第 27 卷第 6 期
2010 年 6 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 27 No. 6
Jun. 2010

文章编号: 1000−8152(2010)06−0786−07

一类具有区间时变多状态时滞系统的稳定性分析

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摘要: 研究一类具有区间时变多状态时滞且时滞导数不确定的系统的稳定性问题. 通过选择合理的Lyapunov-Krasovskii函数结合辅助变量和广义状态法, 以LMI的形式给出了时滞相关的稳定性充分条件. 文中的结论对时滞的导数没有任何限制, 可用于具有快时变时滞系统. 与已有的相关研究成果相比, 结论更具有一般性, 保守性也更低. 最后通过仿真及数值算例说明了本文方法的有效性和优越性.

关键词: 区间时变; 多状态时滞系统; 线性矩阵不等式

中图分类号: TP273　　　**文献标识码:** A

Stabilization analysis for systems with multiple interval time-varying time-delays

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Abstract: The stabilization of multiple interval time-varying time-delay systems is considered，in which the derivatives of the time-varying delay is unknown. By choosing appropriate Lyapunov-Krasovskii functional, the auxiliary variables and the generalized states, we derive for these systems the sufficient conditions of delay-dependent stabilization in terms of LMI (linear matrix inequality). The results are free from the restriction on the derivatives of the time-varying delays; thus, can be applied to systems with fast time-varying time-delay. Compared with existing relevant results, our results are more general and less conservative. Simulation and numerical examples validate the effectiveness and advantages of the proposed method.

Key words: interval time-varying; multiple time-delays systems; LMI(linear matrix inequality)

1　引言(Introduction)

时滞存在于大量的动态系统中, 比如远程控制系统、网络传输系统、电力系统、化学系统以及生物系统等, 若忽略时滞将会对系统的性能产生很大的影响. 现有的研究时滞系统稳定性的成果基本分为两大类: 时滞相关和时滞独立, 由于时滞相关准则利用了时滞信息, 所以它们的保守性相对较低, 而时滞相关稳定条件的得出, 通常都是通过变换得出的, 比如中立变换、参数变换等等, 还有一种就是利用Leibniz-Newton公式, 通过在$\dot{V}(t)$中适当的添加一些零项、引入辅助变量并且利用广义的状态变量, 也可以得出保守性较小的结果[1~3]. 在目前的研究成果中, 很多结果要求系统的时滞参数的导数小于1, 这在很大程度上带来了一定的保守性.

在很多实际系统中, 可能无法预先知道时滞参数导数的情况, 如遥操作系统和网络控制系统中的时滞参数变化很快, 有时时滞参数的导数甚至可能不存在, 已有的方法可能就不适用了. 然而, 这些系统中时滞参数的上界和下界往往可以知道, 所以研究这种具有区间时变时滞系统的稳定性问题已经成为时滞系统研究的一个新方向. 现有针对区间时变时滞系统的研究成果[2,4]分别研究了一类具有区间时变时滞不确定T-S模糊系统的鲁棒H_∞控制以及一类区间时变状态时滞系统的H_∞性能分析及控制.

本文对一类具有区间时变多状态时滞且时滞导数不确定的系统进行稳定性的研究. 并通过证明找到了该方法得到的稳定性准则的一般规律, 通过合理的选择Lyapunov-Krasovskii函数, 结合近年来常用于研究时变时滞鲁棒镇定问题的辅助变量和广义状态法[1,3], 以LMI的形式给出了时滞相关的稳定性充分条件. 本文的最大特点是文中结论对时滞的导数没有任何要求, 可用于具有快时变时滞系统. 数值算

收稿日期: 2008−11−14; 收修改稿日期: 2009−04−28.
基金项目: 国家自然科学基金重点资助项目(60736024,60574004).

例及仿真结果表明了本文方法的有效性和优越性, 同时也给时滞系统的稳定性研究提供了一个新思路.

2 预备知识及问题陈述(System descriptions and preliminaries)

2.1 问题描述(System descriptions)

考虑如下形式的时滞系统:

$$\begin{aligned} &\dot{x}(t)=A_0x(t)+\sum_{i=1}^{k}A_ix(t-\tau_i(t)),\\ &x(t)=\phi(t),\ t\in[-\tau_{\max},0]. \end{aligned} \tag{1}$$

其中: $x(t)\in\mathbb{R}^m$为状态变量, $\phi(t)$是初始条件, $\tau_i(t)$为区间时变的状态时滞且满足

$0<\tau_{i\min}\leqslant\tau_i(t)\leqslant\tau_{i\max},\ \tau_{\max}=\max(\tau_{i\max})$, $A_0, A_i(i=1,\cdots,k)$是合适维数的常数实矩阵. 做如下定义:

$$\tau_{i,1/2}=\frac{1}{2}(\tau_{i\max}+\tau_{i\min}),\ \delta_i=\frac{1}{2}(\tau_{i\max}-\tau_{i\min}),$$

则$\tau_i(t)=\tau_{i,1/2}+\delta_i m_i(t)$, 这里:

$$m_i(t)=\begin{cases}\dfrac{2\tau_i(t)-(\tau_{i\max}+\tau_{i\min})}{\tau_{i\max}-\tau_{i\min}}, & \tau_{i\max}>\tau_{i\min},\\ 0, & \tau_{i\max}=\tau_{i\min},\end{cases}$$

则

$$|m_i(t)|\leqslant 1, \tau_i(t)\in[\tau_{i,1/2}-\delta_i,\tau_{i,1/2}+\delta_i]. \tag{2}$$

本文将会依据参数$\tau_{i,1/2},\delta_i$得到该类系统的稳定性充分条件.

2.2 相关引理(The lemmas)

引理 1　对具有适当维数的任意向量x,y及对称正定矩阵X有

$$\pm 2x^{\mathrm{T}}y\leqslant x^{\mathrm{T}}X^{-1}x+y^{\mathrm{T}}Xy.$$

引理 2　给定适当维数的矩阵Y,D,E, 其中Y是对称的, 则

$$Y+DFE+E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}}<0$$

对所有的满足$F^{\mathrm{T}}F\leqslant I$的矩阵$F$成立, 当且仅当存在一个常数$\varepsilon>0$, 使得

$$Y+\varepsilon DD^{\mathrm{T}}+\varepsilon^{-1}E^{\mathrm{T}}E<0.$$

引理 3　存在不等式:

$$\Big(\int_{t-h}^{t}x(\alpha)\mathrm{d}\alpha\Big)^{\mathrm{T}}\Big(\int_{t-h}^{t}x(\alpha)\mathrm{d}\alpha\Big)\leqslant h\int_{t-h}^{t}x^{\mathrm{T}}(\alpha)x(\alpha)\mathrm{d}\alpha.$$

2.3 相关标注(Some remarks)

这里申明一些标注记号:

1) $\varOmega_{i,j}$ 对应的角标为(i,j).

2) W^v为对应角标i或j的某些运算所对应的辅助向量(v为对应角标i或j的某些运算). 举例来说, 若辅助变量依次为M,N,O, 取$i=2$时, 则W^{i-1} 就是辅助向量M.

3 主要结果(Main results)

定理 1　考虑具有区间时变多状态时滞的系统(1), 假设$\tau_{i,1/2},\delta_i$是已知正数, 若存在合适维数的正定对称阵$P,Q_i,R_i,S_i(i=1,\cdots,k)$以及$k+1$个辅助变量$M_j,N_j,\cdots,O_j,N'_j(j=0,1,\cdots,k+1)$, 使得下面不等式成立:

$$\psi=\begin{bmatrix}\psi_{1,1} & \psi_{1,2} & \cdots & \psi_{1,k+1} & \psi_{1,k+2} & \psi_1\\ * & \psi_{2,2} & \cdots & \psi_{2,k+1} & \psi_{2,k+2} & \psi_2\\ * & * & \cdots & \cdots & \cdots & \cdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & \psi_{k+1,k+1} & \psi_{k+1,k+2} & \psi_{k+1}\\ * & * & * & * & \psi_{k+2,k+2} & \psi_{k+2}\\ * & * & * & * & * & \psi_{k+3}\end{bmatrix}<0. \tag{3}$$

这里:

$$\begin{aligned}\psi_{1,1}=\ &M_0^{\mathrm{T}}+N_0^{\mathrm{T}}+\cdots+O_0^{\mathrm{T}}+M_0+N_0+\cdots+\\ &O_0+\sum_{i=1}^{k}Q_i+{N'_0}^{\mathrm{T}}A_0+A_0^{\mathrm{T}}N'_0,\end{aligned}$$

$$\begin{aligned}\psi_{1,l}=\ &-{W_0^{l-1}}^{\mathrm{T}}+M_{l-1}+N_{l-1}+\cdots+O_{l-1}+\\ &{N'_0}^{\mathrm{T}}A_{l-1}+A_0^{\mathrm{T}}N'_{l-1},\ l\in[2,k+1],\end{aligned}$$

$$\begin{aligned}\psi_{1,k+2}=\ &M_{k+1}+N_{k+1}+\cdots+O_{k+1}+\\ &A_0^{\mathrm{T}}N'_{k+1}+P-{N'_0}^{\mathrm{T}},\end{aligned}$$

$$\begin{aligned}\psi_{i,i}=\ &-{W_{i-1}^{i-1}}^{\mathrm{T}}-W_{i-1}^{i-1}-Q_{i-1}+{N'_{i-1}}^{\mathrm{T}}A_{i-1}+\\ &A_{i-1}^{\mathrm{T}}N'_{i-1}, i\in[2,k+1],\end{aligned}$$

$$\psi_{i,j}=-{W_{i-1}^{j-1}}^{\mathrm{T}}-W_{j-1}^{i-1}+{N'_{i-1}}^{\mathrm{T}}A_{j-1}+A_{i-1}^{\mathrm{T}}N'_{j-1}.$$

这里:

$i\in[2,k],\ j\in[3,k+1],\ i<j$,

$$\psi_{r,k+2}=-W_{k+1}^{r-1}-{N'_{r-1}}^{\mathrm{T}}+A_{r-1}^{\mathrm{T}}N'_{k+1},$$

$r\in[2,k+1]$,

$$\psi_{k+2,k+2}=\sum_{i=1}^{k}\tau_{i,1/2}R_i+\sum_{i=1}^{k}2\delta_iS_i-{N'_{k+1}}^{\mathrm{T}}-N'_{k+1},$$

$$\begin{aligned}\psi_1=[&\tau_{1,1/2}M_0^{\mathrm{T}}\ \ \tau_{2,1/2}N_0^{\mathrm{T}}\ \ \cdots\ \ \tau_{k,1/2}O_0^{\mathrm{T}}\\ &\delta_1{N'_0}^{\mathrm{T}}A_1\ \ \cdots\ \ \delta_k{N'_0}^{\mathrm{T}}A_k],\end{aligned}$$

$$\vdots$$

$$\begin{aligned}\psi_{k+2}=[&\tau_{1,1/2}M_{k+1}^{\mathrm{T}}\ \ \tau_{2,1/2}N_{k+1}^{\mathrm{T}}\ \ \cdots\ \ \tau_{k,1/2}O_{k+1}^{\mathrm{T}}\\ &\delta_1{N'_{k+1}}^{\mathrm{T}}A_1\ \ \cdots\ \ \delta_k{N'_{k+1}}^{\mathrm{T}}A_k],\end{aligned}$$

$$\begin{aligned}\psi_{k+3}=\mathrm{diag}\{&-\tau_{1,1/2}R_1\ \ -\tau_{2,1/2}R_2\ \ \cdots\ \ -\tau_{k,1/2}R_k\\ &-\delta_1S_1\ \ -\delta_2S_2\ \ \cdots\ \ -\delta_kS_k\},\end{aligned}$$

那么对于任意时滞参数满足(2)的系统(1),系统渐近稳定.

证 由Leibniz-Newton公式

$$x(t-\tau_{i,1/2})-x(t-\tau_i(t))=\int_{t-\tau_i(t)}^{t-\tau_{i,1/2}}\dot{x}(s)\mathrm{d}s,$$

结合系统(1)得

$$\dot{x}(t)=A_0x(t)+\sum_{i=1}^{k}A_i[x(t-\tau_{i,1/2})-\int_{t-\tau_i(t)}^{t-\tau_{i,1/2}}\dot{x}(s)\mathrm{d}s].$$

定义Lyapunov-Krasovskii泛函为

$$V(x_t)=x^{\mathrm{T}}(t)Px(t)+\sum_{i=1}^{k}[\int_{t-\tau_{i,1/2}}^{t}x^{\mathrm{T}}(s)Q_ix(s)\mathrm{d}s+\int_{-\tau_{i,1/2}}^{0}\mathrm{d}s\int_{t+s}^{t}\dot{x}^{\mathrm{T}}(\theta)R_i\dot{x}(\theta)\mathrm{d}\theta+\int_{-\tau_{i,1/2}-\delta_i}^{-\tau_{i,1/2}+\delta_i}\mathrm{d}s\int_{t+s}^{t}\dot{x}^{\mathrm{T}}(\theta)S_i\dot{x}(\theta)\mathrm{d}\theta].$$

这里: $P>0,\ Q_i>0,\ R_i>0,\ S_i>0(i=1,\cdots,k)$. 该函数关于时间$t$求导得

$$\begin{aligned}\dot{V}(x_t)=&2x^{\mathrm{T}}(t)P\dot{x}(t)+\sum_{i=1}^{k}[x^{\mathrm{T}}(t)Q_ix(t)-\\&x^{\mathrm{T}}(t-\tau_{i,1/2})Q_ix(t-\tau_{i,1/2})+\\&\tau_{i,1/2}\dot{x}^{\mathrm{T}}(t)R_i\dot{x}(t)-\\&\int_{t-\tau_{i,1/2}}^{t}\dot{x}^{\mathrm{T}}(\theta)R_i\dot{x}(\theta)\mathrm{d}\theta+\\&2\delta_i\dot{x}^{\mathrm{T}}(t)S_i\dot{x}(t)-\\&\int_{t-\tau_{i,1/2}-\delta_i}^{t-\tau_{i,1/2}+\delta_i}\dot{x}^{\mathrm{T}}(\theta)S_i\dot{x}(\theta)\mathrm{d}\theta].\end{aligned}\tag{4}$$

由式(2), $\tau_i(t)-\tau_{i,1/2}\in[-\delta_i,\delta_i]$, 令

$$\Delta\tau_i(t)=\tau_i(t)-\tau_{i,1/2},$$

则

$$\begin{aligned}&-\int_{t-\tau_{i,1/2}-\delta_i}^{t-\tau_{i,1/2}+\delta_i}\dot{x}^{\mathrm{T}}(\theta)S_i\dot{x}(\theta)\mathrm{d}\theta\leqslant\\&-\mathrm{sgn}(\Delta\tau_i(t))\int_{t-\tau_i(t)}^{t-\tau_{i,1/2}}\dot{x}^{\mathrm{T}}(\theta)S_i\dot{x}(\theta)\mathrm{d}\theta.\end{aligned}\tag{5}$$

引入广义状态

$$\xi^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-\tau_{1,1/2})\ \ \cdots\ \ x^{\mathrm{T}}(t-\tau_{k,1/2})\ \ \dot{x}^{\mathrm{T}}(t)]$$

和$k+1$个辅助向量

$$\begin{cases}M=[M_0\ \ M_1\ \ \cdots\ \ M_k\ \ M_{k+1}],\\N=[N_0\ \ N_1\ \ \cdots\ \ N_k\ \ N_{k+1}],\\\quad\vdots\\O=[O_0\ \ O_1\ \ \cdots\ \ O_k\ \ O_{k+1}],\\N'=[N'_0\ \ N'_1\ \ \cdots\ \ N'_k\ \ N'_{k+1}].\end{cases}\tag{6}$$

添加零项

$$\begin{cases}2\xi^{\mathrm{T}}(t)M^{\mathrm{T}}[x(t)-x(t-\tau_{1,1/2})-\int_{t-\tau_{1,1/2}}^{t}\dot{x}(s)\mathrm{d}s]=0,\\2\xi^{\mathrm{T}}(t)N^{\mathrm{T}}[x(t)-x(t-\tau_{2,1/2})-\int_{t-\tau_{2,1/2}}^{t}\dot{x}(s)\mathrm{d}s]=0,\\\quad\vdots\\2\xi^{\mathrm{T}}(t)O^{\mathrm{T}}[x(t)-x(t-\tau_{k,1/2})-\int_{t-\tau_{k,1/2}}^{t}\dot{x}(s)\mathrm{d}s]=0,\\2\xi^{\mathrm{T}}(t)N'^{\mathrm{T}}[A_0x(t)+\sum_{i=1}^{k}A_ix(t-\tau_{i,1/2})-\sum_{i=1}^{k}A_i\int_{t-\tau_i(t)}^{t-\tau_{i,1/2}}\dot{x}(s)\mathrm{d}s-\dot{x}(t)]=0.\end{cases}\tag{7}$$

由引理1及引理3, 同时令

$$X^{-1}=\tau_{1,1/2}R_1^{-1},\cdots,X^{-1}=\tau_{k,1/2}R_k^{-1},$$

得到以下不等式:

$$\begin{cases}-2\xi^{\mathrm{T}}(t)M^{\mathrm{T}}\int_{t-\tau_{1,1/2}}^{t}\dot{x}(s)\mathrm{d}s\leqslant\\\tau_{1,1/2}\xi^{\mathrm{T}}(t)M^{\mathrm{T}}R_1^{-1}M\xi(t)+\int_{t-\tau_{1,1/2}}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\mathrm{d}s,\\\quad\vdots\\-2\xi^{\mathrm{T}}(t)O^{\mathrm{T}}\int_{t-\tau_{k,1/2}}^{t}\dot{x}(s)\mathrm{d}s\leqslant\\\tau_{k,1/2}\xi^{\mathrm{T}}(t)O^{\mathrm{T}}R_k^{-1}O\xi(t)+\int_{t-\tau_{k,1/2}}^{t}\dot{x}^{\mathrm{T}}(s)R_k\dot{x}(s)\mathrm{d}s.\end{cases}\tag{8}$$

同时令$\Delta\tau_i(t)=\tau_i(t)-\tau_{i,1/2}$, 则

$$\begin{aligned}&-2\xi^{\mathrm{T}}(t)N'^{\mathrm{T}}A_k\int_{t-\tau_k(t)}^{t-\tau_{k,1/2}}\dot{x}(s)\mathrm{d}s\leqslant\\&\delta_k\xi^{\mathrm{T}}(t)N'^{\mathrm{T}}A_kS_k^{-1}A_k^{\mathrm{T}}N'\xi(t)+\\&\mathrm{sgn}(\Delta\tau_k(t))\int_{t-\tau_k(t)}^{t-\tau_{k,1/2}}\dot{x}^{\mathrm{T}}(s)S_k\dot{x}(s)\mathrm{d}s\end{aligned}\tag{9}$$

结合式(4)~(9), 合并整理得

$$\begin{aligned}&\dot{V}(x_t)\leqslant\\&2x^{\mathrm{T}}(t)P\dot{x}(t)+\sum_{i=1}^{k}[x^{\mathrm{T}}(t)Q_ix(t)-\\&x^{\mathrm{T}}(t-\tau_{i,1/2})Q_ix(t-\tau_{i,1/2})+\\&\tau_{i,1/2}\dot{x}^{\mathrm{T}}(t)R_i\dot{x}(t)+2\delta_i\dot{x}^{\mathrm{T}}(t)S_i\dot{x}(t)]+\\&2\xi^{\mathrm{T}}(t)M^{\mathrm{T}}[x(t)-x(t-\tau_{1,1/2})]+\cdots+\\&2\xi^{\mathrm{T}}(t)O^{\mathrm{T}}[x(t)-x(t-\tau_{k,1/2})]+\end{aligned}$$

$$\tau_{1,1/2}\xi^{\mathrm T}(t)M^{\mathrm T}R_1^{-1}M\xi(t)+\cdots+$$
$$\tau_{k,1/2}\xi^{\mathrm T}(t)O^{\mathrm T}R_k^{-1}O\xi(t)+$$
$$2\xi^{\mathrm T}(t)N'^{\mathrm T}[A_0x(t)+\sum_{i=1}^{k}A_ix(t-\tau_{i,1/2})-\dot x(t)]+$$
$$\delta_1\xi^{\mathrm T}(t)N'^{\mathrm T}A_1S_1^{-1}A_1^{\mathrm T}N'\xi(t)+\cdots+$$
$$\delta_k\xi^{\mathrm T}(t)N'^{\mathrm T}A_kS_k^{-1}A_k^{\mathrm T}N'\xi(t).$$

利用Schur补定理可得$\dot V(x_t)\leqslant\xi^{\mathrm T}(t)\psi\xi(t)$.

系统满足渐近稳定需$\psi<0$.　　证毕.

本文后续部分讨论在系统(1)的模型基础上添加不确定性, 即系统模型为

$$\begin{cases}\dot x(t)=(A_0+\Delta A_0(t))x(t)+\\ \qquad\sum_{i=1}^{k}(A_i+\Delta A_i(t))x(t-\tau_i(t)),\\ x(t)=\phi(t),t\in[-\tau_{\max},0].\end{cases}\tag{10}$$

这里: $\Delta A_0(t),\Delta A_i(t)$是实矩阵函数, 表示时变参数不确定性并且假设可以分解为如下形式:

$$\Delta A_i(t)=D_iF(t)E_i,\ i=0,1,\cdots,k.\tag{11}$$

其中: $D_0,\cdots,D_k,E_0,\cdots,E_k$为适当维数的常数矩阵, $F(t)$是具有Lebesgue可测元的未知矩阵且满足$\|F_i(t)\|\leqslant1$. 其他变量申明同第一部分的问题描述. 这里将根据定理1的结论继续讨论系统(10)的稳定性问题.

定理 2　考虑具有区间时变多状态时滞的不确定系统(10), 假设$\tau_{i,1/2},\delta_i,r_1,\cdots,r_{k+1}$是已知正数, 若存在合适维数的正定对称阵$\mathcal P,\mathcal Q_i,\mathcal R_i,\mathcal S_i(i=1,\cdots,k)$以及变量$X,\mathcal M_j,\mathcal N_j,\cdots,\mathcal O_j(j=0,1,\cdots,k+1)$、正数$\varepsilon$, 使得下面LMI成立:

$$\begin{bmatrix}\varphi' & \phi_1' & \phi_2' & \dots & \phi_k'\\ * & -\varepsilon I & 0 & \dots & 0\\ * & * & -\varepsilon I & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & \dots & -\varepsilon I\end{bmatrix}<0.\tag{12}$$

式中:

$$\varphi'=\begin{bmatrix}\varphi_{1,1}' & \varphi_{1,2}' & \cdots & \varphi_{1,k+1}' & \varphi_{1,k+2}' & \varphi_1'\\ * & \varphi_{2,2}' & \cdots & \varphi_{2,k+1}' & \varphi_{2,k+2}' & \varphi_2'\\ * & * & \cdots & \cdots & \cdots & \cdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ * & * & \cdots & \varphi_{k+1,k+1}' & \varphi_{k+1,k+2}' & \varphi_{k+1}'\\ * & * & * & \cdots & \varphi_{k+2,k+2}' & \varphi_{k+2}'\\ * & * & * & * & \cdots & \varphi_{k+3}'\end{bmatrix}.$$

这里:

$$\phi_1'=[0\ 0\ \cdots\ 0\ E_1\ 0\ 0\ \cdots\ 0]^{\mathrm T},$$
$$\phi_2'=[0\ 0\ \cdots\ 0\ E_2\ 0\ 0\ \cdots\ 0]^{\mathrm T},$$
$$\vdots$$
$$\phi_k'=[0\ 0\ \cdots\ 0\ E_k]^{\mathrm T}.$$

这里: ϕ_1'表达式中E_1的前面有$(2k+2)$个0, 后面有$(k-1)$个0. ϕ_2'表达式中E_2的前面有$(2k+3)$个0, 后面有$(k-2)$个0. 以此类推, ϕ_k'表达式中E_k的前面有$(3k+1)$个0.

$$\begin{aligned}\varphi_{1,1}'={}&\mathcal M_0^{\mathrm T}+\mathcal N_0^{\mathrm T}+\cdots+\mathcal O_0^{\mathrm T}+\mathcal M_0+\\ &\mathcal N_0+\cdots+\mathcal O_0+\sum_{i=1}^{k}\mathcal Q_i+A_0X+\\ &X^{\mathrm T}A_0^{\mathrm T}+\varepsilon^{-1}X^{\mathrm T}E_0^{\mathrm T}E_0X+\\ &\varepsilon[(D_0D_0^{\mathrm T}+\cdots+D_kD_k^{\mathrm T})+\\ &\delta_1^2D_1D_1^{\mathrm T}+\cdots+\delta_k^2D_kD_k^{\mathrm T}],\end{aligned}$$

$$\begin{aligned}\varphi_{1,l}'={}&-\mathcal W_0^{l-1\,\mathrm T}+\mathcal M_{l-1}+\mathcal N_{l-1}+\cdots+\\ &\mathcal O_{l-1}+r_{l-1}X^{\mathrm T}A_0^{\mathrm T}+A_{l-1}X+\\ &\varepsilon[r_{l-1}(D_0D_0^{\mathrm T}+\cdots+\\ &D_kD_k^{\mathrm T})+r_{l-1}(\delta_1^2D_1D_1^{\mathrm T}+\cdots+\\ &\delta_k^2D_kD_k^{\mathrm T})],\ l\in[2,k+1],\end{aligned}$$

$$\begin{aligned}\varphi_{1,k+2}'={}&\mathcal M_{k+1}+\mathcal N_{k+1}+\cdots+\mathcal O_{k+1}+\\ &\mathcal P-X+r_{k+1}X^{\mathrm T}A_0^{\mathrm T}+\\ &\varepsilon[r_{k+1}(D_0D_0^{\mathrm T}+\cdots+D_kD_k^{\mathrm T})+\\ &r_{k+1}(\delta_1^2D_1D_1^{\mathrm T}+\cdots+\delta_k^2D_kD_k^{\mathrm T})],\end{aligned}$$

$$\begin{aligned}\varphi_{i,i}'={}&-\mathcal W_{i-1}^{i-1\,\mathrm T}-\mathcal W_{i-1}^{i-1}-\mathcal Q_{i-1}+r_{i-1}A_{i-1}X+\\ &r_{i-1}X^{\mathrm T}A_{i-1}^{\mathrm T}+\varepsilon[r_{i-1}^2(D_0D_0^{\mathrm T}+\cdots+\\ &D_kD_k^{\mathrm T})+r_{i-1}^2(\delta_1^2D_1D_1^{\mathrm T}+\cdots+\\ &\delta_k^2D_kD_k^{\mathrm T})]+\varepsilon^{-1}X^{\mathrm T}E_{i-1}^{\mathrm T}E_{i-1}X,\end{aligned}$$

$i\in[2,k+1]$,

$$\begin{aligned}\varphi_{i,j}'={}&-\mathcal W_{i-1}^{j-1\,\mathrm T}-\mathcal W_{j-1}^{i-1}+r_{i-1}A_{j-1}X+\\ &r_{j-1}X^{\mathrm T}A_{i-1}^{\mathrm T}+\varepsilon[r_{i-1}(D_0D_0^{\mathrm T}+\cdots+\\ &D_kD_k^{\mathrm T})r_{j-1}+r_{i-1}(\delta_1^2D_1D_1^{\mathrm T}+\cdots+\\ &\delta_k^2D_kD_k^{\mathrm T})r_{j-1}].\end{aligned}$$

这里:

$$i\in[2,k],\ j\in[3,k+1],\ i<j,$$

$$\begin{aligned}&\varphi_{n,k+2}'=\\ &-\mathcal W_{k+1}^{n-1}+r_{k+1}X^{\mathrm T}A_{n-1}^{\mathrm T}-r_{n-1}X+\\ &\varepsilon[r_{n-1}(D_0D_0^{\mathrm T}+\cdots+D_kD_k^{\mathrm T})r_{k+1}+\\ &r_{n-1}(\delta_1^2D_1D_1^{\mathrm T}+\cdots+\delta_k^2D_kD_k^{\mathrm T})r_{k+1}],\end{aligned}$$

$n\in[2,k+1]$,

$$\varphi_{k+2,k+2}'=$$

$$\sum_{i=1}^{k}\tau_{i,1/2}\mathcal{R}_i+\sum_{i=1}^{k}2\delta_i\mathcal{S}_i-r_{k+1}X-$$
$$r_{k+1}X^{\mathrm{T}}+\varepsilon[r_{k+1}(D_0D_0^{\mathrm{T}}+\cdots+$$
$$D_kD_k^{\mathrm{T}})r_{k+1}+r_{k+1}(\delta_1^2D_1D_1^{\mathrm{T}}+\cdots+$$
$$\delta_k^2D_kD_k^{\mathrm{T}})r_{k+1}],$$
$$\varphi'_1=[\tau_{1,1/2}\mathcal{M}_0^{\mathrm{T}}\ \ \tau_{2,1/2}\mathcal{N}_0^{\mathrm{T}}\ \ \cdots\ \ \tau_{k,1/2}\mathcal{O}_0^{\mathrm{T}}$$
$$\delta_1A_1X\ \ \cdots\ \ \delta_kA_kX],$$
$$\vdots$$
$$\varphi'_{k+2}=[\tau_{1,1/2}\mathcal{M}_{k+1}^{\mathrm{T}}\ \ \tau_{2,1/2}\mathcal{N}_{k+1}^{\mathrm{T}}\ \ \cdots\ \ \tau_{k,1/2}\mathcal{O}_{k+1}^{\mathrm{T}}$$
$$\delta_1r_{k+1}A_1X\ \ \cdots\ \ \delta_kr_{k+1}A_kX],$$
$$\varphi'_{k+3}=\mathrm{diag}\{-\tau_{1,1/2}\mathcal{R}_1\ \ -\tau_{2,1/2}\mathcal{R}_2\ \cdots\ -\tau_{k,1/2}\mathcal{R}_k$$
$$-\delta_1\mathcal{S}_1\ \ -\delta_2\mathcal{S}_2\ \ \cdots\ \ -\delta_k\mathcal{S}_k\}.$$

那么对于任意时滞参数满足(2)且不确定性满足式(11)的系统(10)渐近稳定.

证 在定理1的基础上将不确定性(11)代入定理1中的LMI式(3):

$$\varphi=\psi+HF(t)L+L^{\mathrm{T}}F^{\mathrm{T}}(t)H^{\mathrm{T}}.\tag{13}$$

ψ的各项同定理1. 这里:

$$H=\begin{bmatrix}
{N'_0}^{\mathrm{T}}D_0 & \cdots & {N'_0}^{\mathrm{T}}D_k & 0 & \cdots & 0 & \delta_1{N'_0}^{\mathrm{T}}D_1 & \cdots & \delta_k{N'_0}^{\mathrm{T}}D_k\\
{N'_1}^{\mathrm{T}}D_0 & \cdots & {N'_1}^{\mathrm{T}}D_k & 0 & \cdots & 0 & \delta_1{N'_1}^{\mathrm{T}}D_1 & \cdots & \delta_k{N'_1}^{\mathrm{T}}D_k\\
\vdots & \vdots & \vdots & & & & & & \\
{N'_{k+1}}^{\mathrm{T}}D_0 & \cdots & {N'_{k+1}}^{\mathrm{T}}D_k & 0 & \cdots & 0 & \delta_1{N'_{k+1}}^{\mathrm{T}}D_1 & \cdots & \delta_k{N'_{k+1}}^{\mathrm{T}}D_k\\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \vdots & & & & & & \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0
\end{bmatrix},$$

$$L=\begin{bmatrix}
E_0 & 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & 0 & 0\\
0 & E_1 & \ldots & 0 & 0 & \ldots & 0 & \ldots & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & \ldots & E_{k-1} & 0 & \ldots & 0 & \ldots & 0 & 0\\
0 & 0 & \ldots & 0 & E_k & \ldots & 0 & \ldots & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & \ldots & 0 & 0 & \ldots & E_1 & \ldots & 0 & 0\\
0 & 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & 0 & 0\\
0 & 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & E_{k-1} & 0\\
0 & 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & 0 & E_k
\end{bmatrix},$$

H,L为$(3k+2)\times(3k+2)$的矩阵. 若式(13)中的$\varphi<0$则系统(10)是渐近稳定的.

由引理2, 式(13)中的$\varphi<0$等价于

$$\varphi=\psi+\varepsilon HH^{\mathrm{T}}+\varepsilon^{-1}L^{\mathrm{T}}L<0.$$

每项的具体运算结果这里不再单独给出, 但是运算过程中将会出现诸如${N'_{i-1}}^{\mathrm{T}}(D_0D_0^{\mathrm{T}}+\cdots+D_kD_k^{\mathrm{T}})N'_{j-1}$等项, 它们并不是标准的LMI项, 必须将其进行转换. 定义

$$N'_0=V,\ N'_1=r_1V,\ \cdots,\ N'_{k+1}=r_{k+1}V,\ X=V^{-1},$$

并在

$$\varphi=\psi+\varepsilon HH^{\mathrm{T}}+\varepsilon^{-1}L^{\mathrm{T}}L$$

所得式的两侧左右分别乘以$\mathrm{diag}\{X^{\mathrm{T}}\ \ \cdots\ \ X^{\mathrm{T}}I\ \ \cdots\ \ I\}$(这里包含$X^{\mathrm{T}}$的项共有$(3k+2)$个, 包含$I$的项共有$k$个)及其转置矩阵. 令

$$\mathcal{P}=X^{\mathrm{T}}PX,\ \mathcal{Q}_i=X^{\mathrm{T}}Q_iX,$$
$$\mathcal{R}_i=X^{\mathrm{T}}R_iX,\ \mathcal{S}_i=X^{\mathrm{T}}S_iX,$$
$$i=1,\cdots,k;$$
$$\mathcal{M}_j=X^{\mathrm{T}}M_jX,$$
$$\mathcal{N}_j=X^{\mathrm{T}}N_jX,\cdots,\mathcal{O}_j=X^{\mathrm{T}}O_jX,$$
$$j=0,\cdots,k+1,$$

由Schur补定理可以得到定理2.

注 1 从定理1和定理2可以看出, 本文的结果对时滞参数的导数没有做任何限制, 同文献[1,5,6]中的时滞参数为正弦或者余弦函数相比, 对时滞参数没有要求, 更接近于实际情况.

注 2 从定理1,2及后面的仿真数据比较表明, 本文提出的方法保守性更低, 适用范围更广.

4 仿真与算例(The simulation and numeric example)

算例 1 为了和近年来的相关研究作比较, 本文运用定理1进行相同模型的仿真, 即针对的系统模型为

$$\dot{x}(t)=Ax(t)+A_1x(t-\tau_1(t)),$$

式中:

$$A=\begin{bmatrix}-2.0 & 0\\ 0 & -0.9\end{bmatrix},\ A_1=\begin{bmatrix}-1.0 & 0\\ -1.0 & -1.0\end{bmatrix}.$$

本文结果可以看作时滞导数未知, 下面一组比较数据表明本文结果的优越性(时滞导数未知).

表 1　近年来相关研究结果的数据比较

Table 1　Date compare in recently relevant research results

方法	时滞上限
Niculescu et al.[7]	0.3440
Su[8]	0.4045
Li and de Souza[9]	0.7218
Yue and Won[10]	—
Park[11]	—
Fridman and Shaked[12]	—
Huaicheng Yan *, Xinhan Huang[13]	0.9999

本算例仿真取$\tau_1(t)\in[0,3.9996]$, 对比新近研究成果[13]的时滞上限放大了4倍, 仿真结果如图1所示.

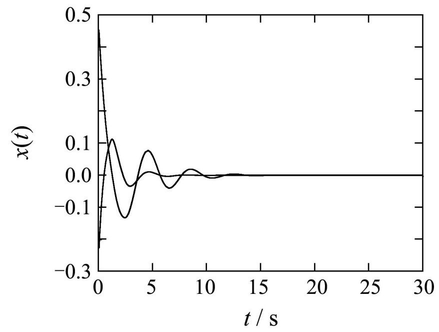

图 1　系统状态曲线

Fig. 1　The state curve of system

这里系统状态初始值为$x(0)=[-0.3\quad 0.5]^{\mathrm{T}}$. 由仿真结果可以看出, 本文的结果较前述文章的结果具有更低的保守性.

算例 2　假设系统模型如下:

$$\dot{x}(t)=(A_0+\Delta A_0(t))x(t)+\sum_{i=1}^{3}(A_i+\Delta A_i(t))x(t-\tau_i(t)).$$

其中:

$$A_0=\begin{bmatrix}0 & 1\\ -2 & 1\end{bmatrix},\ A_1=\begin{bmatrix}0.2 & 0.3\\ 0.2 & 0.1\end{bmatrix},$$

$$A_2=\begin{bmatrix}0 & -1\\ 1 & -2\end{bmatrix},\ A_3=\begin{bmatrix}0.3 & 0.1\\ 0.2 & 1\end{bmatrix},$$

$$D_0=D_1=D_2=D_3=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},$$

$$E_0=E_1=E_2=E_3=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},$$

$$F=[\sin(0.01t)\quad \cos(0.01t)].$$

$\varepsilon=0.1, r_1=r_2=r_3=r_4=0.1, \tau_{1,1/2}=1, \tau_{2,1/2}=1.5, \tau_{3,1/2}=2, \delta_1=\delta_2=\delta_3=0.5$. 这里, 令$\tau_1(t)\in[0.5,1.5], \tau_2(t)\in[1,2], \tau_3(t)\in[1.5,2.5]$. 利用本文中的定理2进行仿真实验. 假设系统状态的初始值为$x(0)=[-0.3\quad 0.5]^{\mathrm{T}}$. 下面给出部分实验数据以及仿真曲线(见图2):

$$\mathcal{P}=\begin{bmatrix}0.0162 & 0.0013\\ 0.0013 & 0.0170\end{bmatrix},\ \mathcal{Q}_1=\begin{bmatrix}0.0085 & 0.0161\\ 0.0161 & 0.0480\end{bmatrix},$$

$$\mathcal{Q}_2=\begin{bmatrix}0.0306 & 0.0135\\ 0.0135 & 0.0342\end{bmatrix},\ \mathcal{Q}_3=\begin{bmatrix}0.0101 & 0.0181\\ 0.0181 & 0.0553\end{bmatrix},$$

$$\mathcal{R}_1=\begin{bmatrix}3.3160 & -0.0123\\ -0.0123 & 3.3008\end{bmatrix},\ \mathcal{R}_2=\begin{bmatrix}2.4468 & -0.0051\\ -0.0051 & 2.4403\end{bmatrix},$$

$$\mathcal{R}_3=\begin{bmatrix}2.0342 & -0.0102\\ -0.0102 & 2.0013\end{bmatrix},\ \mathcal{S}_1=\begin{bmatrix}4.6653 & 0.0143\\ 0.0143 & 4.6062\end{bmatrix},$$

$$\mathcal{S}_2=\begin{bmatrix}4.6637 & 0.0106\\ 0.0106 & 4.6346\end{bmatrix},\mathcal{S}_3=\begin{bmatrix}4.6675 & 0.0142\\ 0.0142 & 4.6360\end{bmatrix},$$

$$X=\begin{bmatrix}-0.0144 & -0.0028\\ -0.0028 & 0.0036\end{bmatrix}.$$

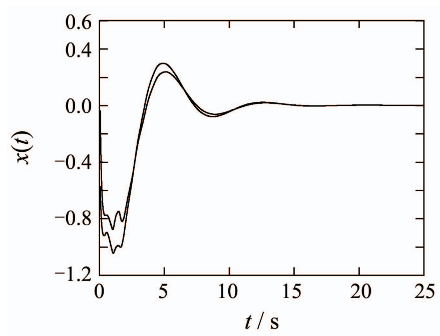

图 2　系统状态曲线

Fig. 2　The state curve of system

5　结论(Conclusion)

本文研究了一类具有区间时变多状态时滞且时滞导数不确定的系统的稳定性问题. 通过选择合理的Lyapunov-Krasovskii函数以及辅助变量和广义状态法, 以LMI的形式给出了时滞相关的稳定性充分条件. 本文结果对时滞的导数没有任何限制, 可用于具有快时变时滞系统. 并且从本文的结论中可以十分容易的得到相关的增强模型稳定性结果. 与已有的相关研究成果相比, 结论更具有一般性, 适用范围更广, 保守性也更低. 最后通过

数值比较及仿真表明了本文方法的有效性和优越性.

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