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时变关联系统的分散自适应输出反馈控制

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摘要: 针对一类具有动静态关联项和未建模动态的时变关联系统, 通过引入输入滤波器及一系列坐标变换, 给出了一种分散自适应输出反馈控制器的设计方案. 当时变参数的变化率属于$\mathrm{L}_1\bigcap\mathrm{L}_\infty$, 外界干扰属于$\mathrm{L}_2\bigcap\mathrm{L}_\infty$, 未建模动态的幅值在某些范围内变化时, 证明了闭环系统的稳定性, 且每一个子系统的输出收敛于零. 仿真例子验证了这一控制方案的有效性.

关键词: 时变关联系统; 投影算法; 分散自适应控制

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Decentralized adaptive output feedback control for time-varying interconnected systems

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Abstract: For a class of time-varying interconnected systems with dynamic and static interconnections and unmodelled dynamics, by introducing input filters and a series of coordinate changes, a design scheme of decentralized adaptive output feedback controller is given. It is proved that when the variation rates of time-varying parameters belong to $\mathrm{L}_1\bigcap\mathrm{L}_\infty$, disturbances will belong to $\mathrm{L}_2\bigcap\mathrm{L}_\infty$, and the magnitude of the unmodeled dynamics is varying in some ranges. The stability of the closed-loop system is proved, and the output of each subsystem converges to zero. A simulation example demonstrates the effectiveness of the control scheme.

Key words: time-varying interconnected systems; projection algorithm; decentralized adaptive control

1 引言(Introduction)

对于关联系统, 分散自适应控制是一个有效的控制方案. 它的开创性工作是文献[1], 最近的研究主要集中在分散自适应输出反馈控制上. 文献[2]针对具有动态、静态关联项和未建模动态的关联系统, 利用MT-滤波器和反推设计方法, 证明了除参数估计外闭环系统的所有信号的渐近收敛性.

然而, 上述工作仅限于每个子系统的参数都是未知常数. 由于在许多实际控制问题中, 受控对象的数学模型的参数往往是时变的, 因此无论从理论还是实际应用上, 研究时变关联系统的分散自适应控制器的设计和分析是一项很有意义的工作. 基于文献[2], 文献[3]针对一类时变关联系统, 利用滤波器, 反推和$\sigma-$修正等设计方法, 保证了闭环系统的所有信号有界. 然而文献[3]中存在如下缺陷: 1) 时变参数的变化率限制在较小的范围内; 2) 闭环系统的输出依赖于时变参数的变化率, 不能渐近收敛到零.

本文针对具有动静态关联项、未建模动态及外界干扰的时变关联系统, 通过引入滤波器及一系列的坐标变换得到一闭环系统. 基于此系统, 给出了一种分散自适应输出反馈控制的设计方法. 当时变参数的变化率属于$\mathrm{L}_1\bigcap\mathrm{L}_\infty$, 外界干扰属于$\mathrm{L}_2\bigcap\mathrm{L}_\infty$, 未建模动态的幅值在某些范围内变化时, 证明了闭环系统的全局稳定性, 且每一个闭环子系统的输出渐近收敛到零. 仿真例子验证了控制方案的有效性.

2 问题的提出(Problem statement)

考虑由N个时变子系统组成的关联系统, 第i个子系统表示如下:

$$y_i(t)=\big(1+\mu_{ii}\Delta_{ii}(s)\big)A_i(s,t)^{-1}\Big(B_i(s,t)u_i(t)+$$

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$$D_i(s)\Big(\sum_{j=1,j\neq i}^{N} f'_{ij}(t,y_j)+w_i(t)\Big)\Big)+$$
$$\sum_{j=1,j\neq i}^{N}\mu_{ij}\Delta_{ij}(s)y_j(t),\ i=1,\cdots,N. \quad (1)$$

其中: $u_i(t)$, $y_i(t)\in\mathbb{R}$分别表示第i个子系统的输入和输出,

$$A_i(s,t)=s^{n_i}-s^{n_i-1}a_{i1}(t)-\cdots-sa_{i,n_i-1}(t)-a_{i,n_i}(t),$$
$$B_i(s,t)=s^{n_i-\rho_i}b'_{i,\rho_i}(t)+\cdots+sb'_{i,n_i-1}(t)+b'_{i,n_i}(t),$$

$a_{ij}(t)$和$b'_{ik}(t)(j=1,\cdots,n_i,k=\rho_i,\cdots,n_i)$为未知时变参数, s表示微分算子, $D_i(s)=(s^{n_i-1},\cdots,s,1)$. $f'_{ij}(t,y_j)\in\mathbb{R}^{n_i}$, $\Delta_{ij}(s)y_j(i\neq j)$分别表示第$j$个子系统到第$i$个子系统的静态关联项和动态关联项, $\Delta_{ii}(s)$表示第i个子系统的未建模动态, $\mu_{ij},\mu_{ii}>0\ (i,j=1,\cdots,N)$分别表示动态关联项和未建模动态的幅值, $w_i(t)\in\mathbb{R}^{n_i}$表示第$i$个子系统的外界干扰.

系统(1)有以下状态空间实现[3]:

$$\begin{cases}\dot x_i(t)=A_{ci}x_i(t)+a_i(t)x_{i1}(t)+\\ \qquad b_i(t)\dfrac{1}{\gamma_i(t)}u_i(t)+w_i(t)+f_i(t),\\ y_i(t)=x_{i1}(t)+\aleph_i(t).\end{cases} \quad (2)$$

其中:

$$A_{ci}=\begin{pmatrix}0 & I_{n_i-1}\\ \vdots & \\ 0\ 0 & \cdots 0\end{pmatrix},$$
$$a_i(t)=(a_{i1},\cdots,a_{i,n_i})^{\mathrm T},$$
$$b_i(t)=(0,\cdots,1,b_{i,\rho_i+1},\cdots,b_{i,n_i})^{\mathrm T},$$
$$f_i(t)=\sum_{j=1,j\neq i}^{N} f'_{ij}(t,y_j),$$
$$\aleph_i=\mu_{ii}\Delta_{ii}(s)x_{i1}+\sum_{j=1,j\neq i}^{N}\mu_{ij}\Delta_{ij}(s)y_j.$$

对系统(2)作以下假设:

假设 1 $a_i(t)$, $b_i(t)$, $\gamma_i(t)$光滑且一致有界, 且对$\forall t\geqslant 0$, 其导数一致有界. 对$\forall t\geqslant 0$, 设$|\gamma_i(t)|\leqslant h_i$, $\|a_i(t)\|\leqslant\bar h_i$, 其中$h_i,\bar h_i$为有界正实数.

假设 2 $w_i(t)=0$, $f_i(t)=0$, $\aleph_i(t)=0$时, 系统(2)的零动态指数稳定.

假设 3 对 $\forall t\geqslant 0$, $w_i(t)$ 有界. $f'_{ij}(t,y_j)$ 满足 $\|f'_{ij}(t,y_j)\|\leqslant\sum\limits_{k=1}^{K_{ij}} r'_{ij}|y_j|^k$, 其中$r'_{ij}>0$, $K_{ij}>0$是已知常数.

假设 4 每个子系统的阶次n_i和相对阶ρ_i已知. 设$\gamma_i(t)$的符号大于零.

假设 5 $\Delta_{ij}(s)(i,j=1,2,\cdots,N)$稳定且严格正则.

控制目标: 设计分散自适应输出反馈控制器, 使得闭环系统的所有信号有界, 且输出y_i收敛于零.

3 分散自适应控制器的设计(Design of decentralized adaptive controller)

首先考虑相对阶$\rho_i=1$的情况. 引入输入滤波

$$\dot{\bar\mu}_i[j]=\begin{pmatrix}-\lambda_{i1} & 1\cdots & 0\\ \vdots & \vdots & \vdots\\ 0 & 0\cdots & 1\\ 0 & 0\cdots & -\lambda_{ij}\end{pmatrix}\bar\mu_i[j]+\begin{pmatrix}0\\ \vdots\\ 0\\ 1\end{pmatrix}u_i\triangleq$$
$$\Lambda_{ij}\bar\mu_i[j]+b_{cij}u_i,\ 1\leqslant j\leqslant n_i-1, \quad (3)$$

其中: $\lambda_{ij}(1\leqslant j\leqslant n_i-1)$是正常数, $\bar\mu_i[j]\in\mathbb{R}^j$. 由假设2知当$w_i=0$, $f_i=0$, $\aleph_i=0$时, 由式(2)和(3)组成的系统的零动态也指数稳定. 引入以下坐标变换:

$$\begin{cases}z_i=x_i-\sum\limits_{j=2}^{n_i}\delta_{ij}(t)\sum\limits_{k=2}^{j}d_i[k]\bar\mu_{i,k-1}[j-1],\\ \bar\mu_i[j]=\bar\mu_i[j],\ 1\leqslant j\leqslant n_i-1.\end{cases} \quad (4)$$

其中:

$$d_i[n_i]=(0,\cdots,0,1)^{\mathrm T},$$
$$d_i[j-1]=A_{ci}d_i[j]+\lambda_{i,j-1}d_i[j],\ j=n_i,\cdots,2,$$
$$(\delta_{i1}(t),\cdots,\delta_{i,n_i}(t))^{\mathrm T}=(d_i[1],\cdots,d_i[n_i])^{-1}\frac{b_i(t)}{\gamma_i(t)},$$
$$\delta_{i1}(t)=\frac{1}{\gamma_i(t)},$$

且

$$d_i=d_i[1]=(1,d_{i2},\cdots,d_{i,n_i})^{\mathrm T}$$

满足

$$s^{n_i-1}+d_{i2}s^{n_i-2}+\cdots+d_{i,n_i-1}s+d_{i,n_i}=\prod_{j=1}^{n_i-1}(s+\lambda_{ij}).$$

记

$$d_i[j]=(0,\bar d_i[j]^{\mathrm T})^{\mathrm T},\ 2\leqslant j\leqslant n_i,$$

引入新的坐标变换:

$$\begin{cases}y_i=z_{i1}+\aleph_i,\\ \eta_{ij}=z_{i,j+1}-d_{i,j+1}z_{i1},\\ \qquad 1\leqslant j\leqslant n_i-1,\\ \mu'_i[j]=\bar\mu_i[j]-\begin{pmatrix}0\\ \vdots\\ 1\end{pmatrix}\gamma_i(t)y_i.\end{cases} \quad (5)$$

由式(2)(4)和(5)得

$$\begin{cases} \dot{y}_i = \eta_{i1} + \dfrac{1}{\gamma_i(t)} u_i + \sum\limits_{j=2}^{n_i} \delta_{ij}(t)\mu'_{i1}[j-1] + \\ \qquad d_{i2}y_i + a_{i1}(t)y_i + \delta_{i2}(t)\gamma_i(t)y_i + w_{i1} + \\ \qquad f_{i1} - \big(d_{i2} + a_{i1}(t)\big)\aleph_i + \dot{\aleph}_i, \\ \dot{\eta}_i = \varGamma_i\eta_i - \sum\limits_{j=2}^{n_i} \dot{\delta}_{ij}(t) \sum\limits_{k=2}^{j} \bar{d}_i[k]\mu'_{i,k-1}[j-1] - \\ \qquad \gamma_i(t)y_i \sum\limits_{j=2}^{n_i} \dot{\delta}_{ij}(t)\bar{d}_i[j] + \beta_i y_i + \\ \qquad \bar{a}_i(t)y_i + \bar{w}_i + \bar{f}_i - \big(\beta_i + \bar{a}_i(t)\big)\aleph_i, \\ \dot{\mu}'_i[j] = \varLambda_{ij}\mu'_i[j] - b_{cij}\gamma_i(t)\sum\limits_{k=2}^{n_i} \delta_{ik}\mu'_{i1}[k-1] + \\ \qquad \big(b_{\lambda ij}\gamma_i(t) - b_{cij}(\dot{\gamma}_i(t) + \gamma_i(t)d_{i2} + \\ \qquad \gamma_i^2(t)\delta_{i2}(t) + \gamma_i(t)a_{i1}(t))\big)y_i - \\ \qquad b_{cij}\gamma_i(t)(\eta_{i1} + w_{i1} + f_{i1}) + b_{cij}\gamma_i(t) \times \\ \qquad (a_{i1}(t) + d_{i2})\aleph_i - b_{cij}\gamma_i(t)\dot{\aleph}_i. \end{cases} \tag{6}$$

其中:

$$\varGamma_i = \begin{pmatrix} -d_{i2} & \\ \vdots & I_{n_i-2} \\ -d_{i,n_i} & 0\cdots 0 \end{pmatrix},\ \beta_i = \begin{pmatrix} d_{i3} - d_{i2}^2 \\ \vdots \\ d_{i,n_i} - d_{i2}d_{i,n_i-1} \\ -d_{i2}d_{i,n_i} \end{pmatrix},$$

$$\bar{w}_i = \begin{pmatrix} w_{i2} - d_{i2}w_{i1} \\ \vdots \\ w_{i,n_i} - d_{i,n_i}w_{i1} \end{pmatrix},\ \bar{f}_i = \begin{pmatrix} f_{i2} - d_{i2}f_{i1} \\ \vdots \\ f_{i,n_i} - d_{i,n_i}f_{i1} \end{pmatrix},$$

$$\bar{a}_i(t) = \begin{pmatrix} a_{i2}(t) - d_{i2}a_{i1}(t) \\ \vdots \\ a_{i,n_i}(t) - d_{i,n_i}a_{i1}(t) \end{pmatrix},$$

$$b_{\lambda ij} = (0,\cdots,0,1,-\lambda_{ij})^{\mathrm{T}} \in \mathbb{R}^j.$$

由于系统(2)和(3)的零动态指数稳定, 且式(4)和(5)是一致有界的Lyapunov变换, 则系统(6)的零动态指数稳定.

由Krasowsky定理[5]知存在函数W_{i1}满足

$$\begin{cases} g_{i1}\left\|\begin{pmatrix}\eta_i\\ \mu'_i\end{pmatrix}\right\|^2 \leqslant W_{i1}(\eta_i,\mu'_i,t) \leqslant g_{i2}\left\|\begin{pmatrix}\eta_i\\ \mu'_i\end{pmatrix}\right\|^2, \\ \dot{W}_{i1}(\eta_i,\mu'_i,t) \leqslant -g_{i3}\left\|\begin{pmatrix}\eta_i\\ \mu'_i\end{pmatrix}\right\|^2, \\ \left\|\begin{pmatrix}\dfrac{\partial W_{i1}}{\partial \eta_i} & \dfrac{\partial W_{i1}}{\partial \mu'_i}\end{pmatrix}\right\| \leqslant g_{i4}\left\|\begin{pmatrix}\eta_i\\ \mu'_i\end{pmatrix}\right\|. \end{cases} \tag{7}$$

其中: $\mu'_i = (\mu_i'^{\mathrm{T}}[1],\cdots,\mu_i'^{\mathrm{T}}[n_i-1])^{\mathrm{T}}$, $g_{ij}(1 \leqslant j \leqslant 4)$为适当的正实数. 选取控制律为

$$\begin{cases} u_i = \hat{\gamma}_i\phi_i - \dfrac{1}{4}y_i\phi_i^2, \\ \phi_i = -d_{i2}y_i - \sum\limits_{j=2}^{n_i}\hat{\delta}_{ij}\mu'_{i1}[j-1] - \hat{\theta}_{i1}y_i - \\ \qquad \dfrac{1}{4}y_i(\nu_i^{\mathrm{T}}\nu_i + 4) - c_{i0}y_i - \bar{c}_{i0}y_i^{2K-1} - \\ \qquad \bar{k}_{i0}y_i - k_iy_i - k_{i0}y_i. \end{cases} \tag{8}$$

其中: $c_{i0}, \bar{c}_{i0}, \bar{k}_{i0}, k_i, k_{i0} > 0$为控制器设计参数, $\theta_{i1} = a_{i1} + \delta_{i2}\gamma_i$, $\nu_i = (y_i, \mu'_{i1}[1], \cdots, \mu'_{i1}[n_i-1], y_i)^{\mathrm{T}}$, $\hat{\gamma}_i$, $\hat{\delta}_{ij}$, $\hat{\theta}_{i1}$分别表示γ_i, δ_{ij}和θ_{i1}的估计. 由式(6)和(8)得

$$\begin{cases} \dot{y}_i = \eta_{i1} - \dfrac{\tilde{\gamma}_i}{\gamma_i}\phi_i - \dfrac{1}{4\gamma_i}y_i\phi_i^2 - \dfrac{1}{4}y_i(\nu_i^{\mathrm{T}}\nu_i + 4) + \\ \qquad \tilde{\theta}_i^{\mathrm{T}}\nu_i - c_{i0}y_i - \bar{k}_{i0}y_i - k_iy_i - k_{i0}y_i + w_{i1} - \\ \qquad \bar{c}_{i0}y_i^{2K-1} + f_{i1} - (d_{i2} + a_{i1})\aleph_i + \dot{\aleph}_i, \\ \dot{\eta}_i = \varGamma_i\eta_i - \sum\limits_{j=2}^{n_i}\dot{\delta}_{ij}\sum\limits_{k=2}^{j}\bar{d}_i[k]\mu'_{i,k-1}[j-1] + \\ \qquad y_iS_{\eta_i} + \bar{w}_i + \bar{f}_i - (\beta_i + \bar{a}_i)\aleph_i, \\ \dot{\mu}'_i[j] = \varLambda_{ij}\mu'_i[j] - b_{cij}\gamma_i\sum\limits_{k=2}^{n_i}\mu'_{i1}[k-1] - \\ \qquad b_{cij}\gamma_i\eta_{i1} + y_iS_{\mu'_{ij}} - b_{cij}\gamma_i(w_{i1} + f_{i1}) + \\ \qquad b_{cij}\gamma_i(a_{i1} + d_{i2})\aleph_i - b_{cij}\gamma_i\dot{\aleph}_i, \end{cases} \tag{9}$$

其中:

$$\theta_i = (\theta_{i1}, \delta_{i2}, \cdots, \delta_{i,n_i}, k_{i0})^{\mathrm{T}},$$

$$s_{\eta_i} = (\bar{a}_i - \gamma_i\sum_{j=2}^{n_i}\dot{\delta}_{ij}\bar{d}_i[j] + \beta_i),$$

$$S_{\mu'_{ij}} = b_{\lambda ij}\gamma_i - b_{cij}(\dot{\gamma}_i + \gamma_id_{i2} + \gamma_i^2\delta_{i2} + \gamma_ia_{i1}),$$

$$\tilde{\gamma}_i = \gamma_i - \hat{\gamma}_i,\ \tilde{\theta}_i = \theta_i - \hat{\theta}_i.$$

选取自适应律为

$$\begin{cases} \dot{\hat{\theta}}_i = \mathrm{Proj}(y_i\nu_i, \hat{\theta}_i, \varepsilon_{i1}, r_{\Omega_{i1}}), & \|\hat{\theta}_i(t_0)\| \leqslant r_{\Omega_{i1}}, \\ \dot{\hat{\gamma}}_i = \mathrm{Proj}(-y_i\phi_i, \hat{\gamma}_i, \varepsilon_{i2}, r_{\Omega_{i2}}), & |\hat{\gamma}_i(t_0)| \leqslant r_{\Omega_{i2}}. \end{cases} \tag{10}$$

其中当$p(\hat{\theta}_i) \leqslant 0$, $p(\hat{\theta}_i) \geqslant 0$且$\langle \mathrm{grad}\,p(\hat{\theta}_i), \psi_i\rangle \leqslant 0$时, $\mathrm{Proj}(\psi_i, \hat{\theta}_i, \varepsilon_i, r_{\Omega_i}) = \psi_i$; 当$p(\hat{\theta}_i) > 0$且$\langle \mathrm{grad}p(\hat{\theta}_i), \psi_i\rangle > 0$时, $\mathrm{Proj}(\psi_i, \hat{\theta}_i, \varepsilon_i, r_{\Omega_i}) = \left[I - \frac{p(\hat{\theta}_i)\mathrm{grad}p(\hat{\theta}_i)^{\mathrm{T}}\mathrm{grad}p(\hat{\theta}_i)}{\|\mathrm{grad}p(\hat{\theta}_i)\|^2}\right]\psi_i$. grad表示梯度算法, $\langle\cdot\rangle$表示内积符号, $p(\hat{\theta}_i) = \dfrac{\|\hat{\theta}_i\|^2 - r_{\Omega_i}^2}{\varepsilon_i^2 + 2\varepsilon_i r_{\Omega_i}}$, r_{Ω_i}是以原点为球心的球的半径, ε_i为任意正实数. 因此若$\|\hat{\theta}_i(t_0)\| \leqslant r_{\Omega_i}$, 则由文献[4]知:

1) $\|\hat{\theta}_i(t)\| \leqslant r_{\Omega_i} + \varepsilon_i, \forall t \geqslant t_0$;

2) $\mathrm{Proj}(\psi_i, \hat{\theta}_i, \varepsilon_i, r_{\Omega_i})$是Lipschitz连续的;

3) $\|\text{Proj}(\psi_i,\hat{\theta}_i,\varepsilon_i,r_{\Omega_i})\| \leqslant \|\psi_i\|$;

4) 若 $\|\theta_i\| \leqslant r_{\Omega_i}$, 则 $\tilde{\theta}_i^{\mathrm{T}}\text{Proj}(\psi_i,\hat{\theta}_i,\varepsilon_i,r_{\Omega_i}) \geqslant \tilde{\theta}_i^{\mathrm{T}}\psi_i$.

由假设5可得$\Delta_{ii}(s)x_{i1}$, $\Delta_{ij}(s)y_j$的状态实现为

$$\begin{cases}\begin{cases}\dot{\xi}_{ii} = A_{ii}\xi_{ii} + B_{ii}x_{i1},\\ \Delta_{ii}(s)x_{i1} = (1,0,\cdots,0)\xi_{ii},\end{cases}\\ \begin{cases}\dot{\xi}_{ij} = A_{ij}\xi_{ij} + B_{ij}y_j,\\ \Delta_{ij}(s)y_j = (1,0,\cdots,0)\xi_{ij},\ j\neq i.\end{cases}\end{cases} \tag{11}$$

其中A_{ii}, A_{ij}满足

$$(1,0\cdots,0)(sI_i - A_{ii})^{-1}B_{ii} = \Delta_{ii}(s),$$
$$(1,0\cdots,0)(sI_i - A_{ij})^{-1}B_{ij} = \Delta_{ij}(s).$$

4 主要结果(Main results)

定理 1 对由系统(2), 自适应律(10)和控制律(8)所组成的自适应控制系统, 当$\rho_i = 1$时, 如果假设条件1~5成立, 且$\|\hat{\theta}_i(t_0)\| \leqslant r_{\Omega_{i1}}$, $|\hat{\gamma}_i(t_0)| \leqslant r_{\Omega_{i2}}$, 则一定存在常数$\mu_i^* > 0$, 使得对所有的$\mu_i \in [0,\mu_i^*)$,

1) 闭环系统的所有信号有界;

2) 如果$\|\theta_i\| \leqslant r_{\Omega_{i1}}$, $|\gamma_i| \leqslant r_{\Omega_{i2}}$, 且对$\forall t \geqslant t_1 \geqslant t_0$, $w_i \in \mathrm{L}_2$, $\dot{\theta}_i \in \mathrm{L}_1$, $\dot{\gamma}_i \in \mathrm{L}_1$, 则$\lim\limits_{t\to\infty} y_i(t) = 0$, $\lim\limits_{t\to\infty}\eta_i(t) = 0$, $\lim\limits_{t\to\infty}\mu_i'(t) = 0$.

证 因为A_{ij}稳定, 则存在$P_{ij} > 0$使$P_{ij}A_{ij} + A_{ij}^{\mathrm{T}}P_{ij} = -I$. 考虑函数

$$W_i = \frac{1}{2}y_i^2 + \frac{1}{g_{i3}}W_{i1} + \sum_{j=1}^{N} r_{ij}\xi_{ij}^{\mathrm{T}}P_{ij}\xi_{ij} \tag{12}$$

其中$r_{ij} > 0$待定, W_{i1}满足式(7). 由式(9)对(12)求导得

$$\begin{aligned}\dot{W}_i \leqslant\ & y_i\eta_{i1} - c_{i0}y_i^2 - \bar{k}_{i0}y_i^2 - k_iy_i^2 - k_{i0}y_i^2 -\\ & \bar{c}_{i0}y_i^{2K} - \frac{1}{4\gamma_i}y_i^2\phi_i^2 - \frac{\tilde{\gamma}_i}{\gamma_i}y_i\phi_i + y_i\tilde{\theta}_i^{\mathrm{T}}\nu_i -\\ & \frac{1}{4}y_i^2(\nu_i^{\mathrm{T}}\nu_i + 4) + y_iw_{i1} + y_if_{i1} -\\ & (d_{i2} + a_{i1})y_i\aleph_i + y_i\dot{\aleph}_i - \left\|\begin{pmatrix}\eta_i\\ \mu_i'\end{pmatrix}\right\|^2 +\\ & \frac{g_{i4}}{g_{i3}}\left\|\begin{pmatrix}\eta_i\\ \mu_i'\end{pmatrix}\right\|\Big(W_i + F_i + |y_i|S_i + H_i|\aleph_i| +\\ & G_i|\dot{\aleph}_i|\Big) - r_{ii}\|\xi_{ii}\|^2 + 2r_{ii}\xi_{ii}^{\mathrm{T}}P_{ii}B_{ii}x_{i1} +\\ & \sum_{j=1,j\neq i}^{N}\big(-r_{ij}\|\xi_{ij}\|^2 + 2r_{ij}\xi_{ij}^{\mathrm{T}}P_{ij}B_{ij}y_j\big).\end{aligned}$$

其中:

$$W_i = \left\|\begin{pmatrix}\bar{w}_i\\ b_{ci1}\gamma_iw_{i1}\\ \vdots\\ b_{cin_i-1}\gamma_iw_{i1}\end{pmatrix}\right\|,\ F_i = \left\|\begin{pmatrix}\bar{f}_i\\ b_{ci1}\gamma_if_{i1}\\ \vdots\\ b_{cin_i-1}\gamma_if_{i1}\end{pmatrix}\right\|,$$

$$S_i = \left\|\begin{pmatrix}S_{\eta_i}\\ S_{\mu_{i1}}\\ \vdots\\ S_{\mu_{i,n_i-1}}\end{pmatrix}\right\|,\ G_i = \left\|\begin{pmatrix}b_{ci1}\gamma_i\\ \vdots\\ b_{cin_i-1}\gamma_i\end{pmatrix}\right\|,$$

$$H_i = \left\|\begin{pmatrix}\beta_i + \bar{a}_i\\ b_{ci1}\gamma_i(a_{i1} + d_{i2})\\ \vdots\\ b_{cin_i-1}\gamma_i(a_{i1} + d_{i2})\end{pmatrix}\right\|.$$

取$r_{ij} = \dfrac{e_{ij}}{\|P_{ij}B_{ij}\|^2}$, $\mu_i = \max\limits_{1\leqslant j\leqslant N}\{\mu_{ij}\}$, 其中$e_{ij}$为任意正常数. 经过一系列计算知存在$\mu_i^* > 0$, 当 $\mu_i \in [0,\mu_i^*)$时, 函数$W = \sum\limits_{i=1}^{N}W_i$满足

$$\begin{aligned}\dot{W} \leqslant & \sum_{i=1}^{N}\Big(-k_iy_i^2 - \frac{1}{6}\left\|\begin{pmatrix}\eta_i\\ \mu_i'\end{pmatrix}\right\|^2 -\\ & \frac{1}{4}\sum_{j=1}^{N}r_{ij}\|\xi_{ij}\|^2\Big) + \sum_{i=1}^{N}\Big(\frac{\tilde{\gamma}_i^2}{\gamma_i} + \|\tilde{\theta}_i\|^2 + \bar{W}_i\Big).\end{aligned} \tag{13}$$

由假设条件及投影算法的性质知$\tilde{\gamma}_i$,$\tilde{\theta}_i$有界. 又w_i有界, 则由W的定义和式(13)知y_i,η_i,μ_i',$\xi_{i1},\cdots,\xi_{iN}$有界, 从而可得x_i, u_i有界, 故结论1)成立.

考虑函数

$$V = W + \sum_{i=1}^{N}\big(\frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \frac{\tilde{\gamma}_i^2}{2\gamma_i}\big),$$

经过一系列求导, 求积可得当$w_i \in \mathrm{L}_2$, $\dot{\theta}_i \in \mathrm{L}_1$, $\dot{\gamma}_i \in \mathrm{L}_1$时,

$$\begin{aligned}\lim_{t\to\infty}\int_{t_1}^{t}\sum_{i=1}^{N}\Big(k_iy_i^2 + \frac{1}{6}\left\|\begin{pmatrix}\eta_i\\ \mu_i'\end{pmatrix}\right\|^2 +\\ \frac{1}{4}\sum_{j=1}^{N}r_{ij}\|\xi_{ij}\|^2\Big)\mathrm{d}\tau < \infty,\end{aligned}$$

故 $y_i,\eta_i,\mu_i',\xi_{ij} \in \mathrm{L}_2$. 又 $\dot{y}_i$, $\dot{\eta}_i,\dot{\mu}_i',\dot{\xi}_{ij} \in \mathrm{L}_\infty$, 利用Barbalat引理得结论2)成立. 证毕.

对于系统(2), 当$\rho_i > 1$时, 引入输入滤波

$$\begin{aligned}\dot{\zeta}_i = & \begin{pmatrix}-\bar{\lambda}_{i1} & 1\cdots & 0\\ \vdots & \vdots & \vdots\\ 0 & 0\cdots & 1\\ 0 & 0\cdots & -\bar{\lambda}_{\rho_i-1}\end{pmatrix}\zeta_i + \begin{pmatrix}0\\ \vdots\\ 0\\ 1\end{pmatrix}u_i \triangleq\\ & \Lambda_i\zeta_i + b_{c,\rho_i-1}u_i\end{aligned} \tag{14}$$

及新的坐标变换

$$\begin{cases}\bar{x}_{i1} = x_{i1},\\ \bar{x}_{ij} = x_{ij} - \sum\limits_{k=2}^{\rho_i}\bar{b}_{ij}[k]\zeta_{i,k-1},\ 2\leqslant j\leqslant n_i,\\ \zeta_i = \zeta_i.\end{cases} \tag{15}$$

其中: $\bar{b}_i[\rho_i]=\dfrac{b_i}{\gamma_i}$, $\bar{b}_i[j-1]=A_{ci}\bar{b}_i[j]+\bar{\lambda}_{i,j-1}\bar{b}_i[j]-\dot{\bar{b}}_i[j]$, $2\leqslant j\leqslant\rho_i$. 则系统(2)和(14)转化为

$$\begin{cases}\dot{\bar{x}}_i=A_{ci}\bar{x}_i+\bar{b}_i[1]\zeta_{i1}+a_i\bar{x}_{i1}+w_i+f_i\triangleq\\ \qquad A_{ci}\bar{x}_i+\bar{b}'_i\dfrac{1}{\gamma_i}\zeta_{i1}+a_i\bar{x}_{i1}+w_i+f_i,\\ \zeta_i=\Lambda_i\zeta_i+b_{c,\rho_i-1}u_i,\\ y_i=\bar{x}_{i1}+\aleph_i.\end{cases}\tag{16}$$

系统(16)与(2)相似, 只是ζ_{i1}替代了u_i.

5 仿真例子(A simulation example)

考虑如下的关联系统:

$$\begin{cases}\dot{x}_i(t)=\begin{pmatrix}a_{i1}(t) & 1\\ a_{i2}(t) & 0\end{pmatrix}x_i(t)+\begin{pmatrix}0\\ b_{i1}(t)\end{pmatrix}u_i(t)+\\ \qquad f_i(t,y_j),\\ y_i(t)=x_{i1}(t)+\aleph_i(t),\ i=1,2,\end{cases}\tag{17}$$

其中:

$$\begin{aligned}&x_i=(x_{i1},x_{i2})^{\mathrm{T}},\\ &f_1=\big(0.1y_2^2-y_2\cos y_2,\ 0.1y_2^2-y_2\big)^{\mathrm{T}},\\ &f_2=\big(0.1y_1^2-y_1\cos y_1,\ 0.1y_1^2-y_1\big)^{\mathrm{T}},\\ &\aleph_1=\mu_{11}\frac{0.125}{s+1}x_{11}+\mu_{12}\frac{0.125}{s+1}y_2,\\ &\aleph_2=\mu_{21}\frac{0.125}{s+1}x_{21}+\mu_{22}\frac{0.125}{s+1}y_1.\end{aligned}$$

由式(3)~(5)得

$$\begin{cases}\dot{y}_i=\eta_{i1}+d_{i2}y_i+\dfrac{1}{\gamma_i}u_i+\delta_{i2}\mu'_{i1}[1]+a_{i1}y_i+\\ \qquad \delta_{i2}\gamma_iy_i+w_{i1}+f_{i1}-\big(d_{i2}+a_{i1}\big)\aleph_i+\dot{\aleph}_i,\\ \dot{\eta}_i=\Gamma_i\eta_i+\beta_iy_i-\dot{\delta}_{i2}\bar{d}_i[2]\mu'_{i1}[1]-\gamma_iy_i\dot{\delta}_{i2}\bar{d}_i[2]+\\ \qquad \bar{a}_iy_i+\bar{w}_i+\bar{f}_i-\big(\beta_i+\bar{a}_i\big)\aleph_i,\\ \dot{\mu}'_i[1]=\Lambda_{i1}\bar{\mu}_i[1]-b_{ci1}\gamma_i\mu'_{i1}[1]+(b_{\lambda i1}\gamma_i-\\ \qquad b_{ci1}(\dot{\gamma}_i+\gamma_id_{i2}+\gamma_i^2\delta_{i2}+\gamma_ia_{i1}))y_i-\\ \qquad b_{ci1}\gamma_i(\eta_{i1}+w_{i1}+f_{i1})+\\ \qquad b_{ci1}\gamma_i(a_{i1}+d_{i2})\aleph_i-b_{ci1}\gamma_i\dot{\aleph}_i.\end{cases}\tag{18}$$

其中$d_i[2]=(0,1)^{\mathrm{T}}$, $d_i[1]=(1,\lambda_{i1})^{\mathrm{T}}$, $(\delta_{i1},\delta_{i1})^{\mathrm{T}}=(d_i[1],d_i[2])^{-1}(0,b_{i1})^{\mathrm{T}}$. 选取控制律为

$$\begin{cases}u_i=\hat{\gamma}_i\phi_i-\dfrac{1}{4}y_i\phi_i^2,\\ \phi_i=-d_{i2}y_i-\hat{\delta}_{i2}\mu'_{i1}[1]-\hat{\theta}_{i1}y_i-\\ \qquad \dfrac{1}{4}y_i(\nu_i^{\mathrm{T}}\nu_i+4)-c_{i0}y_i-\\ \qquad \bar{c}_{i0}y_i^{2K-1}-\bar{k}_{i0}y_i.\end{cases}\tag{19}$$

自适应律为

$$\begin{cases}\dot{\hat{\theta}}_i=\mathrm{Proj}(y_i\nu_i,\hat{\theta}_i,\varepsilon_{i1},r_{\Omega_{i1}}),\quad \|\hat{\theta}_i(t_0)\|\leqslant r_{\Omega_{i1}},\\ \dot{\hat{\gamma}}_i=\mathrm{Proj}(-y_i\phi_i,\hat{\gamma}_i,\varepsilon_{i2},r_{\Omega_{i2}}),\ |\hat{\gamma}_i(t_0)|\leqslant r_{\Omega_{i2}}.\end{cases}\tag{20}$$

由式(11)知$A_{ij}=-1, B_{ij}=1/8$, 选取$e_{i1}=e_{i2}=1/16(i,j=1,2)$, 利用MATLAB编程得$\mu_1^*=0.5214$, $\mu_2^*=0.4655$.

选取系统参数:

$$a_{11}(t)=a_{12}(t)=\frac{1}{4}\cos t,\ a_{21}(t)=a_{22}(t)=\frac{1}{4}\sin t,$$
$$b_{11}(t)=b_{21}(t)=2+\sin t,\ \lambda_{11}=2,\ \lambda_{21}=2.5,$$

控制器参数:

$$K=2,\ \bar{k}_{10}=1.8,\ \bar{k}_{20}=2,\ \bar{c}_{10}=1.5,$$
$$\bar{c}_{20}=1.2,\ c_{10}=2, c_{20}=2.2.$$

取初始值:

$$\begin{aligned}&x_{11}(0)=0.6,\ x_{12}(0)=0.5,\\ &x_{21}(0)=0.5,\ x_{22}(0)=0.31;\\ &\theta_{11}(0)=1.5,\ \theta_{12}(0)=1.3,\\ &\theta_{21}(0)=1,\ \theta_{22}(0)=1.2;\\ &\gamma_1(0)=0.7,\ \gamma_2(0)=0.7.\end{aligned}$$

图1给出了闭环系统的响应曲线.

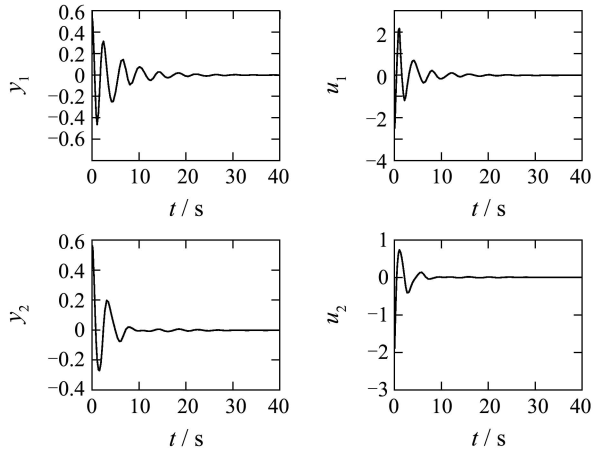

图 1 闭环系统的响应曲线

Fig. 1 Responses of closed-loop system

6 结论(Conclusions)

针对更广的一类时变关联系统, 通过引入输入滤波, 坐标变换以及投影算法, 设计和分析了一种分散自适应输出反馈控制器. 仍有一些问题需要进一步考虑, 如如何设计其他分散自适应控制器, 使其适用于更一般的时变关联系统.

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