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势平衡多目标多伯努利滤波器高斯混合实现的收敛性分析

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摘要: 研究了势平衡多目标多伯努利 (cardinality balanced multi-target multi-Bernoulli, CBMeMBer) 滤波器高斯混合(Gaussian mixture, GM)实现的收敛性问题. 证明在线性高斯条件下, 若 GM–CBMeMBer 滤波器的高斯项足够多, 则它一致收敛于真实的CBMeMBer滤波器. 并且证明在弱非线性条件下, GM–CBMeMBer滤波器的扩展卡尔曼(extended Kalman, EK)滤波近似实现 —— EK–GM–CBMeMBer滤波器, 若每个高斯项的协方差足够小, 也一致收敛于真实的 CBMeMBer 滤波器. 本文的研究目的是从理论上给出 CBMeMBer 滤波器GM实现的收敛结果, 以完善 CBMeMBer 滤波器对多目标跟踪的理论研究.

关键词: 多目标跟踪; 随机有限集; 多伯努利; 高斯混合; 收敛性分析

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Convergence analysis for the Gaussian mixture implementation of the CBMeMBer filter

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Abstract: The convergence for the Gaussian mixture (GM) implementation of the cardinality balanced multi-target multi-Bernoulli (CBMeMBer) filter is studied. This paper proves that the GM–CBMeMBer filter converges uniformly to the true CBMeMBer filter in the linear Gaussian model as the number of Gaussians in the mixture tends to infinity. In addition, this paper proves the extended Kalman (EK) filter approximations of the GM–CBMeMBer filter in weak nonlinear condition — EK–GM–CBMeMBer filter, converges uniformly to the true CBMeMBer filter as the covariance of each Gaussian term tends to zero. The purpose of this paper is to theoretically present the convergence results of the CBMeMBer filter's GM implementation, perfecting the theoretical research of the CBMeMBer filter for the multi-target tracking problem.

Key words: multi-target tracking; random finite set; multi-Bernoulli; Gaussian mixture; convergence analysis

1 引言(Introduction)

基于随机有限集(random finite set, RFS)的多目标跟踪算法[1–2], 从集值估计的角度来解决多目标跟踪问题, 避免了传统跟踪算法中的数据关联过程. 该算法采用RFS建模多目标的状态和观测, 在Bayes滤波框架下通过递推多目标状态的后验分布来解决多目标跟踪问题. 由于多目标密度固有的组合特性和在多目标状态与测量空间上的多重积分运算, 该算法在实际应用中难以进行数值计算. Mahler提出的概率假设密度(probability hypothesis density, PHD)[3]、势概率假设密度(cardinality PHD, CPHD)[4]和多目标多伯努利(multi-target multi-Bernoulli, MeMBer)[1]滤波器, 极大地简化了基于RFS的多目标跟踪算法的求解. 不同于PHD和CPHD滤波器递推强度和势分布估计, MeMBer滤波器直接近似递推多目标状态的后验概率密度, 使得多目标跟踪问题的求解显得更为直观. 由于MeMBer滤波器对势的估计存在偏差, Vo等提出了无偏的势平衡多目标多伯努利(cardinality balanced MeMBer, CBMeMBer)滤波器[5]. 近年来, 在某些实际应用中, 利用CBMeMBer滤波器已取得了一些研究成果[6–10].

本文是CBMeMBer滤波器收敛性问题理论研究的

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一部分, 文献[11]给出了CBMeMBer滤波器(sequential Monte Carlo, SMC)实现的收敛结果, 本文对它的(Gaussian mixture, GM)实现的收敛性进行分析, 从理论上给出CBMeMBer滤波器GM实现的收敛结果, 以完善CBMeMBer滤波器对多目标跟踪的理论研究. 首先, 证明在线性高斯条件下, 若GM–CBMeMBer滤波器的高斯项足够多, 则它一致收敛于真实的CBMeMBer滤波器. 随后, 证明了在弱非线性条件下, GM–CBMeMBer滤波器的扩展卡尔曼(extended Kalman, EK)滤波的近似实现——EK–GM–CBMeMBer滤波器, 若每个高斯项的协方差趋于0, 也一直收敛于真实的CBMeMBer滤波器. 本文的研究目的和意义在于从理论上给出了CBMeMBer滤波器GM实现的收敛结果, 以完善CBMeMBer滤波器对多目标跟踪的理论研究.

2 GM–CBMeMBer滤波器的收敛性分析(Covergence analysis of the GM–CBMeMBer filter)

文献[5]给出了CBMeMBer滤波器的递推过程和实现. 本节主要对GM–CBMeMBer滤波器的收敛性进行分析, 给出它的一致性收敛结果和证明过程.

有限GM密度能以任意精度逼近任意的概率密度[12], 由此可以得到GM–CBMeMBer滤波器初始化的一致性收敛结果.

1) 初始化. 假设$k=0$时刻后验多伯努利密度已知, 则对于任意函数$\varphi \in C_{\rm b}\left(R^{\rm d}\right)$, $C_{\rm b}\left(R^{\rm d}\right)$为Borel可测空间上的$d$维有界函数集, 存在实数$c_0>0$, $d_0>0$满足

$$|r_0^{(i)} - r_0^{(i),J_0^{(i)}}| \leqslant c_0, \tag{1}$$

$$|\langle p_0^{(i)},\varphi\rangle - \langle p_0^{(i),J_0^{(i)}},\varphi\rangle| \leqslant d_0\|\varphi\|_\infty, \tag{2}$$

其中$r^{(i),J^{(i)}}$和$p^{(i),J^{(i)}}$分别表示高斯混合实现过程中第i个目标的存在概率和概率密度.

2) 预测步. 假设$k-1$时刻后验多伯努利密度的GM实现一致收敛于真实的后验密度, 即对任意$\varphi \in C_{\rm b}(R^{\rm d})$, 存在实数$c_{k-1}>0$, $d_{k-1}>0$满足

$$|r_{k-1}^{(i)} - r_{k-1}^{(i),J_{k-1}^{(i)}}| \leqslant c_{k-1}, \tag{3}$$

$$|\langle p_{k-1}^{(i)},\varphi\rangle - \langle p_{k-1}^{(i),J_{k-1}^{(i)}},\varphi\rangle| \leqslant d_{k-1}\|\varphi\|_\infty, \tag{4}$$

则预测步后, 存在实数$c_{{\rm P},k|k-1}>0$, $d_{{\rm P},k|k-1}>0$, $c_{\varGamma,k}>0$和$d_{\varGamma,k}>0$满足

$$|r_{{\rm P},k|k-1}^{(i)} - r_{{\rm P},k|k-1}^{(i),J_{k-1}^{(i)}}| \leqslant c_{{\rm P},k|k-1}, \tag{5}$$

$$|\langle p_{{\rm P},k|k-1}^{(i)},\varphi\rangle - \langle p_{{\rm P},k|k-1}^{(i),J_{k-1}^{(i)}},\varphi\rangle| \leqslant d_{{\rm P},k|k-1}\|\varphi\|_\infty, \tag{6}$$

$$|r_{\varGamma,k}^{(i)} - r_{\varGamma,k}^{(i),J_{\varGamma,k}^{(i)}}| \leqslant c_{\varGamma,k}, \tag{7}$$

$$|\langle p_{\varGamma,k}^{(i)},\varphi\rangle - \langle p_{\varGamma,k}^{(i),J_{\varGamma,k}^{(i)}},\varphi\rangle| \leqslant d_{\varGamma,k}\|\varphi\|_\infty, \tag{8}$$

其中:

$$c_{{\rm P},k|k-1} = p_{{\rm S},k}(c_{k-1}+d_{k-1}), \tag{9}$$

$$d_{{\rm P},k|k-1} = 2d_{k-1}. \tag{10}$$

证明过程参见附录B.

3) 更新步. 假设k时刻预测多伯努利密度的GM实现一致收敛于真实的预测密度, 即对于任意的$\varphi \in C_{\rm b}(R^{\rm d})$, 存在实数$c_{k|k-1}>0$, $d_{k|k-1}>0$满足:

$$|r_{k|k-1}^{(i)} - r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}| \leqslant c_{k|k-1}, \tag{11}$$

$$|\langle p_{k|k-1}^{(i)},\varphi\rangle - \langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},\varphi\rangle| \leqslant d_{k|k-1}\|\varphi\|_\infty, \tag{12}$$

则测量更新后, 存在实数$c_{{\rm L},k}>0$, $d_{{\rm L},k}>0$, $c_{{\rm U},k}>0$和$d_{{\rm U},k}>0$满足:

$$|r_{{\rm L},k}^{(i)} - r_{{\rm L},k}^{(i),J_{k|k-1}^{(i)}}| \leqslant c_{{\rm L},k}, \tag{13}$$

$$|\langle p_{{\rm L},k}^{(i)},\varphi\rangle - \langle p_{{\rm L},k}^{(i),J_{k|k-1}^{(i)}},\varphi\rangle| \leqslant d_{{\rm L},k}\|\varphi\|_\infty, \tag{14}$$

$$|r_{{\rm U},k}(z) - r_{{\rm U},k}^{J_{k|k-1}^{(i)}}(z)| \leqslant c_{{\rm U},k}, \tag{15}$$

$$|\langle p_{{\rm U},k}(\cdot;z),\varphi\rangle - \langle p_{{\rm U},k}^{J_{k|k-1}^{(i)}}(\cdot;z),\varphi\rangle| \leqslant d_{{\rm U},k}\|\varphi\|_\infty, \tag{16}$$

其中:

$$c_{{\rm L},k} = \frac{(1+2p_{{\rm D},k})\,c_{k|k-1} + 2p_{{\rm D},k}d_{k|k-1}}{1-p_{{\rm D},k}}, \tag{17}$$

$$d_{{\rm L},k} = \frac{d_{k|k-1}}{1-p_{{\rm D},k}}, \tag{18}$$

$$c_{{\rm U},k} = \frac{M_{k|k-1}\|\psi_{k,z}\|_\infty}{(1-p_{{\rm D},k})^3\inf(\psi_{k,z})\,n_{k|k-1}}\big[(6-11p_{{\rm D},k}+5p_{{\rm D},k}^2)\,c_{k|k-1} + 0.5\,(1-p_{{\rm D},k}^2)\,d_{k|k-1}\big], \tag{19}$$

$$d_{{\rm U},k} = \frac{2M_{k|k-1}\|\psi_{k,z}\|_\infty\,(c_{k|k-1}+d_{k|k-1})}{(1-p_{{\rm D},k})\inf(\psi_{k,z})\,n_{k|k-1}}, \tag{20}$$

其中: $\inf(\cdot)$表示下确界, $n_{k|k-1} = \sum\limits_{i=1}^{M_{k|k-1}} r_{k|k-1}^{(i)}$表示预测步目标的期望个数. 证明过程参见附录C.

若GM–CBMeMBer滤波器的高斯项趋于无穷多, 则它一致收敛于真实的CBMeMBer滤波器. 该结论与GM–PHD滤波器收敛结果一致. 区别之处在于GM–PHD滤波器是将多目标强度函数整体用GM来表示, 而GM–CBMeMBer滤波器则是将每一个目标的密度函数分别用GM来表示, 包含了多个GM形式.

3 EK–GM–CBMeMBer滤波器的收敛性分析(Covergence analysis of the EK–GM– CBMeMBer filter)

本节主要对EK–GM–CBMeMBer滤波器的收敛性进行分析, 给出它的收敛结果和证明过程. 由于目标状态转移或观测模型为非线性模型, 理论上后验多

伯努利密度不能再描述为GM形式, 进而不能进行递推运算. 但在弱非线性条件下, 每个高斯项的均值和方差可以采用EK滤波近似计算.

1) 预测步. 假设$k-1$时刻后验多伯努利密度$\pi_{k-1} = \{(r_{k-1}^{(i)}, p_{k-1}^{(i)})\}_{i=1}^{M_{k-1}}$已知, $p_{k-1}^{(i)}$为GM形式:

$$p_{k-1}^{(i)}(x) = \sum_{j=1}^{J_{k-1}^{(i)}} w_{k-1}^{(i,j)} N(x; m_{k-1}^{(i,j)}, P_{k-1}^{(i,j)}). \tag{21}$$

若$P_{k-1}^{(i,j)}$趋于$0(j=1,\cdots,J_{k-1}^{(i)})$, 则预测多目标多伯努利密度一致收敛于如下GM形式:

$$\begin{aligned}\pi_{k|k-1}^{\rm EK} \to \{(r_{{\rm P},k|k-1}^{(i)}, p_{{\rm P},k|k-1}^{(i)})\}_{i=1}^{M_{k-1}} \cup \\ \{(r_{\varGamma,k}^{(i)}, p_{\varGamma,k}^{(i)})\}_{i=1}^{M_{\varGamma,k}},\end{aligned} \tag{22}$$

其中存活目标多伯努利密度的GM形式为

$$r_{{\rm P},k|k-1}^{(i),{\rm EK}} \to r_{k-1}^{(i)} p_{{\rm S},k}, \tag{23}$$

$$p_{{\rm P},k|k-1}^{(i),{\rm EK}} \to \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{k-1}^{(i,j)} N(x; m_{{\rm P},k|k-1}^{(i,j)}, P_{{\rm P},k|k-1}^{(i,j)}). \tag{24}$$

具体参数由EK滤波获得

$$m_{{\rm P},k|k-1}^{(i,j)} = \phi_k(m_{k-1}^{(i,j)}, 0), \tag{25}$$

$$P_{{\rm P},k|k-1}^{(i,j)} = F_{k-1}^{(i,j)} P_{k-1}^{(i,j)} (F_{k-1}^{(i,j)})^{\rm T} + G_{k-1}^{(i,j)} Q_{k-1} (G_{k-1}^{(i,j)})^{\rm T}, \tag{26}$$

$$F_{k-1}^{(i,j)} = \frac{\partial \phi_k(x_{k-1}, 0)}{\partial x_{k-1}}\Big|_{x_{k-1}=m_{k-1}^{(i,j)}}, \tag{27}$$

$$G_{k-1}^{(i,j)} = \frac{\partial \phi_k(x_{k-1}, w_{k-1})}{\partial w_{k-1}}\Big|_{w_{k-1}=0}, \tag{28}$$

而新生目标多伯努利密度为已知的GM形式, 见式(15). 证明过程参见附录D.

2) 更新步. 假设k时刻预测多伯努利密度$\pi_{k|k-1}^{(i)} = \{(r_{k|k-1}^{(i)}, p_{k|k-1}^{(i)})\}_{i=1}^{M_{k|k-1}}$已知, $p_{k|k-1}^{(i)}$为GM形式

$$p_{k|k-1}^{(i)}(x) = \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} N(x; m_{k|k-1}^{(i,j)}, P_{k|k-1}^{(i,j)}). \tag{29}$$

若$P_{k|k-1}^{(i,j)}$趋于$0(j=1,\cdots,J_{k-1}^{(i)})$, 则EK–GM–CBMeMBer滤波器的后验密度一致收敛于如下GM形式:

$$\begin{aligned}\pi_k^{\rm EK} \to \{(r_{{\rm L},k}^{(i)}, p_{{\rm L},k}^{(i)})\}_{i=1}^{M_{k|k-1}} \cup \\ \{(r_{{\rm U},k}(z), p_{{\rm U},k}(\cdot; z))\}_{z\in Z_k},\end{aligned} \tag{30}$$

其中:

$$r_{{\rm L},k}^{(i),{\rm EK}} \to r_{k|k-1}^{(i)} \frac{1-p_{{\rm D},k}}{1-r_{k|k-1}^{(i)} p_{{\rm D},k}}, \tag{31}$$

$$p_{{\rm L},k}^{(i),{\rm EK}}(x) \to \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} N(x; m_{k|k-1}^{(i,j)}, P_{k|k-1}^{(i,j)}), \tag{32}$$

$$r_{{\rm U},k}^{\rm EK} \to \frac{\displaystyle\sum_{i=1}^{M_{k|k-1}} \frac{p_{{\rm D},k} r_{k|k-1}^{(i)} (1-r_{k|k-1}^{(i)}) \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} q_k^{(i,j)}(z)}{(1-r_{k|k-1}^{(i)} p_{{\rm D},k})^2}}{\displaystyle\kappa_k(z) + \sum_{i=1}^{M_{k|k-1}} \frac{p_{{\rm D},k} r_{k|k-1}^{(i)} \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} q_k^{(i,j)}(z)}{1-r_{k|k-1}^{(i)} p_{{\rm D},k}}}, \tag{33}$$

$$p_{{\rm U},k}^{\rm EK}(x; z) \to \frac{\displaystyle\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)} p_{{\rm D},k} \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} q_k^{(i,j)}(z) N(x; m_{{\rm U},k}^{(i,j)}, P_{{\rm U},k}^{(i,j)})}{1-r_{k|k-1}^{(i)} p_{{\rm D},k}}}{\displaystyle\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)} p_{{\rm D},k} \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} q_k^{(i,j)}(z)}{1-r_{k|k-1}^{(i)} p_{{\rm D},k}}}. \tag{34}$$

具体参数如下:

$$\begin{aligned}q_k^{(i,j)}(z) = N(z; h_k(m_{k|k-1}^{(i,j)}), \\ (H_k^{(i,j)})^{\rm T} P_{k|k-1}^{(i,j)} H_k^{(i,j)} + R_k),\end{aligned} \tag{35}$$

$$w_{{\rm U},k}^{(i,j)}(z) = \frac{r_{k|k-1}^{(i),{\rm EK}} p_{{\rm D},k} w_{k|k-1}^{(i,j)} q_k^{(i,j)}(z)}{1-r_{k|k-1}^{(i),{\rm EK}}}, \tag{36}$$

$$m_{{\rm U},k}^{(i,j)}(z) = m_{k|k-1}^{(i,j)} + K_{{\rm U},k}^{(i,j)}(z - h_k(m_{k|k-1}^{(i,j)})), \tag{37}$$

$$P_{{\rm U},k}^{(i,j)} = (I - K_{{\rm U},k}^{(i,j)}) P_{k|k-1}^{(i,j)}, \tag{38}$$

$$K_{{\rm U},k}^{(i,j)} = P_{k|k-1}^{(i,j)} (H_k^{(i,j)})^{\rm T} (S_{{\rm U},k}^{(i,j)})^{-1}, \tag{39}$$

$$S_{{\rm U},k}^{(i,j)} = H_k^{(i,j)} P_{k|k-1}^{(i,j)} (H_k^{(i,j)})^{\rm T} + U_k^{(i,j)} R_k (U_k^{(i,j)})^{\rm T}, \tag{40}$$

$$H_k^{(i,j)} = \frac{\partial h_k(x_k, 0)}{\partial x_k}\Big|_{x_k=m_{k|k-1}^{(i,j)}}, \tag{41}$$

$$U_k^{(i,j)} = \frac{\partial h_k(m_{k|k-1}^{(i,j)}, v_k)}{\partial v_k}\Big|_{v_k=0}. \tag{42}$$

证明过程参见附录E.

可知, 若EK–GM–CBMeMBer滤波器的每个高斯项的方差趋于0, 也一致收敛于真实的CBMeMBer滤波器. 该结论与EK–GM–PHD滤波器一致, 是基于高斯和滤波器[13]得到的. 即在低噪声环境下, EK–GM–CBMeMBer滤波器和EK–GM–PHD滤波器几乎是最优的. 它们的本质区别在于PHD滤波器是对多目标状态的强度进行递推估计, 而CBMeMBer滤波器直接近似递推多目标状态的后验概率密度.

与EK滤波器对非线性模型进行线性化处理不同, UK滤波器通过对一簇σ点进行unscented变换来近似计算GM项的均值和方差. 可以证明其预测密度收敛

于具有二阶精度的估计值, 比EK滤波器获得的估计更精确. 详细过程参见文献[12].

4 结论(Conclusions)

有限GM密度能够以任意精度逼近任意的密度函数. 本文首先对GM–CBMeMBer滤波器的收敛性进行分析, 给出它一致性收敛的结果和证明过程. 随后, 证明在弱非线性条件下, 采用EK或UK滤波近似技术, 对应的GM–CBMeMBer滤波器也一致收敛于真实的CBMeMBer滤波器. 上述结论从收敛性的角度证明CBMeMBer滤波器的GM实现在实际应用中是可行的, 进一步完善了CBMeMBer滤波器对多目标跟踪的理论研究.

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附录 数学证明(Appendix Mathematical proofs)

附录 A 引理1(Appendix A Lemma 1)

根据Minkowski不等式, 对于任意$\varphi \in C_{\rm b}(R^{\rm d})$, 多目标多伯努利密度的GM近似$\{(r^{(i),J^{(i)}}, p^{(i),J^{(i)}})\}_{i=1}^{M}$和真实密度$\{(r^{(i)}, p^{(i)})\}_{i=1}^{M}$满足如下不等式:

$$\begin{aligned}&|r^{(i)}\langle p^{(i)},\varphi\rangle - r^{(i),J^{(i)}}\langle p^{(i),J^{(i)}},\varphi\rangle| \leqslant\\&|r^{(i)}\langle p^{(i)},\varphi\rangle - r^{(i)}\langle p^{(i),J^{(i)}},\varphi\rangle| +\\&|r^{(i)}\langle p^{(i),J^{(i)}},\varphi\rangle - r^{(i),J^{(i)}}\langle p^{(i),J^{(i)}},\varphi\rangle| \leqslant\\&r^{(i)}|\langle p^{(i)},\varphi\rangle - \langle p^{(i),J^{(i)}},\varphi\rangle| + \|\varphi\|_\infty |r^{(i)} - r^{(i),J^{(i)}}|. \end{aligned} \tag{A1}$$

附录 B GM–CBMeMBer滤波器预测步收敛性分析(Appendix B Convergence analysis of the prediction step of the GM–CBMeMBer filter)

首先是式(3)的证明.

$$\begin{aligned}&|r_{{\rm P},k|k-1}^{(i)} - r_{{\rm P},k|k-1}^{(i),J_{k-1}^{(i)}}| =\\&|r_{k-1}^{(i)}\langle p_{k-1}^{(i)}, p_{{\rm S},k}\rangle - r_{k-1}^{(i),J_{k-1}^{(i)}}\langle p_{k-1}^{(i),J_{k-1}^{(i)}}, p_{{\rm S},k}\rangle|. \end{aligned} \tag{A2}$$

由引理1可知上式

$$\begin{aligned}&\leqslant r_{k-1}^{(i)}|\langle p_{k-1}^{(i)}, p_{{\rm S},k}\rangle - \langle p_{k-1}^{(i),J_{k-1}^{(i)}}, p_{{\rm S},k}\rangle| +\\&p_{{\rm S},k}|r_{k-1}^{(i)} - r_{k-1}^{(i),J_{k-1}^{(i)}}|. \end{aligned} \tag{A3}$$

由式(1)–(2)及$0 \leqslant r_{k-1}^{(i)} \leqslant 1$可知上式

$$\leqslant p_{{\rm S},k}\,(c_{k-1} + d_{k-1})\,. \tag{A4}$$

因此式(3)成立, 其中$c_{{\rm P},k|k-1}$见式(9).

然后是式(4)的证明.

$$\begin{aligned}&|\langle p_{{\rm P},k|k-1}^{(i)},\varphi\rangle - \langle p_{{\rm P},k|k-1}^{(i),J_{k-1}^{(i)}},\varphi\rangle| =\\&|\langle \frac{\langle f_{k|k-1}(x|\cdot), p_{k-1}^{(i)} p_{{\rm S},k}\rangle}{\langle p_{k-1}^{(i)}, p_{{\rm S},k}\rangle},\varphi\rangle -\\&\langle \frac{\langle f_{k|k-1}(x|\cdot), p_{k-1}^{(i),J_{k-1}^{(i)}} p_{{\rm S},k}\rangle}{\langle p_{k-1}^{(i),J_{k-1}^{(i)}}, p_{{\rm S},k}\rangle},\varphi\rangle|. \end{aligned} \tag{A5}$$

加一项、减一项, 且由Minkowski不等式可知上式

$$\leqslant 2\frac{\|\langle f_{k|k-1}(x|\cdot),\varphi\rangle\|_\infty}{\langle p_{k-1}^{(i)}, p_{{\rm S},k}\rangle}|\langle p_{k-1}^{(i),J_{k-1}^{(i)}}, p_{{\rm S},k}\rangle - \langle p_{k-1}^{(i)}, p_{{\rm S},k}\rangle|. \tag{A6}$$

由式(2)和文献[12]中式(37)可知上式

$$\leqslant 2 d_{k-1}\|\varphi\|_\infty. \tag{A7}$$

因此式(4)成立, 其中$d_{{\rm P},k|k-1}$见式(10).

最后是式(5)–(6)的证明. 由于假设新生目标模型已知, 由式(1)–(2)可知存在常数$c_{\varGamma,k}$和$d_{\varGamma,k}$使得式(5)–(6)成立. 至此, GM–CBMeMBer滤波器预测步收敛性证明完毕.

附录 C　GM–CBMeMBer滤波器更新步收敛性分析(Appendix C Convergence analysis of the update step of the GM-CBMeMBer filter)

首先是式(13)的证明.

$$|r_{\mathrm{L},k}^{(i)}-r_{\mathrm{L},k}^{(i),J_{k|k-1}^{(i)}}|=|r_{k|k-1}^{(i)}\frac{1-\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\frac{1-\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}{1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}|. \tag{A8}$$

加一项、减一项, 且由Minkowski不等式可知上式

$$\leqslant\frac{|(r_{k|k-1}^{(i)}-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}})+(r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle)|}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}+$$
$$|\frac{r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}{1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}\frac{r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}|. \tag{A9}$$

一方面, 对于式(A9)前一项中的分子, 由Minkowski不等式可得

$$\leqslant|r_{k|k-1}^{(i)}-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}|+$$
$$|r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle|. \tag{A10}$$

由式(11)–(12)和引理1可知上式

$$\leqslant(1+p_{\mathrm{D},k})c_{k|k-1}+r_{k|k-1}^{(i)}p_{\mathrm{D},k}d_{k|k-1}. \tag{A11}$$

另一方面, 对于式(A9)后一项, 由$0\leqslant r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\leqslant 1$可知

$$\leqslant\frac{|r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle|}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}. \tag{A12}$$

由式(11)–(12)和引理1可知上式

$$\leqslant\frac{p_{\mathrm{D},k}c_{k|k-1}+r_{k|k-1}^{(i)}p_{\mathrm{D},k}d_{k|k-1}}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}. \tag{A13}$$

将式(A11)(A13)代入式(A9), 且由$0\leqslant r_{k|k-1}^{(i)}\leqslant 1$可得

$$|r_{\mathrm{L},k}^{(i)}-r_{\mathrm{L},k}^{(i),J_{k|k-1}^{(i)}}|\leqslant$$
$$\frac{(1+2p_{\mathrm{D},k})c_{k|k-1}+2p_{\mathrm{D},k}d_{k|k-1}}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}\leqslant$$
$$\frac{(1+2p_{\mathrm{D},k})c_{k|k-1}+2p_{\mathrm{D},k}d_{k|k-1}}{1-p_{\mathrm{D},k}}. \tag{A14}$$

因此式(13)成立, 其中$c_{\mathrm{L},k}$见式(17).

然后是式(14)的证明.

$$|\langle p_{\mathrm{L},k}^{(i)},\varphi\rangle-\langle p_{\mathrm{L},k}^{(i),J_{k|k-1}^{(i)}},\varphi\rangle|=$$
$$|\frac{\langle p_{k|k-1}^{(i)},(1-p_{\mathrm{D},k})\varphi\rangle}{1-\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}-\frac{\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},(1-p_{\mathrm{D},k})\varphi\rangle}{1-\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}|. \tag{A15}$$

加一项、减一项, 且由Minkowski不等式可知上式

$$\leqslant\frac{1}{1-\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}$$
$$|\langle p_{k|k-1}^{(i)},(1-p_{\mathrm{D},k})\varphi\rangle-\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},(1-p_{\mathrm{D},k})\varphi\rangle|+$$
$$\frac{\|\varphi\|_\infty|\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle-\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle|}{1-\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}. \tag{A16}$$

由式(12)可得上式

$$\leqslant\frac{d_{k|k-1}}{1-p_{\mathrm{D},k}}\|\varphi\|_\infty. \tag{A17}$$

因此式(14)成立, 其中$d_{\mathrm{L},k}$见式(18).

接着是式(15)的证明.

$$|r_{\mathrm{U},k}(z)-r_{\mathrm{U},k}^{J_{k|k-1}^{(i)}}(z)|=|\frac{\sum\limits_{i=1}^{M_{k|k-1}}\frac{(1-r_{k|k-1}^{(i)})r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},\psi_{k,z}\rangle}{(1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle)^2}}{\kappa_k(z)+\sum\limits_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},\psi_{k,z}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}}-\frac{\sum\limits_{i=1}^{M_{k|k-1}}\frac{(1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}})r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},\psi_{k,z}\rangle}{(1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle)^2}}{\kappa_k(z)+\sum\limits_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},\psi_{k,z}\rangle}{1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}}|. \tag{A18}$$

由文献[11]中式(A.39)可知上式

$$\leqslant 2\frac{\sum_{i=1}^{M_{k|k-1}}\left|\frac{(1-r_{k|k-1}^{(i)})r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},\psi_{k,z}\rangle}{(1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle)^2}-\frac{(1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}})r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},\psi_{k,z}\rangle}{(1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle)^2}\right|}{\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},\psi_{k,z}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}}. \tag{A19}$$

一方面, 式(A19)中分子部分, 加一项、减一项, 且由Minkowski不等式可知

$$\leqslant \frac{(1-r_{k|k-1}^{(i)})r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},\psi_{k,z}\rangle}{(1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle)^2}\frac{|(1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle)^2-(1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle)^2|}{(1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle)^2}+$$

$$\frac{|(1-r_{k|k-1}^{(i)})r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},\psi_{k,z}\rangle-(1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}})r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},\psi_{k,z}\rangle|}{(1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle)^2}. \tag{A20}$$

由式(11)–(12)和引理1, 整理可得上式

$$\leqslant \frac{0.5p_{\mathrm{D},k}\left(1-p_{\mathrm{D},k}\right)\left\|\psi_{k,z}\right\|_\infty}{\left(1-p_{\mathrm{D},k}\right)^4}\left(c_{k|k-1}+d_{k|k-1}\right)+\frac{0.25d_{k|k-1}+3c_{k|k-1}}{\left(1-p_{\mathrm{D},k}\right)^2}\left\|\psi_{k,z}\right\|_\infty. \tag{A21}$$

另一方面, 式(A19)中分母部分由文献[11]中式(A.45)获得, 故

$$|r_{\mathrm{U},k}(z)-r_{\mathrm{U},k}^{J_{k|k-1}^{(i)}}(z)|\leqslant\frac{(6-11p_{\mathrm{D},k}+5p_{\mathrm{D},k}^2)c_{k|k-1}+0.5(1-p_{\mathrm{D},k}^2)d_{k|k-1}}{(1-p_{\mathrm{D},k})^3\inf(\psi_{k,z})n_{k|k-1}}M_{k|k-1}\|\psi_{k,z}\|_\infty. \tag{A22}$$

因此式(15)成立, 其中$c_{\mathrm{U},k}$见式(19).

最后是式(16)的证明.

$$|\langle p_{\mathrm{U},k}(\cdot;z),\varphi\rangle-\langle p_{\mathrm{U},k}^{J_{k|k-1}^{(i)}}(\cdot;z),\varphi\rangle|=|\langle\frac{\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}p_{k|k-1}^{(i)}\psi_{k,z}}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}}{\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},\psi_{k,z}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}},\varphi\rangle-\langle\frac{\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}p_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\psi_{k,z}}{1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}}{\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},\psi_{k,z}\rangle}{1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}},\varphi\rangle|. \tag{A23}$$

加一项、减一项, 由Minkowski不等式和文献[11]中式(A.59)可知上式

$$\leqslant\frac{2\|\varphi\|_\infty}{\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},\psi_{k,z}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}}\left|\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},\psi_{k,z}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)},p_{\mathrm{D},k}\rangle}-\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},\psi_{k,z}\rangle}{1-r_{k|k-1}^{(i),J_{k|k-1}^{(i)}}\langle p_{k|k-1}^{(i),J_{k|k-1}^{(i)}},p_{\mathrm{D},k}\rangle}\right|. \tag{A24}$$

将文献[11]中式(A.44)和(A.45)的结论代入上式可得

$$|\langle p_{\mathrm{U},k}(\cdot;z),\varphi\rangle-\langle p_{\mathrm{U},k}^{J_{k|k-1}^{(i)}}(\cdot;z),\varphi\rangle|\leqslant\frac{2M_{k|k-1}\|\psi_{k,z}\|_\infty(c_{k|k-1}+d_{k|k-1})}{(1-p_{\mathrm{D},k})\inf(\psi_{k,z})n_{k|k-1}}\|\varphi\|_\infty. \tag{A25}$$

因此式(16)成立, 其中$d_{\mathrm{U},k}$见式(20). 至此, GM–CBMeMBer 滤波器更新步收敛性证明完毕.

附录 D EK–GM–CBMeMBer滤波器预测步收敛性分析(Appendix D Convergence analysis of the prediction step of the EK–GM–CBMeMBer filter)

$$r_{\mathrm{P},k|k-1}^{(i),\mathrm{EK}}=r_{k-1}^{(i)}\int\sum_{j=1}^{J_{k-1}^{(i)}}w_{k-1}^{(i,j)}N(x;m_{k-1}^{(i,j)},P_{k-1}^{(i,j)})p_{\mathrm{S},k}\mathrm{d}x=r_{k-1}^{(i)}p_{\mathrm{S},k}, \tag{A26}$$

$$p_{\mathrm{P},k|k-1}^{(i),\mathrm{EK}}(x)=\frac{\int N(x;\phi_k(\zeta),Q_{k-1})\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k-1}^{(i,j)}N(\zeta;m_{k-1}^{(i,j)},P_{k-1}^{(i,j)})p_{\mathrm{S},k}\mathrm{d}\zeta}{\int\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k-1}^{(i,j)}N(x;m_{k-1}^{(i,j)},P_{k-1}^{(i,j)})p_{\mathrm{S},k}\mathrm{d}x}. \tag{A27}$$

故式(23)–(24)成立, 具体参数见式(25)–(28), 由EK滤波得到. 至此, EK–GM–CBMeMBer滤波器预测步收敛性证明完毕.

附录 E　EK–GM–CBMeMBer滤波器更新步收敛性分析(Appendix E　Convergence analysis of the update step of the EK–GM–CBMeMBer filter)

$$r_{\mathrm{L},k}^{(i),\mathrm{EK}}=r_{k|k-1}^{(i)}\frac{1-\int\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})p_{\mathrm{D},k}\mathrm{d}x}{1-r_{k|k-1}^{(i),\mathrm{EK}}\int\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})p_{\mathrm{D},k}\mathrm{d}x}=r_{k|k-1}^{(i)}\frac{1-p_{\mathrm{D},k}}{1-r_{k|k-1}^{(i)}p_{\mathrm{D},k}},\tag{A28}$$

$$p_{\mathrm{L},k}^{(i),\mathrm{EK}}(x)=\frac{\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})(1-p_{\mathrm{D},k})}{1-\int\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})p_{\mathrm{D},k}\mathrm{d}x}=\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)}).\tag{A29}$$

故式(31)–(32)成立.

$$r_{\mathrm{U},k}^{\mathrm{EK}}=\frac{\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}(1-r_{k|k-1}^{(i)})\int\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})N(z;h_k(x),R_k)p_{\mathrm{D},k}\mathrm{d}x}{(1-r_{k|k-1}^{(i)}\int\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})p_{\mathrm{D},k}\mathrm{d}x)^2}}{\kappa_k(z)+\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}\int\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})N(z;h_k(x),R_k)p_{\mathrm{D},k}\mathrm{d}x}{1-r_{k|k-1}^{(i)}\int\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})p_{\mathrm{D},k}\mathrm{d}x}},\tag{A30}$$

$$p_{\mathrm{U},k}^{\mathrm{EK}}(x;z)=\frac{\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})N(z;h_k(x),R_k)p_{\mathrm{D},k}}{1-r_{k|k-1}^{(i)}\int w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})p_{\mathrm{D},k}\mathrm{d}x}}{\sum_{i=1}^{M_{k|k-1}}\frac{r_{k|k-1}^{(i)}\int\sum_{j=1}^{J_{k|k-1}^{(i)}}w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})N(z;h_k(x),R_k)p_{\mathrm{D},k}\mathrm{d}x}{1-r_{k|k-1}^{(i)}\int w_{k|k-1}^{(i,j)}N(x;m_{k|k-1}^{(i,j)},P_{k|k-1}^{(i,j)})p_{\mathrm{D},k}\mathrm{d}x}}.\tag{A31}$$

故式(33)–(34)成立, 具体参数见式(35)–(42). 至此, EK–GM–CBMeMBer滤波器更新步收敛性证明完毕.

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