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凸多面体不确定随机时滞系统的参数依赖状态反馈控制

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摘要: 考虑了凸多面体不确定随机时滞系统的参数依赖状态反馈控制问题. 把参数相关的Lyapunov-Krasovskii泛函方法和自由权矩阵方法相结合, 得到了基于线性矩阵不等式(LMI)的时滞相关及参数相关的鲁棒镇定的充分条件. 由于在引入自由权矩阵时, 减少了所用的自由矩阵数目, 使得给出的参数依赖的控制器更易于实现. 最后用例子说明了结果的有效性.

关键词: 随机系统; 时滞; 凸多面体不确定性; 鲁棒镇定; 参数依赖控制器

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Parameter-dependent state feedback control for stochastic delay-varying systems with polytopic-type uncertainties

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Abstract: The parameter-dependent state feedback control problem for stochastic delay-varying systems with polytopic-type uncertainties is discussed. Based on parameter-dependent Lyapunov-Krasovskii functional and free-weighting matrix method, some delay-dependent and parameter-dependent stabilization conditions are presented in terms of linear matrix inequalities. Since the number of free-weighting matrices is reduced when introducing free-weighting matrices, the given parameter-dependent controller is easier to be implemented. Finally, the simulation example shows that the result is efficient.

Key words: stochastic systems; time-delay; polytopic-type uncertainties; robust stabilization; parameter-dependent controller

1 引言(Introduction)

模型不确定系统具有广泛的实际工程背景, 模型不确定系统可分为参数不确定性模型和动态不确定性模型. 对于参数不确定性模型, 可以采用凸多面体或仿射参数描述, 而对于动态不确定性模型, 则主要是采用范数有界条件描述. 对于凸多面体不确定的时滞系统的鲁棒稳定性研究, 已有大量研究结果, 比如文献[1, 2]以及其后的参考文献. 而对于凸多面体不确定的时滞系统的鲁棒镇定问题的研究, 参数无关的状态反馈控制也有许多结果[3∼7]. 文献[3]运用松弛矩阵和广义模型变换方法, 得到了凸多面体不确定时滞系统的静态反馈镇定条件, 改进了文献[4∼6]结果, 相比文献[7], 所用自由矩阵数目减少. 对于线性参数变化系统(LPV)以及具有凸多面体不确定的系统, 参数依赖的状态反馈控制问题得到了广泛关注, 可查看文献[8∼10]及其后的参考文献. 文献[9]中的一些例子表明, 有些系统不能用固定增益(参数无关的控制器)来镇定, 但是可以用参数依赖的控制器镇定. 对于凸多面体不确定随机系统, 也有文献讨论了时滞相关的稳定性条件[11, 12], 但是时滞相关的镇定条件, 特别是参数依赖的状态反馈控制问题的研究还不多见.

对凸多面体不确定系统, 通常采用参数相关的Lyapunov-Krasovskii泛函, 由于凸多面体不同顶点采用了不同的Lyapunov-Krasovskii泛函, 这样可以降低所得到的稳定性条件或镇定性条件的保守性. 把参数相关的Lyapunov-Krasovskii泛函方法和自由权矩阵方法相结合, 使得所得到的结果中不存在Lyapunov矩阵变量和系统矩阵的乘积项, 这样就可以研究凸多面体不确定系统的镇定问题. 本文通过构造合适的与参数相关的Lyapunov-Krasovskii泛函, 并灵活运用自由加权矩阵, 减少了所用的自由矩

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阵数目, 给出了参数依赖的状态反馈控制器, 可以通过求解线性矩阵不等式(LMI)来获得, 在该控制器下, 系统达到指数稳定. 最后用例子说明了结果的有效性.

2 问题描述(Problem description)

考虑如下凸多面体不确定随机时滞系统:

$$\mathrm{d}x(t)=(A_\lambda x(t)+A_{1\lambda}x(t-d(t))+B_\lambda u(t))\mathrm{d}t+ (C_\lambda x(t)+C_{1\lambda}x(t-d(t)))\mathrm{d}w(t), \tag{1}$$

$$x(t)=\phi(t),\ t\in[-\tau,0], \tag{2}$$

其中: $x(t)\in\mathbb{R}^n$为状态向量; $u(t)\in\mathbb{R}^m$为输入向量; $A_\lambda, A_{1\lambda}, C_\lambda, C_{1\lambda}\in\mathbb{R}^{n\times n}, B_\lambda\in\mathbb{R}^{n\times m}$为凸多面体不确定系统矩阵, 即满足

$$[A_\lambda\ \ A_{1\lambda}\ \ B_\lambda\ \ C_\lambda\ \ C_{1\lambda}]=\sum_{i=1}^{s}\lambda_i[A_i\ \ A_{1i}\ \ B_i\ \ C_i\ \ C_{1i}];$$

$\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_s)^{\mathrm{T}}$为不确定向量且满足$\sum\limits_{i=1}^{s}\lambda_i=1, \lambda_i\geqslant 0(i=1,2,\cdots,s)$; $w(t)$为定义在概率空间$(\Omega,F,\{F_t\}_{t\geqslant0},P)$上的布朗运动; $\phi(t)$为定义在$L^2_{\mathrm{F}_0}([-\tau,0];\mathbb{R}^n)$上的初值函数; $d(t)$为时变时滞, 并满足$0\leqslant d(t)\leqslant\tau,\ \dot{d}(t)\leqslant\mu$; τ,μ为常数.

由文献[13]可知系统(1)(2)存在惟一解, 并记为$x(t,\phi)$. 显然, $x(t,0)\equiv0$为平凡解.

在本文中矩阵$M>(<,\leqslant,\geqslant)$分别表示正定、负定、半负定、半正定. 在对称矩阵中用*表示的是根据对称性所引出的项. 上标“T”代表矩阵的转置. $\mathrm{E}\{\cdot\}$表示对括号中的项取期望.

定义 1 如果存在$\alpha>0,\ \beta>0$使得对$t\geqslant t_0$有

$$\mathrm{E}\|x(t,\phi)\|^2\leqslant\alpha\mathrm{e}^{-\beta(t-t_0)}\sup_{-\tau\leqslant s\leqslant0}\mathrm{E}\|\phi(s)\|^2,$$

称系统(1)(2)为鲁棒指数稳定.

本文的目标是设计参数依赖的状态反馈控制器

$$u(t)=K(\lambda)x(t), \tag{3}$$

$K(\lambda)$是与参数λ相关的矩阵, 使得在该控制器下, 凸多面体不确定随机时滞系统(1)(2)达到指数稳定.

引理 1[14] 对$\forall a,b\in\mathbb{R}^n$, 矩阵$Q>0$, 有

$$\pm2a^{\mathrm{T}}b\leqslant a^{\mathrm{T}}Qa+b^{\mathrm{T}}Q^{-1}b.$$

引理 2[15] 已知矩阵$\Omega_1,\Omega_2,\Omega_3$, 其中: $\Omega_1=\Omega_1^{\mathrm{T}}, 0<\Omega_2=\Omega_2^{\mathrm{T}}$, 则

$$\Omega_1+\Omega_3^{\mathrm{T}}\Omega_2^{-1}\Omega_3<0$$

成立的充分必要条件是下式成立:

$$\begin{bmatrix}\Omega_1 & \Omega_3^{\mathrm{T}}\\ \Omega_3 & -\Omega_2\end{bmatrix}<0.$$

3 主要结果(Main results)

在控制器(3)作用下, 式(1)~(3)所构成的闭环系统为:

$$\mathrm{d}x(t)=((A_\lambda+B_\lambda K(\lambda))x(t)+A_{1\lambda}x(t-d(t))(\mathrm{d}t+ (C_\lambda x(t)+C_{1\lambda}x(t-d(t)))\mathrm{d}w(t), \tag{4}$$

$$x(t)=\phi(t),\ t\in[-\tau,0]. \tag{5}$$

为使符号简化, 记:

$$y(t)=(A_\lambda+B_\lambda K(\lambda))x(t)+A_{1\lambda}x(t-d(t),$$
$$g(t)=C_\lambda x(t)+C_{1\lambda}x(t-d(t)).$$

于是系统(4)变为

$$\mathrm{d}x(t)=y(t)\mathrm{d}t+g(t)\mathrm{d}w(t). \tag{6}$$

为设计系统(4)(5)的参数依赖的状态反馈控制器, 首先讨论当$u(t)=0$时系统(1)(2)的鲁棒稳定性条件.

定理 1 对给定的常数τ,μ, 如果存在适当维数矩阵$P_i,Q_i,R_i,S_i,U_i,X_i>0, M_i,N_i$及$\Phi_{ij}$满足线性矩阵不等式(7)~(9), 则系统(1)(2)是鲁棒指数稳定的.

$$\Sigma_{ij}+\Sigma_{ji}-\Phi_{ij}-\Phi_{ij}^{\mathrm{T}}<0,\ 1\leqslant i\leqslant j\leqslant s. \tag{7}$$

$$\Pi=\begin{bmatrix}\Phi_{11} & \Phi_{12} & \cdots & \Phi_{1s}\\ * & \Phi_{22} & \cdots & \Phi_{2s}\\ * & * & \ddots & \vdots\\ * & * & \cdots & \Phi_{ss}\end{bmatrix}<0. \tag{8}$$

$$\begin{bmatrix}X_i & M_i\\ M_i^{\mathrm{T}} & S_i\end{bmatrix}>0,\ \begin{bmatrix}X_i & N_i\\ N_i^{\mathrm{T}} & S_i\end{bmatrix}>0,\ i=1,2,\cdots s. \tag{9}$$

其中:

$$\Sigma_{ij}=\begin{bmatrix}\Sigma_{11ij} & \Sigma_{12ij} & \Sigma_{13ij} & \Sigma_{14ij} & \Sigma_{15ij} & 0 & 0\\ * & \Sigma_{22ij} & \Sigma_{23ij} & \Sigma_{24ij} & \Sigma_{25ij} & N_i & 0\\ * & * & \Sigma_{33ij} & \Sigma_{34ij} & \Sigma_{35ij} & 0 & M_i\\ * & * & * & \Sigma_{44ij} & \Sigma_{45ij} & 0 & 0\\ * & * & * & * & \Sigma_{55ij} & 0 & 0\\ * & * & * & * & * & -U_i & 0\\ * & * & * & * & * & * & -U_i\end{bmatrix},$$

$$\Sigma_{11ij}=Q_i+R_i+T_jA_i+A_i^{\mathrm{T}}T_j^{\mathrm{T}},$$
$$\Sigma_{12ij}=N_i^{\mathrm{T}}+T_jA_{1i},\ \Sigma_{13ij}=0,$$
$$\Sigma_{14ij}=P_i-T_j+A_i^{\mathrm{T}}T_j^{\mathrm{T}},\Sigma_{15ij}=C_i^{\mathrm{T}}T_j^{\mathrm{T}},$$
$$\Sigma_{22ij}=-(1-\mu)Q_i-N_i^{\mathrm{T}}-N_i+\tau X_i,$$
$$\Sigma_{23ij}=M_i^{\mathrm{T}},\ \Sigma_{24ij}=A_{1i}^{\mathrm{T}}T_j^{\mathrm{T}},\ \Sigma_{25ij}=C_{1i}^{\mathrm{T}}T_j^{\mathrm{T}},$$
$$\Sigma_{33ij}=-R_i-M_i-M_i^{\mathrm{T}}+\tau X_i,$$
$$\Sigma_{34ij}=\Sigma_{35ij}=0,\ \Sigma_{44ij}=\tau S_i-T_j-T_j^{\mathrm{T}},$$

$\varSigma_{45ij}=0,\ \varSigma_{55ij}=\tau U_i+P_i-T_j-T_j^{\mathrm{T}}.$

证　在系统(1)中令$u(t)=0$, 此时记:

$$\tilde{y}(t)=A_\lambda x(t)+A_{1\lambda}x(t-d(t)),$$
$$g(t)=C_\lambda x(t)+C_{1\lambda}x(t-d(t)).$$

于是系统(1)变为

$$\mathrm{d}x(t)=\tilde{y}(t)\mathrm{d}t+g(t)\mathrm{d}w(t). \tag{10}$$

选择与参数相关的Lyapunov-Krasovskii泛函

$$V(t,\lambda)=\sum_{i=1}^{5}V_i(t,\lambda),$$

其中:

$$V_1(t,\lambda)=x^{\mathrm{T}}(t)P_\lambda x(t),$$
$$V_2(t,\lambda)=\int_{t-d(t)}^{t}x^{\mathrm{T}}(s)Q_\lambda x(s)\mathrm{d}s,$$
$$V_3(t,\lambda)=\int_{t-\tau}^{t}x^{\mathrm{T}}(s)R_\lambda x(s)\mathrm{d}s,$$
$$V_4(t,\lambda)=\int_{-\tau}^{0}\int_{t+\theta}^{t}\tilde{y}^{\mathrm{T}}(s)S_\lambda\tilde{y}(s)\mathrm{d}s\mathrm{d}\theta,$$
$$V_5(t,\lambda)=\int_{-\tau}^{0}\int_{t+\theta}^{t}g^{\mathrm{T}}(s)U_\lambda g(s)\mathrm{d}s\mathrm{d}\theta.$$

这里$P_\lambda,Q_\lambda,R_\lambda,S_\lambda,U_\lambda$均为正定阵, 且满足

$$[P_\lambda\ \ Q_\lambda\ \ R_\lambda\ \ S_\lambda\ \ U_\lambda]=\sum_{i=1}^{s}\lambda_i[P_i\ \ Q_i\ \ R_i\ \ S_i\ \ U_i].$$

根据Itô微分法则, 沿式(10)的解可得:

$$LV_1=2x^{\mathrm{T}}(t)P_\lambda\tilde{y}(t)+g^{\mathrm{T}}(t)P_\lambda g(t), \tag{11}$$
$$LV_2\leqslant x^{\mathrm{T}}(t)Q_\lambda x(t)-(1-\mu)x^{\mathrm{T}}(t-d(t))\cdot Q_\lambda x(t-d(t)), \tag{12}$$
$$LV_3=x^{\mathrm{T}}(t)R_\lambda x(t)-x^{\mathrm{T}}(t-\tau)R_\lambda x(t-\tau), \tag{13}$$
$$LV_4=\tau\tilde{y}^{\mathrm{T}}(t)S_\lambda\tilde{y}(t)-\int_{t-d(t)}^{t}\tilde{y}^{\mathrm{T}}(s)S_\lambda\tilde{y}(s)\mathrm{d}s-\int_{t-\tau}^{t-d(t)}\tilde{y}^{\mathrm{T}}(s)S_\lambda\tilde{y}(s)\mathrm{d}s, \tag{14}$$
$$LV_5=\tau g^{\mathrm{T}}(t)U_\lambda g(t)-\int_{t-d(t)}^{t}g^{\mathrm{T}}(s)U_\lambda g(s)\mathrm{d}s-\int_{t-\tau}^{t-d(t)}g^{\mathrm{T}}(s)U_\lambda g(s)\mathrm{d}s. \tag{15}$$

对矩阵

$$N_\lambda=\sum_{i=1}^{s}\lambda_iN_i,\ M_\lambda=\sum_{i=1}^{s}\lambda_iM_i,\ T_\lambda=\sum_{i=1}^{s}\lambda_iT_i$$

有下列等式成立:

$$2x^{\mathrm{T}}(t-d(t))N_\lambda(x(t)-x(t-d(t))-\int_{t-d(t)}^{t}\tilde{y}(s)\mathrm{d}s-\int_{t-d(t)}^{t}g(s)\mathrm{d}w(s))=0, \tag{16}$$
$$2x^{\mathrm{T}}(t-\tau)M_\lambda(x(t-d(t))-x(t-\tau)-\int_{t-\tau}^{t-d(t)}\tilde{y}(s)\mathrm{d}s-\int_{t-\tau}^{t-d(t)}g(s)\mathrm{d}w(s))=0, \tag{17}$$
$$2x^{\mathrm{T}}(t)T_\lambda(A_\lambda x(t)+A_{1\lambda}x(t-d(t))-\tilde{y}(t))=0, \tag{18}$$
$$2\tilde{y}^{\mathrm{T}}(t)T_\lambda(A_\lambda x(t)+A_{1\lambda}x(t-d(t))-\tilde{y}(t))=0, \tag{19}$$
$$2g^{\mathrm{T}}(t)T_\lambda(C_\lambda x(t)+C_{1\lambda}x(t-d(t))-g(t))=0. \tag{20}$$

由引理1可得:

$$-2x^{\mathrm{T}}(t-d(t))N_\lambda\int_{t-d(t)}^{t}g(s)\mathrm{d}w(s)\leqslant x^{\mathrm{T}}(t-d(t))N_\lambda U_\lambda^{-1}N_\lambda^{\mathrm{T}}x(t-d(t))+\int_{t-d(t)}^{t}g^{\mathrm{T}}(s)\mathrm{d}w(s)U_\lambda\int_{t-d(t)}^{t}g(s)\mathrm{d}w(s), \tag{21}$$
$$-2x^{\mathrm{T}}(t-\tau)M_\lambda\int_{t-\tau}^{t-d(t)}g(s)\mathrm{d}w(s)\leqslant x^{\mathrm{T}}(t-\tau)M_\lambda U_\lambda^{-1}M_\lambda^{\mathrm{T}}x(t-\tau)+\int_{t-\tau}^{t-d(t)}g^{\mathrm{T}}(s)\mathrm{d}w(s)U_\lambda\int_{t-\tau}^{t-d(t)}g(s)\mathrm{d}w(s). \tag{22}$$

对矩阵$X_\lambda=\sum\limits_{i=1}^{s}\lambda_iX_i\geqslant 0$, 有下式成立:

$$\tau x^{\mathrm{T}}(t-\tau)X_\lambda x(t-\tau)-\int_{t-d(t)}^{t}x^{\mathrm{T}}(t-\tau)X_\lambda x(t-\tau)\mathrm{d}s-\int_{t-\tau}^{t-d(t)}x^{\mathrm{T}}(t-\tau)X_\lambda x(t-\tau)\mathrm{d}s=0, \tag{23}$$
$$\tau x^{\mathrm{T}}(t-d(t))X_\lambda x(t-d(t))-\int_{t-d(t)}^{t}x^{\mathrm{T}}(t-d(t))X_\lambda x(t-d(t))\mathrm{d}s-\int_{t-\tau}^{t-d(t)}x^{\mathrm{T}}(t-d(t))X_\lambda x(t-d(t))\mathrm{d}s=0, \tag{24}$$
$$\mathrm{E}[\int_{t-d(t)}^{t}g^{\mathrm{T}}(s)\mathrm{d}w(s)U_\lambda\int_{t-d(t)}^{t}g(s)\mathrm{d}w(s)]=\mathrm{E}[\int_{t-d(t)}^{t}g^{\mathrm{T}}(s)U_\lambda g(s)\mathrm{d}s], \tag{25}$$
$$\mathrm{E}[\int_{t-\tau}^{t-d(t)}g^{\mathrm{T}}(s)\mathrm{d}w(s)U_\lambda\int_{t-\tau}^{t-d(t)}g(s)\mathrm{d}w(s)]=\mathrm{E}[\int_{t-\tau}^{t-d(t)}g^{\mathrm{T}}(s)U_\lambda g(s)\mathrm{d}s]. \tag{26}$$

由式(11)～式(26), 可得

$$\mathrm{E}(LV)\leqslant\mathrm{E}\left\{\xi^{\mathrm{T}}(t)\varLambda_{1\lambda}\xi(t)\right\}-\int_{t-d(t)}^{t}\mathrm{E}\left\{\eta^{\mathrm{T}}(t,s)\varLambda_{2\lambda}\eta(t,s)\right\}\mathrm{d}s-\int_{t-\tau}^{t-d(t)}\mathrm{E}\left\{\eta^{\mathrm{T}}(t,s)\varLambda_{3\lambda}\eta(t,s)\right\}\mathrm{d}s, \tag{27}$$

其中:

$$\xi^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-d(t))\ \ x^{\mathrm{T}}(t-\tau)\ \ \tilde{y}^{\mathrm{T}}(t)\ \ g^{\mathrm{T}}(t)],$$
$$\eta^{\mathrm{T}}(t,s)=[x^{\mathrm{T}}(t-d(t))\ \ x^{\mathrm{T}}(t-\tau)\ \ \tilde{y}^{\mathrm{T}}(s)],$$
$$\varLambda_{1\lambda}=\begin{bmatrix}\varPsi_{11}&\varPsi_{12}&\varPsi_{13}&\varPsi_{14}&\varPsi_{15}\\ *&\varPsi_{22}&\varPsi_{23}&\varPsi_{24}&\varPsi_{25}\\ *&*&\varPsi_{33}&\varPsi_{34}&\varPsi_{35}\\ *&*&*&\varPsi_{44}&\varPsi_{45}\\ *&*&*&*&\varPsi_{55}\end{bmatrix},$$

$$\Lambda_{2\lambda}=\begin{bmatrix}X_\lambda & 0 & N_\lambda\\ 0 & X_\lambda & 0\\ N_\lambda^{\mathrm T} & 0 & S_\lambda\end{bmatrix},\ \Lambda_{3\lambda}=\begin{bmatrix}X_\lambda & 0 & 0\\ 0 & X_\lambda & M_\lambda\\ 0 & M_\lambda^{\mathrm T} & S_\lambda\end{bmatrix}.$$

这里:

$$\begin{aligned}&\Psi_{11}=Q_\lambda+R_\lambda+T_\lambda A_\lambda+A_\lambda^{\mathrm T}T_\lambda^{\mathrm T},\\&\Psi_{12}=N_\lambda^{\mathrm T}+T_\lambda A_{1\lambda},\ \Psi_{13}=0,\\&\Psi_{14}=P_\lambda-T_\lambda+A_\lambda^{\mathrm T}T_\lambda^{\mathrm T},\ \Psi_{15}=C_\lambda^{\mathrm T}T_\lambda^{\mathrm T},\\&\Psi_{22}=-(1-\mu)Q_\lambda-N_\lambda^{\mathrm T}-N_\lambda+\tau X_\lambda+\\&\qquad N_\lambda U_\lambda^{-1}N_\lambda^{\mathrm T},\\&\Psi_{23}=M_\lambda^{\mathrm T},\ \Psi_{24}=A_{1\lambda}^{\mathrm T}T_\lambda^{\mathrm T},\ \Psi_{25}=C_{1\lambda}^{\mathrm T}T_\lambda^{\mathrm T},\\&\Psi_{33}=-R_\lambda-M_\lambda-M_\lambda^{\mathrm T}+\tau X_\lambda+M_\lambda U_\lambda^{-1}M_\lambda^{\mathrm T},\\&\Psi_{34}=\Psi_{35}=0,\ \Psi_{44}=\tau S_\lambda-T_\lambda-T_\lambda^{\mathrm T},\\&\Psi_{45}=0,\ \Psi_{55}=\tau U_\lambda+P_\lambda-T_\lambda-T_\lambda^{\mathrm T}.\end{aligned}$$

现在记

$$\Sigma_\lambda=\begin{bmatrix}\Sigma_{11}&\Sigma_{12}&\Sigma_{13}&\Sigma_{14}&\Sigma_{15}&0&0\\ *&\Sigma_{22}&\Sigma_{23}&\Sigma_{24}&\Sigma_{25}&N_\lambda&0\\ *&*&\Sigma_{33}&\Sigma_{34}&\Sigma_{35}&0&M_\lambda\\ *&*&*&\Sigma_{44}&\Sigma_{45}&0&0\\ *&*&*&*&\Sigma_{55}&0&0\\ *&*&*&*&*&-U_\lambda&0\\ *&*&*&*&*&*&-U_\lambda\end{bmatrix},$$

这里:

$$\begin{aligned}&\Sigma_{11}=\Psi_{11},\ \Sigma_{12}=\Psi_{12},\ \Sigma_{13}=0,\ \Sigma_{14}=\Psi_{14},\\&\Sigma_{15}=\Psi_{15},\ \Sigma_{22}=-(1-\mu)Q_\lambda-N_\lambda^{\mathrm T}-N_\lambda+\tau X_\lambda,\\&\Sigma_{23}=\Psi_{23},\ \Sigma_{24}=\Psi_{24},\ \Sigma_{25}=\Psi_{25},\\&\Sigma_{33}=-R_\lambda-M_\lambda-M_\lambda^{\mathrm T}+\tau X_\lambda,\ \Sigma_{34}=\Sigma_{35}=0,\\&\Sigma_{44}=\Psi_{44},\ \Sigma_{45}=0,\ \Sigma_{55}=\Psi_{55}.\end{aligned}$$

由引理2, 线性矩阵不等式$\Lambda_{1\lambda}<0$与$\Sigma_\lambda<0$等价. 因为

$$\begin{aligned}&\Sigma_\lambda=\sum_{j=1}^{s}\sum_{i=1}^{s}\lambda_i\lambda_j\Sigma_{ij}=\\&\quad\sum_{i=1}^{s}\lambda_i^2\Sigma_{ii}+\sum_{i=1}^{s-1}\sum_{j=i+1}^{s}\lambda_i\lambda_j(\Sigma_{ij}+\Sigma_{ji}).\end{aligned}$$

另一方面式(7)等价于:

$$\begin{aligned}&\Sigma_{ii}<\Phi_{ii},\ i=1,\cdots,s,\\&\Sigma_{ij}+\Sigma_{ji}\leqslant\Phi_{ij}+\Phi_{ij}^{\mathrm T},\ 1\leqslant i<j\leqslant s.\end{aligned}$$

所以得

$$\Sigma_\lambda\leqslant\sum_{i=1}^{s}\lambda_i^2\Phi_{ii}+\sum_{i=1}^{s-1}\sum_{j=i+1}^{s}\lambda_i\lambda_j(\Phi_{ij}+\Phi_{ij}^{\mathrm T})=\zeta^{\mathrm T}\Pi\zeta,$$

其中$\zeta=(\lambda_1I,\lambda_2I,\cdots,\lambda_sI)^{\mathrm T}$, Π见式(8). 又显然式(7)~式(9)满足时, $\Lambda_{2\lambda}>0,\Lambda_{3\lambda}>0$, 所以若令$\varpi=\lambda_{\min}(-\Lambda_{1\lambda})$, 则得到

$$\mathrm E(LV)\leqslant-\varpi\mathrm E\{\|x(t)\|^2+\|x(t-d(t))\|^2\}.$$

于是类似于文献[16], 可证系统(1)(2)为鲁棒指数稳定.

定理 2 对给定的常数τ,μ, 如果存在适当维数矩阵$\tilde P_i,\tilde Q_i,\tilde R_i,\tilde S_i,\tilde U_i,\tilde X_i>0,\tilde M_i,\tilde N_i,V_i,\tilde\Phi_{ij}$及可逆矩阵$W_i$, 满足线性矩阵不等式(28)~(30), 则系统(1)(2)在控制律(3)作用下是鲁棒指数稳定的, 控制增益矩阵为$K(\lambda)=V_\lambda(W_\lambda^{\mathrm T})^{-1}$, V_λ,W_λ满足:

$$[V_\lambda\ \ W_\lambda]=\sum_{i=1}^{s}\lambda_i[V_i\ \ W_i].$$

$$\Xi_{ij}+\Xi_{ji}-\tilde\Phi_{ij}-\tilde\Phi_{ij}^{\mathrm T}<0,\ 1\leqslant i\leqslant j\leqslant s.\tag{28}$$

$$\tilde\Pi=\begin{bmatrix}\tilde\Phi_{11}&\tilde\Phi_{12}&\cdots&\tilde\Phi_{1s}\\ *&\tilde\Phi_{22}&\cdots&\tilde\Phi_{2s}\\ *&*&\ddots&\vdots\\ *&*&\cdots&\tilde\Phi_{ss}\end{bmatrix}<0,\tag{29}$$

$$\begin{bmatrix}X_i&M_i\\ M_i^{\mathrm T}&S_i\end{bmatrix}>0,\ \begin{bmatrix}X_i&N_i\\ N_i^{\mathrm T}&S_i\end{bmatrix}>0,\ i=1,2,\cdots s.\tag{30}$$

其中:

$$\Xi_{ij}=\begin{bmatrix}\tilde\Sigma_{11ij}&\tilde\Sigma_{12ij}&\tilde\Sigma_{13ij}&\tilde\Sigma_{14ij}&\tilde\Sigma_{15ij}&0&0\\ *&\tilde\Sigma_{22ij}&\tilde\Sigma_{23ij}&\tilde\Sigma_{24ij}&\tilde\Sigma_{25ij}&\tilde N_i&0\\ *&*&\tilde\Sigma_{33ij}&\tilde\Sigma_{34ij}&\tilde\Sigma_{35ij}&0&\tilde M_i\\ *&*&*&\tilde\Sigma_{44ij}&\tilde\Sigma_{45ij}&0&0\\ *&*&*&*&\tilde\Sigma_{55ij}&0&0\\ *&*&*&*&*&-\tilde U_i&0\\ *&*&*&*&*&*&-\tilde U_i\end{bmatrix},$$

$$\begin{aligned}&\tilde\Sigma_{11ij}=\tilde Q_i+\tilde R_i+A_iW_j^{\mathrm T}+B_iV_j+W_jA_i^{\mathrm T}+V_j^{\mathrm T}B_i^{\mathrm T},\\&\tilde\Sigma_{12ij}=\tilde N_i^{\mathrm T}+A_{1i}W_j^{\mathrm T},\ \tilde\Sigma_{13ij}=0,\\&\tilde\Sigma_{14ij}=\tilde P_i-W_j^{\mathrm T}+W_jA_i^{\mathrm T}+V_j^{\mathrm T}B_i^{\mathrm T},\ \tilde\Sigma_{15ij}=W_jC_i^{\mathrm T},\\&\tilde\Sigma_{22ij}=-(1-\mu)\tilde Q_i-\tilde N_i^{\mathrm T}-\tilde N_i+\tau\tilde X_i,\\&\tilde\Sigma_{23ij}=\tilde M_i^{\mathrm T},\ \tilde\Sigma_{24ij}=W_jA_{1i}^{\mathrm T},\ \tilde\Sigma_{25ij}=W_jC_{1i}^{\mathrm T},\\&\tilde\Sigma_{33ij}=-\tilde R_i-\tilde M_i-\tilde M_i^{\mathrm T}+\tau\tilde X_i,\\&\tilde\Sigma_{34ij}=\tilde\Sigma_{35ij}=0,\ \tilde\Sigma_{44ij}=\tau\tilde S_i-W_j-W_j^{\mathrm T},\\&\tilde\Sigma_{45ij}=0,\ \tilde\Sigma_{55ij}=\tau\tilde U_i+\tilde P_i-W_j-W_j^{\mathrm T}.\end{aligned}$$

证 选择与定理1证明过程中同样的Lyapunov-krasovskii泛函, 除了要把$V_4(t,\lambda)$中的$\tilde y$换成y. 即

$$V_4(t,\lambda)=\int_{-\tau}^{0}\int_{t+\theta}^{t}y^{\mathrm T}(s)S_\lambda y(s)\mathrm ds\mathrm d\theta.$$

同样地, 在式(11)~(20)中也把$\tilde y$换成y, 结合式(21)~(26), 可得

$$\mathrm E(LV)\leqslant\mathrm E\{\tilde\xi^{\mathrm T}(t)\tilde\Lambda_{1\lambda}\tilde\xi(t)\}-$$

$$\int_{t-d(t)}^{t} \mathrm{E}\{\tilde{\eta}^{\mathrm{T}}(t,s)\tilde{\Lambda}_{2\lambda}\tilde{\eta}(t,s)\}\mathrm{d}s - \int_{t-\tau}^{t-d(t)} \mathrm{E}\{\tilde{\eta}^{\mathrm{T}}(t,s)\tilde{\Lambda}_{3\lambda}\tilde{\eta}(t,s)\}\mathrm{d}s, \quad (31)$$

其中:

$$\tilde{\xi}^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-d(t))\ \ x^{\mathrm{T}}(t-\tau)\ \ y^{\mathrm{T}}(t)\ \ g^{\mathrm{T}}(t)],$$

$$\tilde{\eta}^{\mathrm{T}}(t,s) = [x^{\mathrm{T}}(t-d(t))\ \ x^{\mathrm{T}}(t-\tau)\ \ y^{\mathrm{T}}(s)],$$

$$\tilde{\Lambda}_{1\lambda} = \begin{bmatrix} \tilde{\Psi}_{11} & \tilde{\Psi}_{12} & \tilde{\Psi}_{13} & \tilde{\Psi}_{14} & \tilde{\Psi}_{15} \\ * & \tilde{\Psi}_{22} & \tilde{\Psi}_{23} & \tilde{\Psi}_{24} & \tilde{\Psi}_{25} \\ * & * & \tilde{\Psi}_{33} & \tilde{\Psi}_{34} & \tilde{\Psi}_{35} \\ * & * & * & \tilde{\Psi}_{44} & \tilde{\Psi}_{45} \\ * & * & * & * & \tilde{\Psi}_{55} \end{bmatrix},$$

$$\tilde{\Lambda}_{2\lambda} = \begin{bmatrix} X_\lambda & 0 & N_\lambda \\ 0 & X_\lambda & 0 \\ N_\lambda^{\mathrm{T}} & 0 & S_\lambda \end{bmatrix},\ \tilde{\Lambda}_{3\lambda} = \begin{bmatrix} X_\lambda & 0 & 0 \\ 0 & X_\lambda & M_\lambda \\ 0 & M_\lambda^{\mathrm{T}} & S_\lambda \end{bmatrix},$$

这里:

$$\tilde{\Psi}_{11} = Q_\lambda + R_\lambda + T_\lambda(A_\lambda + B_\lambda K(\lambda)) + (A_\lambda + B_\lambda K(\lambda))^{\mathrm{T}} T_\lambda^{\mathrm{T}},$$

$$\tilde{\Psi}_{12} = N_\lambda^{\mathrm{T}} + T_\lambda A_{1\lambda},\ \tilde{\Psi}_{13} = 0,\ \tilde{\Psi}_{15} = C_\lambda^{\mathrm{T}} T_\lambda^{\mathrm{T}},$$

$$\tilde{\Psi}_{14} = P_\lambda - T_\lambda + (A_\lambda + B_\lambda K(\lambda))^{\mathrm{T}} T_\lambda^{\mathrm{T}},$$

$$\tilde{\Psi}_{22} = -(1-\mu)Q_\lambda - N_\lambda^{\mathrm{T}} - N_\lambda + \tau X_\lambda + N_\lambda U_\lambda^{-1} N_\lambda^{\mathrm{T}},$$

$$\tilde{\Psi}_{23} = M_\lambda^{\mathrm{T}},\ \tilde{\Psi}_{24} = A_{1\lambda}^{\mathrm{T}} T_\lambda^{\mathrm{T}},\ \tilde{\Psi}_{25} = C_{1\lambda}^{\mathrm{T}} T_\lambda^{\mathrm{T}},$$

$$\tilde{\Psi}_{33} = -R_\lambda - M_\lambda - M_\lambda^{\mathrm{T}} + \tau X_\lambda + M_\lambda U_\lambda^{-1} M_\lambda^{\mathrm{T}},$$

$$\tilde{\Psi}_{34} = \tilde{\Psi}_{35} = 0,\ \tilde{\Psi}_{44} = \tau S_\lambda - T_\lambda - T_\lambda^{\mathrm{T}},\ \tilde{\Psi}_{45} = 0,$$

$$\tilde{\Psi}_{55} = \tau U_\lambda + P_\lambda - T_\lambda - T_\lambda^{\mathrm{T}}.$$

现在记:

$$\hat{\Sigma} = \begin{bmatrix} \hat{\Sigma}_{11} & \hat{\Sigma}_{12} & \hat{\Sigma}_{13} & \hat{\Sigma}_{14} & \hat{\Sigma}_{15} & 0 & 0 \\ * & \hat{\Sigma}_{22} & \hat{\Sigma}_{23} & \hat{\Sigma}_{24} & \hat{\Sigma}_{25} & N_\lambda & 0 \\ * & * & \hat{\Sigma}_{33} & \hat{\Sigma}_{34} & \hat{\Sigma}_{35} & 0 & M_\lambda \\ * & * & * & \hat{\Sigma}_{44} & \hat{\Sigma}_{45} & 0 & 0 \\ * & * & * & * & \hat{\Sigma}_{55} & 0 & 0 \\ * & * & * & * & * & -U_\lambda & 0 \\ * & * & * & * & * & * & -U_\lambda \end{bmatrix},$$

$$\hat{\Sigma}_{11} = Q_\lambda + R_\lambda + T_\lambda(A_\lambda + B_\lambda K(\lambda)) + (A_\lambda + B_\lambda K(\lambda))^{\mathrm{T}} T_\lambda^{\mathrm{T}},$$

$$\hat{\Sigma}_{12} = N_\lambda^{\mathrm{T}} + T_\lambda A_{1\lambda},\ \hat{\Sigma}_{13} = 0,\ \hat{\Sigma}_{15} = C_\lambda^{\mathrm{T}} T_\lambda^{\mathrm{T}},$$

$$\hat{\Sigma}_{14} = P_\lambda - T_\lambda + (A_\lambda + B_\lambda K(\lambda))^{\mathrm{T}} T_\lambda^{\mathrm{T}},$$

$$\hat{\Sigma}_{22} = -(1-\mu)Q_\lambda - N_\lambda^{\mathrm{T}} - N_\lambda + \tau X_\lambda,$$

$$\hat{\Sigma}_{23} = M_\lambda^{\mathrm{T}},\ \hat{\Sigma}_{24} = A_{1\lambda}^{\mathrm{T}} T_\lambda^{\mathrm{T}},\ \hat{\Sigma}_{25} = C_{1\lambda}^{\mathrm{T}} T_\lambda^{\mathrm{T}},$$

$$\hat{\Sigma}_{33} = -R_\lambda - M_\lambda - M_\lambda^{\mathrm{T}} + \tau X_\lambda,$$

$$\hat{\Sigma}_{34} = \hat{\Sigma}_{35} = 0,\ \hat{\Sigma}_{44} = \tau S_\lambda - T_\lambda - T_\lambda^{\mathrm{T}},$$

$$\hat{\Sigma}_{45} = 0,\ \hat{\Sigma}_{55} = \tau U_\lambda + P_\lambda - T_\lambda - T_\lambda^{\mathrm{T}}.$$

由引理2, 线性矩阵不等式$\tilde{\Lambda}_{1\lambda} < 0$与$\hat{\Sigma} < 0$等价. 下面来讨论控制器的设计问题. 假设$T_\lambda$为可逆矩阵, 对$\hat{\Sigma}$左边乘以矩阵$\mathrm{diag}\{T_\lambda^{-1}, T_\lambda^{-1}, T_\lambda^{-1}, T_\lambda^{-1}, T_\lambda^{-1}, T_\lambda^{-1}, T_\lambda^{-1}\}$, 右边乘以矩阵$\mathrm{diag}\{(T_\lambda^{-1})^{\mathrm{T}}, (T_\lambda^{-1})^{\mathrm{T}}, (T_\lambda^{-1})^{\mathrm{T}}, (T_\lambda^{-1})^{\mathrm{T}}, (T_\lambda^{-1})^{\mathrm{T}}, (T_\lambda^{-1})^{\mathrm{T}}, (T_\lambda^{-1})^{\mathrm{T}}\}$, 并记:

$$T_\lambda^{-1} = W_\lambda,\ V_\lambda = K(\lambda) W_\lambda^{\mathrm{T}},\ \tilde{P}_\lambda = W_\lambda P_\lambda W_\lambda^{\mathrm{T}},$$

$$\tilde{Q}_\lambda = W_\lambda Q_\lambda W_\lambda^{\mathrm{T}},\ \tilde{R}_\lambda = W_\lambda R_\lambda W_\lambda^{\mathrm{T}},\ \tilde{S}_\lambda = W_\lambda S_\lambda W_\lambda^{\mathrm{T}},$$

$$\tilde{U}_\lambda = W_\lambda U_\lambda W_\lambda^{\mathrm{T}},\ \tilde{N}_\lambda = W_\lambda N_\lambda W_\lambda^{\mathrm{T}},$$

$$\tilde{M}_\lambda = W_\lambda M_\lambda W_\lambda^{\mathrm{T}},\ \tilde{X}_\lambda = W_\lambda X_\lambda W_\lambda^{\mathrm{T}},$$

可得:

$$\Xi_\lambda = \begin{bmatrix} \tilde{\Sigma}_{11} & \tilde{\Sigma}_{12} & \tilde{\Sigma}_{13} & \tilde{\Sigma}_{14} & \tilde{\Sigma}_{15} & 0 & 0 \\ * & \tilde{\Sigma}_{22} & \tilde{\Sigma}_{23} & \tilde{\Sigma}_{24} & \tilde{\Sigma}_{25} & \tilde{N}_\lambda & 0 \\ * & * & \tilde{\Sigma}_{33} & \tilde{\Sigma}_{34} & \tilde{\Sigma}_{35} & 0 & \tilde{M}_\lambda \\ * & * & * & \tilde{\Sigma}_{44} & \tilde{\Sigma}_{45} & 0 & 0 \\ * & * & * & * & \tilde{\Sigma}_{55} & 0 & 0 \\ * & * & * & * & * & -\tilde{U}_\lambda & 0 \\ * & * & * & * & * & * & -\tilde{U}_\lambda \end{bmatrix},$$

其中:

$$\tilde{\Sigma}_{11} = \tilde{Q}_\lambda + \tilde{R}_\lambda + A_\lambda W_\lambda^{\mathrm{T}} + B_\lambda V_\lambda + W_\lambda A_\lambda^{\mathrm{T}} + V_\lambda^{\mathrm{T}} B_\lambda^{\mathrm{T}},$$

$$\tilde{\Sigma}_{12} = \tilde{N}_\lambda^{\mathrm{T}} + A_{1\lambda} W_\lambda^{\mathrm{T}},\ \tilde{\Sigma}_{13} = 0,$$

$$\tilde{\Sigma}_{14} = \tilde{P}_\lambda - W_\lambda^{\mathrm{T}} + W_\lambda A_\lambda^{\mathrm{T}} + V_\lambda^{\mathrm{T}} B_\lambda^{\mathrm{T}},\ \tilde{\Sigma}_{15} = W_\lambda C_\lambda^{\mathrm{T}},$$

$$\tilde{\Sigma}_{22} = -(1-\mu)\tilde{Q}_\lambda - \tilde{N}_\lambda^{\mathrm{T}} - \tilde{N}_\lambda + \tau \tilde{X}_\lambda,$$

$$\tilde{\Sigma}_{23} = \tilde{M}_\lambda^{\mathrm{T}},\ \tilde{\Sigma}_{24} = W_\lambda A_{1\lambda}^{\mathrm{T}},\ \tilde{\Sigma}_{25} = W_\lambda C_{1\lambda}^{\mathrm{T}},$$

$$\tilde{\Sigma}_{33} = -\tilde{R}_\lambda - \tilde{M}_\lambda - \tilde{M}_\lambda^{\mathrm{T}} + \tau \tilde{X}_\lambda,$$

$$\tilde{\Sigma}_{34} = \tilde{\Sigma}_{35} = 0,\ \tilde{\Sigma}_{44} = \tau \tilde{S}_\lambda - W_\lambda - W_\lambda^{\mathrm{T}},$$

$$\tilde{\Sigma}_{45} = 0,\ \tilde{\Sigma}_{55} = \tau \tilde{U}_\lambda + \tilde{P}_\lambda - W_\lambda - W_\lambda^{\mathrm{T}}.$$

由于

$$\Xi_\lambda = \sum_{i=1}^{s}\sum_{j=1}^{s} \lambda_i \lambda_j \Xi_{ij} = \sum_{i=1}^{s} \lambda_i^2 \Xi_{ii} + \sum_{i=1}^{s-1}\sum_{j=i+1}^{s} \lambda_i \lambda_j (\Xi_{ij} + \Xi_{ji}),$$

另一方面式(28)等价于

$$\Xi_{ii} < \tilde{\Phi}_{ii},\ i = 1, \cdots, s.$$

$$\Xi_{ij} + \Xi_{ji} \leqslant \tilde{\Phi}_{ij} + \tilde{\Phi}_{ij}^{\mathrm{T}},\ 1 \leqslant i < j \leqslant s,$$

所以得

$$\Xi_\lambda \leqslant \sum_{i=1}^{s} \lambda_i^2 \tilde{\Phi}_{ii} + \sum_{i=1}^{s-1}\sum_{j=i+1}^{s} \lambda_i \lambda_j (\tilde{\Phi}_{ij} + \tilde{\Phi}_{ij}^{\mathrm{T}}) = \zeta^{\mathrm{T}} \tilde{\Pi} \zeta,$$

其中$\zeta = [\lambda_1 I \quad \lambda_2 I \quad \cdots \quad \lambda_S I]^{\mathrm{T}}$, $\tilde{\Pi}$见式(29). 又显然式(28)~(30)满足时, $\tilde{\Lambda}_{2\lambda} > 0, \tilde{\Lambda}_{3\lambda} > 0$, 所以令$K = \lambda_{\min}(-\tilde{\Lambda}_{1\lambda})$, 则得到

$$\mathrm{E}(LV) \leqslant -K\mathrm{E}\{\|x(t)\|^2 + \|x(t-d(t))\|^2\}.$$

于是类似于文献[16], 可证系统(1), 式(2)为鲁棒指数稳定. 并可得控制增益矩阵为$K(\lambda) = V_\lambda(W_\lambda^{\mathrm{T}})^{-1}$, V_λ, W_λ满足$[V_\lambda \ \ W_\lambda] = \sum\limits_{i=1}^{s} \lambda_i [V_i \ \ W_i]$.

注 1 本文的方法同时适用于带有输入时滞的不确定的随机系统的镇定问题[17].

4 例子(Example)

考虑如下凸多面体不确定随机时滞系统(1), 已知:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 1 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix},\ A_{12} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix},\ C_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

根据定理2, 当μ取不同值时, 上述系统镇定所允许的最大时滞界τ见表1.

表 1 不同μ时系统(1)最大时滞

Table 1 Maximum allowable delay for different μ in systems (1)

μ	1.2	1.0	0.8	0.7	$0 \leqslant \mu \leqslant 0.6$
τ	1.42	1.42	1.5	32	∞

当$\mu = 1.2, \tau = 1.2$, 利用MATLAB的LMI工具箱, 可求得:

$$V_1 = \begin{bmatrix} -48.9210 & -210.7806 \\ -210.7806 & 281.9689 \end{bmatrix},$$

$$V_2 = \begin{bmatrix} -148.4849 & 83.0851 \\ 83.0851 & 121.0507 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 149.0288 & 3.0289 \\ 3.0289 & 3.8400 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 156.6916 & 25.9773 \\ 25.9773 & 131.1887 \end{bmatrix}.$$

从而可选择λ_1, λ_2的值来获得控制增益矩阵$K(\lambda) = V_\lambda(W_\lambda^{\mathrm{T}})^{-1}, V_\lambda, W_\lambda$满足$[V_\lambda \quad W_\lambda] = \sum\limits_{i=1}^{2} \lambda_i [V_i \quad W_i]$. 由于控制器与参数相关, 使得设计的控制器具有较大的灵活性. 并且由于在引入自由权矩阵时, 减少了所用的自由矩阵数目, 使得给出的控制器更易于实现.

5 结论(Conclusions)

本文讨论了凸多面体不确定时滞随机系统的参数相关的鲁棒镇定问题. 利用参数相关的Lyapunov-Krasovskii泛函方法及自由加权矩阵方法, 特别在运用自由加权矩阵方法时, 适当减少了自由矩阵的数目, 并运用Schur补定理, 给出了完全基于线性矩阵不等式(LMI)的参数依赖的鲁棒镇定条件.

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下 期 要 目