第 27 卷第 5 期
2010 年 5 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 27 No. 5
May 2010

文章编号: 1000−8152(2010)05−0668−07

一类非线性系统全局有限时间观测器设计

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摘要: 本文首先讨论了非线性系统的有限时间稳定, 并给出了其全局有限时间稳定的一个充分条件. 然后, 利用几何齐次理论、Lyapunov稳定性理论, 并通过构造一个增益适应律, 对一类具有下三角结构的非线性系统, 讨论其全局有限时间稳定状态观测器的设计问题, 所设计的观测器是连续非光滑的, 能够在有限时间段内实现状态的精确重构.

关键词: 有限时间稳定; 观测器; 非光滑; 非线性系统

中图分类号: O231　　　**文献标识码**: A

Global finite-time observers for a class of nonlinear systems

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Abstract: Finite-time stability is investigated for a class of nonlinear systems and a new sufficient condition for global finite-time stability is presented. By using homogeneous theory and Lyapunov theory, together with an update law for the gain, we develop a global finite-time observer for a class of nonlinear systems with a lower-triangular structure. The observer is continuous but nonsmooth, which can reconstruct the states precisely in a finite-time interval.

Key words: finite-time stable; observer; nonsmooth; nonlinear system

1 引言(Introduction)

近来有限时间稳定成了控制界研究的一个热点, Haimo首先给出了有限时间稳定的充分条件[1]. 之后, Bhat和Bernstein提出了一个充分必要条件[2]. 利用这个充分必要条件, 有限时间稳定控制问题得到了研究[3~8].

另一方面, 有限时间观测器能在给定的时间段内实现状态的精确重构, 具有快速性和精确性等特征. 然而其实现手段不多, 目前有: 滑模观测器[9]、滞后法[10~12]以及文献[13]提出的方法. 这些观测器中, 滑模观测器是不连续的. 连续性和其重要性在有限时间稳定得以体现[2,14]. 沈艳军等[15]在Moulay等[16]研究基础上, 利用高增益观测器技术得到一个一般性的结论: 满足一致可测和Lipschitz条件的非线性系统存在半全局的连续的有限时间观测器.

本文首先给出非线性系统有限时间稳定一个的充分条件, 通过构造一个增益适应律, 利用几何齐次理论, 讨论一类非线性系统全局有限时间状态观测器的设计问题.

2 预备知识(Preliminaries)

考虑如下非线性系统:

$$\dot{x} = f(x),\ f(0) = 0,\ x \in \mathbb{R}^n,\ x(0) = x_0, \quad (1)$$

其中$f\colon D \to \mathbb{R}^n$是连续的.

定义 1[2]　系统(1)的零解是有限时间收敛的, 如果存在原点开领域$U \subseteq D$和函数$T\colon U \setminus \{0\} \to (0,\infty)$, 使得 $\forall x_0 \in U$ 系统 (1) 的解$\Psi(t,x_0) \in U \setminus \{0\}$, $t \in [0, T(x_0))$, 且 $\lim\limits_{t\to T(x_0)} \Psi(t,x_0) = 0$. $T(x_0)$称为设定时间.

定义 2[2]　系统(1)的零解如果是Lyapunov稳定和有限时间收敛的, 则它有限时间稳定的. 如果$U = D = \mathbb{R}^n$, 则它是全局有限时间稳定的.

关于系统(1), Bhat和Bernstein给出了如下有限时间稳定的结论.

收稿日期: 2008−08−04; 收修改稿日期: 2009−07−21.

基金项目: 国家自然科学基金资助项目(60773190, 50779029); 湖北省自然科学基金资助项目(2008CDZ046,2008CDZ047); 湖北省教育厅重点资助项目(D20091305); 湖北省教育厅杰出青年团队资助项目(D200809).

引理 1[2] 如果存在正定函数$V\colon D\to\mathbb{R}$, 实数$c>0,\alpha\in(0,1)$和原点开领域$\varOmega\in D$使得

$$\dot{V}(x)+c(V(x))^{\alpha}\leqslant 0, x\in\varOmega\setminus\{0\}. \tag{2}$$

则, 系统(1)的零解是有限时间稳定的. 设定时间T依赖初始状态x_0且满足

$$T(x_0)\leqslant\frac{1}{c(1-\alpha)}V(x_0^{1-\alpha}), x_0\in\varOmega. \tag{3}$$

如果$D=\mathbb{R}^n$, 式(2)在$\mathbb{R}^n\setminus\{0\}$上成立, 则原点是全局有限时间稳定的.

下面给出系统(1)有限时间稳定的一个新的充分条件.

例 1 下面标量系统:

$$\dot{y}(t)=-c\lceil y(t)\rfloor^{\alpha}-ly(t), y(0)=x, \tag{4}$$

是处处连续的, 在去掉原点的任意领域满足局部Lipschitz条件, 其中$c,l>0,\alpha\in(0,1),\lceil x\rfloor^{\alpha}=|x|^{\alpha}\mathrm{sgn}\, x$. 从而对$x\in\mathbb{R}\setminus\{0\}$的解具有唯一性. 对式(4)两边乘以$\mathrm{e}^{lt}$, 得

$$\frac{\mathrm{d}(\mathrm{e}^{lt}y(t))}{\mathrm{d}t}=-c|y(t)\mathrm{e}^{lt}|^{\alpha}\mathrm{e}^{(1-\alpha)lt}\mathrm{sgn}\, y(t),$$

则, 式(4)的解为

$$\mu(t,x)=\begin{cases}\mathrm{sgn}\, x\mathrm{e}^{-lt}[|x|^{1-\alpha}+\dfrac{c}{l}-\dfrac{c}{l}\mathrm{e}^{(1-\alpha)lt}]^{\frac{1}{1-\alpha}},\\ \quad t<\dfrac{\ln(1+\frac{l}{c}|x|^{1-\alpha})}{l(1-\alpha)}, x\neq 0,\\ 0, t\geqslant\dfrac{\ln(1+\frac{l}{c}|x|^{1-\alpha})}{l(1-\alpha)},\\ 0, t\geqslant 0, x=0.\end{cases} \tag{5}$$

显然, 所有解在有限时间收敛到平衡点$y=0$.

引理 2 如果存在正定函数$V\colon D\to\mathbb{R}$, 实数$c,l>0,\alpha\in(0,1)$和原点开领域$\varOmega\in D$使得

$$\dot{V}(x)\leqslant -c(V(x))^{\alpha}-lV(x), x\in\varOmega\setminus\{0\}, \tag{6}$$

则, 系统(1)的零解是有限时间稳定的. 设定时间T依赖初始状态x_0且满足

$$T(x_0)\leqslant\frac{\ln(1+\frac{l}{c}V(x_0)^{1-\alpha})}{l(1-\alpha)}, x_0\in\varOmega, \tag{7}$$

如果$D=\mathbb{R}^n$, 式(6)在$\mathbb{R}^n\setminus\{0\}$上成立, 则原点是全局有限时间稳定的.

证 注意下面不等式成立:

$$\dot{V}(x)\leqslant -cV(x)^{\alpha}(1+\frac{l}{c}V(x)^{1-\alpha})<0,\\ x\in\varOmega\setminus\{0\}. \tag{8}$$

因为V正定且$\dot{V}<0$, 则$x=0$是系统满足$x(0)=0$的唯一解[2], 这样初始条件$x_0\in\varOmega$的解具有唯一性. 设$\varPsi(t,x_0)$是系统(1)的解, 则

$$\dot{V}(\varPsi(t,x_0))\leqslant -cV(\varPsi(t,x_0))^{\alpha}-lV(\varPsi(t,x_0)). \tag{9}$$

应用比较原理, 得

$$V(\varPsi(t,x_0))\leqslant\mu(t,V(x_0)), \tag{10}$$

其中$\mu(\cdot)$由式(5)给出, 则有

$$\varPsi(t,x_0)=0, t\geqslant\frac{\ln(1+\frac{l}{c}|V(x_0)|^{1-\alpha})}{l(1-\alpha)},\\ \forall x_0\in\varOmega. \tag{11}$$

结论成立. 证毕.

注 1 引理2, 如果令$l=0$, 就是引理1. 如果$l\neq 0$, 显然有$\frac{\ln(1+\frac{l}{c}|V(x_0)|^{1-\alpha})}{l(1-\alpha)}<\frac{V(x_0)^{1-\alpha}}{c(1-\alpha)}$, 这意味着$-lV(x)$可以加快收敛速度, 降低设定时间.

3 主要结果(Main results)

考虑下面非线性系统:

$$\begin{cases}\dot{x}_1=x_2+f_1(x_1,u),\\ \quad\vdots\\ \dot{x}_{n-1}=x_n+f_{n-1}(x_1,\cdots,x_{n-1},u),\\ \dot{x}_n=f_n(x_1,\cdots,x_n,u),\\ y=x_1,\end{cases} \tag{12}$$

其中: $x\in\mathbb{R}^n,u\in\mathbb{R},y\in\mathbb{R}$分别是系统状态、控制输入和输出, $f_j(\cdot)$是连续的, $f_j(0)=0,(j=1,\cdots,n)$, 且满足

$$|f_i(x_1,\cdots,x_i,u)-f_i(\hat{x}_1,\cdots,\hat{x}_i,u)|\leqslant\\ \varPsi(y)(|x_1-\hat{x}_1|+\cdots+|x_i-\hat{x}_i|). \tag{13}$$

假设 1 $\varPsi(y)$是有界光滑函数, 且存在$b_1,b_2>0$, 使得$0<b_2\leqslant\varPsi(y)\leqslant b_1$.

系统(12)的观测器设计如下:

$$\begin{cases}\dot{\hat{x}}_1=\hat{x}_2+f_1(y,u)+La_1\lceil e_1\rfloor^{\alpha_1},\\ \quad\vdots\\ \dot{\hat{x}}_{n-1}=\hat{x}_n+f_{n-1}(y,\hat{x}_2,\cdots,\hat{x}_{n-1},\\ \qquad u)+L^{n-1}a_{n-1}\lceil e_1\rfloor^{\alpha_{n-1}},\\ \dot{\hat{x}}_n=f_n(y,\hat{x}_2,\cdots,\hat{x}_n,u)+L^na_n\lceil e_1\rfloor^{\alpha_n}.\end{cases} \tag{14}$$

增益L满足

$$\begin{cases}\dot{L}=-L[\varphi_1(L-\varphi_2)-\varphi_3\varPsi(y)],\\ L(0)\geqslant\varphi_2\geqslant 1,\end{cases} \tag{15}$$

其中:

$$e_i = x_i - \hat{x}_i, i = 1, \cdots, n,$$

$a_i(i = 1, \cdots, n)$是Hurwitz多项式$s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$的系数, 常数$\varphi_1, \varphi_3 \geqslant 0$,

$$\alpha_i = i\alpha - (i-1), i = 1, \cdots, n, \alpha \in (1 - \frac{1}{n-1}, 1).$$

由式(15)可以得到$L(t) \geqslant \varphi_2 \geqslant 1$, 这是因为当$L(t) = \varphi_2$时, $\dot{L}(t) \geqslant 0$. 此外,

$$\frac{\mathrm{d}}{\mathrm{d}t}[L - (\varphi_2 + \frac{\varphi_3}{\varphi_1}\Psi(y))] = -\frac{\varphi_3}{\varphi_1}\dot{\Psi}(y) - \varphi_1 L[L - (\varphi_2 + \frac{\varphi_3}{\varphi_1}\Psi(y))]. \quad (16)$$

如果$L(t) \geqslant \varphi_2 + \frac{\varphi_3}{\varphi_1}\Psi(y)$. 由式(16)可得

$$\frac{\mathrm{d}}{\mathrm{d}t}[L - (\varphi_2 + \frac{\varphi_3}{\varphi_1}\Psi(y))] = -\frac{\varphi_3}{\varphi_1}\dot{\Psi}(y) - \varphi_1[L - (\varphi_2 + \frac{\varphi_3}{\varphi_1}\Psi(y))].$$

从而得到

$$L(t) \leqslant \varphi_2 + \frac{\varphi_3}{\varphi_1}\Psi(y) + \mathrm{e}^{-\varphi_1 t}[L(0) - (\varphi_2 + \frac{\varphi_3}{\varphi_1}\Psi(y(0)))] - \mathrm{e}^{-\varphi_1 t}\int_0^t \frac{\varphi_3}{\varphi_1}\dot{\Psi}(y)\mathrm{e}^{\varphi_1 s}\mathrm{d}s \leqslant L(0) + \frac{\varphi_3}{\varphi_1}b_1.$$

显然, 可以得到如下结论.

引理 3 如果$\Psi(y)$满足假设1, 则

$$L(t) \leqslant L(0) + \frac{\varphi_3}{\varphi_1}b_1. \quad (17)$$

由式(12)(14)可得误差方程为

$$\begin{cases} \dot{e}_1 = e_2 + \tilde{f}_1 - La_1\lceil e_1 \rfloor^{\alpha_1}, \\ \quad \vdots \\ \dot{e}_{n-1} = e_n + \tilde{f}_{n-1} - L^{n-1}a_{n-1}\lceil e_1 \rfloor^{\alpha_{n-1}}, \\ \dot{e}_n = \tilde{f}_n - L^n a_n \lceil e_1 \rfloor^{\alpha_n}, \end{cases} \quad (18)$$

及

$$\begin{cases} \dot{L} = -L[\varphi_1(L - \varphi_2) - \varphi_3\Psi(y)], \\ L(0) \geqslant \varphi_2 \geqslant 1, \end{cases} \quad (19)$$

其中:

$$\tilde{f}_i = f_i(x_1, \cdots, x_i, u) - f_i(y, \hat{x}_2, \cdots, \hat{x}_i, u), \quad i = 1, \cdots, n.$$

考虑如下坐标变换:

$$\varepsilon_i = \frac{e_i}{L^{i-1+\sigma}}. \quad (20)$$

则式(19)变为

$$\begin{cases} \dot{\varepsilon}_1 = L\varepsilon_2 - L^{(\alpha_1 - 1)\sigma + 1}a_1\lceil \varepsilon_1 \rfloor^{\alpha_1} - \frac{\dot{L}}{L}\sigma\varepsilon_1 + \frac{\tilde{f}_1}{L^{\sigma}}, \\ \quad \vdots \\ \dot{\varepsilon}_{n-1} = L\varepsilon_n - L^{(\alpha_{n-1}-1)\sigma+1}a_{n-1}\lceil \varepsilon_1 \rfloor^{\alpha_{n-1}} - \\ \qquad \frac{\dot{L}}{L}(n-2+\sigma)\varepsilon_{n-1} + \frac{\tilde{f}_{n-1}}{L^{(n-2+\sigma)}}, \\ \dot{\varepsilon}_n = -L^{(\alpha_n - 1)\sigma + 1}a_n\lceil \varepsilon_1 \rfloor^{\alpha_n} - \\ \qquad \frac{\dot{L}}{L}(n-1+\sigma)\varepsilon_n + \frac{\tilde{f}_n}{L^{(n-1+\sigma)}}, \end{cases} \quad (21)$$

其中常数$\sigma \geqslant 0$. 与文献[18]相似, 选取Hurwitz矩阵A的系数$a_i(i = 1, \cdots, n)$, 使得存在$P^{\mathrm{T}} = P > 0$满足

$$\begin{cases} A^{\mathrm{T}}P + PA \leqslant -I, \\ h_1 I \leqslant D_\sigma P + PD_\sigma \leqslant h_2 I, \end{cases} \quad (22)$$

其中h_1, h_2是常数,

$$A = \begin{pmatrix} -a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ -a_{n-1} & 0 & \cdots & 1 \\ -a_n & 0 & \cdots & 0 \end{pmatrix},$$

$$D_\sigma = \begin{pmatrix} \sigma & 0 & \cdots & 0 \\ 0 & \frac{1+\sigma}{\alpha_1} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \frac{n-1+\sigma}{\alpha_{n-1}} \end{pmatrix}.$$

现在给出本文主要结论.

定理 1 非线性系统(12)如果满足假设1和条件(13), 则其存在全局有限时间稳定的观测器.

定理1的证明分为下面几个部分.

首先考虑如下系统:

$$\begin{cases} \dot{\varepsilon}_1 = L\varepsilon_2 - L^{(\alpha_1 - 1)\sigma + 1}a_1\lceil \varepsilon_1 \rfloor^{\alpha_1}, \\ \quad \vdots \\ \dot{\varepsilon}_{n-1} = L\varepsilon_n - L^{(\alpha_{n-1}-1)\sigma+1}a_{n-1}\lceil \varepsilon_1 \rfloor^{\alpha_{n-1}}, \\ \dot{\varepsilon}_n = -L^{(\alpha_n - 1)\sigma+1}a_n\lceil \varepsilon_1 \rfloor^{\alpha_n}. \end{cases} \quad (23)$$

引理 4 对于$\alpha \in (1 - \frac{1}{n-1}, 1)$, 系统(23)关于权系数$\{(i-1)\alpha - (i-2)\}_{1 \leqslant i \leqslant n}$是$\alpha - 1$阶齐次的.

引理 5 存在$\delta \in (1 - \frac{1}{n-1}, 1]$, 使得当$\alpha \in (1-\delta, 1)$, 系统(23)是全局有限时间稳定的.

证 考虑如下Lyapunov函数:

$$V_\alpha(\varepsilon)=\tilde{\varepsilon}^{\mathrm T}P\tilde{\varepsilon}, \tag{24}$$

其中:

$$\begin{aligned}&\tilde{\varepsilon}=[\lceil\varepsilon_1\rfloor^{\frac{1}{r}}\ \cdots\ \lceil\varepsilon_1\rfloor^{\frac{1}{\alpha_{n-1}r}})]^{\mathrm T},\\&\varepsilon=[\varepsilon_1\ \cdots\ \varepsilon_n],\\&r=\prod_{i=1}^{n-1}[(i-1)\alpha-(i-2)].\end{aligned}$$

令f_α表示系统(23)的向量场. 当$\alpha=1$时, 式(23)变为

$$\dot{\varepsilon}=LA\varepsilon. \tag{25}$$

则

$$\frac{\mathrm dV_1}{\mathrm dt}=2\varepsilon^{\mathrm T}P\dot{\varepsilon}=L\varepsilon^{\mathrm T}(A^{\mathrm T}P+PA)\varepsilon<0. \tag{26}$$

设$\varLambda=V_\alpha^{-1}([0,1]),\nabla=V_1^{-1}(\{1\})$, 那么$\varLambda$和$\nabla$是紧集. 定义$\theta$: $(0,1]\times\nabla\to\mathbb{R}$, $\theta(\alpha,\varepsilon,L)=\boldsymbol{L}_{f_\alpha}V_\alpha(\varepsilon)$. 那么$\theta$是连续的, 且满足$\theta(1,\varepsilon,L)<0$, $\varepsilon\times L\in\nabla$, 即有$\theta(\{1\}\times\nabla)\subset(-\infty,0)$. 因为$\nabla$是紧集, 那么存在$\xi>0$使得$\theta((1-\xi]\times\nabla)\subset(-\infty,0)$. 因此, 对$\alpha\in(1-\xi,1]$, $\varepsilon\times L\in\nabla$, $\boldsymbol{L}_{f_\alpha}V_\alpha(\varepsilon)<0$. 从而对$\forall\alpha\in(1-\xi,1]$, $\varLambda$是正的不变集. 故对$\alpha\in(1-\xi,1]$, 系统的原点是渐近稳定的平衡点[14]. 注意到$\alpha-1<0$, 由定理6.1[14]可知, 系统(23)是有限时间稳定的.

证毕.

此外, 由文献[14]引理4.2可得

$$-c_1(\alpha,L)[V_\alpha(\varepsilon)]^{\frac{2/r+\alpha-1}{2/r}}\leqslant\boldsymbol{L}_{f_\alpha}V_\alpha(\varepsilon)\leqslant -c_2(\alpha,L)[V_\alpha(\varepsilon)]^{\frac{2/r+\alpha-1}{2/r}}. \tag{27}$$

引理 6 $c_2(\alpha,L)$满足$\lim\limits_{\alpha\to1}c_2(\alpha,L)=-L\bar{c}$, 其中$\bar{c}=\max\limits_{\{\varepsilon:\,V_1(\varepsilon)=1\}}\{\varepsilon^{\mathrm T}(A^{\mathrm T}P+PA)\varepsilon\}$.

证 注意集合$\{z\colon V_\alpha(z)=1\}$和$\{z\colon V_1(z)=1\}$存在一一对应关系, 即, 任意

$$z=[z_1\ \cdots\ z_n]^{\mathrm T}\in\{z\colon V_\alpha(z)=1\},$$

有

$$\bar{z}=[\lceil z_1\rfloor^{\frac{1}{r}}\ \cdots\ \lceil z_n\rfloor^{\frac{1}{\alpha_{n-1}r}})]^{\mathrm T}\in\{z\colon V_1(z)=1\}$$

与之对应, 且$\lim\limits_{\alpha\to1}\|\bar{z}-z\|_2=0$. 因为$\boldsymbol{L}_{f_\alpha}V_\alpha(z)$是连续的, 故对任意$\delta_2>0$有$\delta_1,\eta>0$, 使得当$|\alpha-1|<\eta,\|z-\bar{z}\|<\delta_1$有下式成立:

$$\boldsymbol{L}_{f_1}V_1(\bar{z})-\delta_2<\boldsymbol{L}_{f_\alpha}V_\alpha(z)<\boldsymbol{L}_{f_1}V_1(\bar{z})+\delta_2.$$

从而

$$\max_{\{z:\,V_\alpha(z)=1\}}\boldsymbol{L}_{f_\alpha}V_\alpha(\bar{z})<\max_{\{z:\,V_\alpha(z)=1\}}\boldsymbol{L}_{f_1}V_1(\bar{z})+\delta_2.$$

则

$$\overline{\lim_{\alpha\to1}}\max_{\{z:\,V_\alpha(z)=1\}}\boldsymbol{L}_{f_\alpha}V_\alpha(\bar{z})\leqslant\max_{\{z:\,V_\alpha(z)=1\}}\boldsymbol{L}_{f_1}V_1(\bar{z}).$$

同理可证

$$\max_{\{z:\,V_\alpha(z)=1\}}\boldsymbol{L}_{f_1}V_1(\bar{z})\leqslant\underline{\lim_{\alpha\to1}}\max_{\{z:\,V_\alpha(z)=1\}}\boldsymbol{L}_{f_\alpha}V_\alpha(\bar{z}).$$

则结论成立.

证毕.

定理1的证明: 利用$\dfrac{\mathrm d}{\mathrm dt}\lceil e_i\rfloor^{\alpha_1}=\alpha_i|e_i|^{\alpha_i-1}$[4], 计算Lyapunov函数(24)沿系统(21)状态的导数.

$$\begin{aligned}&\left.\frac{\mathrm dV_\alpha(\varepsilon)}{\mathrm dt}\right|_{(21)}=\\&\left.\frac{\mathrm dV_\alpha(\varepsilon)}{\mathrm dt}\right|_{(23)}+2\tilde{\varepsilon}^{\mathrm T}P\begin{bmatrix}\dfrac{|\varepsilon_1|^{1/r-1}}{r}\dfrac{\tilde{f}_1}{L^\sigma}\\\dfrac{|\varepsilon_2|^{1/(\alpha_1r)-1}}{\alpha_1r}\dfrac{\tilde{f}_2}{L^{1+\sigma}}\\\vdots\\\dfrac{|\varepsilon_n|^{1/(\alpha_{n-1}r)-1}}{\alpha_{n-1}r}\dfrac{\tilde{f}_n}{L^{n-1+\sigma}}\end{bmatrix}-2\tilde{\varepsilon}^{\mathrm T}P\begin{bmatrix}\dfrac{|\varepsilon_1|^{1/r-1}}{r}\dfrac{\dot{L}}{L}\sigma\varepsilon_1\\\dfrac{|\varepsilon_2|^{1/(\alpha_1r)-1}}{\alpha_1r}\dfrac{\dot{L}}{L}(\sigma+1)\varepsilon_2\\\vdots\\\dfrac{|\varepsilon_n|^{1/(\alpha_{n-1}r)-1}}{\alpha_{n-1}r}\dfrac{\dot{L}}{L}(n-1+\sigma)\varepsilon_n\end{bmatrix}\leqslant\\&-c_2(\alpha,L)[V_\alpha(\varepsilon)]^{\frac{2/r+\alpha-1}{2/r}}+2\varPsi(y)[\tilde{\varepsilon}^{\mathrm T}P\varepsilon]^{1/2}\Big(\sum_{i,j}\frac{|P_{i,j}|}{\alpha_{n-1}^2r^2}\frac{|\varepsilon_i|^{1/(\alpha_{i-1}r)-1}\sum\limits_{k=1}^{i}|e_k|}{L^{(i-1+\sigma)}}\frac{|\varepsilon_i|^{1/(\alpha_{j-1}r)-1}\sum\limits_{k=1}^{j}|e_k|}{L^{(j-1+\sigma)}}\Big)^{1/2}-\\&\frac{2\varphi_3\varPsi(y)}{r}\tilde{\varepsilon}^{\mathrm T}PD_\sigma\tilde{\varepsilon}+\frac{2\varphi_1(L-\varphi_2)}{r}\tilde{\varepsilon}^{\mathrm T}PD_\sigma\tilde{\varepsilon}.\end{aligned}$$

令$\bar{p} = \max\limits_{i,j}|P_{i,j}|$, 注意存在$\lambda_1, \lambda_2 > 0$使得$\lambda_1 I \leqslant P \leqslant \lambda_2 I$和$L > 1$, 则

$$\frac{\mathrm{d}V_\alpha(\varepsilon)}{\mathrm{d}t}\Big|_{(21)} \leqslant$$
$$-c_2(\alpha, L)[V_\alpha(\varepsilon)]^{\frac{2/r+\alpha-1}{2/r}} - \frac{\varphi_3 h_1 \Psi(y)}{\lambda_2 r}V_\alpha(\varepsilon) +$$
$$\frac{\varphi_1 h_2(L-\varphi_2)}{\lambda_1 r}V_\alpha(\varepsilon) + \frac{2\Psi(y)\bar{p}}{\alpha_{n-1} r}V_\alpha^{1/2}(\varepsilon) \times$$
$$(\sum_{i,j}|\varepsilon_i|^{1/(\alpha_{i-1}r)-1}\sum_{k=1}^{i}|\varepsilon_k||\varepsilon_j|^{1/(\alpha_{j-1}r)-1}\sum_{k=1}^{j}|\varepsilon_k|)^{1/2}.$$

由文献[18]引理2.4, 则存在$\bar{c}_i, \bar{d}_i > 0(1 \leqslant i \leqslant n)$使得

$$|\varepsilon_i|^{1/(\alpha_{i-1}r)-1}\sum_{k=1}^{i} \leqslant$$
$$\sum_{k=1}^{i}[\bar{c}_i|\varepsilon_i|^{1/(\alpha_{i-1}r)} + \bar{d}_i|\varepsilon_k|^{1/(\alpha_{i-1}r)}] \triangleq$$
$$\sum_{k=1}^{i} z_{i,k}|\varepsilon_k|^{1/(\alpha_{i-1}r)}.$$

令$\bar{z} = \max\limits_{i,k} z_{i,k}$, 则有

$$\frac{\mathrm{d}V_\alpha(\varepsilon)}{\mathrm{d}t}\Big|_{(21)} \leqslant$$
$$-c_2(\alpha, L)\big[V_\alpha(\varepsilon)\big]^{\frac{2/r+\alpha-1}{2/r}} - \frac{\varphi_3 h_1 \Psi(y)}{\lambda_2 r}V_\alpha(\varepsilon) +$$
$$\frac{\varphi_1 h_2(L-\varphi_2)}{\lambda_1 r}V_\alpha(\varepsilon) + \frac{2\Psi(y)\bar{p}n\bar{z}}{\alpha_{n-1} r\lambda_1}V_\alpha(\varepsilon).$$

选取$\varphi_1, \varphi_2, \varphi_3, L(0)$使得

$$\varphi_2 = \varphi_3 b_1, L(0) \geqslant \varphi_2,$$
$$\frac{\varphi_3 h_1 b_2}{\lambda_2} > \frac{\varphi_1 h_2 L(0)}{\lambda_1} + \frac{2b_1\bar{p}n\bar{z}}{\alpha_{n-1}\lambda_1}.$$

则有

$$\frac{\mathrm{d}V_\alpha(\varepsilon)}{\mathrm{d}t}\Big|_{(22)} \leqslant$$
$$-c_2(\alpha, L)\big[V_\alpha(\varepsilon)\big]^{\frac{2/r+\alpha-1}{2/r}} -$$
$$(\frac{\varphi_3 h_1 b_2}{\lambda_2 r} - \frac{\varphi_1 h_2 L(0)}{\lambda_1 r} - \frac{2b_1\bar{p}n\bar{z}}{\alpha_{n-1} r})V_\alpha(\varepsilon).$$

由引理2可得系统(22)是有限时间稳定的.

证毕.

注 2 本文有限时间观测器的设计思路主要来自文献[16], 但是文献[16]仅考虑了一类误差可线性化非线性系统的有限时间观测器的设计, 即对应的非线性函数$f_i(\cdot)$仅与输出y和输入u有关, 而本文考虑的非线性函数$f_i(\cdot)$不仅与y, u, 还和状态变量$x_1, \cdots, x_{i-1}$有关, 从而更具广泛性. 然而, 本文所设计的观测器需要知道系统的精确模型, 很难推广到模型中存在参数不确定或结构不确定性的系统; 而基于滑模技术的观测器则具有很强的鲁棒性, 但是滑模观测器是不连续的, 两者设计方法也是不同的, 文献[19,20]指出连续性观测器具有唯一性.

4 数值实验(Numerical experiment)

考虑如下非线性系统[21]:

$$\begin{cases} \dot{\eta}_1 = \dfrac{\eta_1\eta_2}{1+\eta_2} - u\eta_1, \\ \dot{\eta}_2 = -\dfrac{\eta_1\eta_2}{\eta_1+\eta_2} + u(1-\eta_2), \\ y = \eta_1, \end{cases}$$

其中控制$u \in M_\mathrm{u} = [u_{\min}, u_{\max}] \in (0,1)$, 则下面集合

$$M_\eta = \{(\eta_1, \eta_2) \in \mathbb{R}^2 \colon \eta_1 \geqslant \varepsilon_1, \eta_2 \geqslant \varepsilon_2, \eta_1 + \eta_2 \leqslant 1\}$$

是前向不变集, 其中$\varepsilon_1 = \dfrac{(1-u_{\max})\varepsilon_2}{u_{\max}}$和$u_{\min} \geqslant \varepsilon_2$. 这保证非线性系统的解是有界的. 考虑如下变换:

$$M_\eta \to M_x = F(M_\eta),$$
$$(\eta_1, \eta_2) \to (x_1, x_2) = (\eta_1, \frac{\eta_1\eta_2}{\eta_1+\eta_2}),$$

则有

$$\begin{cases} \dot{x}_1 = x_2 - ux_1, \\ \dot{x}_2 = f_2(x_1, x_2, u), \\ y = \eta_1, \end{cases} \tag{28}$$

其中

$$f_2(x_1, x_2, u) = u + (-u - 1 - \frac{2u}{x_1})x_2 + (\frac{2}{x_1} + \frac{u}{x_1^2})x_2^2.$$

注意对于$(x_1, x_2, u) \in M_x \times M_\mathrm{u}$, 有

$$\underline{x}_2(x_1) = x_1\frac{\varepsilon_2}{x_1+\varepsilon_2} \leqslant x_2 \leqslant x_1 = \bar{x}_2(x_1).$$

从而当给定$(u, x_1) \in [u_{\min}, u_{\max}] \times [\varepsilon_1, 1-\varepsilon_2]$, 则$x_2 \in [\underline{x}_2(x_1), \bar{x}_2(x_1)]$, 与文献[21]估计$f_2(\cdot)$的方法相同, 用$(x_{1s}, x_{2s})$代替$(x_1, x_2)$, 其中:

$$x_{1s} = \max\{\varepsilon_1, \min\{1-\varepsilon_2, x_1\}\},$$
$$x_{2s} = \max\{\underline{x}_2(x_{1s}), \min\{\bar{x}_2(x_{1s}), x_2\}\},$$

则

$$|f_2(x_1, x_2, u) - f_2(x_1, \hat{x}_2, u)| \leqslant \varOmega(u, x_1, \hat{x}_2)|x_2 - \hat{x}_2|,$$

其中

$$\varOmega(u, x_1, \hat{x}_2) =$$
$$\max_{x_2 \in [\underline{x}_2(x_{1s}), \bar{x}_2(x_{1s})]}|-u-1-\frac{2u}{x_1}+(\frac{2}{x_1}+\frac{u}{x_1^2})(\hat{x}_2+x_2)|.$$

由于$\hat{x}_2$是x_2的估计值, 可以用$\hat{x}_{2s}$ 替代$\hat{x}_2$, 即

$$x_{2s} = \max\{\underline{x}_2(x_{1s}), \min\{\bar{x}_2(x_{1s}), \hat{x}_{2s}\}\},$$

根据定理1, 设计如下观测器

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - ux_1 - 3L\lceil e_1\rfloor^{0.98}, \\ \dot{\hat{x}}_2 = f_2(x_1, \hat{x}_2, u) - 2L\lceil e_1\rfloor^{0.96}, \\ \dot{L} = -L(0.01(L-1)) - 0.1\Omega(u, x_1, \hat{x}_2)). \end{cases} \tag{29}$$

系统(29)初始值为$x(0) = [0.7, 0.3]^{\mathrm{T}}$, 观测器初始值为$\hat{x}(0) = [0.3, 0.5]^{\mathrm{T}}, L(0) = 1$, 仿真结果如图1所示. 另外, 根据文献[22], 设计如下滑模观测器:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + ux_1 + \lambda_1 \mathrm{sgn}(x_1 - \hat{x}_1), \\ \dot{\hat{x}}_2 = f_2(x_1, \hat{x}_2, u) + \lambda_2 \mathrm{sgn}(\lambda_1 \mathrm{sgn}(x_1 - \hat{x}_1))_{\mathrm{eq}}, \end{cases} \tag{30}$$

其中$\lambda_1 = 0.08, \lambda_2 = 1.2$, 观测器初始值也为$\hat{x}(0) = [0.3, 0.5]^{\mathrm{T}}$, 仿真结果如图2所示.

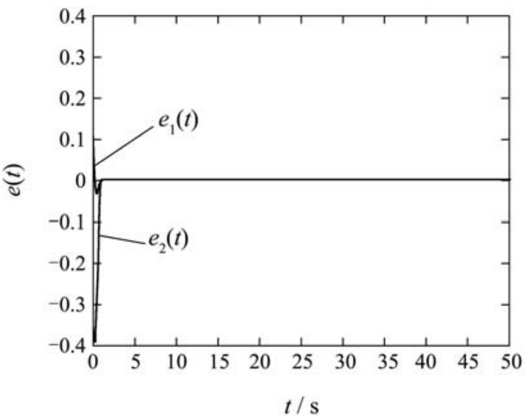

图 1 高增益有限时间观测器状态误差图

Fig. 1 Trajectories of error states with high-gain finite-time observer

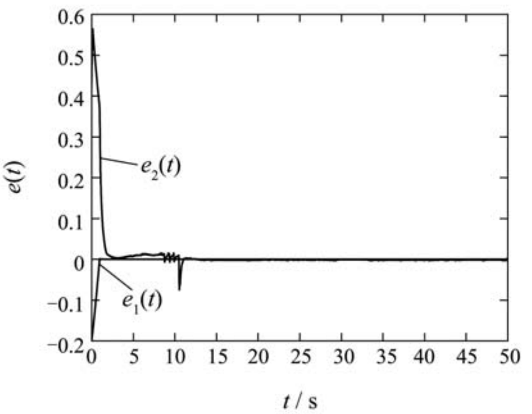

图 2 滑模观测器状态误差图

Fig. 2 Trajectories of error states with sliding mode observer

显然, 本文给出的设计方法具有很好的收敛性能.

5 总结(Summary)

本文给出了非线性系统有限时间稳定一个新的充分条件, 在此基础上, 讨论了一类非线性系统有限时间观测器的设计, 仿真实验验证本方案的有效性.

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