文章编号: 1000−8152(2011)07−1015−06

具概率分布变时滞随机系统鲁棒稳定性

李亚军[1,2], 邓飞其[1], 彭云建[1]

(1. 华南理工大学 自动化科学与工程学院, 广东 广州 510640; 2. 顺德职业技术学院 电子与信息工程系, 广东 佛山 528300)

摘要: 本文研究了一类有非线性时变随机时滞的线性不确定系统的鲁棒稳定性. 其中时变随机时滞表征为伯努利随机过程, 具有已知的概率分布和变化范围. 通过构造新泛函, 建立了基于线性矩阵不等式的鲁棒均方指数稳定的充分条件, 此条件易于用MATLAB工具箱来验证. 本文所获得结果的主要特征是稳定性条件依赖时滞的概率分布和时滞导数的上界. 同时也证明了允许时变随机时滞的时滞比之传统的确定性时滞有更大的变化范围, 因此我们的条件比确定性时滞更为保守. 算例表明了文中所提方法的有效性.

关键词: 变时滞概率分布; 不确定随机系统; 自由权矩阵; 鲁棒稳定; 线性矩阵不等式

中图分类号: TP13 **文献标识码**: A

Robust delay-probability-distribution stability of linear stochastic systems with time-varying delay

LI Ya-jun[1,2], DENG Fei-qi[1], PENG Yun-jian[1]

(1. College of Automation Science and Engineering, South China University of Technology, Guangzhou Guangdong 510640, China;
2. The Department of Electronic and Information Engineering, Shunde Polytechnic, Foshan Guangdong 528300, China)

Abstract: This paper is concerned with the robust stability of a class of linear uncertain stochastic systems with nonlinear time-varying stochastic time-delay which is characterized by a Bernoulli stochastic process with given distribution probability in a given variation range. By constructing a new Lyapunov-Krasovskii functional, we derive for the system the sufficient conditions of mean-square exponential stability in terms of the linear matrix inequalities(LMIs), which can be checked readily by using MATLAB toolbox. The feature of our results is the conclusion of stability conditions being dependent not only on the probability distribution of the time-delay, but also on the upper bound of the its derivative. Meanwhile, we also show that the allowable variation range of the time-varying stochastic time-delay can be greater than that of a deterministic time-delay in ensuring the same stability; this demonstrates the less conservativeness of our requirements than the traditional ones. An example is given to illustrate the effectiveness of the proposed method.

Key words: varying delay-probability-distribution; uncertain stochastic system; free weight matrix; robust stability; linear matrix inequality(LMI)

1 引言(Introduction)

时滞现象广泛存在于各种如电子、生物、化学、网络控制等动态系统中, 是使系统性能变坏和不稳定的主要来源之一, 因此对时滞系统的稳定性研究在过去几十年来一直是热点话题[1~5], 形成的稳定标准主要分为时滞依赖和时滞独立两种, 时滞依赖系统主要利用时滞长度信息, 通常认为比时滞独立系统有更小的保守性, 因此对时滞依赖系统的研究引起了更多学者的广泛关注[6~8], 当研究依赖时滞系统时, 对时滞的求导信息通常被考虑, 但是一些文献中时滞导数的上界规定必须小于1, 因此怎么样突破此限制是广大学者研究的热门话题.

另外系统在建模时, 系统不确定和随机干扰是不可避免的, 因此研究随机不确定时滞系统的稳定性一直是稳定性研究领域里感兴趣的话题. 已出现有大量关于不确定随机时滞系统的研究文献[9~12], 大部分结果以线性矩阵不等式的方法给出. 其中文献[4, 5, 7]利用自由权矩阵的方法研究了变时滞稳定性问题, 通过跟其他文献的比较, 降低了保守性.

本文主要研究了一类具有不确定性和随机非线性干扰时滞系统的鲁棒性, 采用新的方法, 将时滞分段并考虑其概率分布, 更多地利用时滞的有用信息, 此方法可以处理其他方法不能处理的概率比较小而时滞比较大的情形, 而且突破时滞导数的上界条件限制, 使得具有变时滞概率分布的随机系统在所有容许不确定下是随机鲁棒渐近稳定的. 文中所有结果以线性矩阵不等式给出, 可以通过MATLAB中LMI工具箱求解, 算例表明本文的结

收稿日期: 2009−12−16; 收修改稿日期: 2010−10−12.
基金项目: 国家自然科学基金资助项目(60874114).

果跟其他文献的比较降低了系统保守性.

2 系统描述(System description)

符号说明: I为适当维数单位矩阵, $*$表示矩阵对称项, $\mathrm{E}\{\cdot\}$ 表示求期望, $\mathrm{diag}\{\cdot\}$表示对角矩阵.

考虑以下不确定随机时滞系统:

$$\begin{cases} \mathrm{d}x(t)=[A(t)x(t)+B(t)(x(t-\tau(t)))]\mathrm{d}t+\\ \qquad g(t,x(t),x(t-\tau(t)))\mathrm{d}\omega(t),\\ x(t)=\xi(t),\ t\in[-\tau_{\mathrm{M}},0]. \end{cases} \tag{1}$$

其中: $x(t)\in\mathbb{R}^n$为系统状态变量; $\tau(t)$为时滞变量, 且满足$0<\tau(t)<\tau_{\mathrm{M}}$, τ_{M}为系统最大时滞, $A(t)=A+\Delta A(t)$, $B(t)=B+\Delta B(t)$; A,B 为已知适当维数矩阵, $\Delta A(t),\Delta B(t)$表示系统参数不确定项且满足

$$[\Delta A(t)\ \ \Delta B(t)]=EF(t)[H_1\ \ H_2], \tag{2}$$

这里: E,H_1,H_2为已知适当维数矩阵, $F(t)$为适当维数时变未知矩阵且满足

$$F^{\mathrm{T}}(t)F(t)\leqslant I,\ \forall t\in\mathbb{R}. \tag{3}$$

$g(t,x(t),x(t-\tau(t)))\in\mathbb{R}^{n\times n}$为非线性函数, 且满足

$$\mathrm{tr}[g^{\mathrm{T}}(t,x(t),x(t-\tau(t)))][g(t,x(t),x(t-\tau(t)))]\leqslant \|G_0x(t)\|^2+\|G_1x(t-\tau(t)))\|^2, \tag{4}$$

其中G_0, G_1为已知常数适当维数矩阵. $\omega(t)$为定义在完整概率空间上的m维布朗运动且满足

$$\mathrm{E}\{\mathrm{d}\omega(t)\}=0,\ \mathrm{E}\{\mathrm{d}\omega^2(t)\}=\mathrm{d}t.$$

假设 1 考虑时滞的概率分布信息, 定义以下两个集合:

$$\Omega_1=\{t:\tau(t)\in[0,\tau_0)\}$$

以及

$$\Omega_2=\{t:\tau(t)\in[\tau_0,\tau_{\mathrm{M}})\},$$

$$\tau_1(t)=\begin{cases}\tau(t),\ t\in\Omega_1,\\ \overline{\tau}_1,\quad t\in\Omega_2.\end{cases}$$

$$\tau_2(t)=\begin{cases}\tau(t),\ t\in\Omega_2,\\ \overline{\tau}_2,\quad t\in\Omega_1.\end{cases}$$

而时滞的导数满足

$$\dot{\tau}_1(t)\leqslant\mu_1<\infty,\ \dot{\tau}_2(t)\leqslant\mu_2<\infty, \tag{5}$$

其中: $\tau_0\in[0,\tau_{\mathrm{M}}],\ \bar{\tau}_1\in[0,\tau_0],\ \bar{\tau}_2\in[\tau_0,\ \tau_{\mathrm{M}}]$.

因此可以定义随机变量:

$$\alpha(t)=\begin{cases}1,\ t\in\Omega_1,\\ 0,\ t\in\Omega_2.\end{cases}$$

这样当$t\in\Omega_1$ 时, 变时滞$\tau(t)$概率分布在区间$[0,\tau_0]$, 当$t\in\Omega_2$ 时, 变时滞$\tau(t)$概率分布在区间$[\tau_0,\tau_{\mathrm{M}}]$.

假设 2 $\alpha(t)$是一Bernoulli分布序列且满足

$$\mathrm{P}\{\alpha(t)=1\}=\mathrm{E}\{\alpha(t)\}=\alpha_0,$$
$$\mathrm{P}\{\alpha(t)=0\}=1-\mathrm{E}\{\alpha(t)\}=1-\alpha_0,$$

这里$0\leqslant\alpha_0\leqslant1$为一常数.

注 1 由假设2, 容易知道:

$$\mathrm{E}\{\alpha(t)-\alpha_0\}=0,$$
$$\mathrm{E}\{\alpha(t)-\alpha_0\}^2=\alpha_0(1-\alpha_0).$$

首先考虑以下没有不确定项的随机系统:

$$\begin{cases} \mathrm{d}x(t)=[Ax(t)+B(x(t-\tau(t)))]\mathrm{d}t+\\ \qquad g(t,x(t),x(t-\tau(t)))\mathrm{d}\omega(t),\\ x(t)=\xi(t),\ t\in[-\tau_{\mathrm{M}},0]. \end{cases} \tag{6}$$

由假设1和2, 可以推出系统(6)等价于

$$\begin{aligned} &\mathrm{d}x(t)=\\ &[Ax(t)+\alpha_0B(x(t-\tau_1(t)))+\\ &(1-\alpha_0)B(x(t-\tau_2(t)))+(\alpha(t)-\\ &\alpha_0)B(x(t-\tau_1(t))-x(t-\tau_2(t)))]\mathrm{d}t+\\ &g(t,x(t),x(t-\tau_1(t)),x(t-\tau_2(t)))\mathrm{d}\omega(t). \end{aligned} \tag{7}$$

为了证明方便, 令

$$\begin{aligned} y(t)=&\ Ax(t)+\alpha_0B(x(t-\tau_1(t)))+\\ &(1-\alpha_0)B(x(t-\tau_2(t)))+(\alpha(t)-\\ &\alpha_0)B(x(t-\tau_1(t))-x(t-\tau_2(t))),\\ g(t)=&\ g(t,x(t),x(t-\tau_1(t)),x(t-\tau_2(t))). \end{aligned}$$

定义 1 不确定随机时滞系统(1) 为鲁棒随机稳定的, 如果存在正标量$\varepsilon>0$, 使得下式成立:

$$\lim_{T\to\infty}\mathrm{E}\{\int_0^T\|x(t)\|^2\}<\varepsilon\sup_{s\in[-\tau_{\mathrm{M}},0]}\mathrm{E}\{\|\xi(s)\|^2\}.$$

引理 1[4] 假定U,V,W以及M为适当维数实矩阵, 且矩阵M满足$M=M^{\mathrm{T}}$, 则对所有$V^{\mathrm{T}}V\leqslant I$, 不等式

$$M+UVW+W^{\mathrm{T}}V^{\mathrm{T}}U^{\mathrm{T}}<0$$

成立, 当且仅当存在标量$\varepsilon>0$ 使得

$$M+\varepsilon^{-1}UU^{\mathrm{T}}+\varepsilon W^{\mathrm{T}}W<0.$$

引理 2[4] 对任意正对称常矩阵$M\in\mathbb{R}^{n\times n}$和标量$\gamma>0$, 如果存在矢量函数$\omega:[0,\gamma]\to\mathbb{R}^n$使得积分$\int_0^\gamma\omega^{\mathrm{T}}(s)M\omega(s)\mathrm{d}s$和$\int_0^\gamma\omega(s)\mathrm{d}s$有意义, 则下面不等式成立:

$$\gamma\int_0^\gamma\omega^{\mathrm{T}}(s)M\omega(s)\mathrm{d}s\geqslant (\int_0^\gamma\omega^{\mathrm{T}}(s)\mathrm{d}s)M(\int_0^\gamma\omega(s)\mathrm{d}s).$$

3 主要结果(Main results)

定理 1 给定标量$\lambda^*>0,\tau_0\geqslant0,\tau_{\mathrm{M}}>0,\mu_1>0,\ \alpha_0>0$满足$\alpha_0\mu_1<1$, 系统(6) 为随机稳定的. 如果存在正定矩阵$P>0,\ Q_j>0(j=1,2,3),\ R_1>0,\ R_2>0,\ S_1>0,\ S_2>0$, 以及适当维数实矩阵$M_i,N_i(i=1,2,\cdots,6)$使得以下矩阵不等式成立:

$$P+\tau_0S_1+(\tau_{\mathrm{M}}-\tau_0)S_2<\lambda^*I, \tag{8}$$

$$\Omega=\begin{bmatrix}\Psi & M & \bar{M} & \tilde{M} & N & \bar{N}\\ * & -S_1 & 0 & 0 & 0 & 0\\ * & * & -S_1 & 0 & 0 & 0\\ * & * & * & -S_1 & 0 & 0\\ * & * & * & * & -S_2 & 0\\ * & * & * & * & * & -S_2\end{bmatrix}<0, \quad (9)$$

这里Ψ为12行12列矩阵. 定义$\Psi=\Psi_{i\times j}(i,j=1,\cdots,12)$, 其中:

$\Psi_{1,1}=Q_1+Q_2+Q_3+\lambda^*G_0^{\mathrm{T}}G_0+M_1+M_1^{\mathrm{T}}+N_5A+A^{\mathrm{T}}N_5^{\mathrm{T}}$,

$\Psi_{1,2}=-M_1+M_2^{\mathrm{T}}$, $\Psi_{1,3}=\alpha_0N_5B$, $\Psi_{1,4}=0$,

$\Psi_{1,5}=(1-\alpha_0)N_5B$, $\Psi_{1,6}=0$,

$\Psi_{1,7}=P-A^{\mathrm{T}}N_6^{\mathrm{T}}-N_5$, $\Psi_{1,8}=-M_1$,

$\Psi_{1,9}=\Psi_{1,10}=\Psi_{1,11}=\Psi_{1,12}=0$,

$\Psi_{2,2}=-(1-\alpha_0\mu_1)Q_1-M_2-M_2^{\mathrm{T}}+M_3+M_3^{\mathrm{T}}$,

$\Psi_{2,3}=-M_3+M_4^{\mathrm{T}}$, $\Psi_{2,4}=\Psi_{2,5}=\Psi_{2,6}=0$,

$\Psi_{2,7}=0$, $\Psi_{2,8}=-M_2$, $\Psi_{2,9}=-M_3$,

$\Psi_{2,10}=\Psi_{2,11}=\Psi_{2,12}=0$,

$\Psi_{3,3}=\lambda^*\alpha_0G_1^{\mathrm{T}}G_1-M_4-M_4^{\mathrm{T}}+M_5+M_5^{\mathrm{T}}$,

$\Psi_{3,4}=-M_5+M_6^{\mathrm{T}}$, $\Psi_{3,5}=\Psi_{3,6}=\Psi_{3,8}=0$,

$\Psi_{3,7}=\alpha_0B^{\mathrm{T}}N_6^{\mathrm{T}}$, $\Psi_{3,9}=-M_4$,

$\Psi_{3,10}=-M_5$, $\Psi_{3,11}=\Psi_{3,12}=0$,

$\Psi_{4,4}=-Q_2-M_6-N_6^{\mathrm{T}}+N_1+N_1^{\mathrm{T}}$,

$\Psi_{4,5}=-N_1+N_2^{\mathrm{T}}$, $\Psi_{4,6}=\Psi_{4,7}=\Psi_{4,8}=\Psi_{4,9}=0$,

$\Psi_{4,10}=-M_6$, $\Psi_{4,11}=-N_1$, $\Psi_{4,12}=0$,

$\Psi_{5,5}=\lambda^*(1-\alpha_0)G_1^{\mathrm{T}}G_1-N_2-N_2^{\mathrm{T}}+N_3+N_3^{\mathrm{T}}$,

$\Psi_{5,6}=-N_3+N_4^{\mathrm{T}}$, $\Psi_{5,7}=(1-\alpha_0)B^{\mathrm{T}}N_6^{\mathrm{T}}$,

$\Psi_{5,8}=\Psi_{5,9}=\Psi_{5,10}=0$, $\Psi_{5,11}=-N_2$,

$\Psi_{5,12}=-N_3$, $\Psi_{6,6}=-Q_3-N_4-N_4^{\mathrm{T}}$,

$\Psi_{6,7}=\Psi_{6,8}=\Psi_{6,9}=\Psi_{6,10}=\Psi_{6,11}=0$, $\Psi_{6,12}=-N_4$,

$\Psi_{7,7}=\tau_0R_1+(\tau_{\mathrm{M}}-\tau_0)R_2-N_6-N_6^{\mathrm{T}}$,

$\Psi_{7,8}=\Psi_{7,9}=\Psi_{7,10}=\Psi_{7,11}=\Psi_{7,12}=0$,

$\Psi_{8,9}=\Psi_{8,10}=\Psi_{8,11}=\Psi_{8,12}=0$,

$\Psi_{8,8}=-\dfrac{1}{\alpha_0\tau_0}R_1$, $\Psi_{9,9}=-\dfrac{1}{\tau_0(1-\alpha_0)}R_1$,

$\Psi_{9,10}=\Psi_{9,11}=\Psi_{9,12}=0$,

$\Psi_{10,10}=-\dfrac{1}{\tau_0}R_1$, $\Psi_{10,11}=\Psi_{10,12}=\Psi_{11,12}=0$,

$\Psi_{11,11}=\Psi_{12,12}=-\dfrac{1}{\tau_{\mathrm{M}}-\tau_0}R_2$,

$M^{\mathrm{T}}=[M_1^{\mathrm{T}}\ \ M_2^{\mathrm{T}}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$,

$\bar{M}^{\mathrm{T}}=[0\ \ M_3^{\mathrm{T}}\ \ M_4^{\mathrm{T}}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$,

$\tilde{M}^{\mathrm{T}}=[0\ \ 0\ \ M_5^{\mathrm{T}}\ \ M_6^{\mathrm{T}}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$,

$N^{\mathrm{T}}=[0\ \ 0\ \ 0\ \ N_1^{\mathrm{T}}\ \ N_2^{\mathrm{T}}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$,

$\bar{N}^{\mathrm{T}}=[0\ \ 0\ \ 0\ \ 0\ \ N_3^{\mathrm{T}}\ \ N_4^{\mathrm{T}}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$.

证 取如下Lyapunov-Krasovskii范函:

$$V(x_t,t)=\sum_{i=1}^{4}V_i(x_t,t),$$

其中:

$$V_1(x_t,t)=x(t)^{\mathrm{T}}Px(t),$$

$$V_2(x_t,t)=\int_{t-\alpha_0\tau_1(t)}^{t}x^{\mathrm{T}}(s)Q_1x(s)\mathrm{d}s+\int_{t-\tau_0}^{t}x^{\mathrm{T}}(s)Q_2x(s)\mathrm{d}s+\int_{t-\tau_{\mathrm{M}}}^{t}x^{\mathrm{T}}(s)Q_3x(s)\mathrm{d}s,$$

$$V_3(x_t,t)=\int_{-\tau_0}^{0}\int_{t+s}^{t}y^{\mathrm{T}}(\theta)R_1y(\theta)\mathrm{d}\theta\mathrm{d}s+\int_{-\tau_{\mathrm{M}}}^{-\tau_0}\int_{t+s}^{t}y^{\mathrm{T}}(\theta)R_2y(\theta)\mathrm{d}\theta\mathrm{d}s,$$

$$V_4(x_t,t)=\int_{-\tau_0}^{0}\int_{t+s}^{t}\mathrm{tr}[g^{\mathrm{T}}(\theta)S_1g(\theta)]\mathrm{d}\theta\mathrm{d}s+\int_{-\tau_{\mathrm{M}}}^{-\tau_0}\int_{t+s}^{t}\mathrm{tr}[g^{\mathrm{T}}(\theta)S_2g(\theta)]\mathrm{d}\theta\mathrm{d}s.$$

利用伊藤公式[6], $V(x_t,t)$对系统(6)的随机微分为

$$\mathrm{d}V(x_t,t)=\mathcal{L}V(x_t,t)\mathrm{d}t+2x(t)^{\mathrm{T}}Pg(t)\mathrm{d}\omega(t), \quad (10)$$

其中:

$$\mathcal{L}V(x_t,t)=\sum_{i=1}^{4}\mathcal{L}V_i(x_t,t), \quad (11)$$

$$\mathcal{L}V_1(x_t,t)=2x(t)^{\mathrm{T}}Py(t)+g^{\mathrm{T}}(t)Pg(t), \quad (12)$$

$$\begin{aligned}&\mathcal{L}V_2(x_t,t)\leqslant\\&x^{\mathrm{T}}(t)(Q_1+Q_2+Q_3)x(t)-(1-\\&\alpha_0\mu_1)x^{\mathrm{T}}(t-\alpha_0\tau_1(t))Q_1x(t-\alpha_0\tau_1(t))-\\&x^{\mathrm{T}}(t-\tau_0)Q_2x(t-\tau_0)-\\&x^{\mathrm{T}}(t-\tau_{\mathrm{M}})Q_3x(t-\tau_{\mathrm{M}}),\end{aligned} \quad (13)$$

$$\begin{aligned}&\mathcal{L}V_3(x_t,t)=\\&y(t)^{\mathrm{T}}[\tau_0R_1+(\tau_{\mathrm{M}}-\tau_0)R_2]y(t)-\\&\int_{t-\alpha_0\tau_1(t)}^{t}y^{\mathrm{T}}(s)R_1y(s)\mathrm{d}s-\\&\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}y^{\mathrm{T}}(s)R_1y(s)\mathrm{d}s-\\&\int_{t-\tau_0}^{t-\tau_1(t)}y^{\mathrm{T}}(s)R_1y(s)\mathrm{d}s-\\&\int_{t-\tau_2(t)}^{t-\tau_0}y^{\mathrm{T}}(s)R_2y(s)\mathrm{d}s-\\&\int_{t-\tau_{\mathrm{M}}}^{t-\tau_2(t)}y^{\mathrm{T}}(s)R_2y(s)\mathrm{d}s\leqslant\\&y(t)^{\mathrm{T}}[\tau_0R_1+(\tau_{\mathrm{M}}-\tau_0)R_2]y(t)-\\&\frac{1}{\alpha_0\tau_0}\int_{t-\alpha_0\tau_1(t)}^{t}y^{\mathrm{T}}(s)\mathrm{d}sR_1\int_{t-\alpha_0\tau_1(t)}^{t}y(s)\mathrm{d}s-\end{aligned}$$

$$\frac{1}{\tau_0(1-\alpha_0)}\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)} y^{\mathrm T}(s)\mathrm ds \times$$
$$R_1\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)} y(s)\mathrm ds -$$
$$\frac{1}{\tau_0}\int_{t-\tau_0}^{t-\tau_1(t)} y^{\mathrm T}(s)\mathrm dsR_1\int_{t-\tau_0}^{t-\tau_1(t)} y(s)\mathrm ds -$$
$$\frac{1}{\tau_{\mathrm M}-\tau_0}\int_{t-\tau_2(t)}^{t-\tau_0} y^{\mathrm T}(s)\mathrm dsR_2\int_{t-\tau_2(t)}^{t-\tau_0} y(s)\mathrm ds -$$
$$\frac{1}{\tau_{\mathrm M}-\tau_0}\int_{t-\tau_{\mathrm M}}^{t-\tau_2(t)} y^{\mathrm T}(s)\mathrm dsR_2\int_{t-\tau_{\mathrm M}}^{t-\tau_2(t)} y(s)\mathrm ds,\quad (14)$$

$$\mathcal LV_4(x_t,t)\leqslant$$
$$\mathrm{tr}[g^{\mathrm T}(t)(\tau_0S_1+(\tau_{\mathrm M}-\tau_0)S_2)]g(t)-$$
$$\int_{t-\alpha_0\tau_1(t)}^{t}\mathrm{tr}[g^{\mathrm T}(s)S_1g(s)]\mathrm ds-$$
$$\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}\mathrm{tr}[g^{\mathrm T}(s)S_1g(s)]\mathrm ds-$$
$$\int_{t-\tau_0}^{t-\tau_1(t)}\mathrm{tr}[g^{\mathrm T}(s)S_1g(s)]\mathrm ds-$$
$$\int_{t-\tau_2(t)}^{t-\tau_0}\mathrm{tr}[g^{\mathrm T}(s)S_2g(s)]\mathrm ds-$$
$$\int_{t-\tau_{\mathrm M}}^{t-\tau_2(t)}\mathrm{tr}[g^{\mathrm T}(s)S_2g(s)]\mathrm ds.\quad (15)$$

对任意适当维数矩阵$M_i,N_i(i=1,2,\cdots,6)$, 有

$$[x^{\mathrm T}(t)M_1+x^{\mathrm T}(t-\alpha_0\tau_1(t))M_2][x(t)-$$
$$x^{\mathrm T}(t-\alpha_0\tau_1(t))-\int_{t-\alpha_0\tau_1(t)}^{t}y(s)\mathrm ds-$$
$$\int_{t-\alpha_0\tau_1(t)}^{t}g(s)\mathrm dw(s)]=0,\quad (16)$$

$$[x^{\mathrm T}(t-\alpha_0\tau_1(t))M_3+x^{\mathrm T}(t-\alpha_1\tau_1(t))M_4]\cdot$$
$$[x(t-\alpha_0\tau_1(t))-x^{\mathrm T}(t-\tau_1(t))-$$
$$\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}y(s)\mathrm ds-\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}g(s)\mathrm dw(s)]=0,\quad (17)$$

$$[x^{\mathrm T}(t-\tau_1(t))M_5+x^{\mathrm T}(t-\tau_0)M_6]\cdot$$
$$[x(t-\tau_1(t))-x^{\mathrm T}(t-\tau_0)-$$
$$\int_{t-\tau_0}^{t-\tau_1(t)}y(s)\mathrm ds-\int_{t-\tau_0}^{t-\tau_1(t)}g(s)\mathrm dw(s)]=0,\quad (18)$$

$$[x^{\mathrm T}(t-\tau_0)N_1+x^{\mathrm T}(t-\tau_2(t))N_2]\cdot$$
$$[x(t-\tau_0)-x^{\mathrm T}(t-\tau_2(t))-\int_{t-\tau_2(t)}^{t-\tau_0}y(s)\mathrm ds-$$
$$\int_{t-\tau_2(t)}^{t-\tau_0}g(s)\mathrm dw(s)]=0,\quad (19)$$

$$[x^{\mathrm T}(t-\tau_2(t))N_3+x^{\mathrm T}(t-\tau_{\mathrm M})N_4]\cdot$$
$$[x(t-\tau_2(t))-x^{\mathrm T}(t-\tau_{\mathrm M})-$$
$$\int_{t-\tau_{\mathrm M}}^{t-\tau_2(t)}y(s)\mathrm ds-\int_{t-\tau_{\mathrm M}}^{t-\tau_2(t)}g(s)\mathrm dw(s)]=0,\quad (20)$$

$$[x^{\mathrm T}(t)N_5+y^{\mathrm T}(t)N_6][Ax(t)+\alpha_0B(x(t-$$
$$\tau_1(t)))+(1-\alpha_0)B(x(t-\tau_2(t)))+$$
$$(\alpha(t)-\alpha_0)B(x(t-\tau_1(t))-x(t-$$
$$\tau_2(t)))-y(t)]=0.\quad (21)$$

对式(16)~(21), 进一步可以得到

$$-2\zeta^{\mathrm T}(t)M\int_{t-\alpha_0\tau_1(t)}^{t}g(t)\mathrm dw(s)\leqslant$$
$$\zeta^{\mathrm T}(t)MS_1^{-1}M^{\mathrm T}\zeta(t)+\varSigma_1^{\mathrm T}S_1\varSigma_1,\quad (22)$$
$$-2\zeta^{\mathrm T}(t)\bar M\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}g(t)\mathrm dw(s)\leqslant$$
$$\zeta^{\mathrm T}(t)\bar MS_1^{-1}\bar M^{\mathrm T}\zeta(t)+\varSigma_2^{\mathrm T}S_1\varSigma_2,\quad (23)$$
$$-2\zeta^{\mathrm T}(t)\tilde M\int_{t-\tau_0}^{t-\tau_1(t)}g(t)\mathrm dw(s)\leqslant$$
$$\zeta^{\mathrm T}(t)\tilde MS_1^{-1}\widetilde M^{\mathrm T}\zeta(t)+\varSigma_3^{\mathrm T}S_1\varSigma_3,\quad (24)$$
$$-2\zeta^{\mathrm T}(t)N\int_{t-\tau_2(t)}^{t-\tau_0}g(t)\mathrm dw(s)\leqslant$$
$$\zeta^{\mathrm T}(t)NS_2^{-1}N^{\mathrm T}\zeta(t)+\varSigma_4^{\mathrm T}S_2\varSigma_4,\quad (25)$$
$$-2\zeta^{\mathrm T}(t)\bar N\int_{t-\tau_{\mathrm M}}^{t-\tau_2(t)}g(t)\mathrm dw(s)\leqslant$$
$$\zeta^{\mathrm T}(t)\bar NS_2^{-1}\bar N^{\mathrm T}\zeta(t)+\varSigma_5^{\mathrm T}S_2\varSigma_5.\quad (26)$$

这里:

$$\zeta^{\mathrm T}(t)=$$
$$[x^{\mathrm T}(t)\quad x^{\mathrm T}(t-\alpha_0\tau_1(t))\quad x^{\mathrm T}(t-\tau_1(t))$$
$$x^{\mathrm T}(t-\tau_0)\quad x^{\mathrm T}(t-\tau_2(t))\quad x^{\mathrm T}(t-\tau_{\mathrm M})$$
$$y^{\mathrm T}(t)\quad \int_{t-\alpha_0\tau_1(t)}^{t}y^{\mathrm T}(s)\mathrm ds$$
$$\int_{t-\tau_1(t)}^{t-\alpha_0}y^{\mathrm T}(s)\mathrm ds\quad \int_{t-\tau_0}^{t-\tau_1(t)}y^{\mathrm T}(s)\mathrm ds$$
$$\int_{t-\tau_2(t)}^{t-\tau_0(t)}y^{\mathrm T}(s)\mathrm ds\quad \int_{t-\tau_{\mathrm M}}^{t-\tau_2}y^{\mathrm T}(s)\mathrm ds],$$
$$\varSigma_1=\int_{t-\alpha_0\tau_1(t)}^{t}g(s)\mathrm dw(s),$$
$$\varSigma_2=\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}g(s)\mathrm dw(s),$$
$$\varSigma_3=\int_{t-\tau_0}^{t-\tau_1(t)}g(s)\mathrm dw(s),$$
$$\varSigma_4=\int_{t-\tau_2}^{t-\tau_0}g(s)\mathrm dw(s),$$
$$\varSigma_5=\int_{t-\tau_{\mathrm M}}^{t-\tau_2}g(s)\mathrm dw(s).$$

利用假设1、假设2以及式子(4)和式子(8)可以得到

$$\mathrm E\{\mathrm{tr}[g^{\mathrm T}(t)(P+\tau_0S_1+(\tau_{\mathrm M}-\tau_0)S_2g(t))]\}\leqslant$$
$$\lambda_{\max}(\bar P)\mathrm E\{\mathrm{tr}[g^{\mathrm T}(t)g(t)]\}=$$
$$\lambda^*x^{\mathrm T}(t)G_0^{\mathrm T}G_0x(t)+\lambda^*\alpha_0x^{\mathrm T}(t-$$
$$\tau_1(t))G_1^{\mathrm T}G_1x(t-\tau_1(t))+\lambda^*(1-\alpha_0)x^{\mathrm T}(t-$$
$$\tau_2(t))G_1^{\mathrm T}G_1x(t-\tau_2(t)),\quad (27)$$

其中:

$$\bar P=P+\tau_0S_1+(\tau_{\mathrm M}-\tau_0)S_2,\ \lambda^*=\lambda_{max}\{\bar P\}.$$

又由伊藤积分性质可以得到

$$\mathrm E\{\int_{t-\alpha_0\tau_1(t)}^{t}g^{\mathrm T}(s)\mathrm dw(s)S_1\int_{t-\alpha_0\tau_1(t)}^{t}g(s)\mathrm dw(s)\}=$$

$$\mathrm{E}\{\int_{t-\alpha_0\tau_1(t)}^{t} g^{\mathrm{T}}(s)S_1 g(s)\mathrm{d}s\},\tag{28}$$

$$\mathrm{E}\{\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)} g^{\mathrm{T}}(s)\mathrm{d}w(s)S_1\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)} g(s)\mathrm{d}w(s)\}=$$
$$\mathrm{E}\{\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)} g^{\mathrm{T}}(s)S_1 g(s)\mathrm{d}s\},\tag{29}$$

$$\mathrm{E}\{\int_{t-\tau_0}^{t-\tau_1(t)} g^{\mathrm{T}}(s)\mathrm{d}w(s)S_1\int_{t-\tau_0}^{t-\tau_1(t)} g(s)\mathrm{d}s\}=$$
$$\mathrm{E}\{\int_{t-\tau_0}^{t-\tau_1(t)} g^{\mathrm{T}}(s)S_1 g(s)\mathrm{d}s\},\tag{30}$$

$$\mathrm{E}\{\int_{t-\tau_2(t)}^{t-\tau_0} g^{\mathrm{T}}(s)\mathrm{d}w(s)S_2\int_{t-\tau_2(t)}^{t-\tau_0} g(s)\mathrm{d}w(s)\}=$$
$$\mathrm{E}\{\int_{t-\tau_2(t)}^{t-\tau_0} g^{\mathrm{T}}(s)S_2 g(s)\mathrm{d}s\},\tag{31}$$

$$\mathrm{E}\{\int_{t-\tau_{\mathrm{M}}}^{t-\tau_2(t)} g^{\mathrm{T}}(s)\mathrm{d}w(s)S_2\int_{t-\tau_{\mathrm{M}}}^{t-\tau_2(t)} g(s)\mathrm{d}w(s)\}=$$
$$\mathrm{E}\{\int_{t-\tau_{\mathrm{M}}}^{t-\tau_2(t)} g^{\mathrm{T}}(s)S_2 g(s)\mathrm{d}s\}.\tag{32}$$

利用$\mathcal{L}V(x_t,t)=\sum\limits_{i=1}^{4}\mathcal{L}V_i(x_t,t)$, 把式(12)~(15)代入式(10)中, 然后在式(10)的右边加上式(16)~(21),接着利用式子(22)~(26) 然后对式(10)两端同时取期望, 再利用不等式(27)~(32)可以得到

$$\mathrm{E}\{\mathrm{d}V(x_t,t)\}=\mathrm{E}\{\mathcal{L}V(x_t,t)\mathrm{d}t\}\leqslant$$
$$\zeta^{\mathrm{T}}(t)\mathrm{E}\{\Psi_1\}\zeta(t),\tag{33}$$

这里:

$$\Psi_1=\Psi+MS_1^{-1}M^{\mathrm{T}}+\bar{M}S_1^{-1}\bar{M}^{\mathrm{T}}+$$
$$\tilde{M}S_1^{-1}\tilde{M}^{\mathrm{T}}+NS_2^{-1}N^{\mathrm{T}}+\bar{N}S_2^{-1}\bar{N}^{\mathrm{T}}.$$

由Schur补定理可以得到式(9)等价于$\Psi_1<0$. 令$\lambda_0=\min\lambda_{\min}\{-\Psi_1\}$, 对式(33)两端同时利用伊藤积分公式得到

$$\mathrm{E}(V(x_0,t))-\mathrm{E}(V(x_0,0))=$$
$$\mathrm{E}\{\int_0^t\mathcal{L}V(x_s,s)\mathrm{d}s\}\leqslant-\lambda_0\mathrm{E}\{\int_0^t\|x(s)\|^2\mathrm{d}s\}.$$

由上式进一步可以得到

$$\mathrm{E}\{\int_0^t\|x(s)\|^2\mathrm{d}s\}\leqslant\frac{1}{\lambda_0}V(x_0,0),\ t>0.$$

对上式子从0到T取极限, 可知存在正标量$c>0$使得下式成立:

$$\lim_{0\to T}\mathrm{E}\{\int_0^{\mathrm{T}}\|x(t)\|^2\}\leqslant$$
$$c\sup_{s\in[-\tau_{\mathrm{M}},0]}\mathrm{E}\{\|\xi(s)\|^2\},\ t\in[0,\tau_{\mathrm{M}}].\tag{34}$$

因此由定义1可以得到系统(6)是随机稳定的.

证毕.

定理 2 给定标量$\lambda^*>0,\tau_0\geqslant0,\tau_{\mathrm{M}}>0,\mu_1>0,\alpha_0>0$满足$\alpha_0\mu_1<1$, 系统(1)被称为随机鲁棒稳定的, 如果存在正定矩阵$P>0,\ Q_j>0(j=1,2,3)$, $R_1>0,\ R_2>0,\ S_1>0,\ S_2>0$, 及适当维数实矩阵$M_i,\ N_i(i=1,2,\cdots,6)$和正标量$\gamma$, 使得式(8)和以下矩阵不等式成立, 则系统(1)是鲁棒均方渐进稳定的:

$$\begin{bmatrix}\bar{\Psi} & M & \bar{M} & \tilde{M} & N & \bar{N} & \Sigma\\ * & -S_1 & 0 & 0 & 0 & 0 & 0\\ * & * & -S_1 & 0 & 0 & 0 & 0\\ * & * & * & -S_1 & 0 & 0 & 0\\ * & * & * & * & -S_2 & 0 & 0\\ * & * & * & * & * & -S_2 & 0\\ * & * & * & * & * & * & -\gamma I\end{bmatrix}<0,\tag{35}$$

其中:

$$\bar{\Psi}=$$
$$\Psi+\mathrm{diag}\{\gamma H_1^{\mathrm{T}}H_1,0,\gamma H_2^{\mathrm{T}}H_2,0,\gamma H_2^{\mathrm{T}}H_2,\underbrace{0,\cdots,0}_{7}\},$$

$$\Sigma^{\mathrm{T}}=[\rho E^{\mathrm{T}}N_5^{\mathrm{T}}\ \ \underbrace{0\cdots0}_{5}\ \ \rho E^{\mathrm{T}}N_6^{\mathrm{T}}\ \ \underbrace{0\cdots0}_{5}],$$

$$\rho=\sqrt{1+\alpha_0^2+(1-\alpha_0)^2},$$

这里$M,\bar{M},\tilde{M},N,\bar{N}$跟定理1中同样定义.

把不等式(9)中A,B, 替换为$A+\Delta A,\ B+\Delta B$, 则式子(9)可以重新写成

$$\Omega+\Phi\Theta\Upsilon+\Upsilon^{\mathrm{T}}\Theta^{\mathrm{T}}\Phi^{\mathrm{T}}<0,\tag{36}$$

这里:

$$\Phi=[\Phi_1^{\mathrm{T}}\ \ \underbrace{0\cdots0}_{5}\ \ \Phi_2^{\mathrm{T}}\ \ \underbrace{0\cdots0}_{10}],$$

$$\Phi_1=[N_5E\ \ 0\ \ \alpha_0N_5E\ \ 0\ \ (1-\alpha_0)N_5E\ \ \underbrace{0\cdots0}_{12}],$$

$$\Phi_2=[N_6E\ \ 0\ \ \alpha_0N_6E\ \ 0\ \ (1-\alpha_0)N_6E\ \ \underbrace{0\cdots0}_{12}],$$

$$\Theta=\mathrm{diag}\{F(t),F(t),\cdots,F(t)\},$$
$$\Upsilon=\mathrm{diag}\{H_1\ \ 0\ \ H_2\ \ 0\ \ H_2\ \ \underbrace{0\cdots0}_{12}\}.$$

由Schur补引理, 不等式(35)等价于以下不等式:

$$\Omega+\gamma^{-1}\Phi\Phi^{\mathrm{T}}+\gamma\Upsilon^{\mathrm{T}}\Upsilon<0.\tag{37}$$

由引理2可以推出, 对标量$\gamma>0$, 不等式(37)等价于不等式(36), 根据定理1的证明方法, 定理2得证.

推论 1 当系统(1)没有随机干扰时, 系统变为

$$\begin{cases}\dot{x}(t)=[A(t)x(t)+B(t)(x(t-\tau(t)))]\mathrm{d}t,\\ x(t)=\xi(t),\ t\in[-\tau_{\mathrm{M}},0].\end{cases}\tag{38}$$

此类系统的鲁棒稳定性问题很多文献已经研究过, 见文献[4]及里面的参考文献. 下面的定理给出此系统稳定性标准.

定理 3 给定标量$\lambda^*>0,\tau_0\geqslant0,\tau_{\mathrm{M}}>0,\mu_1>0,\ \alpha_0>0$满足$\alpha_0\mu_1<1$, 系统(38)被称为鲁棒均方稳定的, 如果存在正定矩阵$P>0,\ Q_j>0(j=1,2,3)$, $R_1>0,\ R_2>0,\ S_1>0,\ S_2>0$, 以及适当维数实矩阵$M_i,N_i(i=1,2,\cdots,6)$, 使得式(8)和以下矩阵不等式成立:

$$\begin{bmatrix} \bar{\Psi} & \Sigma \\ * & -\gamma I \end{bmatrix} < 0, \tag{39}$$

其中$\bar{\Psi}, \Sigma$的表达式见定理2.

4 算例(Illustrative examples)

例 1 系统(1)具有如下参数[4]:

$$A = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -0.5 & -1 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, H_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, G_0 = G_1 = \begin{bmatrix} \sqrt{0.1} & 0 \\ 0 & \sqrt{0.1} \end{bmatrix}.$$

利用仿真软件MATLAB中LMI工具箱求解得使系统稳定的最大时滞为2.3347. 其他文献不同方法所获得的最大时滞见表1所示.

表 1 不同方法所获得的最大时滞

Table 1 The maximum delay obtained by different methods

方法	YUE[5]	HUA[7]	CHEN[2]	定理2
最大时滞	2.1411	2.1406	2.1402	2.3347

例 2 对系统(1), 对不同的μ_1, α_0, 计算使系统稳定的最大时滞τ_{M}见表2. 从表2可以看出, 当$\tau_0 = 0.4$时, 不同的μ_1, 以及时滞概率分布α_0, 都可以使得系统(1)鲁棒渐近稳定, 引入时滞概率分布信息, 克服了文献[9]中时滞的导数$\mu_1 < 1$的限制.

表 2 α_0不同时的上界值

Table 2 The different upper value for different α_0

结果	$\mu_1 = 0.2$	$\mu_1 = 0.6$	$\mu_1 = 1.1$
$\alpha_0 = 0.5$	1.0974	1.0972	1.0971
$\alpha_0 = 0.8$	1.6699	1.6582	1.6581
$\alpha_0 = 0.99$	2.3347	2.3095	2.3091

例 3 考虑系统(38), 利用跟例子1同样的参数, 得到的时滞最大值跟其他文献的比较见表3所示.

从表3可以看出本文对系统(38)的研究结果保守性比文献[8, 12]小.

表 3 不同方法所获得的最大时滞

Table 3 The maximum delay obtained by different methods

方法	Miyamura A[8]	Yan[12]	定理3
最大时滞	0.3555	1.0660	1.4906

5 结论(Conclusion)

本文用新的方法研究了一类具有不确定性和变时滞非线性随机干扰系统的鲁棒稳定性, 引入变时滞概率分布, 提出并证明了使得该系统鲁棒均方渐进稳定的充分条件, 该方法克服了以前文献中的一些限制, 数值仿真表明新的方法降低了系统的保守性.

参考文献(References):

[1] MAO X R. Robustness of exponential stability of stochastic differential delay equations[J]. *IEEE Transactions on Automatic Control*, 1996, 41(3): 442 – 447.

[2] CHEN W H, GUAN Z H, LU X M. Delay- dependent exponential stability of uncertain stochastic systems with multiple delays: an LMI approach[J]. *Systems & Control Letters*, 2005, 54(6): 547 – 555.

[3] XIE S, XIE L. Stabilization of a class of uncertain large-scale stochastic systems with time delays[J]. *Automatica*, 2000, 36(1): 161 – 167.

[4] ZHANG Y, HE Y, WU M. Delay-dependent robust stability for uncertain stochastic systems with interval time-varying delay[J]. *Acta Automatica Sinica*, 2009, 32(5): 577 – 582.

[5] YUE D, HAN Q L. Delay-dependent exponential stability of stochastic systems with time-varying delay, nonlinearity, and Markovian switching[J]. *IEEE Transactions on Automatic Control*, 2005, 50(2): 217 – 222.

[6] HE Y, ZHANG Y, WU M, et al. Improved exponential stability for stochastic Markovian jump systems with nonlinearity and time-varying delay[J]. *International Journal Of Robust and Nonlinear Control*, 2010, 20(1): 16 – 26.

[7] 华民刚, 邓飞其, 彭云建. 不确定变时滞随机系统的鲁棒均方指数稳定性[J]. 控制理论与应用, 2009, 26(5): 558 – 561.
(HUA Mingang, DENG Feiqi, PENG Yunjian. Robust mean square exponential stability of uncertain stochastic systems with time-varying delay[J]. *Control Theory & Applications*, 2009, 26(5): 558 – 561.)

[8] Miyamura A, Aihara K. Delay-dependent robust stability of uncertain delayed stochastic systems: an LMI-based approach[C] //*Proceeding of the 5th Asian Control Conference.* [S.l.]: [s.n.], 2004: 449 – 455.

[9] WU M, HE Y, SHE J H, et al. Delay-dependent criteria for robust stability of time-varying delays systems[J]. *Automatica*, 2004, 40(3): 1435 – 1439.

[10] YUE D, WONS. Delay-dependent robust stability of stochastic uncertain systems with time delay and nonlinear uncertainties[J]. *Electronics Letters*, 2001, 37(15): 992 – 993 .

[11] ZHANG Y, HE Y, WU M. Delay-dependent robust stability of uncertain delayed stochastic systems with interval time varying delay[J]. *Acta Automatica Sinica*, 2009, 35(5): 577 – 582.

[12] YAN H C, HUANG X H, ZHANG H, et al. Delay-dependent robust stability criteria of uncertain stochastic systems with time-varying delay[J]. *Chaos Solitons Fractals*, 2009, 40(4): 1668 – 1679.

作者简介:

李亚军 (1976—), 男, 讲师, 博士研究生, 从事不确定随机时滞系统鲁棒稳定性和反馈控制研究, E-mail: lyjfirst@163.com;

邓飞其 (1962—), 男, 教授, 博士, 博士生导师, 从事论动力系统的稳定性分析、随机与非线性系统的控制理论、信息系统的分析与集成研究, E-mail: aufqdeng@scut.edu.cn;

彭云建 (1973—), 男, 博士, 目前研究方向为随机电力系统稳定性研究, E-mail: pengyj@scut.edu.cn.