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四阶时变离散系统的一致渐近稳定性

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摘要: 特征建模的方法为智能控制器设计和一些高阶对象进行低阶控制器设计提供了理论依据. 对于速度跟踪、加速度控制, 基于特征模型设计的自适应控制方案其稳定性问题即为四阶时变离散系统的稳定性问题. 首先给出了变系数二次三项式的系数满足一组较复杂的差分关系式时, 其判别式的简洁表达式, 在此基础上, 利用Lyapunov直接方法定量地给出了四阶线性时变离散系统的一致渐近稳定性判据. 获得的结果仅依赖于系统方程系数变化范围, 从而简化了现有的依赖于系统方程系数函数变化范围的结论.

关键词: 时变离散系统; 渐近稳定性; 特征建模

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Uniformly asymptotic stability of the 4th-order time-varying discrete systems

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Abstract: The principle of characteristic modelling provides a theoretical basis for the design of intelligent controllers and the control of higher-order plants by using lower controllers. For the control of speed tracking or acceleration, the stability of the adaptive control systems based on characteristic modelling is exactly the stability of the 4th-order time-varying discrete systems. The brief expressions of discriminants for the varying coefficient quadratic trinomials are deduced when their coefficients satisfy complicated system of difference equations. Based on the results, the uniformly asymptotic stability criteria of the 4th-order linear time-varying discrete systems are also quantitatively given by employing the Lyapunov's direct method. The obtained results are only related to the variation ranges of coefficients in the system equation, thus simplifying the existing results in their dependence on the variation ranges of some coefficient functions.

Key words: time-varying discrete system; asymptotic stability; characteristic modelling

1 引言(Introduction)

目前, 对于线性时不变离散系统的渐近稳定性研究已比较清楚, 然而, 正如文献[1]所述: 对线性时变系统希望从最一般的形式出发得到有关稳定性的判据, 这种想法至今仍收效甚微. 关于这方面的研究可分为3个方面: 1) 定性研究. 文献[2]研究了慢时变系统的稳定性问题; 文献[3]只是定性地研究了线性时变离散系统的稳定性, 因为该文中用到的上限 $\tilde{u}_i(\tilde{u}_i < 1,\ i = 1, 2, \cdots, n)$只是预先假设存在的值; 2) 当时变离散系统进入稳态, 该系统为定常系统时, 文献[4]给出了稳定性判据; 3) 定量研究. 只研究了阶数较低的情形. 文献[5]用类比法构造Lyapunov函数, 给出了二阶线性时变离散系统渐近稳定的广义Jury判据, 但至今尚未推广到三阶以上的线性时变离散系统; 文献[6]用另外一种方法给出了二阶线性时变离散系统的一致渐近稳定性判据, 所给判据是用系数的有理分式及其变化率来表达的, 没有直接体现稳定性与系数本身的关系, 且迄今尚未推广到三阶以上线性时变离散系统. 新近, 文献[7]只是定量地给出了三阶时变离散系统的一致渐近稳定性判据, 因此, 定量地给出四阶以上时变离散系统的稳定性判据仍是未解决的问题.

鉴于对时变线性系统的研究表明, 在离开系统的一些特性作一般性研究, 目前仍是不现实的[1], 本文只定量地给出4阶时变离散系统的一致渐近稳定性判据. 研究这一问题具有一定的实际意义: 对于

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阶数及参数未知的高阶线性系统, 如何设计一个工程上易于实现的低阶控制器, 以达到高性能控制要求, 这是目前航天等领域急需解决的问题. 文献[8]提出的特征建模方法突破了原有对被控对象建模的框架, 为解决这些问题提供了一种有效途径.

所谓特征建模[8], 就是结合被控对象的动力学特征和控制性能要求进行建模, 一般用时变差分方程来描述. 对于速度跟踪与加速度控制, 基于特征模型设计的自适应控制方案其稳定性问题即为四阶时变离散系统的稳定性问题, 本文定量地研究当四阶时变离散系统的系数满足什么条件时, 系统是一致渐近稳定的, 从而为基于特征建模设计适当的控制律以满足稳定性的要求打下理论基础. 所给判据不仅是对文献[6]所给判据的推广也是其简化; 其难点之一是需要证明所给系数变化范围的合理性(详见文中对式(30)的证明), 而文献[7]讨论三阶时变离散系统的一致渐近稳定性时无须涉及此问题, 因此本文所给结果并非是文献[7]所给结论的直接推广. 由本文定理1的证明可知, 若将所给结论推广到五阶以上时变离散系统, 均需要证明所给系数变化范围的合理性, 因而所给方法为研究五阶以上时变离散系统的一致渐近稳定性打下了基础.

2 问题的描述和主要结果 (Probelm formulation and main results)

对于线性定常高阶对象, 欲实现速度跟踪或加速度控制时, 基于特征模型设计线性黄金分割控制方案所得到的闭环系统具有如下形式:

$$y(k+1)+f_1(k)y(k)+f_2(k)y(k-1)+f_3(k)y(k-2)+f_4(k)y(k-3)=0. \tag{1}$$

为讨论系统(1)的一致渐近稳定性, 从文献[9]引入

引理 1 设$B(k)=(b_{ij}(k))$为对称矩阵, 且

1) $B(k)$的每一元素$b_{ij}(k)$ 和每一主子式均为$f_i(k)$的连续函数或常数;

2) $f_i(k)$ $(i=1,2,\cdots,N)$ 属于有限闭区间, 其中N 为有限正数;

3) $B(k)$的每一主子式均大于某个正数.

那么, $B(k)$ 是一致有界、正定的矩阵.

引理 2 设$B(k+1)$, $C(k+1)$, $D(k)$, $E_0(k)$, $E_1(k)$, $F(k)$, $M(k)\in\mathbb{R}$, 且$0<M(k)\leqslant M$, M为正常数,

$$E_0(k)=M(k)B^2(k+1)+E_1(k)B(k+1)-M(k)C^2(k+1)+F(k)C(k+1)+D(k), \tag{2}$$

$$E_1(k)=-2M(k)B(k+1)-2M(k)C(k+1)+F(k), \tag{3}$$

则关于$Z(k)$的二次三项式

$$-M(k)Z^2(k)+E_1(k)Z(k)+E_0(k)$$

的判别式$\Delta(k)=F^2(k)+4M(k)D(k)$, 且当

$$\begin{cases}\Delta(k)>0,\\ \dfrac{E_1(k)-\sqrt{\Delta(k)}}{2M(k)}<Z(k)<\dfrac{E_1(k)+\sqrt{\Delta(k)}}{2M(k)}\end{cases} \tag{4}$$

时, 该二次三项式大于0; 当

$$\begin{cases}\Delta(k)>\varepsilon,\\ \dfrac{E_1(k)-\sqrt{\Delta(k)-\varepsilon}}{2M(k)}<Z(k)<\dfrac{E_1(k)+\sqrt{\Delta(k)-\varepsilon}}{2M(k)}\end{cases} \tag{5}$$

时, 该二次三项式大于$\varepsilon/4M$, 这里ε为正常数.

证 经计算知, 所给二次三项式的判别式为

$$\Delta(k)=E_1^2(k)+4M(k)E_0(k)=F^2(k)+4M(k)D(k). \tag{6}$$

所给二次三项式的根

$$Z_{1,2}(k)=\frac{-E_1(k)\pm\sqrt{\Delta(k)}}{2(-M(k))}=\frac{E_1(k)\mp\sqrt{\Delta(k)}}{2M(k)},$$

故式(4)成立时该二次三项式大于0.

取$\delta(k)=\dfrac{\sqrt{\Delta(k)}-\sqrt{\Delta(k)-\varepsilon}}{2M(k)}$, 则

$$\frac{E_1(k)-\sqrt{\Delta(k)-\varepsilon}}{2M(k)}=Z_1(k)+\delta(k),$$

$$\frac{E_1(k)+\sqrt{\Delta(k)-\varepsilon}}{2M(k)}=Z_2(k)-\delta(k).$$

易知在$Z_1(k)+\delta(k)$, $Z_2(k)-\delta(k)$处二次三项式的值为$\varepsilon/4M(k)\geqslant\varepsilon/4M$, 故式(5)成立时结论成立.

令$\boldsymbol{Y}(k)=(y(k-2),y(k-1),y(k),y(k+1))^{\mathrm{T}}$, 则系统(1)可写为

$$\boldsymbol{Y}(k+1)=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\-f_4(t)&-f_3(t)&-f_2(t)&-f_1(t)\end{bmatrix}\boldsymbol{Y}(k)=A(k+1)\boldsymbol{Y}(k), \tag{7}$$

式中$t=k+1$.

构造对称矩阵$P(k)=(p_{ij}(k))_{4\times4}$, 其中

$$p_{ii}(k)=p_{ii},\ i=1,2,3,4. \tag{8}$$

p_{11}, p_{22}, p_{33}和p_{44} 是正常数, 且$0<p_{11}<p_{22}<$

$p_{33} < p_{44}$.

$$\begin{cases} p_{12}(k) = p_{21}(k) = \varepsilon_1^2 f_1^2(k), \\ p_{13}(k) = p_{31}(k) = \varepsilon_2^2 f_2^2(k), \\ p_{23}(k) = p_{32}(k) = \varepsilon_4 f_4(k), \\ p_{i4}(k) = p_{4i}(k) = \varepsilon_{4-i} f_{4-i}(k),\ i = 1, 2, 3. \end{cases} \tag{9}$$

这里：$\varepsilon_1, \varepsilon_2, \varepsilon_3$ 和ε_4 也是正常数, 而且$0 < \varepsilon_i \leqslant \dfrac{p_{11}}{M_i\sqrt{7}} (i = 1, 2, 3, 4)$, M_i为$f_i(k)$的界, 即$|f_i(k)| \leqslant M_i$.

首先注意, 对于如上给定的对称矩阵$P(k)$,

$$(Q_{ij}(k))_{4\times 4} \triangleq Q(k) = \\ -A(k+1)^{\mathrm{T}} P(k+1) A(k+1) + P(k). \tag{10}$$

其中：

$$Q_{11}(k) = p_{11} - f_4^2(k+1) p_{44}, \tag{11}$$

$$Q_{22}(k) = \\ p_{22} - p_{11} + 2f_3(k+1) p_{14}(k+1) - f_3^2(k+1) p_{44}, \tag{12}$$

$$Q_{33}(k) = \\ p_{33} - p_{22} + 2f_2(k+1) p_{24}(k+1) - f_2^2(k+1) p_{44}, \tag{13}$$

$$Q_{44}(k) = \\ p_{44} - p_{33} + 2f_1(k+1) p_{34}(k+1) - f_1^2(k+1) p_{44}, \tag{14}$$

$$Q_{12}(k) = Q_{21}(k) = \\ p_{12}(k) + f_4(k+1) p_{14}(k+1) - f_3(k+1) f_4(k+1) p_{44}, \tag{15}$$

$$Q_{13}(k) = Q_{31}(k) = \\ p_{13}(k) + f_4(k+1) p_{24}(k+1) - f_2(k+1) f_4(k+1) p_{44}, \tag{16}$$

$$Q_{14}(k) = Q_{41}(k) = \\ p_{14}(k) + f_4(k+1) p_{34}(k+1) - f_1(k+1) f_4(k+1) p_{44}, \tag{17}$$

$$Q_{23}(k) = Q_{32}(k) = \\ p_{23}(k) - p_{12}(k+1) + f_3(k+1) p_{24}(k+1) + \\ f_2(k+1) p_{14}(k+1) - f_2(k+1) f_3(k+1) p_{44}, \tag{18}$$

$$Q_{24}(k) = Q_{42}(k) = \\ p_{24}(k) - p_{13}(k+1) + f_3(k+1) p_{34}(k+1) + \\ f_1(k+1) p_{14}(k+1) - f_1(k+1) f_3(k+1) p_{44}, \tag{19}$$

$$Q_{34}(k) = Q_{43}(k) = \\ p_{34}(k) - p_{23}(k+1) + f_2(k+1) p_{34}(k+1) + \\ f_1(k+1) p_{24}(k+1) - f_1(k+1) f_2(k+1) p_{44}. \tag{20}$$

本文在给出稳定性定理之前, 首先转述如下的数学符号: 对于给定的矩阵$D(k) \triangleq (d_{ij}(k))_{n\times n}$, 用$D\binom{i_1, i_2, \cdots, i_s}{j_1, j_2, \cdots, j_s}(k)$表示由矩阵$D(k)$的第$i_1, i_2, \cdots, i_s$行与第$j_1, j_2, \cdots, j_s$列交叉处的元素构成的子式, $D(i_1, i_2, \cdots, i_s)(k)$表示由矩阵$D(k)$的第$i_1, i_2, \cdots, i_s$行与第$i_1, i_2, \cdots, i_s$列交叉处的元素构成的子式, 这里$1 \leqslant i_1 < i_2 < \cdots < i_s \leqslant n$; $1 \leqslant j_1 < j_2 < \cdots < j_s \leqslant n$. 记

$$\Delta f_i(k) = f_i(k+1) - f_i(k)\ (i = 1, 2, 3, 4), \\ \Delta f_i^2(k) = f_i^2(k+1) - f_i^2(k)\ (i = 1, 2).$$

引理 3　1) 若时变离散系统(1)的系数$f_1(k)$, $f_2(k)$, $f_3(k)$和$f_4(k)$分别满足

$$|f_i(k)| < \sqrt{\frac{p_{5-i,5-i} - p_{4-i,4-i} - c_i}{p_{44}}} \triangleq M_i, \\ i = 1, 2, 3, 4,\ p_{00} \triangleq 0. \tag{21}$$

其中p_{11}, p_{22}, p_{33}, p_{44}和c_i 均为正常数, 且 $0 < p_{11} < p_{22} < p_{33} < p_{44}$, $c_i < p_{5-i,5-i} - p_{4-i,4-i}$, 则对于由式(10)确定的对称阵$Q(k)$有

$$c_1 < Q_{11}(k) \leqslant p_{11}, \\ c_j < Q_{jj}(k) \leqslant p_{5-j,5-j} - p_{4-j,4-j} + 2\varepsilon_{5-j} M_{5-j}^2 \triangleq Q_j, \\ j = 2, 3, 4.$$

2) 若时变离散系统(1)的系数$f_1(k)$, $f_2(k)$, $f_3(k)$和$f_4(k)$除满足式(21)外, 还满足

$$\frac{N_{1,j-1}(k)}{2} - \sqrt{Q_{11}(k) Q_{jj}(k) - \delta_{1j}} < \\ -\varepsilon_{j-1}^2 \Delta f_{j-1}^2(k) < \\ \frac{N_{1,j-1}(k)}{2} + \sqrt{Q_{11}(k) Q_{jj}(k) - \delta_{1j}},\ j = 2, 3, \tag{22}$$

$$\frac{R_1(k)}{2} - \sqrt{Q_{22}(k) Q_{33}(k) - \delta_{23}} < -\varepsilon_4 \Delta f_4(k) < \\ \frac{R_1(k)}{2} + \sqrt{Q_{22}(k) Q_{33}(k) - \delta_{23}}, \tag{23}$$

其中$\delta_{1j} < c_1 c_j$, $\delta_{23} < c_2 c_3$为正常数.

$$N_{11}(k) = -2p_{12}(k+1) - 2[f_4(k+1) p_{14}(k+1) - \\ f_3(k+1) f_4(k+1) p_{44}], \tag{24}$$

$$N_{12}(k) = -2p_{13}(k+1) - 2[f_4(k+1) p_{24}(k+1) - \\ f_2(k+1) f_4(k+1) p_{44}], \tag{25}$$

$$R_1(k) = \\ -2[p_{23}(k+1) - p_{12}(k+1) + f_3(k+1) p_{24}(k+1) + \\ f_2(k+1) p_{14}(k+1) - f_2(k+1) f_3(k+1) p_{44}], \tag{26}$$

则由式(10)确定的对称阵$Q(k)$的二阶子式

$$\delta_{ij} < Q(ij)(k) \leqslant Q_{ii}(k) Q_{jj}(k) \leqslant Q_i Q_j, \\ i, j = 1, 2, 3,\ i < j.$$

证 1) 因为$|f_4(k)|<\sqrt{\frac{p_{11}-c_1}{p_{44}}}$, 所以$f_4^2(k)<\frac{p_{11}-c_1}{p_{44}}$, 故 $Q_{11}(k)=p_{11}-f_4^2(k+1)p_{44}>c_1$. 且$Q_{11}(k)\leqslant p_{11}$. 对于$j=2,3,4$, 由式(12)~(14)知

$$Q_{jj}(k)=$$
$$p_{jj}-p_{j-1,j-1}+2\varepsilon_{5-j}f_{5-j}^2(k+1)-f_{5-j}^2(k+1)p_{44},$$

从而

$$p_{jj}-p_{j-1,j-1}-f_{5-j}^2(k+1)p_{44}\leqslant$$
$$Q_{jj}(k)\leqslant p_{jj}-p_{j-1,j-1}+2\varepsilon_{5-j}M_{5-j}^2.$$

由式(21)的后三个式子知$Q_{jj}(k)>c_j$.

2) 下面证明第二条结论. $i=1$时, 则$j=2$或3. 于是

$$Q(ij)(k)=Q(1j)(k)=Q_{11}(k)Q_{jj}(k)-Q_{1j}^2(k)=$$
$$-[p_{1j}(k)-p_{1j}(k+1)]^2+$$
$$N_{1,j-1}(k)[p_{1j}(k)-p_{1j}(k+1)]+N_{0,j-1}(k).$$

此为关于$p_{1j}(k)-p_{1j}(k+1)$的二次三项式, 式中的$N_{1,j-1}(k)$由式(24)和(25)给出, $N_{0,j-1}(k)$, $N_{1,j-1}(k)$可分别表为式(2)(3)的形式, 其中: $M(k)=1$, $B(k+1)=p_{1j}(k+1)$, $D(k)=Q_{11}(k)Q_{jj}(k)$, $F(k)=0$, $C(k+1)=f_4(k+1)p_{j-1,4}(k+1)-f_{5-j}(k+1)f_4(k+1)p_{44}$, 由引理2知, 此二次三项式的判别式为

$$F^2(k)+4M(k)D(k)=4Q_{11}(k)Q_{jj}(k).$$

已证$Q_{ii}(k)>c_i$, 于是$F^2(k)+4M(k)D(k)>4c_1c_i$. 取$\varepsilon=4\delta_{1j}$, 由引理2知, 式(22)成立时, $Q(ij)(k)>\delta_{1j}$.

$i=2$时, 则$j=3$, 仿上可证式(23)成立时, $Q(23)(k)>\delta_{23}$.

由上一条结论和$Q(ij)(k)$表达式知$Q(ij)(k)\leqslant Q_{ii}(k)Q_{jj}(k)\leqslant Q_iQ_j$.

定理 1 时变离散系统(1)一致渐近稳定的充分条件为

1) $f_1(k),f_2(k),f_3(k)$和$f_4(k)$满足式(21), 且$0<p_{11}\leqslant 1$;

2) $f_1(k),f_2(k),f_3(k)$, $f_4(k)$, $f_1^2(k)$和$f_2^2(k)$的改变量分别满足

$$B_1(k)<-\varepsilon_1\Delta f_1(k)<B_2(k), \tag{27}$$
$$B_3(k)<-\varepsilon_2\Delta f_2(k)<B_4(k), \tag{28}$$
$$B_5(k)<-\varepsilon_3\Delta f_3(k)<B_6(k), \tag{29}$$
$$B_7(k)<-\varepsilon_4\Delta f_4(k)<B_8(k), \tag{30}$$
$$B_9(k)<-\varepsilon_1^2\Delta f_1^2(k)<B_{10}(k), \tag{31}$$
$$B_{11}(k)<-\varepsilon_2^2\Delta f_2^2(k)<B_{12}(k), \tag{32}$$

$$B_1(k)=\frac{U_1(k)-\sqrt{U_{10}^2(k)+4M_{Q2}(k)U_{00}(k)-\delta_{34}}}{2M_{Q2}(k)},$$
$$B_2(k)=\frac{U_1(k)+\sqrt{U_{10}^2(k)+4M_{Q2}(k)U_{00}(k)-\delta_{34}}}{2M_{Q2}(k)}. \tag{33}$$

$$B_3(k)=(V_1(k)/2-([Q_{14}(k)Q(^{13}_{23})(k)]^2+Q(13)(k)V_{00}(k)-\delta_{33})^{0.5})/Q(13)(k),$$
$$B_4(k)=(V_1(k)/2+([Q_{14}(k)Q(^{13}_{23})(k)]^2+Q(13)(k)V_{00}(k)-\delta_{33})^{0.5})/Q(13)(k). \tag{34}$$

$$B_5(k)=\frac{\frac{W_1(k)}{2}-\sqrt{Q(23)(k)Q_{44}(k)M_{Q3}(k)-\delta_{32}}}{Q(23)(k)},$$
$$B_6(k)=\frac{\frac{W_1(k)}{2}+\sqrt{Q(23)(k)Q_{44}(k)M_{Q3}(k)-\delta_{32}}}{Q(23)(k)}, \tag{35}$$

$$B_7(k)=\max\Big\{R_1(k)/2-(Q_{22}(k)Q_{33}(k)-\delta_{23}^{'})^{0.5},(S_1(k)/2-(M_{Q_2}(k)Q(13)(k)-\delta_{31})^{0.5})/Q_{11}(k)\Big\},$$
$$B_8(k)=\min\Big\{R_1(k)/2+(Q_{22}(k)Q_{33}(k)-\delta_{23}^{'})^{0.5},(S_1(k)/2+(M_{Q_2}(k)Q(13)(k)-\delta_{31})^{0.5})/Q_{11}(k)\Big\}, \tag{36}$$

$$B_9(k)=\frac{N_1(k)}{2}-\sqrt{Q_{11}(k)Q_{22}(k)-\delta_{12}},$$
$$B_{10}(k)=\frac{N_1(k)}{2}+\sqrt{Q_{11}(k)Q_{22}(k)-\delta_{12}}, \tag{37}$$
$$B_{11}(k)=\frac{T_1(k)}{2}-\sqrt{Q_{11}(k)Q_{33}(k)-\delta_{13}},$$
$$B_{12}(k)=\frac{T_1(k)}{2}+\sqrt{Q_{11}(k)Q_{33}(k)-\delta_{13}}. \tag{38}$$

其中:

$$\delta_{23}^{'}\leqslant\min\{\delta_{23},2d\sqrt{c_2c_3}-d^2\},$$
$$\delta_{31}\leqslant\min\{\delta_{12}\delta_{13},3d^2\},$$
$$d=\sqrt{\delta_{12}\delta_{13}}/2p_{11},\ \delta_{32}<c_4\delta_{23}^{'}\delta_{31}/p_{11},$$
$$\delta_{33}<\delta_{13}\delta_{32}/Q_2Q_3,\ \delta_{34}<\delta_{12}\delta_{33}/Q_1Q_3$$

为正常数;

$$N_1(k)=N_{11}(k)=$$
$$-2p_{12}(k+1)-2[f_4(k+1)p_{14}(k+1)-$$

$$f_3(k+1)f_4(k+1)p_{44}], \tag{39}$$

$$R_1(k) = -2[p_{23}(k+1)-p_{12}(k+1)+f_3(k+1)p_{24}(k+1)+ f_2(k+1)p_{14}(k+1) - f_2(k+1)f_3(k+1)p_{44}], \tag{40}$$

$$S_1(k) = -2Q_{11}(k)p_{23}(k+1) - 2Q_{11}(k)[-p_{12}(k+1) + f_3(k+1)p_{24}(k+1) + f_2(k+1)p_{14}(k+1) - f_2(k+1)f_3(k+1)p_{44}] + 2Q_{13}(k)Q_{12}(k), \tag{41}$$

$$T_1(k) = N_{12}(k) = -2p_{13}(k+1) - 2[f_4(k+1)p_{24}(k+1) - f_2(k+1)f_4(k+1)p_{44}], \tag{42}$$

$$U_1(k) = -2M_{Q2}(k)[p_{34}(k+1) - p_{23}(k+1) + f_2(k+1)p_{34}(k+1) + f_1(k+1)p_{24}(k+1) - f_1(k+1)f_2(k+1)p_{44}] + U_{10}(k), \tag{43}$$

$$U_{10}(k) = -[-Q_{23}(k)Q\binom{14}{12}(k) + Q_{13}(k)Q\binom{24}{12}(k) - Q_{24}(k)Q\binom{13}{12}(k) + Q_{14}(k)Q\binom{23}{12}(k)], \tag{44}$$

$$U_{00}(k) = -Q(13)(k)Q_{24}^2(k) + 2Q_{14}(k)Q\binom{13}{23}(k)Q_{24}(k) - Q(23)(k)Q_{14}^2(k) + Q_{44}(k)M_{Q3}(k), \tag{45}$$

$$V_1(k) = -2Q(13)(k)[p_{24}(k+1) - p_{13}(k+1) + f_3(k+1)p_{34}(k+1) + f_1(k+1)p_{14}(k+1) - f_1(k+1)f_3(k+1)p_{44}] + 2Q_{14}(k)Q\binom{13}{23}(k), \tag{46}$$

$$V_{00}(k) = -Q(23)(k)Q_{14}^2(k) + Q_{44}(k)M_{Q3}(k), \tag{47}$$

$$W_1(k) = -2Q(23)(k)[p_{14}(k+1) + f_4(k+1)p_{34}(k+1) - f_1(k+1)f_4(k+1)p_{44}], \tag{48}$$

$$M_{Q_2}(k) = Q(12)(k),\ M_{Q_3}(k) = Q(123)(k). \tag{49}$$

注 1　定理中的p_{ii}是容易取到的, 例如, 取$p_{ii} = i$, 则有$0<p_{11}<p_{22}<p_{33}<p_{44}$. 此时, 式(21)成为$|f_i(k)| < 1/2$.

注 2　由定理证明中的第二步可知, $B_7(k) < B_8(k)$, 即式(30)有意义.

注 3　定理中的条件(1)并非苛刻, 由引理3可见, 这一条件等价于矩阵$Q(k)$的主对角线上的元素全大于零, 这正是$Q(k)$正定的必要条件.

证　1) 首先证明$P(k)$是一致有界、正定的对称矩阵.

事实上, $P(k)$的一阶主子式$M_{p1}(k) = p_{11}(k) = p_{11} > p_{11}/2$.

因为$|f_1^4(k)| \leqslant M_1^4$, $0 < \varepsilon_1 \leqslant \dfrac{p_{11}}{M_1\sqrt{7}}$, $0 < p_{11} \leqslant 1$, 故$P(k)$的二阶主子式$M_{p2}(k) = p_{11}p_{22} - \varepsilon_1^4 f_1^4(k) > p_{11}^2 - \dfrac{p_{11}^4}{7^2} \geqslant p_{11}^2 - \dfrac{p_{11}^2}{7^2}$.

显然,

$$|f_1^2(k)f_2^2(k)f_4(k)| = |f_1^2(k)|\cdot|f_2^2(k)|\cdot|f_4(k)| \leqslant M_1^2M_2^2M_4,$$

于是 $f_1^2(k)f_2^2(k)f_4(k) \geqslant -M_1^2M_2^2M_4$. 又$0 < \varepsilon_i \leqslant \dfrac{p_{11}}{M_i\sqrt{7}}$ $(i = 1,2,3,4)$, $0 < p_{11} \leqslant 1$, 因此, 对于$P(k)$的三阶主子式, 有

$$\begin{aligned}M_{p3}(k) \geqslant\ & \left[\frac{4}{7}p_{11}p_{22}p_{33} - \frac{2p_{11}^3}{7^2\sqrt{7}}\right] + \left[\frac{1}{7}p_{11}p_{22}p_{33} - p_{22}\frac{p_{11}^2}{7^2}\right] + \\ & \left[\frac{1}{7}p_{11}p_{22}p_{33} - \frac{p_{11}^3}{7}\right] + \left[\frac{1}{7}p_{11}p_{22}p_{33} - p_{33}\frac{p_{11}^2}{7^2}\right] > \\ & \left[4\frac{p_{11}^3}{7} - \frac{2p_{11}^3}{7^2\sqrt{7}}\right] + \left[p_{22}\frac{p_{11}^2}{7} - p_{22}\frac{p_{11}^2}{7^2}\right] + \\ & \left[p_{11}\frac{p_{11}^2}{7} - \frac{p_{11}^3}{7}\right] + \left[p_{33}\frac{p_{11}^2}{7} - p_{33}\frac{p_{11}^2}{7^2}\right] > \frac{6p_{22}p_{11}^2}{49}.\end{aligned}$$

$P(k)$的四阶主子式

$$\begin{aligned}M_{p4}(k) =\ & p_{11}p_{22}p_{33}p_{44} - p_{11}p_{22}p_{34}(k)p_{43}(k) - \\ & p_{11}p_{32}(k)p_{23}(k)p_{44} + p_{11}p_{32}(k)p_{24}(k)p_{43}(k) + \\ & p_{11}p_{42}(k)p_{23}(k)p_{34}(k) - p_{11}p_{42}(k)p_{24}(k)p_{33} - \\ & p_{21}(k)p_{12}(k)p_{33}p_{44} + p_{21}^2(k)p_{34}^2(k) + \\ & p_{21}(k)p_{32}(k)p_{13}(k)p_{44} - p_{21}(k)p_{32}(k)p_{14}(k)p_{43}(k) - \\ & p_{21}(k)p_{42}(k)p_{13}(k)p_{34}(k) + p_{21}(k)p_{42}(k)p_{14}(k)p_{33} + \\ & p_{31}(k)p_{12}(k)p_{23}(k)p_{44} - p_{31}(k)p_{12}(k)p_{24}(k)p_{43}(k) - \\ & p_{31}(k)p_{22}p_{13}(k)p_{44} + p_{31}(k)p_{22}p_{14}(k)p_{43}(k) + \\ & p_{31}^2(k)p_{42}^2(k) - p_{31}(k)p_{42}(k)p_{14}(k)p_{23}(k) - \\ & p_{41}(k)p_{12}(k)p_{23}(k)p_{34}(k) + p_{41}(k)p_{12}(k)p_{24}(k)p_{33} + \\ & p_{41}(k)p_{22}p_{13}(k)p_{34}(k) - p_{41}(k)p_{22}p_{14}(k)p_{33} - \\ & p_{41}(k)p_{32}(k)p_{13}(k)p_{24}(k) + p_{41}^2(k)p_{32}^2(k).\end{aligned}$$

因为$0 < \varepsilon_i \leqslant p_{11}/M_i\sqrt{7}$, $|f_i(k)| \leqslant M_i$, 所以, $\varepsilon_i^2 f_i^2(k) \leqslant p_{11}^2/7$, 故有

$$\frac{1}{7}p_{11}p_{22}p_{33}p_{44} + \text{第 } 2,3,6,22 \text{ 项之一} > 0. \tag{50}$$

类似可得

$$\frac{1}{14}p_{11}p_{22}p_{33}p_{44}+\text{第4, 5项} > \frac{1}{14}p_{11}^4-\frac{1}{7\sqrt{7}}p_{11}^4. \tag{51}$$

注意到$0 < p_{11} \leqslant 1$, 则有

$$\frac{1}{49}p_{11}p_{22}p_{33}p_{44} + \text{第7, 9至16项, 第18至21项,}\ \text{第23项之一} > 0. \tag{52}$$

$M_{p4}(k)$展开式的第8, 17, 24项均非负.

综上所述, $M_{p4}(k) > \frac{1}{14}p_{11}^4 - \frac{1}{7\sqrt{7}}p_{11}^4$. 因此, 由引理1知, $P(k)$ 是一致有界、正定的对称矩阵.

2) 在证$Q(k)$ 为正定矩阵之前,需证式(30)有意义. 只需证明确有$\delta_{23}' > 0$及

$$\frac{\frac{S_1(k)}{2} - \sqrt{M_{Q_2}(k)Q(13)(k) - \delta_{31}}}{Q_{11}(k)} < \frac{R_1(k)}{2} + \sqrt{Q_{22}(k)Q_{33}(k) - \delta_{23}'}, \tag{53}$$

$$\frac{R_1(k)}{2} - \sqrt{Q_{22}(k)Q_{33}(k) - \delta_{23}'} < \frac{\frac{S_1(k)}{2} + \sqrt{M_{Q_2}(k)Q(13)(k) - \delta_{31}}}{Q_{11}(k)}. \tag{54}$$

因$0 < \delta_{12} < c_1c_2,\ 0 < \delta_{13} < c_1c_3$, 所以, $\sqrt{\delta_{12}\delta_{13}} < c_1\sqrt{c_2c_3}$, 于是$\sqrt{\delta_{12}\delta_{13}}/p_{11} < c_1\sqrt{c_2c_3}/p_{11}$; 又$0 < c_1 < p_{11}$, 故$\sqrt{\delta_{12}\delta_{13}}/p_{11} < \sqrt{c_2c_3}$, 即$2d < \sqrt{c_2c_3}$, 从而$2d\sqrt{c_2c_3} - d^2 = d(2\sqrt{c_2c_3} - d) > 0$, 故确实可取到正数$\delta_{23}'$. 记

$$a_1(k) = \frac{R_1(k)}{2} - \sqrt{Q_{22}(k)Q_{33}(k)},$$

$$a_2(k) = \frac{\frac{S_1(k)}{2} - \sqrt{M_{Q_2}(k)Q(13)(k)}}{Q_{11}(k)},$$

$$b_1(k) = \frac{R_1(k)}{2} + \sqrt{Q_{22}(k)Q_{33}(k)},$$

$$b_2(k) = \frac{\frac{S_1(k)}{2} + \sqrt{M_{Q_2}(k)Q(13)(k)}}{Q_{11}(k)}.$$

由引理3的第2条结论知, 式(31)(32)成立时,

$$M_{Q_2}(k) = Q(12)(k) = Q_{11}(k)Q_{22}(k) - Q_{12}^2(k) > 0,$$
$$Q(13)(k) = Q_{11}(k)Q_{33}(k) - Q_{13}^2(k) > 0,$$

所以

$$Q_{12}^2(k) < Q_{11}(k)Q_{22}(k),$$
$$Q_{13}^2(k) < Q_{11}(k)Q_{33}(k),$$

从而$Q_{13}^2(k)Q_{12}^2(k) < Q_{11}^2(k)Q_{22}(k)Q_{33}(k)$, 进而

$$|Q_{13}(k)Q_{12}(k)| < Q_{11}(k)\sqrt{Q_{22}(k)Q_{33}(k)}. \tag{55}$$

由上式并注意$S_1(k) = Q_{11}(k)R_1(k)+2Q_{13}(k)Q_{12}(k)$, 则有

$$b_1(k) - a_2(k) = [Q_{11}(k)\sqrt{Q_{22}(k)Q_{33}(k)} - Q_{13}(k)Q_{12}(k)]/Q_{11}(k) + \sqrt{M_{Q_2}(k)Q(13)(k)}/Q_{11}(k) > \sqrt{M_{Q_2}(k)Q(13)(k)}/Q_{11}(k) > \sqrt{\delta_{12}\delta_{13}}/p_{11} = 2d,$$

同理, $b_2(k) - a_1(k) > \sqrt{M_{Q_2}(k)Q(13)(k)}/Q_{11}(k) > 2d$.

由引理3知

$$M_{Q_2}(k) = Q(12)(k) \leqslant Q_{11}(k)Q_{22}(k),$$
$$Q(13)(k) \leqslant Q_{11}(k)Q_{33}(k),$$

故

$$\sqrt{Q_{22}(k)Q_{33}(k)} \geqslant \sqrt{M_{Q_2}(k)Q(13)(k)}/Q_{11}(k) > 2d.$$

因

$$\delta_{23}' \leqslant 2d\sqrt{c_2c_3} - d^2 < 2d\sqrt{Q_{22}(k)Q_{33}(k)} - d^2,$$

故

$$(d - \sqrt{Q_{22}(k)Q_{33}(k)}\ )^2 < Q_{22}(k)Q_{33}(k) - \delta_{23}'.$$

而$d < 2d < \sqrt{Q_{22}(k)Q_{33}(k)}$, 故

$$\sqrt{Q_{22}(k)Q_{33}(k)} - d < \sqrt{Q_{22}(k)Q_{33}(k) - \delta_{23}'},$$

即

$$e_1(k) \triangleq \sqrt{Q_{22}(k)Q_{33}(k)} - \sqrt{Q_{22}(k)Q_{33}(k) - \delta_{23}'} < d,$$

故
$$e_1(k) < \frac{b_1(k) - a_2(k)}{2},$$

于是

$$b_1(k) - e_1(k) > \frac{b_1(k) + a_2(k)}{2}.$$

因

$$\delta_{31} \leqslant 3d^2 = 4d^2 - d^2 = 2d\frac{\sqrt{\delta_{12}\delta_{13}}}{p_{11}} - d^2 < 2d\frac{\sqrt{M_{Q_2}(k)Q(13)(k)}}{Q_{11}(k)} - d^2,$$

故仿上可证

$$e_2(k) \triangleq \frac{\sqrt{M_{Q_2}(k)Q(13)(k)}}{Q_{11}(k)} - \frac{\sqrt{M_{Q_2}(k)Q(13)(k) - \delta_{31}}}{Q_{11}(k)} < d,$$

故$e_2(k) < \dfrac{b_1(k)-a_2(k)}{2}$, 于是$a_2(k)+e_2(k) < \dfrac{b_1(k)+a_2(k)}{2}$.

综上所述, $a_2(k)+e_2(k) < b_1(k)-e_1(k)$, 此即式(53)成立. 仿上可证式(54)成立.

总之, 式(30)有意义.

3) 下证$Q(k)$ 也是一致有界、正定的对称矩阵. 事实上, 由引理 3 第 1 条结论知, 式(21)成立时, $Q(k)$的一阶主子式$M_{Q1}(k)=Q_{11}(k)>c_1$.

由引理3的第2条结论知, 式(31)成立时, $Q(k)$的二阶主子式$M_{Q2}(k)=Q(12)(k)>\delta_{12}$.

将$Q(k)$的三阶主子式按第3列展开可得

$$\begin{aligned}M_{Q3}(k)=&-Q_{11}(k)[p_{23}(k)-p_{23}(k+1)]^2+\\&S_1(k)[p_{23}(k)-p_{23}(k+1)]+S_0(k).\end{aligned}$$

此为关于$p_{23}(k)-p_{23}(k+1)$的二次三项式, 其中$S_1(k)$由式(41)给出, 且$S_0(k)$, $S_1(k)$可分别表示为式(2) (3)的形式, 其中:

$$\begin{aligned}&M(k)=Q_{11}(k),\ B(k+1)=p_{23}(k+1),\\&D(k)=-Q_{13}^2(k)Q_{22}(k)+Q_{33}(k)M_{Q2}(k),\\&F(k)=2Q_{13}(k)Q_{12}(k),\\&C(k+1)=\\&-p_{12}(k+1)+f_3(k+1)p_{24}(k+1)+\\&f_2(k+1)p_{14}(k+1)-f_2(k+1)f_3(k+1)p_{44},\end{aligned}$$

由引理2知, 上述关于$p_{23}(k)-p_{23}(k+1)$的二次三项式的判别式为

$$\begin{aligned}&S_1^2(k)+4Q_{11}(k)S_0(k)=\\&F^2(k)+4M(k)D(k)=4M_{Q2}(k)Q(13)(k).\end{aligned}$$

由引理3的第2条结论知, 式(32)成立时, $Q(13)(k)>\delta_{13}$, 故$F^2(k)+4M(k)D(k)>4\delta_{12}\delta_{13}$. 由引理2知, 当式(30)成立时, $M_{Q3}(k)>\delta_{31}/p_{11}$.

将$Q(k)$的四阶主子式按第4列展开可得

$$\begin{aligned}M_{Q4}(k)=&-M_{Q2}(k)[p_{34}(k)-p_{34}(k+1)]^2+\\&U_1(k)[p_{34}(k)-p_{34}(k+1)]+U_0(k).\end{aligned}$$

此为关于$p_{34}(k)-p_{34}(k+1)$的二次三项式, 其中$U_1(k)$由式(43)给出, 且$U_0(k),U_1(k)$可分别表示为式(2) (3)的形式, 式中:

$$\begin{aligned}&M(k)=M_{Q2}(k),\ B(k+1)=p_{34}(k+1),\\&D(k)=U_{00}(k),\ F(k)=U_{10}(k),\\&C(k+1)=\\&-p_{23}(k+1)+f_2(k+1)p_{34}(k+1)+\\&f_1(k+1)p_{24}(k+1)-f_1(k+1)f_2(k+1)p_{44},\end{aligned}$$

$U_{10}(k),U_{00}(k)$分别由式(44) (45)给出, 由引理2知

$$\begin{aligned}&U_1^2(k)+4M_{Q2}(k)U_0(k)=\\&U_{10}^2(k)+4M_{Q2}(k)U_{00}(k).\end{aligned}$$

可以证明$U_{00}(k)>\delta_{33}/Q_1Q_3$. 事实上,

$$\begin{aligned}U_{00}(k)=&-Q(13)(k)[p_{24}(k)-p_{24}(k+1)]^2+\\&V_1(k)[p_{24}(k)-p_{24}(k+1)]+V_0(k).\end{aligned}$$

此为关于$p_{24}(k)-p_{24}(k+1)$的二次三项式, 其中$V_1(k)$由式(46)给出, 且$V_0(k),V_1(k)$可分别表为式(2) (3)的形式, 式中:

$$\begin{aligned}&M(k)=Q(13)(k),\ B(k+1)=p_{24}(k+1),\\&D(k)=V_{00}(k),\ F(k)=2Q_{14}(k)Q(^{13}_{23})(k),\\&C(k+1)=\\&-p_{13}(k+1)+f_3(k+1)p_{34}(k+1)+\\&f_1(k+1)p_{14}(k+1)-f_1(k+1)f_3(k+1)p_{44}.\end{aligned}$$

由引理2知

$$\begin{aligned}&V_1^2(k)+4Q(13)(k)V_0(k)=\\&F^2(k)+4M(k)D(k)=\\&4[Q_{14}(k)Q(^{13}_{23})(k)]^2+4Q(13)(k)V_{00}(k).\end{aligned}$$

由引理3的第2条结论知, 式(32)成立时, $Q(13)(k)>\delta_{13}$.

下证$V_{00}(k)>\delta_{32}/Q_2Q_3$, 事实上,

$$\begin{aligned}V_{00}(k)=&-Q(23)(k)[p_{14}(k)-p_{14}(k+1)]^2+\\&W_1(k)[p_{14}(k)-p_{14}(k+1)]+W_0(k).\end{aligned}$$

此为关于$p_{14}(k)-p_{14}(k+1)$的二次三项式, 其中$W_1(k)$由式(48)给出, 且$W_0(k),W_1(k)$可分别表示为式(2) (3)的形式, 式中:

$$\begin{aligned}&M(k)=Q(23)(k),\ B(k+1)=p_{14}(k+1),\\&D(k)=Q_{44}(k)M_{Q3}(k),\ F(k)=0,\\&C(k+1)=\\&f_4(k+1)p_{34}(k+1)-f_1(k+1)f_4(k+1)p_{44}.\end{aligned}$$

由引理2知

$$\begin{aligned}&W_1^2(k)+4Q(23)(k)W_0(k)=\\&4Q(23)(k)Q_{44}(k)M_{Q3}(k).\end{aligned}$$

分别由引理3的第2条、第1条结论知, 式(30) (21)成立时, $Q(23)(k)>\delta_{23}'$, $Q_{44}(k)>c_4$, 于是,

$$W_1^2(k)+4Q(23)(k)W_0(k)>4c_4\delta_{23}'\delta_{31}/p_{11}.$$

由引理2, 式(29)成立时, $V_{00}(k)>\delta_{32}/Q_2Q_3$. 所以, $V_1^2(k)+4Q(13)V_0(k)>4\delta_{13}\delta_{32}/Q_2Q_3$, 且当式(28)成立时, $U_{00}(k)>\delta_{33}/Q_1Q_3$, 进而, 当式(27)成立时,

$M_{Q4}(k) > \delta_{34}/Q_1Q_2$. 由引理1, $Q(k)$ 也是一致有界、正定的矩阵.

由Lyapunov渐近稳定性基本定理知结论得证.

说明 1 关于确定定理1中式(27)~(32)所涉及的诸$B_j(k)$的上确界或下确界问题$|f_i(k)| < \alpha_i M_i$ (α_i为常数且$0 < \alpha_i < 1, M_i$ 的定义见式(21))时, 笔者已估计出诸$B_{2j-1}(k)$的上确界及诸$B_{2j}(k)$的下确界, 即得到了诸$f_i(k)$及改变量的确切范围, 并利用计算机做了大量的随机试验, 验证了所得结果的正确性. 但因推导过程过繁且笔者认为结果较保守, 故留待以后作进一步研究.

3 例子(Examples)

对形如系统(1)的系统, 若

$$f_1(k) = (\sin k + \cos k)/5,\ f_2(k) = \sin k/5,$$
$$f_3(k) = \cos(2k)/5,\ f_4(k) = \cos(3k)/5,$$

取$\varepsilon_i = 0.1$, p_{ii}的取值同注1, 取说明1中的$\alpha = 0.4$, 则通过不等式放大运算可验证所给系统满足定理1中的条件, 故为一致渐近稳定. 大量数值试验证实了所给结果的正确性, 图1给出的仅是: $y(k), y(k-1), y(k-2), y(k-3)$的初值均取(–1, 1)中的随机数, 种子为0的$y$的轨迹.

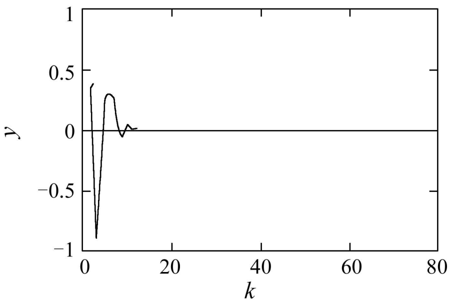

图 1 y的轨迹

Fig. 1 Trajectories of y

4 结论(Conclusion)

基于特征建模设计的控制方案, 其稳定性问题归结为时变离散系统的稳定性问题. 文中利用系统方程中的系数构造了正定矩阵$P(k)$, 据此, 定量地给出了四阶时变离散系统一致渐近稳定性的充分条件, 为基于特征建模设计适当的控制律以满足稳定性的要求打下了理论基础.

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