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广义预测控制中 Diophantine矩阵多项式方程的显式解

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摘要: 直接利用被控对象的离散差分方程推导出多变量广义预测控制中Diophantine 矩阵多项式方程的显式解, 从而避免了其递推求解或迭代求解, 使广义预测控制的应用更加方便.

关键词: 广义预测控制; Diophantine 矩阵多项式方程; 显式解

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Explicit solution of Diophantine matrix polynomial equations in generalized predictive control

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Abstract: Based on the discrete-time difference equation of the plant, an explicit solution of Diophantine matrix polynomial equations in multivariable generalized predictive control is derived without appealing to Diophantine matrix polynomial equations recursions or iterations, which makes the applications of generalized predictive control much more convenient.

Key words: generalized predictive control; Diophantine matrix polynomial equations; explicit solution

1 引言 (Introduction)

从Clarke等在文[1 ∼ 3]中提出单变量广义预测控制算法以来, 由于该算法具有模型参数少, 对扰动、随机噪声、时滞变化等有较强的鲁棒性等特点, 使其在工业过程控制中得到了成功的应用. 但在实际应用中, 许多工业过程是多输入多输出系统, 因此, 对于多变量广义预测控制方法的研究更具实际意义. 如同单变量系统一样, 在计算多变量系统广义预测控制的控制律时, 亦需要根据预测步数的不同, 对 Diophantine 矩阵多项式方程迭代求解或递推求解, 这使该算法的应用不太方便.

本文推导出了直接用被控对象参数矩阵表示的 Diophantine 矩阵多项式方程显式解, 这样避免了其迭代求解或递推求解, 为广义预测控制在工业控制中的应用提供了方便.

2 问题描述 (Problem statement)

被控对象采用如下的的数学模型来描述[4]:

$$A(z^{-1})\Delta y(t) = B(z^{-1})\Delta u(t-1) + \omega(t). \quad (1)$$

其中: $u(t)$ 和$y(t)$分别是$n\times 1$的输入和输出向量, $\omega(t)$是$n\times 1$的互不相关的随机扰动向量, $\Delta = \mathrm{diag}(1-z^{-1})$, $A(z^{-1}) = I + A_1z^{-1} + \cdots + A_{n_A}z^{-n_A}$, $B(z^{-1}) = B_0 + B_1z^{-1} + \cdots + B_{n_B}z^{-n_B}$.

为了求得广义预测控制律, 需要求解如下的Diophantine 矩阵多项式方程[5]:

$$I = E_j(z^{-1})A(z^{-1})\Delta + z^{-j}F_j(z^{-1}), \quad (2)$$

$$E_j(z^{-1})B(z^{-1}) = G_j(z^{-1}) + z^{-j}H_j(z^{-1}). \quad (3)$$

其中:

$$j = 1,\cdots,N,$$
$$E_j(z^{-1}) = E_0 + E_1z^{-1} + \cdots + E_{j-1}z^{-j+1},$$
$$F_j(z^{-1}) = F_0^j + F_1^jz^{-1} + \cdots + F_{n_A}^jz^{-n_A},$$
$$G_j(z^{-1}) = G_0 + G_1z^{-1} + \cdots + G_{j-1}z^{-j+1},$$
$$H_j(z^{-1}) = H_0^j + H_1^jz^{-1} + \cdots + H_{n_B-1}^jz^{-n_B+1}.$$

3 Diophantine 矩阵多项式方程的求解 (Solution of Diophantine matrix polynomial equation)

为研究问题方便, 记

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$$\max\{n_a+1,n_b+1\}=n,$$
$$A(z^{-1})\Delta=I+\overline{A}_1z^{-1}+\cdots+\overline{A}_nz^{-n}=\overline{A}(z^{-1}),$$
$$\overline{B}_0+\overline{B}_1z^{-1}+\cdots+\overline{B}_{n-1}z^{-(n-1)}=\overline{B}(z^{-1}).$$

式中:

$$\overline{A}_i=\begin{cases}A_1-I, & i=1,\\ A_i-A_{i-1}, & 2\leqslant i\leqslant n_A,\\ -A_{n_A}, & i=n_A+1,\\ O, & i>n_A+1,\end{cases}$$
$$\overline{B}_i=\begin{cases}B_i, & 0\leqslant i\leqslant n_B,\\ O, & i>n_B.\end{cases}$$

则式(1)可写为

$$\overline{A}(z^{-1})y(t)=\overline{B}(z^{-1})\Delta u(t-1)+\omega(t). \tag{4}$$

式(2)(3)可写为

$$I=E_j(z^{-1})\overline{A}(z^{-1})+z^{-j}F_j(z^{-1}), \tag{5}$$
$$E_j(z^{-1})\overline{B}(z^{-1})=G_j(z^{-1})+z^{-j}H_j(z^{-1}). \tag{6}$$

式中 $j=1,\cdots,N$. 由式(4)和式(5)(6)可得

$$Y=GU+Fy(t)+H\Delta u(t-1)+E. \tag{7}$$

其中:

$$Y=[y(t+1)^{\mathrm T},\cdots,y(t+N)^{\mathrm T}]^{\mathrm T},$$
$$U=[\Delta u(t)^{\mathrm T},\cdots,\Delta u(t+N_u-1)^{\mathrm T}]^{\mathrm T},$$
$$F=[F_1(z^{-1})^{\mathrm T},\cdots,F_N(z^{-1})^{\mathrm T}]^{\mathrm T},$$
$$H=[H_1(z^{-1})^{\mathrm T},\cdots,H_N(z^{-1})^{\mathrm T}]^{\mathrm T},$$
$$E=[(E_1(z^{-1})\omega(t+1))^{\mathrm T},\cdots,(E_N(z^{-1})\omega(t+N))^{\mathrm T}]^{\mathrm T},$$
$$G=\begin{bmatrix}G_0 & & & \\ G_1 & G_0 & & \\ \vdots & \vdots & \ddots & \\ G_{Nu-1} & G_{Nu-2} & \cdots & G_0\\ \vdots & \vdots & & \vdots\\ G_{N-1} & G_{N-2} & \cdots & G_{N-Nu}\end{bmatrix}.$$

上式中N_u 是控制时域. 由式(4)得

$$y(t+1)=$$
$$\overline{B}_0\Delta u(t)-\sum_{i=1}^{n}\overline{A}_iz^{-(i-1)}y(t)+$$
$$\sum_{i=1}^{n-1}\overline{B}_iz^{-(i-1)}\Delta u(t-1)+\omega(t+1) \tag{8}$$

当$j=2,\cdots,n$时,

$$y(t+j)+\sum_{i=1}^{j-1}\overline{A}_iy(t-i+j)=$$
$$\sum_{i=0}^{j-1}\overline{B}_i\Delta u(t-i+j-1)-\sum_{i=j}^{n}\overline{A}_iz^{-(i-j)}y(t)+$$
$$\sum_{i=j}^{n-1}\overline{B}_iz^{-(i-j)}\Delta u(t-1)+\omega(t+j). \tag{9}$$

记

$$\overline{\overline{A}}=\begin{bmatrix}-\overline{A}_1 & I & O & \cdots & O\\ -\overline{A}_2 & O & I & \cdots & O\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ -\overline{A}_{n-1} & O & O & \cdots & I\\ -\overline{A}_n & O & O & \cdots & O\end{bmatrix},\ \overline{\overline{\boldsymbol c}}=\begin{bmatrix}I\\ O\\ \vdots\\ O\end{bmatrix}.$$

取

$$\varPhi_n=\begin{bmatrix}\overline{\overline{\boldsymbol c}}^{\mathrm T}\\ \overline{\overline{\boldsymbol c}}^{\mathrm T}\overline{\overline{A}}\\ \vdots\\ \overline{\overline{\boldsymbol c}}^{\mathrm T}\overline{\overline{A}}^{n-1}\end{bmatrix},\varPhi=\begin{bmatrix}\overline{\overline{\boldsymbol c}}^{\mathrm T}\\ \overline{\overline{\boldsymbol c}}^{\mathrm T}\overline{\overline{A}}\\ \vdots\\ \overline{\overline{\boldsymbol c}}^{\mathrm T}\overline{\overline{A}}^{N-1}\end{bmatrix}.$$

I) 当$n\geqslant N$时, 式(8)和(9)写成向量形式为

$$R^n_{\overline{A}}Y_n=R^n_{\overline{B}}U_n+M_n(z^{-1})y(t)+$$
$$R_n(z^{-1})\Delta u(t-1)+\varOmega_n \tag{10}$$

其中:

$$Y_n=[y(t+1)^{\mathrm T},\cdots,y(t+n)^{\mathrm T}]^{\mathrm T},$$
$$U_n=[\Delta u(t)^T,\cdots,\Delta u(t+n-1)^{\mathrm T}]^{\mathrm T},$$
$$\varOmega_n=[\omega(t+1)^{\mathrm T},\cdots,\omega(t+n)^{\mathrm T}]^{\mathrm T},$$
$$R^n_{\overline{A}}=\begin{bmatrix}I & O & \cdots & O\\ \overline{A}_1 & I & \cdots & O\\ \vdots & \vdots & \vdots & \vdots\\ \overline{A}_{n-1} & \overline{A}_{n-2} & \cdots & I\end{bmatrix},$$
$$R^n_{\overline{B}}=\begin{bmatrix}\overline{B}_0 & O & \cdots & O\\ \overline{B}_1 & \overline{B}_0 & \cdots & O\\ \vdots & \vdots & \vdots & \vdots\\ \overline{B}_{n-1} & \overline{B}_{n-2} & \cdots & \overline{B}_0\end{bmatrix},$$
$$M_n(z^{-1})=\begin{bmatrix}-\overline{A}_1-\overline{A}_2z^{-1}-\cdots-\overline{A}_nz^{-(n-1)}\\ -\overline{A}_2-\overline{A}_3z^{-1}-\cdots-\overline{A}_nz^{-(n-2)}\\ \vdots\\ -\overline{A}_n\end{bmatrix},$$
$$R_n(z^{-1})=\begin{bmatrix}\overline{B}_1+\overline{B}_2z^{-1}+\cdots+\overline{B}_{n-1}z^{-(n-2)}\\ \overline{B}_2+\overline{B}_3z^{-1}+\cdots+\overline{B}_{n-1}z^{-(n-3)}\\ \vdots\\ \overline{B}_{n-1}\\ O\end{bmatrix}.$$

易知$\varPhi_n$与 $R^n_{\overline{A}}$互为逆矩阵, 由式(10)得

$$Y_n=\varPhi_n[R^n_{\overline{B}}U_n+M_n(z^{-1})y(t)+$$
$$R_n(z^{-1})\Delta u(t-1)+\varOmega_n]. \tag{11}$$

又在广义预测控制中, $\Delta u(t+j)=0(j=N_u,\cdots,N)$, 由式(11)知

$$Y=\varPhi R^n_{N_u}U+\varPhi M_n(z^{-1})y(t)+$$
$$\varPhi R_n(z^{-1})\Delta u(t-1)+\varPhi\varOmega_n. \tag{12}$$

其中$R^n_{N_u}$是$R^n_{\overline{B}}$的前$n\times N_u$列.

比较式(7)和式(12)得

$$\begin{cases} G = \Phi R^n_{N_u}, F(z^{-1}) = \Phi M_n(z^{-1}), \\ H(z^{-1}) = \Phi R_n(z^{-1}), E = \Phi \Omega_n. \end{cases} \tag{13}$$

由式(13)知 $\begin{bmatrix} G_0 \\ G_1 \\ \vdots \\ G_{N-1} \end{bmatrix} = \Phi \begin{bmatrix} \overline{B}_0 \\ \overline{B}_1 \\ \vdots \\ \overline{B}_{n-1} \end{bmatrix}$. 令$\Omega_N = [\omega(t+1)^{\mathrm{T}}, \cdots, \omega(t+N)^{\mathrm{T}}]^{\mathrm{T}}$, 则

$$E = \begin{bmatrix} E_0 & & & \\ E_1 & E_0 & & \\ \vdots & \vdots & \ddots & \\ E_{N-1} & E_{N-2} & \cdots & E_0 \end{bmatrix} \Omega_N.$$

令Φ^N表示Φ的前$n \times N$ 列, 则$\Phi = [\Phi^N \quad O]$, 所以$\Phi\Omega_n = \Phi^N \Omega_N$, 因此

$$\begin{bmatrix} E_0 & & & \\ E_1 & E_0 & & \\ \vdots & \vdots & \ddots & \\ E_{N-1} & E_{N-2} & \cdots & E_0 \end{bmatrix} = \Phi^N,$$

所以

$$\begin{bmatrix} E_0 \\ E_1 \\ \vdots \\ E_{N-1} \end{bmatrix} = \Phi^N \begin{bmatrix} I \\ O \\ \vdots \\ O \end{bmatrix} = \Phi \begin{bmatrix} I \\ O \\ \vdots \\ O \end{bmatrix}.$$

II) 当 $n < N$时, 把以上I)中的n换成N, 然后按照I) 中从式(10)到式(11)的推导思路可得

$$Y = \Phi_N R^N_{N_u} U + \Phi_N M_N(z^{-1}) y(t) + \Phi_N R_N(z^{-1}) \Delta u(t-1) + \Phi_N \Omega_N. \tag{14}$$

其中$\Phi_N, R^N_{\overline{B}}, M_N(z^{-1}), R_N(z^{-1})$ 分别是把以上$\Phi_n, R^n_{\overline{B}}, M_n(z^{-1}), R_n(z^{-1})$中的$n$换成$N$后所得, $R^N_{N_u}$是$R^N_{\overline{B}}$的前$n \times N_u$列.

比较式(7)和式(14)得

$$\begin{cases} G = \Phi_N R^N_{N_u}, F(z^{-1}) = \Phi_N M_N(z^{-1}), \\ H(z^{-1}) = \Phi_N R_N(z^{-1}),\ E = \Phi_N \Omega_N. \end{cases} \tag{15}$$

当$n < N$时, $\Phi_N = [\Phi \ \ *]$, 由式(15)知

$$\begin{bmatrix} G_0 \\ G_1 \\ \vdots \\ G_{N-1} \end{bmatrix} = \Phi \begin{bmatrix} \overline{B}_0 \\ \overline{B}_1 \\ \vdots \\ \overline{B}_{n-1} \end{bmatrix}, \begin{bmatrix} E_0 \\ E_1 \\ \vdots \\ E_{N-1} \end{bmatrix} = \Phi \begin{bmatrix} I \\ O \\ \vdots \\ O \end{bmatrix},$$

$$F(z^{-1}) = [\Phi \ \ *] \begin{bmatrix} M_n(z^{-1}) \\ O \end{bmatrix} = \Phi M_n(z^{-1}),$$

$$H(z^{-1}) = [\Phi \ \ *] \begin{bmatrix} R_n(z^{-1}) \\ O \end{bmatrix} = \Phi R_n(z^{-1}).$$

综上所述, 以上两种情况下所得的$G_j(z^{-1})$, $E_j(z^{-1}), F(z^{-1}), H(z^{-1})$具有同一形式, 又

$$M_n(z^{-1}) = \begin{bmatrix} -\overline{A}_1 & -\overline{A}_2 & \cdots & -\overline{A}_n \\ -\overline{A}_2 & -\overline{A}_3 & \cdots & O \\ \vdots & \vdots & & \vdots \\ -\overline{A}_n & O & \cdots & O \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(n-1)} \end{bmatrix},$$

$$R_n(z^{-1}) = \begin{bmatrix} \overline{B}_1 & \overline{B}_2 & \cdots & \overline{B}_{n-1} & O \\ \overline{B}_2 & \overline{B}_3 & \cdots & O & O \\ \vdots & \vdots & & \vdots & \vdots \\ \overline{B}_{n-1} & O & \cdots & O & O \\ O & O & \cdots & O & O \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(n-1)} \end{bmatrix}.$$

因此有如下定理:

定理 1 在多变量广义预测控制中, 用方程(1)参数矩阵表示的Diophantine 矩阵多项式方程解的系数矩阵为

$$\begin{bmatrix} E_0 \\ E_1 \\ \vdots \\ E_{N-1} \end{bmatrix} = \Phi \begin{bmatrix} I \\ O \\ \vdots \\ O \end{bmatrix}, \begin{bmatrix} G_0 \\ G_1 \\ \vdots \\ G_{N-1} \end{bmatrix} = \Phi \begin{bmatrix} \overline{B}_0 \\ \overline{B}_1 \\ \vdots \\ \overline{B}_{n-1} \end{bmatrix},$$

$$\begin{bmatrix} F^1_0 & F^1_1 & \cdots & F^1_{n-1} \\ F^2_0 & F^2_1 & \cdots & F^2_{n-1} \\ \vdots & \vdots & & \vdots \\ F^N_0 & F^N_1 & \cdots & F^N_{n-1} \end{bmatrix} = \Phi \begin{bmatrix} -\overline{A}_1 & -\overline{A}_2 & \cdots & -\overline{A}_n \\ -\overline{A}_2 & -\overline{A}_3 & \cdots & O \\ \vdots & \vdots & & \vdots \\ -\overline{A}_n & O & \cdots & O \end{bmatrix},$$

$$\begin{bmatrix} H^1_0 & H^1_1 & \cdots & H^1_{n-1} \\ H^2_0 & H^2_1 & \cdots & H^2_{n-1} \\ \vdots & \vdots & & \vdots \\ H^N_0 & H^N_1 & \cdots & H^N_{n-1} \end{bmatrix} = \Phi \begin{bmatrix} \overline{B}_1 & \overline{B}_2 & \cdots & \overline{B}_{n-1} & O \\ \overline{B}_2 & \overline{B}_3 & \cdots & O & O \\ \vdots & \vdots & & \vdots & \vdots \\ \overline{B}_{n-1} & O & \cdots & O & O \\ O & O & \cdots & O & O \end{bmatrix}.$$

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