文章编号: 1000−8152(2011)11−1621−06

不确定时滞切换系统的鲁棒滑模控制

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摘要: 该文研究了一类具有非匹配不确定性和非线性扰动的时滞切换系统的滑模控制问题. 首先, 针对每个子系统设计对应的时滞依赖滑模面, 利用驻留时间方法, 给出了由滑动模态方程组成的切换系统鲁棒渐近稳定的充分条件; 然后设计了滑模控制器, 使得闭环系统的状态能够到达滑模面上, 产生滑动模态. 最后, 仿真实例说明所提出方法的有效性.

关键词: 切换系统; 滑模控制; 不确定; 驻留时间; 线性矩阵不等式

中图分类号: TP273 **文献标识码**: A

Robust sliding-mode control for a class of uncertain switched systems with time-delay

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Abstract: The sliding-mode control problem is studied for a class of time-delayed switched systems with mismatched uncertainties and nonlinear disturbances. The delay-dependent sliding surface is designed for each subsystem by using the dwell-time approach. The sufficient condition is deduced under which the switched system composed by sliding-mode dynamic equations is robustly asymptotically stable. Furthermore, a sliding-mode controller is designed which ensure the reachability of the sliding surface; thus the sliding-mode dynamics is resulted. A numerical example illustrates the effectiveness of the proposed design method.

Key words: switched systems; sliding-mode control; uncertain; dwell-time; linear matrix inequality

1 引言(Introduction)

切换系统是一类重要的混杂系统, 它在电力系统、受限机器人系统以及智能高速公路系统等都有广泛的应用. 文献[1∼5]对简单切换系统的各种稳定的充分条件做了详细的综述, 总结了切换系统早期的控制方法. 文献[6]通过构造多Lyapunov泛函给出了不确定时滞切换系统的时滞相关鲁棒稳定的充分条件并设计了相应的鲁棒镇定控制器.

另一方面, 滑模控制作为消除不确定性影响的有效方法, 具有响应速度快、鲁棒性能好等优点. 文献[7, 8]研究了非线性仿射控制系统的滑模控制问题, 文献[9]研究了不确定离散控制系统准滑模控制问题. 近年来, 对具有不匹配不确定性线性系统的滑模控制研究已取得一些成果[10∼13]. 文献[10]研究了一类系统矩阵中具有非匹配不确定性和匹配干扰的线性系统的滑模控制问题, 设计鲁棒滑模面, 确保闭环系统是鲁棒二次稳定的. 文献[11, 12]分别研究了含有非匹配不确定性和匹配干扰的时滞系统的滑模控制问题, 其中文献[12]设计了时滞依赖的滑模面, 确保闭环系统的鲁棒稳定性. 文献[13]进一步研究了具有非线性扰动的时滞系统的静态输出反馈滑模控制问题. 而对切换系统滑模控制问题的研究成果却十分有限. 文献[14]研究了二阶线性自治切换系统的滑模面存在问题, 但未给出切换律设计方法. 文献[15]讨论了一类布尔输入切换系统的滑模控制问题, 建立了切换系统滑动模态可达的几种不同滑模控制策略. 文献[16]研究了一类马尔可夫跳变不确定系统的滑模控制问题. 文献[17, 18]研究了不确定切换系统的H_∞滑模控制问题. 文献[19]进一步研究了一类不确定时滞切换系统的H_∞滑模控制问题. 设计了H_∞单滑模面并给出了切换律, 使切换系统的闭环系统为鲁棒稳定且具有γ扰动衰减度. 对于切换系统, 平均驻留时间方法是寻求切换律的一个有效工具, 并且设计简便[3∼5], 但对基于驻留时间方法研究不确定时滞切换系统的滑模控制问题的研究成果并不多见.

收稿日期: 2010−04−21; 收修改稿日期: 2011−01−29.
基金项目: 国家自然科学基金资助项目(60774041); 河南省基础与前沿技术研究资助项目(072300410110).

本文针对一类同时含有非匹配不确定性和非线性扰动的时滞切换系统, 研究滑模控制问题. 同时给出子系统滑模面, 滑模控制器和切换律的设计方法. 首先通过求解N个LMIs设计每个子系统的时滞相关滑模面并给出驻留时间, 在满足此驻留时间的条件下可确保由每个子系统的滑模面所组成的切换系统在任意切换下是鲁棒渐近稳定的; 然后设计了滑模控制器, 使得每个子系统的状态能够到达滑模面上, 产生滑动模态. 最后, 仿真实例说明所提出方法的有效性. 由于本文讨论的切换系统的时滞参数是已知的, 因此设计时滞依赖滑模面以提高系统的鲁棒性.

2 系统描述及主要引理(System description and main lemmas)

考虑如下的一类含有非匹配不确定性和具有非线性扰动的时滞切换系统:

$$\begin{cases} \dot{x}(t)=(A_\sigma+\Delta A_\sigma(t))x(t)+(A_{\mathrm{d}\sigma}+\Delta A_{\mathrm{d}\sigma}(t)) \\ \qquad x(t-h)+B_\sigma[u_\sigma(t)+f_\sigma(x,t)], \\ x(t)=\phi(t),\ t\in[-h,0], \end{cases} \tag{1}$$

其中: $x(t)\in\mathbb{R}^n$是系统的状态向量, $u_\sigma(t)\in\mathbb{R}^m$是控制输入向量; $\sigma(t):[t_0,+\infty)\rightarrow\psi=\{1,\cdots,N\}$为切换信号是一个分段右连续的时间函数, 即$\sigma(t_k)=\lim\limits_{t\rightarrow t_k^+}\sigma(t), k=0,1,2,\cdots$, 这里$t_k,\sigma(t_k)$为切换时刻及相应的切换序列取值, 可以随时间的演化规律将其表述为切换序列的形式:

$$\{(t_0,\sigma(t_0)),\cdots,(t_k,\sigma(t_k)),\cdots|\lim_{k\rightarrow\infty}t_k=\infty\}.$$

对系统(1)在每个状态上的驻留时间引入时间数量:

$$T_0=\inf\{t_{k+1}-t_k\},\ k=0,1,2,\cdots, \tag{2}$$

T_0称为切换系统的驻留时间[5]. $\Delta A_\sigma(t),\Delta A_{\mathrm{d}\sigma}(t)$为时变不确定性, 满足如下形式:

$$\Delta A_\sigma(t)=D_\sigma F_\sigma(t)E_\sigma,\ \Delta A_{\mathrm{d}\sigma}(t)=D_{\mathrm{d}\sigma}F_{\mathrm{d}\sigma}(t)E_{\mathrm{d}\sigma},$$

其中: $D_\sigma,D_{\mathrm{d}\sigma},E_\sigma,E_{\mathrm{d}\sigma}$为具有适当维数的常数矩阵; $F_\sigma(t),F_{\mathrm{d}\sigma}(t)$为具有Lebesgue可测的未知矩阵函数, 满足$F_\sigma^{\mathrm{T}}(t)F_\sigma(t)\leqslant I, F_{\mathrm{d}\sigma}^{\mathrm{T}}(t)F_{\mathrm{d}\sigma}(t)\leqslant I$.

对切换系统(1)首先作如下假设:

假设 1 系统(1)是完全可控的, 且$\forall\ i\in\psi=\{1,\cdots,N\},\mathrm{rank}(B_i)=m$.

假设 2 $\forall\ i\in\psi=\{1,\cdots,N\}, f_i(x,t)$是连续的, 且存在已知的正函数$\varphi(x)$, 使得$\|f_i(x,t)\|\leqslant\psi(x), i\in\psi$.

在进行证明主要问题之前, 先引入如下引理.

引理 1[20] 给对任意正定矩阵$M\in\mathbb{R}^{n\times n}$, 标量$\tau>0$, 向量值函数$w:[0,\tau]\rightarrow\mathbb{R}^n$, 使得下面的积分有定义, 则有下述矩阵不等式成立:

$$(\int_0^\tau w(s)\mathrm{d}s)^{\mathrm{T}}M(\int_0^\tau w(s)\mathrm{d}s)\leqslant \\ \tau\int_0^\tau w^{\mathrm{T}}(s)Mw(s)\mathrm{d}s. \tag{3}$$

3 主要结果(Main results)

由文献[12]知道存在非奇异的变换

$$T_i=\begin{bmatrix}U_{i2}^{\mathrm{T}}\\U_{i1}^{\mathrm{T}}\end{bmatrix},\ 使得\ T_iB_i=\begin{bmatrix}0\\B_{i2}\end{bmatrix},$$

其中U_{i1},U_{i2}由B_i的奇异值分解得到:

$$B_i=[U_{i1}\ \ U_{i2}]\begin{bmatrix}\varSigma_i\\0\end{bmatrix}M_i^{\mathrm{T}}.$$

令$z(t)=T_ix(t)$, 则系统(1)化为如下的正则型:

$$\begin{cases}\dot{z}(t)=(\bar{A}_i+\Delta\bar{A}_i(t))z(t)+(\bar{A}_{\mathrm{d}i}+\Delta\bar{A}_{\mathrm{d}i}(t)) \\ \qquad z(t-h)+\begin{bmatrix}0\\B_{i2}\end{bmatrix}[u_i(t)+f_i(T_i^{-1}z,t)], \\ z(t)=\bar{\phi}(t),\ t\in[-h,0],\end{cases} \tag{4}$$

其中: $\bar{A}_i=T_iA_iT_i^{-1},\ \bar{A}_{\mathrm{d}i}=T_iA_{\mathrm{d}i}T_i^{-1},\ \Delta\bar{A}_i(t)=T_i\Delta A_i(t)T_i^{-1},\ \Delta\bar{A}_{\mathrm{d}i}(t)=T_i\Delta A_{\mathrm{d}i}(t)T_i^{-1},\ \bar{\phi}(t)=T_i\phi(t)$, 则系统(4)可被改写为

$$\begin{cases}\dot{z}_1(t)=(\bar{A}_{i11}+\Delta\bar{A}_{i11}(t))z_1(t)+ \\ \qquad(\bar{A}_{\mathrm{d}i11}+\Delta\bar{A}_{\mathrm{d}i11}(t))z_1(t-h)+ \\ \qquad(\bar{A}_{i12}+\Delta\bar{A}_{i12}(t))z_2(t)+ \\ \qquad(\bar{A}_{\mathrm{d}i12}+\Delta\bar{A}_{\mathrm{d}i12}(t))z_2(t-h), \\ \dot{z}_2(t)=(\bar{A}_{i21}+\Delta\bar{A}_{i21}(t))z_1(t)+ \\ \qquad(\bar{A}_{\mathrm{d}i21}+\Delta\bar{A}_{\mathrm{d}i21}(t))z_1(t-h)+ \\ \qquad(\bar{A}_{i22}+\Delta\bar{A}_{i22}(t))z_2(t)+ \\ \qquad(\bar{A}_{\mathrm{d}i22}+\Delta\bar{A}_{\mathrm{d}i22}(t))z_2(t-h)+ \\ \qquad B_{i2}[u_i(t)+f_i(T_i^{-1}z,t)], \\ z_1(t)=\bar{\phi}_1(t),t\in[-h,0], \\ z_2(t)=\bar{\phi}_2(t),t\in[-h,0],\end{cases} \tag{5}$$

其中:

$$z_1\in\mathbb{R}^{n-m},\ z_2\in\mathbb{R}^m,\ B_{i2}=\varSigma_iM_i^{\mathrm{T}}, \\ \bar{A}_{i11}=U_{i2}^{\mathrm{T}}A_iU_{i2},\ \bar{A}_{i12}=U_{i2}^{\mathrm{T}}A_iU_{i1}, \\ \bar{A}_{\mathrm{d}i11}=U_{i2}^{\mathrm{T}}A_{\mathrm{d}i}U_{i2},\ \bar{A}_{\mathrm{d}i12}=U_{i2}^{\mathrm{T}}A_{\mathrm{d}i}U_{i1}, \\ \Delta\bar{A}_{i11}(t)=U_{i2}^{\mathrm{T}}D_iF_i(t)E_iU_{i2}, \\ \Delta\bar{A}_{i12}(t)=U_{i2}^{\mathrm{T}}D_iF_i(t)E_iU_{i1}, \\ \Delta\bar{A}_{\mathrm{d}i11}(t)=U_{i2}^{\mathrm{T}}D_{\mathrm{d}i}F_{\mathrm{d}i}(t)E_{\mathrm{d}i}U_{i2}, \\ \Delta\bar{A}_{\mathrm{d}i12}(t)=U_{i2}^{\mathrm{T}}D_{\mathrm{d}i}F_{\mathrm{d}i}(t)E_{\mathrm{d}i}U_{i1}, \\ \bar{\phi}_1(t)\in\mathbb{R}^{n-m},\ \bar{\phi}_2(t)\in\mathbb{R}^m.$$

选取第i个子系统的滑模面为

$$S_i = [C_i \ \ I]z(t) = C_i z_1(t) + z_2(t) = 0, \tag{6}$$

其中: $C_i \in \mathbb{R}^{m\times(n-m)}$, 将$z_2(t) = -C_i z_1(t)$代入式(4)的第一个方程, 得到第$i$个子系统的滑动模态方程为:

$$\begin{cases} \dot z_1(t) = (\bar A_{i11} + \Delta\bar A_{i11}(t) - \bar A_{i12}C_i - \\ \qquad \Delta\bar A_{i12}(t)C_i)z_1(t) + (\bar A_{\mathrm{d}i11} + \\ \qquad \Delta\bar A_{\mathrm{d}i11}(t) - \bar A_{\mathrm{d}i12}C_i - \\ \qquad \Delta\bar A_{\mathrm{d}i12}(t)C_i)z_1(t-h), \\ z_1(t) = \bar\phi_1(t),\ t\in[-h,0],\ i=1,2,\cdots,N. \end{cases} \tag{7}$$

显然式(7)是含有N个$n-m$维子系统的切换系统.

定理 1　给定$h > 0, \gamma > 1$, 如果存在正定对称矩阵X_i, W_i, 矩阵$Y_i (i\in\psi)$和正常数$\varepsilon_{ji} (j = 1, 2, 3, 4, i \in \psi)$满足如下线性矩阵不等式:

$$\varOmega_i = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & 0 & 0 & \gamma_{16} \\ * & \gamma_{22} & \gamma_{23} & \gamma_{24} & 0 & 0 \\ * & * & \gamma_{33} & 0 & \gamma_{35} & 0 \\ * & * & * & \gamma_{44} & 0 & 0 \\ * & * & * & * & \gamma_{55} & 0 \\ * & * & * & * & * & \gamma_{66} \end{pmatrix}, \tag{8}$$

则由每个子系统的滑模方程所构成的切换系统(7)对驻留时间满足

$$T_0 \geqslant \frac{1}{\rho}\ln\frac{\alpha}{\beta} \tag{9}$$

的任意切换信号都是鲁棒渐近稳定的, 且第i个子系统的滑模面可选为

$$S_i = Y_i X_i^{-1} z_1(t) + z_2(t) = 0, \tag{10}$$

其中:

$$\gamma_{11} = U_{i2}^{\mathrm T}A_iU_{i2}X_i + X_iU_{i2}^{\mathrm T}A_i^{\mathrm T}U_{i2} + U_{i2}^{\mathrm T}A_{\mathrm d i}^{\mathrm T}U_{i2}X_i + X_iU_{i2}^{\mathrm T}A_{\mathrm d i}^{\mathrm T}U_{i2} - U_{i2}^{\mathrm T}A_iU_{i2}Y_i - Y_i^{\mathrm T}U_{i1}^{\mathrm T}A_i^{\mathrm T}U_{i2} + W_i,$$

$$\gamma_{12} = X_iU_{i2}^{\mathrm T}A_i^{\mathrm T}U_{i2} + X_iU_{i2}^{\mathrm T}A_{\mathrm d i}^{\mathrm T}U_{i2} - Y_i^{\mathrm T}U_{i1}^{\mathrm T}A_i^{\mathrm T}U_{i2},$$

$$\gamma_{13} = U_{i2}^{\mathrm T}A_{\mathrm d i}^{\mathrm T}U_{i1}Y_i,$$

$$\gamma_{22} = -h^{-1}(\gamma-1)X_i, \gamma_{23} = U_{i2}^{\mathrm T}A_{\mathrm d i}^{\mathrm T}U_{i1}Y_i,$$

$$\gamma_{33} = -W_i,\ \gamma_{24} = [U_{i2}^{\mathrm T}D_i \ \ U_{i2}^{\mathrm T}D_{\mathrm d i}],$$

$$\gamma_{44} = \mathrm{diag}\{-\varepsilon_{3i}I, -\varepsilon_{4i}I\},$$

$$\gamma_{35} = [\varepsilon_{3i}X_iU_{i2}^{\mathrm T}E_{\mathrm d i}^{\mathrm T} \ -\varepsilon_{3i}Y_i^{\mathrm T}U_{i1}^{\mathrm T}E_{\mathrm d i}^{\mathrm T} \quad \varepsilon_{4i}X_iU_{i2}^{\mathrm T}E_{\mathrm d i}^{\mathrm T} \ -\varepsilon_{4i}Y_i^{\mathrm T}U_{i1}^{\mathrm T}E_{\mathrm d i}^{\mathrm T}],$$

$$\gamma_{55} = \mathrm{diag}\{-\varepsilon_{3i}I, -\varepsilon_{3i}I, -\varepsilon_{4i}I, -\varepsilon_{4i}I\},$$

$$\gamma_{16} = [U_{i2}^{\mathrm T}D_i \ \ \varepsilon_{1i}X_iU_{i2}^{\mathrm T}E_i^{\mathrm T} \ -\varepsilon_{1i}Y_i^{\mathrm T}U_{i1}^{\mathrm T}E_i^{\mathrm T} \ \ U_{i2}^{\mathrm T}D_{\mathrm d i} \quad \varepsilon_{3i}U_{i2}^{\mathrm T}E_i^{\mathrm T} \ -\varepsilon_{3i}Y_i^{\mathrm T}U_{i1}^{\mathrm T}E_i^{\mathrm T} \ \ \gamma hU_{i2}A_{\mathrm d i}^{\mathrm T}U_{i2}^{\mathrm T}X_i],$$

$$\gamma_{66} = \mathrm{diag}\{-\varepsilon_{1i}I, \varepsilon_{1i}I, -\varepsilon_{1i}I, -\varepsilon_{1i}I, -\varepsilon_{2i}I, -\varepsilon_{3i}I, -\varepsilon_{3i}I, \gamma hX_i\},$$

$$\lambda_1 = \max_{i\in\psi}\{\lambda_{\max}(X_i^{-1})\},\ \lambda_2 = \min_{i\in\psi}\{\lambda_{\min}(X_i^{-1})\},$$

$$\lambda_3 = \min_{i\in\psi}\{\lambda_{\min}(X_i^{-1}W_iX_i^{-1})^{-1}\bar A_{\mathrm d i11}^{\mathrm T}\bar A_{\mathrm d i11}\},$$

$$\lambda_4 = \max_{i\in\psi}\{\lambda_{\max}(X_i^{-1}W_iX_i^{-1})\},$$

$$\lambda_5 = \min_{i\in\psi}\{\lambda_{\min}(X_i^{-1}W_iX_i^{-1})\},$$

$$\lambda_6 = \max_{i\in\psi}\{\lambda_{\max}(\bar A_{\mathrm d i11}^{\mathrm T}X_i^{-1}\bar A_{\mathrm d i11})\},$$

$$\lambda_7 = \min_{i\in\psi}\{\lambda_{\min}(\bar A_{\mathrm d i11}^{\mathrm T}X_i^{-1}\bar A_{\mathrm d i11})\},$$

$$\lambda = \max_{i\in\psi}\{\lambda_{\max}(\bar\varOmega_i)\},\ 0<\alpha<1,$$

$$\rho = \frac{1}{2}\min\{\frac{\lambda}{\lambda_1}, \frac{1}{h\gamma}, \lambda_2\lambda_3\},\ \beta = \max\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_4}{\lambda_5}, \frac{\lambda_6}{\lambda_7}\}.$$

证　对每个$i\in\psi$本文考虑Lyapunov泛函$V_i : C_h \to \mathbb{R}^n$

$$V_i = V_{1i} + V_{2i} + V_{3i}, \tag{11}$$

其中:

$$V_{1i}(z_{1t}) = D^{\mathrm T}(z_{1t})P_iD(z_{1t}),$$

$$V_{2i}(z_{1t}) = \gamma\int_{t-h}^{t}\int_{s}^{t} z_1^{\mathrm T}(u)\bar A_{\mathrm d i11}^{\mathrm T}P_i\bar A_{\mathrm d i11}z_1(u)\mathrm du\mathrm ds,$$

$$V_{3i}(z_{1t}) = \int_{t-h}^{t} z_1^{\mathrm T}(s)T_iz_1(s)\mathrm ds,$$

其中$P_i, T_i (i\in\psi)$为正定对称矩阵, 算子$D : C_h \to \mathbb{R}^n$定义为

$$D(z_{1t}) = z_1(t) + \int_{t-h}^{t}\bar A_{\mathrm d i11}z_1(s)\mathrm ds. \tag{12}$$

由V_i的定义, 本文得到

$$V_i(z_{1t}) \leqslant c_2\|z_{1t}\|_{C_h}^2, \tag{13}$$

其中c_2为正常数, 由引理1本文得到

$$\begin{aligned} &(\int_{t-h}^{t}\bar A_{\mathrm d i11}z_1(s)\mathrm ds)^{\mathrm T}(\int_{t-h}^{t}\bar A_{\mathrm d i11}z_1(s)\mathrm ds) \leqslant \\ &h\int_{t-h}^{t}z_1^{\mathrm T}(s)\bar A_{\mathrm d i11}^{\mathrm T}\bar A_{\mathrm d i11}z_1(s)\mathrm ds \leqslant \\ &h\lambda_{\max}(T_i^{-1}\bar A_{\mathrm d i11}^{\mathrm T}\bar A_{\mathrm d i11})\int_{t-h}^{t}z_1^{\mathrm T}(s)T_iz_1(s)\mathrm ds \leqslant \\ &cV_{3i}(z_{1t}), \end{aligned} \tag{14}$$

其中c为正常数, 从而

$$\begin{aligned} V_{1i}(z_{1t}) \geqslant\ & V_{2i}(z_{1t}) + V_{3i}(z_{1t}) \geqslant \\ & \min(\lambda_2, c^{-1})(\|D(z_{1t})\|^2 + \\ & \|\int_{t-h}^{t}\bar A_{\mathrm d i11}z_1(s)\mathrm ds\|^2) \geqslant \\ & \frac{1}{2}\min(\lambda_2, c^{-1})(\|D(z_{1t}) - \\ & \int_{t-h}^{t}\bar A_{\mathrm d i11}z_1(s)\mathrm ds\|^2) = \\ & c_1\|z_1(t)\|^2, \end{aligned} \tag{15}$$

其中c_1为正常数, 定义$\tilde z_1(t) = \int_{t-h}^{t}\bar A_{\mathrm d i11}z_1(s)\mathrm ds$, 沿

式(7)微分则有

$$
\begin{cases}
\dot V_{1i}(z_{1t})=2D^{\mathrm T}(z_{1t})P_i\dot D(z_{1t})= \\
2z_1^{\mathrm T}(t)P_i(\bar A_{i11}-\bar A_{i12}C_i+\bar A_{\mathrm{d}i11})z_1(t)+ \\
2z_1^{\mathrm T}P_i(-\bar A_{\mathrm{d}i12}C_i)z_1(t-h)+ \\
2\tilde z_1^{\mathrm T}(t)P_i(\bar A_{i11}-\bar A_{i12}C_i+\bar A_{\mathrm{d}i11})z_1(t)+ \\
2\tilde z_1^{\mathrm T}P_i(-\bar A_{\mathrm{d}i12}C_i)z_1(t-h)+ \\
2z_1^{\mathrm T}(t)P_i(\Delta\bar A_{i11}-\Delta\bar A_{i12}C_i)z_1(t)+ \\
2z_1^{\mathrm T}(t)P_i(\Delta\bar A_{\mathrm{d}i11}-\Delta\bar A_{\mathrm{d}i12}C_i)z_1(t-h)+ \\
2\tilde z_1^{\mathrm T}(t)P_i(\Delta\bar A_{i11}-\Delta\bar A_{i12}C_i)z_1(t)+ \\
2\tilde z_1^{\mathrm T}(t)P_i(\Delta\bar A_{\mathrm{d}i11}-\Delta\bar A_{\mathrm{d}i12}C_i)z_1(t-h), \\
\dot V_{2i}(z_{1t})= \\
\gamma h z_1^{\mathrm T}(t)\bar A_{\mathrm{d}i11}^{\mathrm T}P_i\bar A_{\mathrm{d}i11}z_1(t)- \\
\gamma\int_{t-h}^{t}z_1^{\mathrm T}(u)\bar A_{\mathrm{d}i11}^{\mathrm T}P_i\bar A_{\mathrm{d}i11}z_1(u)\mathrm{d}u\leqslant \\
\gamma h z_1^{\mathrm T}(t)\bar A_{\mathrm{d}i11}^{\mathrm T}P_i\bar A_{\mathrm{d}i11}z_1(t)- \\
\int_{t-h}^{t}z_1^{\mathrm T}(u)\bar A_{\mathrm{d}i11}^{\mathrm T}P_i\bar A_{\mathrm{d}i11}z_1(u)\mathrm{d}u- \\
h^{-1}(\gamma-1)\tilde z_1^{\mathrm T}(t)P_i\tilde z_1(t), \\
\dot V_{3i}(z_{1t})=z_1^{\mathrm T}(t)T_iz_1(t)-z_1^{\mathrm T}(t-h)T_iz_1(t-h).
\end{cases}
\tag{16}
$$

由前面关于系统不确定性的假设可得

$$
\dot V_i(t)\leqslant\begin{bmatrix}z_1(t)\\ \tilde z_1(t)\\ z_1(t-h)\end{bmatrix}^{\mathrm T}\bar\Omega_i\begin{bmatrix}z_1(t)\\ \tilde z_1(t)\\ z_1(t-h)\end{bmatrix}-\int_{t-h}^{t}z_1^{\mathrm T}(s)\bar A_{\mathrm{d}i11}^{\mathrm T}P_i\bar A_{\mathrm{d}i11}z_1(s)\mathrm{d}s, \tag{17}
$$

其中:

$$
\bar\Omega_i=\begin{bmatrix}\theta_{11}&\theta_{12}&\theta_{13}\\ *&\theta_{22}&\theta_{23}\\ *&*&\theta_{33}\end{bmatrix},
$$

$$
\begin{aligned}
\theta_{11}=\;&P_i[U_{i2}^{\mathrm T}A_iU_{i2}-U_{i2}^{\mathrm T}A_iU_{i1}C_i+U_{i2}^{\mathrm T}A_{\mathrm{d}i}U_{i2}]+\\
&[U_{i2}^{\mathrm T}A_iU_{i2}-U_{i2}^{\mathrm T}A_iU_{i1}C_i+U_{i2}^{\mathrm T}A_{\mathrm{d}i}U_{i2}]^{\mathrm T}P_i+\\
&\varepsilon_{1i}^{-1}P_iU_{i2}^{\mathrm T}D_iD_i^{\mathrm T}U_{i2}P_i+\\
&\varepsilon_{1i}(U_{i2}-U_{i1}C_i)^{\mathrm T}E_i^{\mathrm T}E_i(U_{i2}-U_{i1}C_i)+\\
&\varepsilon_{2i}^{-1}P_iU_{i2}^{\mathrm T}D_{\mathrm{d}i}D_{\mathrm{d}i}^{\mathrm T}U_{i2}P_i+\\
&\varepsilon_{3i}(U_{i2}-U_{i1}C_i)^{\mathrm T}E_i^{\mathrm T}E_i(U_{i2}-U_{i1}C_i)+\\
&\gamma h(U_{i2}^{\mathrm T}A_{\mathrm{d}i}U_{i2})^{\mathrm T}P_i(U_{i2}^{\mathrm T}A_{\mathrm{d}i}U_{i2})+T_i,\\
\theta_{22}=\;&-h^{-1}(\gamma-1)P_i+\varepsilon_{3i}^{-1}P_iU_{i2}^{\mathrm T}D_iD_i^{\mathrm T}U_{i2}P_i+\\
&\varepsilon_{4i}^{-1}P_iU_{i2}^{\mathrm T}D_{\mathrm{d}i}D_{\mathrm{d}i}^{\mathrm T}U_{i2}P_i,\\
\theta_{33}=\;&-T_i+\varepsilon_{2i}(U_{i2}-U_{i1}C_i)^{\mathrm T}E_i^{\mathrm T}E_i(U_{i2}-U_{i1}C_i)+\\
&\varepsilon_{4i}(U_{i2}-U_{i1}C_i)^{\mathrm T}E_{\mathrm{d}i}^{\mathrm T}E_{\mathrm{d}i}(U_{i2}-U_{i1}C_i),\\
\theta_{12}=\;&[U_{i2}^{\mathrm T}A_iU_{i2}-U_{i2}^{\mathrm T}A_iU_{i1}C_i+U_{i2}^{\mathrm T}A_{\mathrm{d}i}U_{i2}]^{\mathrm T}P_i,\\
\theta_{13}=\;&P_i(-U_{i2}^{\mathrm T}A_{\mathrm{d}i}U_{i2}C_i),\ \theta_{23}=P_i(-U_{i2}^{\mathrm T}A_{\mathrm{d}i}U_{i2}C_i).
\end{aligned}
$$

令$X_i=P_i^{-1},W_i=X_iT_iX_i,Y_i=C_iX_i$, 则$\bar\Omega_i<0$等价于$\Omega_i<0$. 因为$\bar\Omega_i<0,P_i>0,i\in\psi$, 得到

$$
\dot V_i(z_{1t})\leqslant -c_3\|z_1(t)\|^2, \tag{18}
$$

其中c_3为正常数. 根据切换信号$\sigma(t)$分段连续, 假设τ为从状态j切换到状态i的切换时间, 即$\sigma(\tau^+)=i$, $\sigma(\tau^-)=j$, 进一步可得

$$
\begin{aligned}
&V_{1i}(z_{1\tau^+})=D^{\mathrm T}(z_{1\tau})P_iD(z_{1\tau})\leqslant\\
&\frac{\lambda_1}{\lambda_2}D^{\mathrm T}(z_{1\tau})P_jD(z_{1\tau})=\frac{\lambda_1}{\lambda_2}V_{1j}(z_{1\tau^-}),
\end{aligned}\tag{19}
$$

$$
V_{2i}(z_{1\tau^+})\leqslant\frac{\lambda_4}{\lambda_5}V_{2j}(z_{1\tau^-}), \tag{20}
$$

$$
V_{3i}(z_{1\tau^+})\leqslant\frac{\lambda_6}{\lambda_7}V_{2j}(z_{1\tau^-}). \tag{21}
$$

由β的定义可得

$$
V_i(z_{1\tau^+})\leqslant\beta V_j(z_{1\tau^-}). \tag{22}
$$

现在用v来表示系统从其他状态k切换到状态j时的时间, 本文来比较$V_j(x_{v^+})$和$V_i(x_{\tau^+})$, 有

$$
\begin{aligned}
&\begin{bmatrix}z_1(t)\\ \tilde z_1(t)\\ z_1(t-h)\end{bmatrix}^{\mathrm T}\bar\Omega_j\begin{bmatrix}z_1(t)\\ \tilde z_1(t)\\ z_1(t-h)\end{bmatrix}\leqslant\\
&-\lambda(\|z_1(t)\|^2+\|\tilde z_1(t)\|^2+\|z_1(t-h)\|^2)\leqslant\\
&-\frac{\lambda}{2}\|z_1(t)+\tilde z_1(t)\|^2\leqslant-\frac{\lambda}{2\lambda_1}V_{1j}(z_{1t}),
\end{aligned}\tag{23}
$$

$$
\begin{aligned}
&-\int_{t-h}^{t}z_1^{\mathrm T}(s)\bar A_{\mathrm{d}j11}^{\mathrm T}P_j\bar A_{\mathrm{d}j11}z_1(s)\mathrm{d}s\leqslant\\
&-\frac{1}{h}\int_{t-h}^{t}\int_{s}^{t}z_1^{\mathrm T}(u)\bar A_{\mathrm{d}j11}^{\mathrm T}P_j\bar A_{\mathrm{d}j11}z_1(u)\mathrm{d}u\mathrm{d}s\leqslant\\
&-\frac{1}{h\gamma}V_{2j}(z_{1t}),
\end{aligned}\tag{24}
$$

$$
\begin{aligned}
&-\int_{t-h}^{t}z_1^{\mathrm T}(s)\bar A_{\mathrm{d}j11}^{\mathrm T}P_j\bar A_{\mathrm{d}j11}z_1(s)\mathrm{d}s\leqslant\\
&-\lambda_2\int_{t-h}^{t}\lambda_{\min}(T_j^{-1}\bar A_{\mathrm{d}j11}^{\mathrm T}\bar A_{\mathrm{d}j11})z_1^{\mathrm T}(s)T_jz_1(s)\mathrm{d}s\leqslant\\
&-\lambda_2\lambda_3V_{3j}(z_{1t}).
\end{aligned}\tag{25}
$$

现在利用式(17)和式(23)~(25), 对$t\in(v,\tau)$, 有

$$
\dot V_j(z_{1t})\leqslant-\frac{1}{2}\min\{\frac{\lambda}{\lambda_1},\frac{1}{h\gamma},\lambda_2\lambda_3\}V_j(z_{1t}), \tag{26}
$$

则有

$$
\frac{1}{V_j(z_{1t})}\mathrm{d}V_j(z_{1t})\leqslant-\rho\mathrm{d}t. \tag{27}
$$

对式(27)式从(v,τ)积分, 可得$V_j(z_{1\tau^-})\leqslant V_j(z_{1v^+})\times\mathrm{e}^{-\rho(\tau-v)}$, 上式结合式(22), 有

$$
V_i(z_{1\tau^+})\leqslant\beta V_j(z_{1v^+})\mathrm{e}^{-\rho(\tau-v)}. \tag{28}
$$

利用式(9), 本文有$\tau-v\geqslant T_0\geqslant\dfrac{1}{\rho}\ln\dfrac{\beta}{\alpha}$, 则式(28)变

为$V_i(z_{1\tau^+})\leqslant\alpha V_j(z_{1v^+})\mathrm{e}^{-\rho(\tau-v)}$, 不失一般性, 本文假设系统式(7)在$[0,\infty)$经过有限时间切换, 用$N(t)$来表示在$[0,t)$上的切换次数, 本文有$\lim\limits_{t\to\infty}N(t)=\infty$. 假设$\sigma(t)=i$和$\sigma(0)=i_0$, 则有

$$c_1\|z_1(t)\|^2\leqslant V_i(z_{1t})\leqslant\alpha^{N(t)}V_{i_0}(z_{10})\to 0.\tag{29}$$

当$t\to\infty$时, 则$\|z_1(t)\|\to 0$, 当$t\to\infty$, 这意味着滑模方程的平凡解是鲁棒渐近稳定的.　　证毕.

注 1　如果条件(8)满足, 那么由每个子系统的滑模方程所构成的切换系统(7), 在驻留时间满足式(9)的任意切换信号下均是鲁棒渐近稳定的, 因此与文献[20]相比, 本文设计的切换律更加简便实用. 另外, 本文讨论的切换系统的时滞参数是已知的, 因此设计时滞依赖滑模面以提高鲁棒性; 同时利用系统的状态矩阵构造Lyapunov泛函也可降低保守性[12], 调节参数γ可给Lyapunov函数的构造增加更大的自由度.

现在, 将设计滑模控制器以确保系统到达滑动模态区并作滑模运动. 系统(1)滑动模态到达条件为

$$\dot V=\sum_{i=1}^{N}S_i^{\mathrm T}S_i<0,\tag{30}$$

其中S_i是各子系统的滑模函数

$$S_i=\bar C_iz(t)=Y_iX_i^{-1}z_1(t)+z_2(t).$$

定理 2　考虑切换系统(4), 对任意$i\in\psi$, 定义滑模控制器为

$$\begin{aligned}u_i(t)=&-(B_{i2})^{-1}[\bar C_i\bar A_iz(t)+\bar C_i\bar A_{\mathrm{d}i}z(t-h)]-\\&(B_{i2})^{-1}[\|\bar C_iD_i\|\|E_iz(t)\|+\\&\|\bar C_iD_{\mathrm{d}i}\|\|E_{\mathrm{d}i}z(t-h)\|+\|B_{i2}\|\varphi(T_i^{-1}z)+\\&\mu]\mathrm{sgn}\,S_i,\end{aligned}\tag{31}$$

其中μ为常数, 则系统(4)在控制器(31)作用下进入滑动模态区并做滑模运动.

证　当$S_i(t)=0$时系统已进入滑动模态区做滑模运动. 当$S_i(t)\neq 0$时, 定义如下Lyapunov函数

$$V=\frac{1}{2}\sum_{i-1}^{N}S_i^{\mathrm T}S_i,$$

由假设2和式(31), 得

$$\sum_{i-1}^{N}S_i^{\mathrm T}\dot S_i\leqslant-\sum_{i=1}^{N}\|S_i\|,$$

因此滑动模到达条件满足, 在控制(31)和给定的切换律的作用下, 系统(1)能够进入滑动模态区并做滑模运动.　　证毕.

4　仿真算例(Simulation example)

考虑切换系统(1), 其中$i=1,2$,

$$A_1=\begin{bmatrix}1.0&2.0\\0.0&1.2\end{bmatrix},\ A_2=\begin{bmatrix}1.0&2.0\\0.1&1.1\end{bmatrix},$$

$$A_{\mathrm{d}1}=\begin{bmatrix}1.0&0.5\\1.0&2.0\end{bmatrix},\ A_{\mathrm{d}2}=\begin{bmatrix}1.2&0.6\\1.1&2.0\end{bmatrix},$$

$$B_1=\begin{bmatrix}0.0\\1.0\end{bmatrix},\ B_2=\begin{bmatrix}0.0\\1.0\end{bmatrix},$$

$$D_1=D_2=\begin{bmatrix}0.5&0.0\\0.0&0.5\end{bmatrix},$$

$$E_i=E_{\mathrm{d}i}=\begin{bmatrix}0.01&0.0\\0.0&0.01\end{bmatrix},$$

$$F_i(t)=\sin t,\ f_1=f_2=0.$$

取初始状态$z(0)=[1\ 1]^{\mathrm T}$, 常数$\mu=10$; 选取调节参数$\gamma=5.5$,

$$\begin{aligned}&\varepsilon_{11}=\varepsilon_{21}=\varepsilon_{31}=\varepsilon_{41}=\\&\varepsilon_{12}=\varepsilon_{22}=\varepsilon_{32}=\varepsilon_{42}=1.\end{aligned}$$

通过求解线性矩阵不等式(8), 可以得到切换系统滑模面为

$$\begin{aligned}&S_1(t)=[7.2251\ \ 1.0]z(t)=0,\\&S_2(t)=[7.7666\ \ 1.0]z(t)=0.\end{aligned}$$

由定理2可求得滑模控制器, 为减弱了高频抖振利用$\tanh(\dfrac{x}{\rho})$设计光滑切换律来逼近sgn函数

$$\begin{aligned}u_1=&7.2251z_1(t)+15.6502z_2(t)+\\&8.2251z_1(t-h)+5.6126z_2(t-h)+\\&[3.647\|0.01z_1(t)+0.01z_2(t)\|+\\&3.647\|0.01z_1(t-h)+0.01z_2(t-h)\|+\\&10]\tanh(7.2251z_1(t)+z_2(t)),\\u_2=&7.8666z_1(t)+16.6332z_2(t)+\\&10.4199z_1(t-h)+6.66z_2(t-h)+\\&[3.9154\|0.01z_1(t)+0.01z_2(t)\|+\\&3.9154\|0.01z_1(t-h)+0.01z_2(t-h)\|+\\&10]\tanh(7.7666z_1(t)+z_2(t)).\end{aligned}$$

MATLAB仿真结果如图1～3所示.

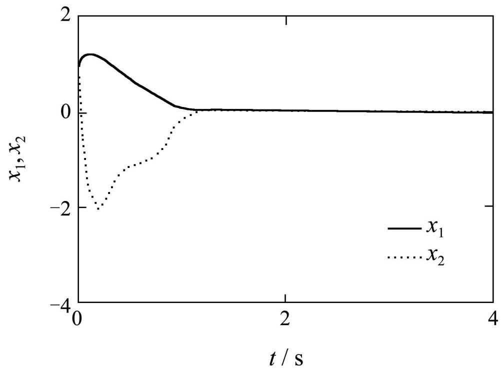

图 1　切换系统(1)状态响应

Fig. 1　The state response of switched system

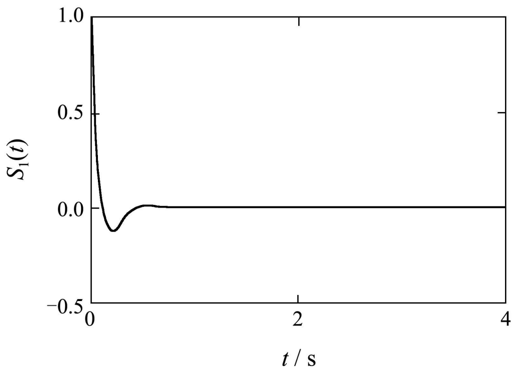

图 2 滑模面S_1的变化曲线

Fig. 2 The trajectory of the sliding surface S_1

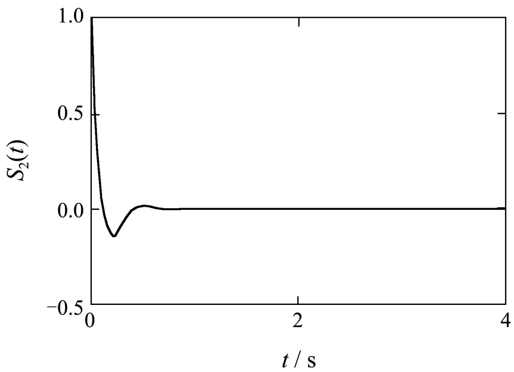

图 3 滑模面S_2的变化曲线

Fig. 3 The trajectory of the sliding surface S_2

5 结论(Conclusions)

本文针对不确定时滞切换系统给出了一种新的滑模控制策略. 首先, 基于驻留时间方法给出每个滑动模态方程鲁棒渐近稳定的充分条件, 且在求解LMIs后得到滑模面系数矩阵; 其次, 设计了滑模控制器, 使得闭环系统的状态能够到达滑模面上, 产生滑动模态; 最后, 仿真实例说明所提出方法的有效性.

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