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带乘性噪声和延时观测广义系统的多步最优预测器

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摘要: 本文研究了状态空间描述的离散广义系统最优预测器的设计问题, 该系统带有即时和延时观测, 所有观测中带有乘性噪声. 论文在两个基本假设条件下采用标准的奇异值分解方法给出了受限等价时滞系统, 对于此类系统没有采用状态增广方法, 而是采用新息重组分析理论给出了多步预测器. 因为延时观测的存在, 所给出的多步预测器包含了两套递推的广义系统黎卡提方程. 本文给出了一个数学算例验证了所提方法的正确性和有效性, 并给出了四幅图片, 根据算例可以看出一般情况下预测的步数越少, 预测的结果越好. 本文方法可以进一步来研究更复杂的一些问题, 如延时广义系统的H_∞滤波和控制问题.

关键词: 广义系统; 多步预测器; 延时观测; 乘性噪声; 新息重组

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Optimal multi-step predictor for descriptor systems with multiplicative noise and delayed measurement

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Abstract: Optimal multi-step predictor problem for the discrete-time descriptor systems is dealt with, where the descriptor systems is given in terms of state-space equations. The state-space equations involve instantaneous measurements and delayed measurements, and multiplicative noises in all measurements. Two standard assumptions are made for the descriptor system case, under which the restricted equivalent delayed system is derived by using standard singular value decomposition. Without resorting to state augmentation which is usually employed for dealing with estimation for delay system, we put forward a reorganized innovation lemma for presenting the result of multi-step predictor. The multi-step predictor involves two sets of recursive Riccati equations of descriptor systems since of the existence of the delayed measurements. The proposed approach is validated being efficient by a numerical example where four figures are presented, demonstrating that the proposed approach can give better predicting estimation when the number of steps is less. Furthermore, the proposed approach can be used to tackle more difficult problems, such as filtering and control for descriptor systems with multiplicative noises.

Key words: descriptor systems; multi-step predictor; delayed measurements; multiplicative noise; re-organized innovation

1 引言(Introduction)

最优化估计问题长期以来一直是控制理论和信号处理领域的一个重要问题[1–5]. 优化估计问题主要包括预测、滤波和平滑问题. 卡尔曼滤波是最经典和最具代表性的滤波方法[6]. 预测是指基于过去和现在的观测数据来估计未来的状态或信号, 它具有很重要的实际意义, 例如, 根据海底潜艇现在和过去的位置、速度等数据, 对它将来一定时间的位置等状态进行预测. 正常不带乘性噪声和延时观测系统的卡尔曼滤波和预测问题近年来已经研究的很好了, 见文献[1–2]和其中的参考文献.

广义系统的最优化滤波和预测问题近年来受到了广泛的关注[7–10]. 一般来说, 为了给出系统的状态估计结果, 在一定的假设条件下, 广义系统需要进行受限等价转换. 对于不带延时和乘性噪声的离散广义系统的最优估计问题, 有很多方法可以进行处理, 如现

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代时间序列分析方法[8,11]、奇异值分解方法[12]、状态增广方法[10,13]、卡尔曼滤波方法[14], 等等. 而在很多的实际应用中观测通道往往夹杂着乘性噪声, 如石油地震勘探、水声处理等[15–16]. 对于这类问题常用的方法一般有贝叶斯估计方法[17]、系统增广方法[18]和希尔伯特空间的射影定理方法[19–21]. 例如, 文献[18]采用系统增广的方法处理了含有多通道和乘性噪声的广义系统滤波问题, 文献[19–21]采用卡尔曼滤波和标准分解方法研究了广义系统的优化估计问题. 几乎没有相关的论文对含有即时观测和延时观测的广义系统进行过研究[22], 其中系统的所有观测中都带有乘性噪声. 对于此类问题, 上面提到的状态增广是一个可行的方法[10,13], 但是该方法容易导致计算量的急剧增加. 对于此类延时问题, 新息重组分析理论是一个有效可行的方法[3]. 本文处理了带有乘性噪声和延时观测的离散广义系统的最优多步预测问题. 与经典的状态增广方法不同, 本文采用了作者早前提出的新息重组分析理论[4]. 标准的奇异值分解将广义系统转换为受限等价时滞系统. 文末给出了一个数学算例证明了所采用方法的正确性和有效性.

本文还包括以下部分: 第2部分给出了所要研究的问题; 第3部分给出了受限等价时滞系统的多步预测器; 第4部分给出了本文的主要部分–带乘性噪声和延时观测广义系统的多步预测器; 第5部分给出了数学算例, 对所提方法的正确性和有效性进行了验证; 最后给出了本文的结论.

2 问题描述(Problem statement)

考虑下面带有乘性噪声、即时和延时观测的离散广义系统

$$F\boldsymbol{x}(t+1)=A\boldsymbol{x}(t)+B\boldsymbol{u}(t), \tag{1}$$

$$\boldsymbol{y}(t)=C\boldsymbol{x}(t)+D\boldsymbol{w}(t)\boldsymbol{x}(t)+\boldsymbol{v}(t), \tag{2}$$

$$\begin{aligned}\boldsymbol{y}_{\rm d}(t)=&C_{\rm d}\boldsymbol{x}(t_{\rm d})+D_d\boldsymbol{w}(t)\boldsymbol{x}(t_{\rm d})+\boldsymbol{v}_{\rm d}(t),\\&d>0,\ t_{\rm d}=t-d,\end{aligned} \tag{3}$$

其中: $\boldsymbol{x}(t)\in\mathbb{R}^n$是系统状态; $\boldsymbol{y}(t)\in\mathbb{R}^q$和$\boldsymbol{y}_{\rm d}(t)\in\mathbb{R}^{q_{\rm d}}$是即时和延时观测; $\boldsymbol{u}(t)\in\mathbb{R}^p$, $\boldsymbol{v}(t)\in\mathbb{R}^q$和$\boldsymbol{v}_{\rm d}(t)\in\mathbb{R}^{q_{\rm d}}$分别是输入干扰、观测噪声和延时观测噪声; $\boldsymbol{w}(t)\in\mathbb{R}^1$是存在于即时和延时观测的乘性噪声; 矩阵$A,\ B,\ C,\ D,\ C_{\rm d},\ D_{\rm d}$假设已知, 并具有适当的维数.

本文首先给出下面两个广义系统的标准假设:

假设 1 $\boldsymbol{u}(t),\boldsymbol{v}(t),\boldsymbol{v}_{\rm d}(t)$和$\boldsymbol{w}(t)$是不相关的零均值白噪声, 并且与$\boldsymbol{x}(0)$也不相关, 并且满足

$$\begin{aligned}&<\boldsymbol{u}(t),\boldsymbol{u}(s)>=Q_u\delta_{t,s},\\&<\boldsymbol{v}(t),\boldsymbol{v}(s)>=Q_v\delta_{t,s},\\&<\boldsymbol{v}_{\rm d}(t),\boldsymbol{v}_{\rm d}(s)>=Q_{v_{\rm d}}\delta_{t,s},\\&<\boldsymbol{w}(t),\boldsymbol{w}(s)>=M\delta_{t,s},\\&<\boldsymbol{x}(0),\boldsymbol{x}(0)>=\Pi(0),\end{aligned}$$

其中$Q_u,\ Q_v,\ Q_{v_{\rm d}},\ M,\ \Pi(0)$是假设已知的.

假设 2 矩阵F是已知和奇异的, 系统是正则的, 即$\operatorname{rank}F=n_1<n$, 而且存在$\lambda$满足$\det(\lambda F-A)\neq 0$.

上述广义系统的最优多步预测器问题描述如下:

问题 P 对于广义系统(1)–(3), 已知观测$\{\boldsymbol{y}(0),\cdots,\boldsymbol{y}(t);\boldsymbol{y}_{\rm d}(d),\cdots,\boldsymbol{y}_{\rm d}(t)\}$和固定的数$l>1$, 求$\boldsymbol{x}(t+l)$的最小均方差预测器$\hat{\boldsymbol{x}}(t+l\mid t)$.

在假设2下, 根据传统广义系统的结果[18–19], 存在非奇异的矩阵$\{J_1,\ J_2\}\in\mathbb{R}^{n\times n}$, $\boldsymbol{x}(t)=J_2\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\end{bmatrix}$, 满足下面的引理:

引理 1 广义系统(1)–(3)若满足假设1–2, 则其等价于

$$\boldsymbol{x}_1(t+1)=A_1\boldsymbol{x}_1(t)+B_1\boldsymbol{u}(t), \tag{4}$$

$$F_1\boldsymbol{x}_2(t+1)=\boldsymbol{x}_2(t)+B_2\boldsymbol{u}(t), \tag{5}$$

$$\begin{aligned}\boldsymbol{y}(t)=&C_1\boldsymbol{x}_1(t)+C_2\boldsymbol{x}_2(t)+D_1\boldsymbol{w}(t)\boldsymbol{x}_1(t)+\\&D_2\boldsymbol{w}(t)\boldsymbol{x}_2(t)+\boldsymbol{v}(t),\end{aligned} \tag{6}$$

$$\begin{aligned}\boldsymbol{y}_{\rm d}(t)=&C_{1\rm d}\boldsymbol{x}_1(t_{\rm d})+C_{2\rm d}\boldsymbol{x}_2(t_{\rm d})+\\&D_{1\rm d}\boldsymbol{w}(t)\boldsymbol{x}_1(t_{\rm d})+\\&D_{2\rm d}\boldsymbol{w}(t)\boldsymbol{x}_2(t_{\rm d})+\boldsymbol{v}_{\rm d}(t),\end{aligned} \tag{7}$$

其中:

$$J_1FJ_2=\begin{bmatrix}I_{n_1}&0\\0&F_1\end{bmatrix},\ J_1AJ_2=\begin{bmatrix}A_1&0\\0&I_{n-n_1}\end{bmatrix},$$

$$J_1B=\begin{bmatrix}B_1\\B_2\end{bmatrix},\ J_2\begin{bmatrix}C\\C_{\rm d}\\D\\D_{\rm d}\end{bmatrix}=\begin{bmatrix}C_1&C_2\\C_{1\rm d}&C_{2\rm d}\\D_1&D_2\\D_{1\rm d}&D_{2\rm d}\end{bmatrix},$$

$\boldsymbol{x}_1\in\mathbb{R}^{n_1}$, $\boldsymbol{x}_2\in\mathbb{R}^{n-n_1}$, F_1是一个λ–幂零矩阵, 即$F_1^{\lambda}=0$, $F_1^{\lambda-1}\neq 0$.

3 多步预测器$\hat{\boldsymbol{x}}_1(t+l|t)$(Multi-step predictor $\hat{\boldsymbol{x}}_1(t+l|t)$)

根据引理1, 为了给出问题P的最优多步预测器, 需要先给出受限等价系统(4)–(7)的多步预测器.

3.1 新息重组分析理论(Reorganized innovation analysis lemma)

采用论文[22]相似的定义或标记$\boldsymbol{e}_1(t,i),\tilde{\boldsymbol{Y}}_i(t)$, $P_1(t,i)$, $L(t,t,i)$, $Q_{\tilde{\boldsymbol{Y}}_i}(t)$, 根据新息重组分析理论[3–4], 可以给出如下引理:

引理 2 对于系统(1)–(3), $\boldsymbol{L}\{\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_{\rm d});\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1),\cdots,\tilde{\boldsymbol{Y}}_1(t)\}$是新息序列, 其张成的空间等价于$\boldsymbol{L}\{\boldsymbol{Y}_2(0),\cdots,\boldsymbol{Y}_2(t_{\rm d});\boldsymbol{Y}_1(t_{\rm d}+1),\cdots,\boldsymbol{Y}_1(t)\}$, 也等价于由观测序列$\{\boldsymbol{y}(0),\cdots,\boldsymbol{y}(t);\boldsymbol{y}_{\rm d}(d),\cdots,\boldsymbol{y}_{\rm d}(t)\}$张成的空间.

在上述引理中:

$$\boldsymbol{Y}_2(t)\triangleq\begin{bmatrix}\boldsymbol{y}(t)\\ \boldsymbol{y}_{\rm d}(t+d)\end{bmatrix},$$

$$\boldsymbol{Y}_1(t)\triangleq\boldsymbol{y}(t),$$

而且

$$\boldsymbol{Y}_i(t)=\boldsymbol{C}_{1i}\boldsymbol{x}_1(t)+\boldsymbol{C}_{2i}\boldsymbol{x}_2(t)+\boldsymbol{D}_{1i}(w,t)\boldsymbol{x}_1(t)+\boldsymbol{D}_{2i}(w,t)\boldsymbol{x}_2(t)+\boldsymbol{V}_i(t),\ i=1,2.$$

通过射影定理, 可以得到

$$\tilde{\boldsymbol{Y}}_i(t)=\boldsymbol{C}_{1i}\boldsymbol{e}_1(t,i)+\boldsymbol{D}_{1i}(w,t)\boldsymbol{x}_1(t)+\tilde{\tilde{\boldsymbol{V}}}_i(t).\quad(8)$$

其中:

$$\boldsymbol{C}_{11}=C_1,\ \boldsymbol{C}_{12}=\begin{bmatrix}C_1\\ C_{1\rm d}\end{bmatrix},\ \boldsymbol{C}_{21}=C_2,\ \boldsymbol{C}_{22}=\begin{bmatrix}C_2\\ C_{2\rm d}\end{bmatrix},$$

$$\boldsymbol{D}_{11}(w,t)=D_1\boldsymbol{w}(t),\ \boldsymbol{D}_{12}(w,t)=\begin{bmatrix}D_1\boldsymbol{w}(t)\\ D_{1\rm d}\boldsymbol{w}(t+d)\end{bmatrix},$$

$$\boldsymbol{D}_{21}(w,t)=D_2\boldsymbol{w}(t),\ \boldsymbol{D}_{22}(w,t)=\begin{bmatrix}D_2\boldsymbol{w}(t)\\ D_{2\rm d}\boldsymbol{w}(t+d)\end{bmatrix},$$

$$\boldsymbol{V}_2(t)=\begin{bmatrix}\boldsymbol{v}(t)\\ \boldsymbol{v}_{\rm d}(t+d)\end{bmatrix},\ \boldsymbol{V}_1(t)=\boldsymbol{v}(t).$$

3.2 黎卡提方程(Riccati equation)

给出了上面的新息重组分析理论, 本节将给出用来计算误差自协方差矩阵$P_1(t+1,i)=<\boldsymbol{e}_1(t+1,i),\boldsymbol{e}_1(t+1,i)>$的黎卡提方程.

通过采用传统的带有乘性噪声系统的卡尔曼滤波方法[18−19], $P_1(t+1,2)$可以递推计算如下:

$$\begin{cases}P_1(t+1,2)=\\ A_1P_1(t,2)A_1^{\rm T}+B_1[Q_u-G_3(t,t|t,2)]B_1^{\rm T}+\\ A_1\left[G_1^{\rm T}(t,t,2)-K(t,2)G^{\rm T}(t,t,2)\right]B_1^{\rm T}+\\ B_1[G_1(t,t,2)-G(t,t,2)K^{\rm T}(t,2)]A_1^{\rm T}-\\ A_1K(t,2)Q_{\tilde{\boldsymbol{Y}}_2}(t)K^{\rm T}(t,2)A_1^{\rm T},\\ P_1(0,2)=\mathit{\Pi}_1(0),\end{cases}\quad(9)$$

其中:

$$\mathit{\Pi}_1(t+1)=A_1\mathit{\Pi}_1(t)A_1^{\rm T}+B_1Q_uB_1^{\rm T},\quad(10)$$

$K(t,2)$和$Q_{\tilde{\boldsymbol{Y}}_2}(t)$见论文[22]中式(3.43)和(3.45), $G(t,t,2),\ G_1(t,t,2),\ G_3(t,t|t,2),\ H_1(t,t,2), H_2(t,t|t-1,2)$ 见论文[22]引理3.4, 初始值$G(0,0,2)=T(0,0,2),\ G_1(0,0,2)=G_3(0,0|-1,2)=H_1(0,0,2)=H_2(0,0|-1,2)=0,\ R(t,t,2)$ 见论文[22]引理3.3(取$s=t$), $\xi^-(t)$是$\xi(t)$的广义逆.

另外, $P_1(t+1,1)$将在下面的定理中给出:

定理 1 在假设1−2下, 对于给出的受限等价系统模型(4)−(7), $P_1(t+1,1)=<\boldsymbol{e}_1(t+1,1),\boldsymbol{e}_1(t+1,1)>$可以按如下公式进行递推计算:

$$\begin{aligned}&P_1(t+1,1)=\\ &A_1P_1(t,1)A_1^{\rm T}+B_1[Q_u-G_3(t,t|t,1)]B_1^{\rm T}+\\ &A_1\left[G_1^{\rm T}(t,t,1)-K(t,1)G^{\rm T}(t,t,1)\right]B_1^{\rm T}+\\ &B_1\left[G_1(t,t,1)-G(t,t,1)K^{\rm T}(t,1)\right]A_1^{\rm T}-\\ &A_1K(t,1)Q_{\tilde{\boldsymbol{Y}}_1}(t)K^{\rm T}(t,1)A_1^{\rm T},\\ &P_1(t_{\rm d}+1,1)=P_1(t_{\rm d}+1,2),\ t\geqslant d,\end{aligned}\quad(11)$$

其中

$$K(t,1)=L(t,t,1)Q_{\tilde{\boldsymbol{Y}}_1}^-(t).\quad(12)$$

在上式中:

$$\begin{aligned}&L(t,t,1)=P_1(t,1)\boldsymbol{C}_{11}^{\rm T}+H_1^{\rm T}(t,t,1),\\ &Q_{\tilde{\boldsymbol{Y}}_1}(t)=\boldsymbol{C}_{11}P_1(t,1)\boldsymbol{C}_{11}^{\rm T}+D_1M\mathit{\Pi}_1(t)D_1^{\rm T}+\\ &\qquad R(t,t,1)-H_2(t,t|t-1,1)+\\ &\qquad \boldsymbol{C}_{11}H_1^{\rm T}(t,t,1)+H_1(t,t,1)\boldsymbol{C}_{11}^{\rm T},\end{aligned}$$

其中:

$$R(t,t,1)=Q_v+\sum_{j=0}^{\lambda-1}\boldsymbol{C}_{21}F_1^jB_2Q_uB_2^{\rm T}(F_1^j)^{\rm T}\boldsymbol{C}_{21}^{\rm T}+M\sum_{j=0}^{\lambda-1}D_2F_1^jB_2Q_uB_2^{\rm T}(F_1^j)^{\rm T}D_2^{\rm T},$$

$G(t,t,1),\ G_1(t,t,1),\ G_3(t,t|t,1),\ H_1(t,t,1), H_2(t,t|t-1,1)$见论文[22]引理3.5, $\mathit{\Pi}_1(t)$在式(10)中.

证 证明可以根据射影定理, 其中一些细节可以参照下面的定理.

3.3 $\boldsymbol{x_1(t)}$ 的多步预测器 (Multi-step predictor for $\boldsymbol{x}_1(t)$)

在本小节中, 首先给出一个重要的引理:

引理 3 当$s<t_{\rm d}+1$时, 有

$$\begin{aligned}&H(t_{\rm d}+1,s,12)=\\ &H_1(t_{\rm d}+1,s,12)\boldsymbol{C}_{12}^{\rm T}+R(t_{\rm d}+1,s,12)-\\ &H_2(t_{\rm d}+1,s|s-1,12),\end{aligned}\quad(13)$$

$$\begin{aligned}&H_1(t_{\rm d}+1,s,12)=\\ &H_1(t_{\rm d}+1,s-1,12)A_1^{\rm T}-\\ &H(t_{\rm d}+1,s-1,12)K^{\rm T}(s-1,2)A_1^{\rm T}+\\ &T^{\rm T}(s-1,t_{\rm d}+1,1)B_1^{\rm T}-\\ &H_3(t_{\rm d}+1,s-1|s-2,12)B_1^{\rm T}-\\ &H(t_{\rm d}+1,s-1,12)Q_{\tilde{\boldsymbol{Y}}_2}^-(s-1)\times\\ &G^{\rm T}(s-1,s-1,2)B_1^{\rm T},\end{aligned}\quad(14)$$

$$\begin{aligned}&H_2(t_{\rm d}+1,s|m,12)=\\ &H_2(t_{\rm d}+1,s|m-1,12)+\\ &H(t_{\rm d}+1,m,12)Q_{\tilde{\boldsymbol{Y}}_2}^-(m)H^{\rm T}(s,m,2),\end{aligned}\quad(15)$$

$$\begin{aligned}&H_3(t_{\rm d}+1,s|m,12)=\\ &H_3(t_{\rm d}+1,s|m-1,12)+\end{aligned}$$

$$H(t_{\rm d}+1,m,12)Q^{-}_{\tilde{\boldsymbol{Y}}_2}(m)G^{\rm T}(s,m,2). \tag{16}$$

在上面的引理基础上, 可以给出$\boldsymbol{x}_1(t)$的最优化多步预测器.

定理 2 在假设1–2下, 考虑受限等价系统模型(4)–(7), 优化多步预测器$\hat{\boldsymbol{x}}_1(t+l|t)$可以按下式计算:

$$\begin{aligned}&\hat{\boldsymbol{x}}_1(t+l|t)=\hat{\boldsymbol{x}}_1(t+l|t,1)=\\&A_1^{l-1}\hat{\boldsymbol{x}}_1(t+1,1)+\\&\sum_{i=0}^{l-2}\sum_{j=0}^{d-1}A_1^iB_1G(t+l-1-i,t-j,1)\times\\&Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t-j)\times\\&[\boldsymbol{Y}_1(t-j)-\boldsymbol{C}_{11}\hat{\boldsymbol{x}}_1(t-j,1)-\hat{\hat{\boldsymbol{V}}}_1(t-j,1)]+\\&\sum_{i=0}^{l-2}\sum_{j=d}^{t}A_1^iB_1G(t+l-1-i,t-j,2)\times\\&Q^{-}_{\tilde{\boldsymbol{Y}}_2}(t-j)\times[\boldsymbol{Y}_2(t-j)-\boldsymbol{C}_{12}\hat{\boldsymbol{x}}_1(t-j,2)-\\&\hat{\hat{\boldsymbol{V}}}_2(t-j,2)],\end{aligned} \tag{17}$$

其中$\hat{\boldsymbol{x}}_1(t+1,1)=\hat{\boldsymbol{x}}_1(t+1|t,1)$, 计算如下:

$$\begin{aligned}&\hat{\boldsymbol{x}}_1(t+1,1)=\\&A_1\hat{\boldsymbol{x}}_1(t,1)+A_1K(t,1)[\boldsymbol{Y}_1(t)-\\&\boldsymbol{C}_{11}\hat{\boldsymbol{x}}_1(t,1)-\hat{\hat{\boldsymbol{V}}}_1(t,1)]+B_1\hat{\boldsymbol{u}}(t|t,1),\end{aligned} \tag{18}$$

而且$\hat{\boldsymbol{x}}_1(t_{\rm d}+1,1)=\hat{\boldsymbol{x}}_1(t_{\rm d}+1,2)$计算如下:

$$\begin{cases}\hat{\boldsymbol{x}}_1(t_{\rm d}+1,2)=\\A_1\hat{\boldsymbol{x}}_1(t_{\rm d},2)+A_1K(t_{\rm d},2)[\boldsymbol{Y}_2(t_{\rm d})-\\\boldsymbol{C}_{12}\hat{\boldsymbol{x}}_1(t_{\rm d},2)-\hat{\hat{\boldsymbol{V}}}_2(t_{\rm d},2)]+B_1\hat{\boldsymbol{u}}(t_{\rm d}|t_{\rm d},2),\\\hat{\boldsymbol{x}}_1(0,2)=\hat{\boldsymbol{x}}_1(0|-1,2)=0.\end{cases} \tag{19}$$

在上式中:

$$\begin{aligned}&\hat{\boldsymbol{u}}(t|t,1)=\\&\hat{\boldsymbol{u}}(t|t-1,1)+G(t,t,1)Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t)[\boldsymbol{Y}_1(t)-\\&\boldsymbol{C}_{11}\hat{\boldsymbol{x}}_1(t,1)-\hat{\hat{\boldsymbol{V}}}_1(t,1)],\end{aligned} \tag{20}$$

$$\begin{aligned}&\hat{\boldsymbol{u}}(t_{\rm d}|t_{\rm d},2)=\\&\hat{\boldsymbol{u}}(t_{\rm d}|t_{\rm d}-1,2)+G(t_{\rm d},t_{\rm d},2)Q^{-}_{\tilde{\boldsymbol{Y}}_2}(t_{\rm d})[\boldsymbol{Y}_2(t_{\rm d})-\\&\boldsymbol{C}_{12}\hat{\boldsymbol{x}}_1(t_{\rm d},2)-\hat{\hat{\boldsymbol{V}}}_2(t_{\rm d},2)],\end{aligned} \tag{21}$$

其中:

$$\begin{cases}\hat{\hat{\boldsymbol{V}}}_1(t-j,1)=\\\hat{\hat{\boldsymbol{V}}}_1(t-j|t-j-2,1)+H(t-j,t-j-1,1)\times\\Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t-j-1)[\boldsymbol{Y}_1(t-j-1)-\\\boldsymbol{C}_{11}\hat{\boldsymbol{x}}_1(t-j-1,1)-\hat{\hat{\boldsymbol{V}}}_1(t-j-1,1)],\\\hat{\hat{\boldsymbol{V}}}_1(t_{\rm d}+1,1)=\hat{\hat{\boldsymbol{V}}}_1(t_{\rm d}+1,2),\ j=0,\cdots,d-1,\end{cases} \tag{22}$$

$$\begin{cases}\hat{\hat{\boldsymbol{V}}}_2(t-j,2)=\\\hat{\hat{\boldsymbol{V}}}_2(t-j|t-j-2,2)+H(t-j,t-j-1,2)\times\\Q^{-}_{\tilde{\boldsymbol{Y}}_2}(t-j-1)[\boldsymbol{Y}_2(t-j-1)-\\\boldsymbol{C}_{12}\hat{\boldsymbol{x}}_1(t-j-1,2)-\hat{\hat{\boldsymbol{V}}}_2(t-j-1,2)],\\\hat{\hat{\boldsymbol{V}}}_2(0,2)=0,\ j=d,\cdots,t.\end{cases} \tag{23}$$

其中$\hat{\hat{\boldsymbol{V}}}_1(t_{\rm d}+1,2)$由下式得到:

$$\begin{aligned}&\hat{\hat{\boldsymbol{V}}}_1(t_{\rm d}+1,2)=\\&\hat{\hat{\boldsymbol{V}}}_1(t_{\rm d}+1|t_{\rm d}-1,2)+H(t_{\rm d}+1,t_{\rm d},12)Q^{-}_{\tilde{\boldsymbol{Y}}_2}(t_{\rm d})\times\\&[\boldsymbol{Y}_2(t_{\rm d})-\boldsymbol{C}_{12}\hat{\boldsymbol{x}}_1(t_{\rm d},2)-\hat{\hat{\boldsymbol{V}}}_2(t_{\rm d},2)],\end{aligned} \tag{24}$$

$K(t,1)$ 见式 (12), $K(t_{\rm d},2)$ 见论文[22] 式 (3.43), $G(t,s,i)(i=1,2)$见论文[22]引理3.4和引理3.5, $H(t_{\rm d}+1,t_{\rm d},12)$见引理3式(13).

证 根据式(4)可得

$$\begin{aligned}&\boldsymbol{x}_1(t+l)=\\&A_1^l\boldsymbol{x}_1(t)+\sum_{i=0}^{l-1}A_1^iB_1\boldsymbol{u}(t+l-1-i)=\\&A_1^{l-1}\boldsymbol{x}_1(t+1)+\sum_{i=0}^{l-2}A_1^iB_1\boldsymbol{u}(t+l-1-i),\end{aligned}$$

所以

$$\begin{aligned}&\hat{\boldsymbol{x}}_1(t+l|t)=\\&A_1^{l-1}\hat{\boldsymbol{x}}_1(t+1|t)+\sum_{i=0}^{l-2}A_1^iB_1\hat{\boldsymbol{u}}(t+l-1-i|t).\end{aligned} \tag{25}$$

根据定理1, 可得

$$\begin{aligned}&\hat{\boldsymbol{x}}_1(t+1,1)=\\&A_1\hat{\boldsymbol{x}}_1(t,1)+A_1K(t,1)\tilde{\boldsymbol{Y}}_1(t)+B_1\hat{\boldsymbol{u}}(t|t,1)=\\&A_1\hat{\boldsymbol{x}}_1(t,1)+B_1\hat{\boldsymbol{u}}(t|t,1)+A_1K(t,1)\times\\&[\boldsymbol{Y}_1(t)-\boldsymbol{C}_{11}\hat{\boldsymbol{x}}_1(t,1)-\hat{\hat{\boldsymbol{V}}}_1(t,1)],\end{aligned}$$

即式(18).

类似地, 根据式(1),

$$\begin{aligned}&\hat{\boldsymbol{u}}(t|t,1)=\\&\hat{\boldsymbol{u}}(t|t-1,1)+G(t,t,1)Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t)\tilde{\boldsymbol{Y}}_1(t)=\\&\hat{\boldsymbol{u}}(t|t-1,1)+G(t,t,1)Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t)\times\\&[\boldsymbol{Y}_1(t)-\boldsymbol{C}_{11}\hat{\boldsymbol{x}}_1(t,1)-\hat{\hat{\boldsymbol{V}}}_1(t,1)],\end{aligned}$$

即式(20).

另外, 根据射影定理可得

$$\begin{aligned}&\hat{\boldsymbol{u}}(t+l-1-i|t,1)=\\&{\rm P}\{\boldsymbol{u}(t+l-1-i)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_{\rm d});\\&\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1),\cdots,\tilde{\boldsymbol{Y}}_1(t)\}=\end{aligned}$$

$$\begin{aligned}
&\mathrm{P}\{\boldsymbol{u}(t+l-1-i)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_{\rm d});\\
&\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1),\cdots,\tilde{\boldsymbol{Y}}_1(t-1)\}+\\
&\mathrm{P}\{\boldsymbol{u}(t+l-1-i)|\tilde{\boldsymbol{Y}}_1(t)\}=\\
&\hat{\boldsymbol{u}}(t+l-1-i|t-1,1)+\\
&<\boldsymbol{u}(t+l-1-i),\tilde{\boldsymbol{Y}}_1(t)>Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t)\tilde{\boldsymbol{Y}}_1(t)=\\
&\hat{\boldsymbol{u}}(t+l-1-i|t-1,1)+\\
&G(t+l-1-i,t,1)Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t)\tilde{\boldsymbol{Y}}_1(t).
\end{aligned}$$

类似地,

$$\begin{aligned}
&\hat{\boldsymbol{u}}(t+l-1-i|t-1,1)=\\
&\mathrm{P}\{\boldsymbol{u}(t+l-1-i)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_{\rm d});\\
&\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1),\cdots,\tilde{\boldsymbol{Y}}_1(t-1)\}=\\
&\mathrm{P}\{\boldsymbol{u}(t+l-1-i)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_{\rm d});\\
&\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1),\cdots,\tilde{\boldsymbol{Y}}_1(t-2)\}+\\
&\mathrm{P}\{\boldsymbol{u}(t+l-1-i)|\tilde{\boldsymbol{Y}}_1(t-1)\}=\\
&\hat{\boldsymbol{u}}(t+l-1-i|t-2,1)+\\
&<\boldsymbol{u}(t+l-1-i),\tilde{\boldsymbol{Y}}_1(t-1)>\times\\
&Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t-1)\tilde{\boldsymbol{Y}}_1(t-1)=\\
&\hat{\boldsymbol{u}}(t+l-1-i|t-2,1)+\\
&G(t+l-1-i,t-1,1)\times\\
&Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t-1)\tilde{\boldsymbol{Y}}_1(t-1).
\end{aligned}\tag{26}$$

按式(26)计算可得

$$\begin{aligned}
&\hat{\boldsymbol{u}}(t+l-1-i|t_{\rm d}+1,1)=\\
&\mathrm{P}\{\boldsymbol{u}(t+l-1-i)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_{\rm d});\\
&\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1)\}=\\
&\mathrm{P}\{\boldsymbol{u}(t+l-1-i)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_{\rm d})\}+\\
&\mathrm{P}\{\boldsymbol{u}(t+l-1-i)|\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1)\}=\\
&\hat{\boldsymbol{u}}(t+l-1-i|t_{\rm d},1)+<\boldsymbol{u}(t+l-1-i),\\
&\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1)>Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t_{\rm d}+1)\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1)=\\
&\hat{\boldsymbol{u}}(t+l-1-i|t_{\rm d},2)+\\
&G(t+l-1-i,t_{\rm d}+1,1)\times\\
&Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t_{\rm d}+1)\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1),
\end{aligned}$$

所以

$$\begin{aligned}
&\hat{\boldsymbol{u}}(t+l-1-i|t,1)=\\
&\sum_{j=d}^{t}G(t+l-1-i,t-j,2)Q^{-}_{\tilde{\boldsymbol{Y}}_2}(t-j)\tilde{\boldsymbol{Y}}_2(t-j)+\\
&\sum_{j=0}^{d-1}G(t+l-1-i,t-j,1)Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t-j)\times\\
&\tilde{\boldsymbol{Y}}_1(t-j),
\end{aligned}$$

再考虑到式(25), 可以得到式(17). 式(19)和式(21)可类似地给出.

根据射影定理可得

$$\begin{aligned}
&\hat{\bar{\boldsymbol{V}}}_1(t-j,1)=\\
&\mathrm{P}\{\bar{\boldsymbol{V}}_1(t-j)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_{\rm d});\\
&\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1),\cdots,\tilde{\boldsymbol{Y}}_1(t-j-1)\}=\\
&\mathrm{P}\{\bar{\boldsymbol{V}}_1(t-j)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_{\rm d});\\
&\tilde{\boldsymbol{Y}}_1(t_{\rm d}+1),\cdots,\tilde{\boldsymbol{Y}}_1(t-j-2)\}+\\
&\mathrm{P}\{\bar{\boldsymbol{V}}_1(t-j)|\tilde{\boldsymbol{Y}}_1(t-j-1)\}=\\
&\hat{\bar{\boldsymbol{V}}}_1(t-j|t-j-2,1)+\\
&H(t-j,t-j-1,1)\times\\
&Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t-j-1)\tilde{\boldsymbol{Y}}_1(t-j-1),
\end{aligned}$$

则可以得到式(22). 式(23)和式(24)可类似地给出.

4 多步预测器$\hat{\boldsymbol{x}}(t+l|t)$(Multi-step predictor $\hat{\boldsymbol{x}}(t+l|t)$)

给定了上述的多步预测器$\hat{\boldsymbol{x}}_1(t+l|t,1)(l>1)$, 本部分将给出优化多步预测器$\hat{\boldsymbol{x}}(t+l|t)$.

定理 3 广义系统模型(1)−(3) 在满足假设1−2的情况下, 若$[0\ \ I]J_2^{-1}\boldsymbol{x}(0)=-\sum\limits_{j=0}^{\lambda-1}F_1^jB_2\boldsymbol{u}(j)$, 则多步预测器估计$\hat{\boldsymbol{x}}(t+l|t)=\hat{\boldsymbol{x}}(t+l|t,1)$可以计算如下:

$$\hat{\boldsymbol{x}}(t+l|t)=J_2\begin{bmatrix}\hat{\boldsymbol{x}}_1(t+l|t,1)\\ \hat{\boldsymbol{x}}_2(t+l|t,1)\end{bmatrix},\ l>1.\tag{27}$$

在上式中: $\hat{\boldsymbol{x}}_1(t+l|t,1)$可从式(17)得到, $\hat{\boldsymbol{x}}_2(t+l|t,1)$按下式给出:

$$\hat{\boldsymbol{x}}_2(t+l|t,1)=-\sum_{j=0}^{\lambda-1}F_1^jB_2\hat{\boldsymbol{u}}(t+l+j|t,1),\tag{28}$$

其中

$$\begin{aligned}
&\hat{\boldsymbol{u}}(t+l+j|t,1)=\\
&\sum_{k=d}^{t}G(t+l+j,t-k,2)Q^{-}_{\tilde{\boldsymbol{Y}}_2}(t-k)\tilde{\boldsymbol{Y}}_2(t-k)+\\
&\sum_{k=0}^{d-1}G(t+l+j,t-k,1)Q^{-}_{\tilde{\boldsymbol{Y}}_1}(t-k)\times\tilde{\boldsymbol{Y}}_1(t-k),
\end{aligned}\tag{29}$$

上式中:

$$\tilde{\boldsymbol{Y}}_i(t)=\boldsymbol{Y}_i(t)-\boldsymbol{C}_{1i}\hat{\boldsymbol{x}}_1(t,i)+\hat{\bar{\boldsymbol{V}}}_i(t,i),$$

$G(t+l+j,t,i)$见论文[22]引理3.4和引理3.5, $\hat{\boldsymbol{x}}_1(t,i)$见式(18)和式(19), $\hat{\bar{\boldsymbol{V}}}_i(t,i)$见式(22)和(23).

证 式(27)根据引理1可得到.

根据式(4)−(5), 可得

$$\boldsymbol{x}_1(t)=A_1^t\boldsymbol{x}_1(0)+\sum_{i=0}^{t-1}A_1^{t-i-1}B_1\boldsymbol{u}(i),$$

其中

$$\boldsymbol{x}_2(t)=-\sum_{i=0}^{\lambda-1}F_1^iB_2\boldsymbol{u}(t+i).\tag{30}$$

根据射影定理和式(30), 可得

$$\begin{aligned}&\hat{\boldsymbol{x}}_2(t+l|t,1)=\\&\mathrm{P}\{\boldsymbol{x}_2(t+l)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_\mathrm{d});\tilde{\boldsymbol{Y}}_1(t_\mathrm{d}+1),\\&\cdots,\tilde{\boldsymbol{Y}}_1(t)\}=\\&\mathrm{P}\{-\sum_{j=0}^{\lambda-1}F_1^jB_2\boldsymbol{u}(t+l+j)|\tilde{\boldsymbol{Y}}_2(0),\cdots,\tilde{\boldsymbol{Y}}_2(t_\mathrm{d});\\&\tilde{\boldsymbol{Y}}_1(t_\mathrm{d}+1),\cdots,\tilde{\boldsymbol{Y}}_1(t)\}=\\&-\sum_{j=0}^{\lambda-1}F_1^jB_2\hat{\boldsymbol{u}}(t+l+j|t,1),\end{aligned}$$

即式(28).

式(29)可根据定理1得到.

5 数学算例(Numerical example)

为了验证所提方法的正确性和有效性, 本节将给出一个数学算例进行验证. 对于不同的步长个数预测器效果是不一样的, 为了更好地表现这一点, 本算例给出了两种情况, 分别是步长数 $l_1=2$和步长数 $l_2=3$, 其中步长是固定的. 系统模型(1)–(3)中的矩阵取为

$$F=\begin{bmatrix}0&1&0\\1&0&0\\0&0&0\end{bmatrix},\ A=\begin{bmatrix}0.8&-0.4&0\\0.4&0&0\\0&0&0.5\end{bmatrix},\ B=\begin{bmatrix}1\\1\\2\end{bmatrix},$$

$$C=[1\ \ 1\ \ 1],\ C_\mathrm{d}=[1\ \ 2\ \ 2],$$

$$D=[1\ \ 2\ \ 1],\ D_\mathrm{d}=[2\ \ 1\ \ 1].$$

另外:

$$n=3,\ n_1=2,\ d=20,\ Q_u=1,$$

$$M=Q_v=Q_{v_\mathrm{d}}=0.01,\ \boldsymbol{x}_1(0)=\begin{bmatrix}1\\0.5\end{bmatrix},$$

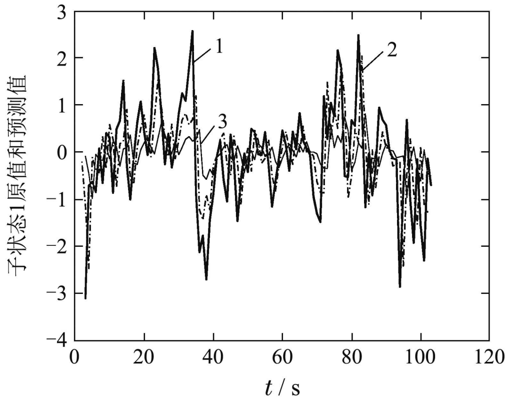
$\boldsymbol{w}(t),\ \boldsymbol{v}(t),\ \boldsymbol{v}_\mathrm{d}(t),\ \boldsymbol{u}(t)$满足假设1,

$$\mathit{\Pi}_1(0)=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ P_1(0,2)=\begin{bmatrix}1&0\\0&1\end{bmatrix}.$$

根据定理3画出图1–4. 对于步长数$l_1=2$和步长数$l_2=3$, 本文给出了3个子状态的原始状态, 它的2步预测器和3步预测器, 其中第1个子状态 $\hat{\boldsymbol{x}}_\mathrm{F}(t+l|t)=\hat{\boldsymbol{x}}_{11}(t+l|t,1)$ 见图1, 第2个子状态 $\hat{\boldsymbol{x}}_\mathrm{S}(t+l|t)=\hat{\boldsymbol{x}}_{11}(t+l|t,1)+\hat{\boldsymbol{x}}_{12}(t+l|t,1)$ 见图2, 第3个子状态 $\hat{\boldsymbol{x}}_\mathrm{T}(t+l|t)=\hat{\boldsymbol{x}}_2(t+l|t,1)$见图3.

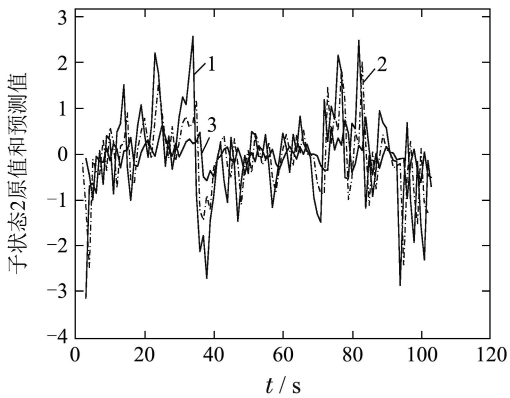
从图1–2可容易看出, 2步和3步最优预测器$\hat{\boldsymbol{x}}_\mathrm{F}(t+l|t)$, $\hat{\boldsymbol{x}}_\mathrm{S}(t+l|t)$可以很好的跟踪原始的子状态. 对于$\hat{\boldsymbol{x}}_\mathrm{T}(t+l|t)$, 它是$\boldsymbol{x}_2(t+l)$的估计$\hat{\boldsymbol{x}}_2(t+l|t)$, 根据定理3中式(28)–(29), 有 $\hat{\boldsymbol{x}}_2(t+l|t)\equiv 0$, 所以在图3中, 2步和3步预测器为零.

为了更好地给出2步和3步预测器的区别, 本文在图4中给出了它们的误差自协方差的波形. 从图1和图2, 可以看出2步的预测要比3步的预测效果更好一些, 这在图4中得到了证实, 因为一般来说步长数越少的预测器所用的信息相对来说越多, 所以估计得越准确.



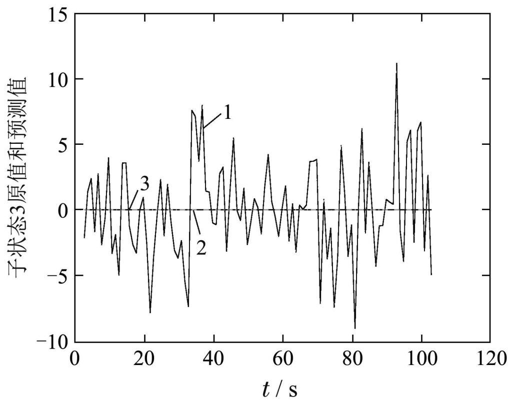
1: 原始状态　2: 步长数为l_1=2的预测器
3: 步长数为l_2=3的预测器

图 1 $\boldsymbol{x}(t+l_i)$的第1个子状态的原始值和预测值$\hat{\boldsymbol{x}}_\mathrm{F}(t+l_i|t)$
Fig. 1 The origin and its predictor $\hat{\boldsymbol{x}}_\mathrm{F}(t+l_i|t)$ for the first sub-state of $\boldsymbol{x}(t+l_i)$



1: 原始状态　2: 步长数为l_1=2的预测器
3: 步长数为l_2=3的预测器

图 2 $\boldsymbol{x}(t+l_i)$的第2个子状态的原始值和预测值$\hat{\boldsymbol{x}}_\mathrm{S}(t+l_i|t)$
Fig. 2 The origin and its predictor $\hat{\boldsymbol{x}}_\mathrm{S}(t+l_i|t)$ for the second sub-state of $\boldsymbol{x}(t+l_i)$



1：原始状态　2：步长数为l_1=2的预测器
3：步长数为l_2=3的预测器

图 3 $\boldsymbol{x}(t+l_i)$的第3个子状态的原始值和预测值$\hat{\boldsymbol{x}}_\mathrm{T}(t+l_i|t)$
Fig. 3 The origin and its predictor $\hat{\boldsymbol{x}}_\mathrm{T}(t+l_i|t)$ for the second sub-state of $\boldsymbol{x}(t+l_i)$

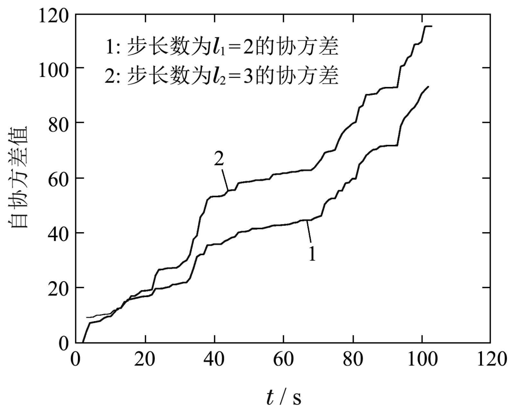


图 4 不同步长数时预测误差自协方差的比较

Fig. 4 Comparison of the covariance of the predictor error for different

6 结论(Conclusions)

本文研究了带有乘性噪声和延时观测的广义系统的优化多步预测器的设计问题. 根据标准奇异值分解方法, 本文给出了受限等价延时系统, 利用新息重组分析理论和射影定理给出了多步预测器. 该预测器是基于黎卡提方程给出的, 根据数学算例可以看出, 本文所用的方法能够很好的估计预测步数少的状态, 当步长数大时效果稍差. 除此以外, 本文所提出的方法结合别的方法还可以研究更复杂的一些问题, 如, 延时广义系统的H_∞控制和滤波问题文献[23–25].

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