK P的不等式避免了$-2a^{\mathrm{T}}b \leqslant a^{\mathrm{T}}G^{-1}a + b^{\mathrm{T}}Gb, G > 0$中$-2a^{\mathrm{T}}b \leqslant 0$的情形[7].因而得到了较好的有效性和较少的保守性的结果.

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作者简介:

马新军 (1972—), 男, 博士, 助理研究员, 研究兴趣为鲁棒控制, 数字图像处理及模式识别以及网络化控制的理论和应用等, E-mail: aumaxj@126.com;

向少华 (1975—), 男, 博士, 研究领域为网络化控制理论与应用等, E-mail: xsh880@163.com;

胥布工 (1956—) , 男, 教授, 博士生导师, 研究领域为时滞动态系统稳定性、鲁棒控制、网络化控制理论与应用等, E-mail: aubgxu@scut.edu.cn;

黄德先 (1958—), 男, 研究员, 研究领域为生产过程建模、控制与优化等, E-mail: huangdx@mail.tsinghua.edu.cn.