

具有模糊监督控制器的积分变结构间接自适应控制

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摘要: 针对一类不确定非线性系统, 基于变结构控制原理, 并利用具有非线性可调参数的模糊系统去逼近过程未知函数, 提出一种具有模糊监督控制器的积分变结构间接自适应控制方案. 该方案通过监督控制器保证闭环系统所有信号有界. 进一步, 通过引入最优逼近误差的自适应补偿项来消除建模误差的影响. 理论分析证明了跟踪误差收敛到零. 仿真结果表明了该方法的有效性.

关键词: 自适应控制; 模糊控制; 积分变结构控制; 非线性系统; 全局稳定性

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Integral variable structure indirect adaptive control with fuzzy supervisory controller

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Abstract: A new integral variable structure indirect adaptive control scheme with fuzzy supervisory controller is developed for a class of uncertain nonlinear systems in this paper. The design is based on the principle of variable structure control. The fuzzy systems with nonlinear adjustable parameters are used to approximate unknown plant functions. With the help of a supervisory controller, the resulting closed-loop system is globally stable in the sense that all signals involved are uniformly bounded. Furthermore, the adaptive compensation term of the optimal approximation error is introduced to minimize the effects of modeling error. By theoretical analysis, it is shown that the tracking error converges to zero. Finally, simulation results demonstrate the effectiveness of the approach.

Key words: adaptive control; fuzzy control; integral variable structure control; nonlinear systems; global stability

1 引言 (Introduction)

近年来,利用神经网络或模糊系统研究不确定非线性系统的自适应控制受到国内外学者的广泛关注,取得了一些研究成果. 文献[1,2]中利用模糊系统的逼近性质,提出了自适应模糊控制的系统设计方案,但文献[1,2]中跟踪误差的收敛性依赖于逼近误差平方可积这一假设. 针对文献[1,2]中的缺点,文献[3,4]分别提出不同的修正方案,但闭环系统的渐近稳定性分析中假设最优逼近误差的上确界已知. 文献[5,6]基于CMAC神经网络逼近性质,提出两种直接自适应控制方案,但文献[5]中参数自适应调节律含有未知控制增益函数,其算法是不可实现的. 此外,文献[5]中还假设了综合误差存在有界上界;文献[6]中的渐近稳定性证明有待商榷. 文献[7]根据目标的不同,利用多模型神经网络,提出一种间接神经网络控制策略,但跟踪误差的收敛性仍依赖于逼

近误差平方可积,此外,假设了逼近误差存在有界上界. 由于模糊系统和神经网络的通用逼近性质只在给定的有界闭区域上有效,因此,在未证明状态有界的条件下假定逼近误差的上确界存在且有界是不合理的. 另外,实际控制中此条件无法验证. 文献[8]在文献[2]基础上,提出一种改进的控制策略,避免了误差平方可积的假设. 变结构控制是控制系统的一种综合设计方法,它对干扰和未建模动态具有较强的鲁棒性. 文献[9]基于滑模控制原理和积分型李雅普诺夫函数,并利用模糊系统的通用逼近能力,提出了一种直接自适应模糊控制器的设计方案. 文献[10]利用具有线性可调参数的模糊系统,提出了一种具有监督控制器和参数投影的间接自适应模糊滑模控制方案. 文献[11]利用输入输出状态稳定方法和小增益定理,对一类具有不确定性干扰的SISO非线性系统提出了一种直接鲁棒自适应控制,并证明了闭环系

统一一致终结有界.

本文在文献[2,8]基础上, 引入积分型切换函数, 并利用第2类模糊逻辑系统逼近的能力, 提出了一种稳定间接自适应模糊控制器设计的新方案. 该方案通过监督控制项保证闭环系统的稳定性, 由此确定出用于建模的有界闭区域. 通过引入最优逼近误差的自适应补偿项, 保证跟踪误差收敛到零. 本文提出的控制策略在稳定性分析中取消了要求逼近误差平方可积的条件, 同时也避免了逼近误差上确界已知的假设. 此外, 该方案无须求解李雅普诺夫方程, 控制结构简单.

2 问题的描述及基本假设(Problem statement and basic assumptions)

考虑下面一类不确定非线性系统

$$\begin{cases} \dot{x}^{(n)} = f(x, \dot{x}, \dots, x^{(n-1)}) + g(x, \dot{x}, \dots, x^{(n-1)})u(t) + d(x, \dot{x}, \dots, x^{(n-1)}, t), \\ y = x_1, \end{cases} \quad (1)$$

其中: $\mathbf{x} = (x_1, x_2, \dots, x_n)^T = (x, \dot{x}, \dots, x^{(n-1)})^T$ 是 n 维状态向量, u 是控制输入, y 是系统输出, f 是未知连续函数, g 是未知函数控制增益, $d(\mathbf{x}, t)$ 是不确定性干扰.

控制目标要求系统输出 y 尽可能好地去跟踪一个指定的期望轨迹 y_d . 因此, 问题是设计一个控制律 u , 使得 $y_d - y$ 收敛到零. 定义跟踪误差向量 $\mathbf{e}$ 如下:

$$\mathbf{e} = (e_1, \dots, e_n)^T = (y_d - x_1, \dots, y_d^{(n-1)} - x_n)^T. \quad (2)$$

为了设计稳定的自适应模糊控制, 对未知连续函数 $f(\mathbf{x})$ 及控制增益 $g(\mathbf{x})$ 作出如下假设:

- 1) $|f(\mathbf{x})| \leq F(\mathbf{x}), \forall \mathbf{x} \in \mathbb{R}^n$;
- 2) $0 < K_1(\mathbf{x}) \leq g(\mathbf{x}) \leq K_2(\mathbf{x}), \mathbf{x} \in \mathbb{R}^n$;
- 3) $|d(\mathbf{x}, t)| \leq D(\mathbf{x}), \mathbf{x} \in \mathbb{R}^n, \forall t \geq 0$;
- 4) $\mathbf{x}_d \in \Omega_d \subset \mathbb{R}^{n+1}$ 且 $\|\mathbf{x}_d\| \leq M_d$,

其中: $F(\mathbf{x}), K_1(\mathbf{x}), K_2(\mathbf{x}), D(\mathbf{x})$ 均是已知正的连续函数, Ω_d 是一个已知的有界闭集, $\mathbf{x}_d = (y_d, \dot{y}_d, \dots, y_d^{(n)})^T$, M_d 是一个已知的正常数.

选取常数 $k_1, k_2, \dots, k_n$, 使得 $h(s) = s^n + k_1 s^{n-1} + \dots + k_{n-1} s + k_n = (s + \lambda)^n, \lambda > 0$, 即, $k_i = C_n^{n-i} \lambda^i, i = 1, \dots, n$. 若 $f(\mathbf{x}), g(\mathbf{x})$ 已知, $d(\mathbf{x}, t) = 0$, 则取

$$u^* = \frac{1}{g(\mathbf{x})} [-f(\mathbf{x}) + y_d^{(n)} + \sum_{i=1}^n k_i e_{n-i+1}]. \quad (3)$$

将上式代入式(1)中不难推出

$$e_1^{(n)} + k_1 e_1^{(n-1)} + k_2 e_1^{(n-2)} + \dots + k_n e_1 = 0,$$

从而可得 $\lim_{t \rightarrow \infty} e_1(t) = 0$. 由于 $f(\mathbf{x}), g(\mathbf{x})$ 未知且 $d(\mathbf{x},$

$t) \neq 0$, 故控制律式(3)不可实现. 下面将采用两个模糊逻辑系统分别对 u^* 中的未知函数 $f(\mathbf{x}), g(\mathbf{x})$ 进行逼近.

定义具有积分的切换函数

$$\sigma = k_n e_0 + \sum_{i=1}^{n-1} k_{n-i} e_i + e_n = \left(\frac{d}{dt} + \lambda \right)^n e_0, \quad (4)$$

其中 $\dot{e}_0 = y_d - x_1 = e_1$ 或 $e_0 = \int e_1 dt$.

引理 1^[12] 若 σ 由式(4)确定, 则

- 1) 当 $\sigma = 0$ 时, $\lim_{t \rightarrow \infty} e_0(t) = 0$,
- 2) 当 $|\sigma| \leq c, \mathbf{E}(0) \in \Omega_c$ 时, $\mathbf{E}(t) \in \Omega_c, \forall t \geq 0$,
- 3) 当 $|\sigma| \leq c, \mathbf{E}(0) \notin \Omega_c$ 时, 存在 $T = n/\lambda, \exists \forall t \geq T$, 有 $\mathbf{E}(t) \in \Omega_c$.

其中: $\mathbf{E}(t) = (e_0, \mathbf{e}^T)^T, c > 0, \Omega_c = \{\mathbf{E}(t) : |e_j| \leq 2^j \lambda^{j-n} c, j = 0, 1, \dots, n\}$.

定义 $\Omega_{\mathbf{x}} = \{\mathbf{x} : \|\mathbf{x}\| \leq M_{\mathbf{x}}\}$, 其中 $M_{\mathbf{x}} \geq M_d$ 是设计常数, 确定方法在后面定理1中给出. 设 $f(\mathbf{x}, \theta_f), g(\mathbf{x}, \theta_g)$ 是两个 Π 型模糊逻辑系统在区域 $\Omega_{\mathbf{x}}$ 上分别对 $f(\mathbf{x}), g(\mathbf{x})$ 的一个逼近, 即

$$f(\mathbf{x}, \theta_f) = \frac{\sum_{l=1}^M y_{fl} [\prod_{j=1}^n \exp(-\frac{(x_j - a_{fj}^l)^2}{(b_{fj}^l)^2 + b_{f0}})]}{\sum_{l=1}^M [\prod_{j=1}^n \exp(-\frac{(x_j - a_{fj}^l)^2}{(b_{fj}^l)^2 + b_{f0}})]}, \quad (5)$$

$$g(\mathbf{x}, \theta_g) = \frac{\sum_{l=1}^N y_{gl} [\prod_{j=1}^n \exp(-\frac{(x_j - a_{gj}^l)^2}{(b_{gj}^l)^2 + b_{g0}})]}{\sum_{l=1}^N [\prod_{j=1}^n \exp(-\frac{(x_j - a_{gj}^l)^2}{(b_{gj}^l)^2 + b_{g0}})]}, \quad (6)$$

而 M, N 是两个模糊系统的规则数目,

$$\begin{aligned} \theta_f &= (y_{f1}, \dots, y_{fM}, b_{f1}^1, \dots, b_{f1}^M, \dots, b_{f1}^M, \dots, \\ &\quad b_{fn}^M, a_{f1}^1, \dots, a_{fn}^1, \dots, a_{f1}^M, \dots, a_{fn}^M)^T, \\ \theta_g &= (y_{g1}, \dots, y_{gN}, b_{g1}^1, \dots, b_{g1}^N, \dots, b_{g1}^N, \dots, \\ &\quad b_{gn}^N, a_{g1}^1, \dots, a_{gn}^1, \dots, a_{g1}^N, \dots, a_{gn}^N)^T \end{aligned}$$

是可调参数, 正常数 b_{f0}, b_{g0} 是设计参数. 令

$$\Omega_{\theta_f} = \{\theta_f : \|\theta_f\| \leq M_{\theta_f}\}, \quad (7)$$

$$\Omega_{\theta_g} = \{\theta_g : \|\theta_g\| \leq M_{\theta_g}, y_{gl} \geq \varepsilon, l = 1, \dots, N\}, \quad (8)$$

$$\theta_f^* = \arg \min_{\theta_f \in \Omega_{\theta_f}} [\sup_{\mathbf{x} \in \Omega_{\mathbf{x}}} |f(\mathbf{x}, \theta_f) - f(\mathbf{x})|],$$

$$\theta_g^* = \arg \min_{\theta_g \in \Omega_{\theta_g}} [\sup_{\mathbf{x} \in \Omega_{\mathbf{x}}} |g(\mathbf{x}, \theta_g) - g(\mathbf{x})|],$$

其中正常数 $M_{\theta_f}, M_{\theta_g}$ 和 ε 是设计参数. 设 $\hat{\theta}_f(t) \in \Omega_{\theta_f}, \hat{\theta}_g(t) \in \Omega_{\theta_g}$ 分别是 θ_f^*, θ_g^* 在 t 时刻的估计值, 将 $f(\mathbf{x}, \theta_f^*), g(\mathbf{x}, \theta_g^*)$ 分别在 $\hat{\theta}_f(t), \hat{\theta}_g(t)$ 的邻域内展开成泰勒式得

$$f(\mathbf{x}, \boldsymbol{\theta}_f^*) - f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f(t)) = -\boldsymbol{\varphi}_f^T(t) \frac{\partial f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)}{\partial \hat{\boldsymbol{\theta}}_f} + O(\|\boldsymbol{\varphi}_f(t)\|^2), \quad (9)$$

$$g(\mathbf{x}, \boldsymbol{\theta}_g^*) - g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g(t)) = -\boldsymbol{\varphi}_g^T(t) \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_g} + O(\|\boldsymbol{\varphi}_g(t)\|^2), \quad (10)$$

其中: $\boldsymbol{\varphi}_f(t) = \hat{\boldsymbol{\theta}}_f(t) - \boldsymbol{\theta}_f^*$, $\boldsymbol{\varphi}_g(t) = \hat{\boldsymbol{\theta}}_g(t) - \boldsymbol{\theta}_g^*$. 令最小逼近误差

$$\nu = f(\mathbf{x}, \boldsymbol{\theta}_f^*) - f(\mathbf{x}) + [g(\mathbf{x}, \boldsymbol{\theta}_g^*) - g(\mathbf{x})]u_f - O(\|\boldsymbol{\varphi}_f(t)\|^2) - O(\|\boldsymbol{\varphi}_g(t)\|^2)u_f, \quad (11)$$

其中

$$u_f = \frac{-f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f) + y_d^{(n)} + \sum_{i=1}^n k_i e_{n-i+1}}{g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}. \quad (12)$$

令 $\varepsilon_\nu = \max_{\mathbf{x} \in \Omega_{\mathbf{x}}, \hat{\boldsymbol{\theta}}_f(t) \in \Omega_{\boldsymbol{\theta}_f}, \hat{\boldsymbol{\theta}}_g(t) \in \Omega_{\boldsymbol{\theta}_g}} |\nu|$, 则 ε_ν 是未知有界常数.

3 自适应模糊控制器的设计(Adaptive fuzzy controller design)

采用如下控制律

$$u(t) = u_f + u_s + u_c, \quad (13)$$

式中 u_f 由式(12)确定,

$$u_c = \frac{1}{K_1(\mathbf{x})} \sigma + \frac{\hat{\varepsilon}_\nu + D(\mathbf{x})}{K_1(\mathbf{x})} \text{sgn}(\sigma), \quad (14)$$

$$u_s = I(\bar{V}) \text{sgn}(\sigma) \frac{1}{K_1(\mathbf{x})} [|f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)| + F(\mathbf{x}) + |g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)u_f| + |K_2(\mathbf{x})u_f|], \quad (15)$$

$$I(\bar{V}) = \begin{cases} 1, & \text{当 } V_\sigma = \sigma^2/2 > \bar{V}, \\ 0, & \text{当 } V_\sigma \leq \bar{V}, \end{cases} \quad (16)$$

$\bar{V}$ 是设计参数; u_f 是自适应模糊控制项, u_s 是监督控制项, u_c 是建模误差和扰动的鲁棒自适应补偿项; $\hat{\boldsymbol{\theta}}_f, \hat{\boldsymbol{\theta}}_g, \hat{\varepsilon}_\nu$ 分别表示 $\boldsymbol{\theta}_f^*, \boldsymbol{\theta}_g^*, \varepsilon_\nu$ 在 t 时刻的估计值.

采用如下自适应律

$$\dot{\hat{\boldsymbol{\theta}}}_f = \begin{cases} -\eta_1 \sigma \frac{\partial f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)}{\partial \hat{\boldsymbol{\theta}}_f}, & \text{当 } \|\hat{\boldsymbol{\theta}}_f\| < M_{\boldsymbol{\theta}_f} \text{ 或 } \|\hat{\boldsymbol{\theta}}_f\| = M_{\boldsymbol{\theta}_f} \\ & \text{且 } \sigma \hat{\boldsymbol{\theta}}_f^T \frac{\partial f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)}{\partial \hat{\boldsymbol{\theta}}_f} \geq 0; \\ -\eta_1 \sigma \frac{\partial f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)}{\partial \hat{\boldsymbol{\theta}}_f} + \eta_1 \sigma \frac{\hat{\boldsymbol{\theta}}_f \hat{\boldsymbol{\theta}}_f^T}{\|\hat{\boldsymbol{\theta}}_f\|^2} \frac{\partial f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)}{\partial \hat{\boldsymbol{\theta}}_f}, & \\ \text{当 } \|\hat{\boldsymbol{\theta}}_f\| = M_{\boldsymbol{\theta}_f} \text{ 且 } \sigma \hat{\boldsymbol{\theta}}_f^T \frac{\partial f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)}{\partial \hat{\boldsymbol{\theta}}_f} < 0. \end{cases} \quad (17)$$

当 $\hat{y}_{gl} = \varepsilon$ 时,

$$\dot{\hat{y}}_{gl} = \begin{cases} -\eta_2 \sigma u_f \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{y}_{gl}}, & \text{当 } \sigma u_f \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{y}_{gl}} < 0, \\ 0, & \text{当 } \sigma u_f \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{y}_{gl}} \geq 0, \end{cases} \quad (18)$$

否则

$$\dot{\hat{\boldsymbol{\theta}}}_{g1} = \begin{cases} -\eta_2 \sigma u_f \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g1}}, & \\ \text{当 } \|\hat{\boldsymbol{\theta}}_g\| < M_{\boldsymbol{\theta}_g} \text{ 或 } \|\hat{\boldsymbol{\theta}}_g\| = M_{\boldsymbol{\theta}_g} \text{ 且} \\ \sigma u_f [\hat{\boldsymbol{\theta}}_{g1}^T \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g1}} + \hat{\boldsymbol{\theta}}_{g2}^T \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g2}}] \geq 0; \\ -\eta_2 \sigma u_f \left\{ \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g1}} \right. \\ \left. - \frac{\hat{\boldsymbol{\theta}}_{g1}}{\|\hat{\boldsymbol{\theta}}_{g1}\|^2} [\hat{\boldsymbol{\theta}}_{g1}^T \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g1}} + \hat{\boldsymbol{\theta}}_{g2}^T \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g2}}] \right\}, & \\ \text{当 } \|\hat{\boldsymbol{\theta}}_g\| = M_{\boldsymbol{\theta}_g} \text{ 且} \\ \sigma u_f [\hat{\boldsymbol{\theta}}_{g1}^T \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g1}} + \hat{\boldsymbol{\theta}}_{g2}^T \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g2}}] < 0, \end{cases} \quad (19)$$

$$\dot{\hat{\varepsilon}}_\nu = \eta_3 |\sigma|, \quad (20)$$

其中: $\eta_1 > 0, \eta_2 > 0, \eta_3 > 0$ 是自适应率, $\hat{\boldsymbol{\theta}}_{g1}$ 表示 $\hat{\boldsymbol{\theta}}_g$ 中删去满足式(18)的分量后所得参数估计向量, $\hat{\boldsymbol{\theta}}_{g2}$ 表示 $\hat{\boldsymbol{\theta}}_g$ 中满足式(18)第1个条件的所有分量所构成的列向量.

4 稳定性分析(Stability analysis)

由式(1)~(3) 得误差方程

$$\begin{aligned} \dot{e}_n = & -\sum_{i=1}^n k_i e_{n-i+1} + f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f) - f(\mathbf{x}) + \\ & [g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g) - g(\mathbf{x})]u_f - \\ & g(\mathbf{x})u_s - g(\mathbf{x})u_c - d(\mathbf{x}, t) = \\ & -\sum_{i=1}^n k_i e_{n-i+1} + [\boldsymbol{\varphi}_f^T(t) \frac{\partial f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)}{\partial \hat{\boldsymbol{\theta}}_f} + \\ & u_f \boldsymbol{\varphi}_g^T(t) \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_g}] - \\ & g(\mathbf{x})u_s - g(\mathbf{x})u_c - \nu - d(\mathbf{x}, t). \end{aligned} \quad (21)$$

所以

$$\begin{aligned} \dot{\sigma} = & f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f) - f(\mathbf{x}) + [g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g) - g(\mathbf{x})]u_f - \\ & g(\mathbf{x})u_s - g(\mathbf{x})u_c - d(\mathbf{x}, t) = \\ & [\boldsymbol{\varphi}_f^T(t) \frac{\partial f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)}{\partial \hat{\boldsymbol{\theta}}_f} + u_f \boldsymbol{\varphi}_g^T(t) \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_g}] - \\ & g(\mathbf{x})u_s - g(\mathbf{x})u_c - \nu - d(\mathbf{x}, t). \end{aligned} \quad (22)$$

由式(13)~(16)构成的控制律, 提出如下稳定性定理:

定理 1 考虑过程(1), 其控制律由式(13)~(16)

确定, 自适应律由式(17)~(20)确定, 并满足假设1)~4). 若取 $M_x = M_d + \sum_{j=1}^n 2^{j+1} \lambda^{j-n} \sqrt{\bar{V}}$, $\mathbf{E}(0) \in$

Ω_c , $\hat{\boldsymbol{\theta}}_f(0) \in \Omega_{\theta_f}$, $\hat{\boldsymbol{\theta}}_g(0) \in \Omega_{\theta_g}$, 则

1) $\|\hat{\boldsymbol{\theta}}_f\| \leq M_{\theta_f}$, $\|\hat{\boldsymbol{\theta}}_g\| \leq M_{\theta_g}$ 且 $\hat{y}_{gl} \geq \varepsilon$, $l = 1, \dots, N$, $\|\mathbf{x}\| \leq M_d + \sum_{j=1}^n 2^{j+1} \lambda^{j-n} \sqrt{\bar{V}}$;

2) $\lim_{t \rightarrow \infty} |e_i(t)| = 0, i = 0, 1, \dots, n-1$,
其中:

$\Omega_c = \{\mathbf{E}(t) : |e_j| \leq 2^j \lambda^{j-n} c, j = 0, 1, \dots, n\}$,
 $c = 2\sqrt{\bar{V}}$.

证 1) 类似于文献[2, 8]中的讨论, 易得 $\|\hat{\boldsymbol{\theta}}_f\| \leq M_{\theta_f}$, $\|\hat{\boldsymbol{\theta}}_g\| \leq M_{\theta_g}$ 且 $\hat{y}_{gl} \geq \varepsilon, l = 1, \dots, N$.

由式(22)的第1个等式得

$$\dot{V}_\sigma \leq |\sigma| [|f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)| + F(\mathbf{x}) + |g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)| |u_f| + K_2(\mathbf{x}) |u_f|] - \sigma g(\mathbf{x}) u_s - \sigma g(\mathbf{x}) u_c + |\sigma| D(\mathbf{x}). \quad (23)$$

由式(16)可知, 当 $V_\sigma > \bar{V}$ 时, $I(\bar{V}) = 1$, 所以

$$\dot{V}_\sigma \leq -\sigma^2 < 0. \quad (24)$$

由于 V_σ 是时间 t 的连续函数, 故在采样间隔充分小的条件下, 有 $V_\sigma \leq 2\bar{V}$. 因此, $|\sigma| \leq 2\sqrt{\bar{V}}, t \geq 0$. 根据引理1可知 $|e_j| \leq 2^{j+1} \lambda^{j-n} \sqrt{\bar{V}}$, 从而 $e \in L_\infty$. 根据假设4)及 $\mathbf{x} = \mathbf{e} + (y_d, \dot{y}_d, \dots, y_d^{(n-1)})^T$ 得

$$\|\mathbf{x}\| \leq M_d + \sum_{j=1}^n 2^{j+1} \lambda^{j-n} \sqrt{\bar{V}}, \forall t \geq 0. \quad (25)$$

2) 令

$$V(t) = V_\sigma + \frac{1}{2\eta_1} \boldsymbol{\varphi}_f^T \boldsymbol{\varphi}_f + \frac{1}{2\eta_2} \boldsymbol{\varphi}_g^T \boldsymbol{\varphi}_g + \frac{1}{2\eta_3} (\hat{\varepsilon}_\nu - \varepsilon_\nu)^2. \quad (26)$$

将 $V(t)$ 对时间 t 求导得

$$\dot{V}(t) = \dot{V}_\sigma + \frac{1}{\eta_1} \boldsymbol{\varphi}_f^T \dot{\boldsymbol{\theta}}_f + \frac{1}{\eta_2} \boldsymbol{\varphi}_g^T \dot{\boldsymbol{\theta}}_g + \frac{1}{\eta_3} (\hat{\varepsilon}_\nu - \varepsilon_\nu) \dot{\hat{\varepsilon}}_\nu. \quad (27)$$

将式(13)~(20)及式(22)的第2个等式代入式(27), 类似于文献[8]中的讨论易得

$$\begin{aligned} \dot{V}(t) \leq & -\sigma^2 + I_1 \sigma \boldsymbol{\varphi}_f^T \frac{\hat{\boldsymbol{\theta}}_f \hat{\boldsymbol{\theta}}_f^T}{\|\hat{\boldsymbol{\theta}}_f\|} \frac{\partial f(\mathbf{x}, \hat{\boldsymbol{\theta}}_f)}{\partial \hat{\boldsymbol{\theta}}_f} + I_2 \sigma \boldsymbol{\varphi}_{g1}^T \hat{\boldsymbol{\theta}}_{g1} \cdot \\ & [\hat{\boldsymbol{\theta}}_{g1}^T \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g1}} + \hat{\boldsymbol{\theta}}_{g2}^T \frac{\partial g(\mathbf{x}, \hat{\boldsymbol{\theta}}_g)}{\partial \hat{\boldsymbol{\theta}}_{g2}}] / \|\hat{\boldsymbol{\theta}}_{g1}\|^2. \end{aligned} \quad (28)$$

其中: $\boldsymbol{\varphi}_{g1}$ 是向量 $\boldsymbol{\varphi}_g$ 对应于 $\hat{\boldsymbol{\theta}}_{g1}$ 的分量所构成的列向量; $I_1 = 0(1)$, 当式(17)的第1(2)个条件成立时; $I_2 = 0(1)$, 当式(19)的第1(2)个条件成立时. 类似于

文献[8]中的讨论不难推出

$$\dot{V}(t) \leq -\sigma^2 \leq 0, \forall t \geq 0. \quad (29)$$

因此, $V(t) \in L_\infty$ 且 $V(+\infty)$ 存在, 进一步有 $\int_0^\infty \sigma^2 dt \leq V(0) - V(+\infty) < +\infty$. 又因为 $\sigma, \dot{\sigma} \in L_\infty$, 所以由Barbalat's 引理可知 $\lim_{t \rightarrow \infty} \sigma(t) = 0$. 又

因为 $\frac{s^i}{h(s)} = \frac{s^i}{(s+\lambda)^n} = \sum_{j=1}^n \frac{l_{ij}}{(s+\lambda)^j}$, 所以其脉

冲响应为 $\sum_{j=1}^n \frac{l_{ij}}{(j-1)!} t^{j-1} e^{-\lambda t}$. 进一步有 $e_i(t) = \int_0^t \sum_{j=1}^n \frac{l_{ij}}{(j-1)!} \tau^{j-1} e^{-\lambda \tau} \sigma(t-\tau) d\tau \rightarrow 0, t \rightarrow \infty, i = 0, 1, \dots, n-1$.

证毕.

5 仿真结果(Simulation results)

例1 考虑如下非线性系统

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = (1-x_1^2)x_2 - x_1 + (1+x_1^2+x_2^2)u + 0.1x_1 \sin(0.5t), \\ y = x_1. \end{cases}$$

控制目标是使系统状态 x_1 跟踪参考轨迹 $y_d = 0.5(\sin t + \sin(0.5t))$.

仿真中取: $F(\mathbf{x}) = (1+x_1^2)^2/2 + x_2^2/2 + |x_1|$, $K_1(\mathbf{x}) = 1$, $K_2(\mathbf{x}) = 1 + x_1^2 + x_2^2$, $k_1 = 2$, $k_2 = 1$, $D(\mathbf{x}) = 0.1|x_1|$, $\mathbf{x}(0) = (0, 0)^T$, $M_{\theta_f} = M_{\theta_g} = 4$, 模糊规则数 $M = N = 5$, $\bar{V} = 3$, $\hat{\varepsilon}_\nu(0) = 0.5$, 初始可调参数 $\hat{\boldsymbol{\theta}}_f(0) \in \mathbb{R}^{25}$ 在区间 $[-0.5, 0.5]$ 上随机选取, 初始可调参数 $\hat{\boldsymbol{\theta}}_g(0) \in \mathbb{R}^{25}$ 及 $e_0(0)$ 在区间 $[0, 1]$ 上随机选取, $\eta_1 = 4$, $\eta_2 = 2$, $\eta_3 = 2$, $\varepsilon = 0.01$, $b_{f0} = b_{g0} = 0.01$, 仿真结果如图1~3所示.

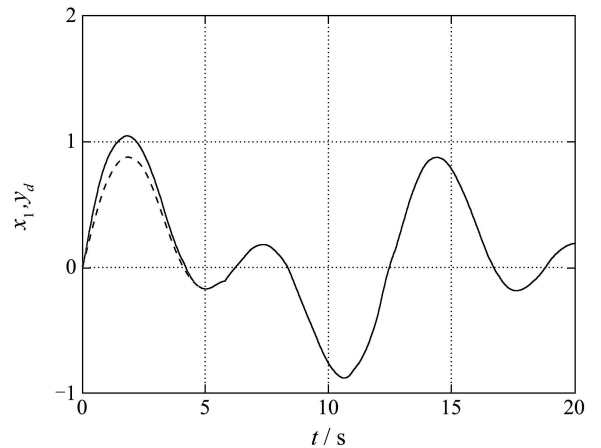


图1 状态 x_1 (实线) 和期望跟踪信号 y_d (虚线) 的曲线
Fig. 1 State x_1 (solid line) and its desired tracking signal y_d (dashed line)

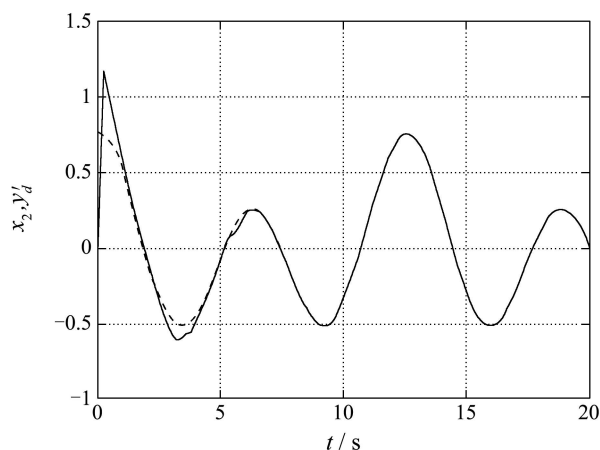


图2 状态 x_2 (实线)和期望跟踪信号 y_d' (虚线)的曲线
Fig. 2 State x_2 (solid line) and its desired tracking signal y_d' (dashed line)

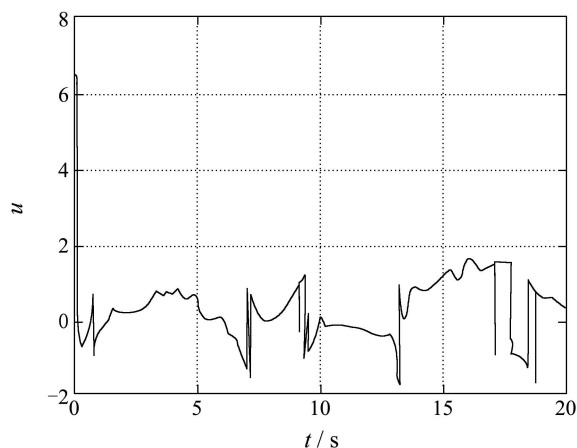


图3 控制信号
Fig. 3 Control signal

6 结论(Conclusions)

本文利用具有非线性可调参数的模糊系统去逼近过程未知函数,提出了一种具有误差补偿的积分变结构间接自适应模糊控制器的设计方案.根据李雅普诺夫方法,确定了模糊系统中可调参数和逼近误差的自适应律.通过理论分析,证明了闭环控制系统的渐近稳定性.

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