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# 不确定时滞关联大系统的全局稳定模糊容错控制

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**摘要:**研究了一类带有时变时滞的不确定非线性关联大系统的自适应模糊容错控制问题.用有界的参考信号代 换模糊逼近器输入中的未知时滞信号,使得控制器的设计与应用不再依赖于时滞假设条件,使得控制器的设计和控 制方法的应用更为方便.容错反推控制技术和自适应技术相结合来处理代换误差和逼近误差.所提出的方案能有 效补偿所有4种类型的执行器故障,同时还可保证闭环系统的全局稳定性.仿真结果进一步验证了本文方法的有效 性.

关键词:容错控制;时滞;全局稳定性;模糊逼近;关联大系统

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# Fuzzy fault-tolerant control for global stabilization of uncertain time-delay large-scale systems

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Abstract: The fuzzy fault-tolerant control problem is studied for global stabilization of a class of uncertain nonlinear interconnected large-scale systems with unknown time varying time delays. The unknown delayed input signals of the fuzzy approximator are substituted by the bounded references signals, and as a result, the control design and application are not dependent on the delay assumptions any more, so that the convenience of the controller design and application is greatly improved. The fault-tolerant backstepping control and the adaptive control technique are combined to deal with the errors of the replacement and the approximation. The proposed control scheme can compensate for all of the four types of actor faults efficiently, and global stability of the closed-loop system is guaranteed. Simulation results are provided to show the effectiveness of the control approach.

Key words: fault-tolerant control; time-delay; global stability; fuzzy approximation; interconnected large-scale systems

#### 1 引言(Introduction)

非线性系统控制中,反推控制(backstepping control, BC)是一种重要的设计方法.近年来,对不确定非 线性时滞系统反推控制的研究,受到了众多学者的广 泛关注.

通过构造 Lyapunov-Krasovskii 或 Lyapunov-Razumikhin 泛函,可以消除这类系统中时滞对闭环系统 稳定性的影响,如文献[1-6]以及实际应用如二阶化学 反应器<sup>[7]</sup>等.值得指出的是,时滞系统控制器的设计

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常依赖于对系统未知时滞所做的假设条件,如系统时 滞为已知常数、未知常数、有界的未知时变时滞和时 变时滞 d(t) 的导数满足 d(t) < d\* < 1等.如何判定 某一实际系统的时滞是否满足时滞假设条件,是这类 控制方法在实际应用时面临的主要困难.这类系统中 不确定项对闭环系统稳定性的影响,可通过引入模糊 逻辑系统或神经网络逼近器予以消除.由于自适应方 法常常被用来处理逼近误差,因此这类控制方法也被 称为基于逼近器的自适应反推控制.在这个研究领域,

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最近发展出两种可得全局稳定结果的方法.一种方法 采用代换技术<sup>[4,8-9]</sup>,将未知函数的输入替换为有界的 系统参考信号,从而使得逼近器对未知函数的逼近始 终成立.另一种方法采用复合切换技术<sup>[10-12]</sup>,通过在 逼近器所成立的紧集外部设置额外的控制律保证闭 环系统的全局稳定性.基于代换的方法结构简单,但 主要适用于系统不确定项仅含系统输出变量y的系统; 基于切换的方法可适用于系统不确定项含任意状态 变量,但控制器结构复杂.

针对日益复杂的控制系统,反推控制也与容错控制 (fault-tolerant control, FTC)相结合,发展出基于FTC的 反推控制方法,显著提高了复杂非线性控制系统的可 靠性,如文献[13–17]等.值得注意的是,针对时滞非线 性系统容错控制的研究结果较少,仅有文献如[15–16] 给出了基于BC和FTC的控制结果,但控制器的设计需 依赖于时滞假设条件0  $\leq \tau_i(t) \leq \bar{\tau}_i, \dot{\tau}(t) < \vartheta_i^* < 1.$ 

本文首先采用模糊逼近器和时滞代换技术处理系统中的不确定项和未知时滞,并基于FTC的理论构建 了全局稳定的自适应容错控制器.控制器的设计过程 不再依赖于时滞假设条件,大大增加了控制器设计与 应用的便易性.所考虑的4种执行器故障模型均可得 到有效补偿.

本文中,对于未知常数a、向量B和矩阵C,  $\hat{a}$ 表示a的估计值,  $\tilde{a}$ 表示a与 $\hat{a}$ 之间的差, 即 $\tilde{a} = a - \hat{a}$ ; ||B||表示B的2-范数,  $\lambda_{\max}(C)$ 表示矩阵C的最大特征值.

### 2 问题描述(Problem formulation)

考虑N个非线性子系统相互关联的大系统,其第 *i*个子系统为

$$\begin{cases} \dot{x}_{i,j}(t) = x_{i,j+1}(t) + f_{i,j}(\bar{x}_{i,j}(t)) + \\ \sum_{k=1}^{N} h_{i,j,k}(y_k(t - d_k(t))), \\ j = 1, \cdots, n_i - 1, \ i = 1, \cdots, N, \\ \dot{x}_{i,n_i}(t) = \omega_i^{\mathrm{T}} u_i(t) + f_{i,n_i}(x(t)) + \\ \sum_{k=1}^{N} h_{i,n_i,k}(y_k(t - d_k(t))), \\ y_i(t) = x_{i,1}(t), \end{cases}$$
(1)

其中:  $x_i = [x_{i,1} \cdots x_{i,n_i}]^T$ 为第*i*个子系统的系统状态,  $y_i$ 为该子系统的系统输出,  $\bar{x}_{i,j} = [x_{i,1} \cdots x_{i,j}]^T$ ;  $\omega_i^T = [\omega_{i,1} \ \omega_{i,2} \cdots \ \omega_{i,m_i}] \in \mathbb{R}^{m_i}$ 为常数向量,  $u_i = [u_{i,1} \ u_{i,2} \cdots \ u_{i,m_i}]^T \in \mathbb{R}^{m_i}$ 为系统输入, 同时也是执行器的输出, 下标 $m_i$ 表示执行器的个数, 注意本文考虑了可能发生的执行器失效情况; 第k, *i*个子系统之间的关联项 $h_{i,j,k}(y_k(t-d_k(t)))$ 为未知光滑函数,  $d_k(t)$ 为未知时变时滞.

控制目标:所考虑的执行器失效情况均可得到有效的补偿,同时保证闭环系统全局一致最终有界(glo-

bal uniformly ultimately bounded, GUUB), 跟踪误差可以收敛到原点附近的一个小邻域内.

**假设1** 在区间  $[0, +\infty)$ 上, 参考信号  $y_{i,r}(t)$  及 其前 $n_i$ 阶导数已知, 分段连续且有界.

**注1** 一般来说, 控制系统存在的时滞会影响系统的性能, 因而对时滞系统控制问题的研究, 具有很强的理论与现实意义.

文中式(1)所示为一类含有信号传递延迟的关联大系统的模型,许多实际的控制系统具有或可转化为这种结构,如冷 轧机、互联双倒立摆等.然而现有的研究结果一般依赖于对 系统时滞所做的假设条件,如文献[1-4]等.与之前研究不同 的是,本文在去除对系统时滞假设条件的基础上,对系统(1) 设计了全局稳定的容错控制器.

本文研究的另一个突出意义在于,若仅考虑子系统间信号传递的延迟,可以去除文献[4,8-9]中在应用变量代换技巧时对未知函数所作的限制,从而扩大这一方法的应用范围,即 便系统可能存在执行器故障.

#### 3 预处理(Preliminaries)

为了便于表达,将式(1)中的时滞关联项写为

$$\sum_{k=1}^{N} h_{i,j,k} \big( y_k (t - d_k(t)) \big) = h_{i,j}(y_d), \qquad (2)$$

其中 $y_{d} = [y_{1}(t - d_{1}(t)) \cdots y_{N}(t - d_{N}(t))]^{\mathrm{T}}$ ,于是

$$h_{i,j}(y_{\rm d}) = [h_{i,j}(y_{\rm d}) - h_{i,j}(y_{\rm rd})] + [h_{i,j}(y_{\rm rd}) - h_{i,j}(y_{\rm r})] + h_{i,j}(y_{\rm r}) = p_{i,j} + q_{i,j} + h_{i,j}(y_{\rm r}),$$
(3)

其中:

$$y_{\rm rd} = [y_{1,\rm r}(t - d_1(t)), \cdots, y_{N,\rm r}(t - d_N(t))]^{\rm T}, y_{\rm r} = [y_{1,\rm r}(t), \cdots, y_{N,\rm r}(t)]^{\rm T}.$$

代换误差 $p_{i,i}$ 和 $q_{i,i}$ 为

$$p_{i,j} = h_{i,j}(y_{\rm d}) - h_{i,j}(y_{\rm rd}),$$
  

$$q_{i,j} = h_{i,j}(y_{\rm rd}) - h_{i,j}(y_{\rm r}).$$
(4)

对于光滑函数
$$f$$
, 有 $|f(x) - f(y)| \leq l||x - y||$ , 于是有

$$p_{i,j} \leq ||y_{\rm d} - y_{\rm rd}|| l_{i,j,1},$$
 (5)

$$|q_{i,j}| \leq ||y_{\rm rd} - y_{\rm r}|| l_{i,j,2},$$
 (6)

这里的 $l_{i,j,1}$ 和 $l_{i,j,2}$ 为未知的Lipschitz常数. 对于式(5), 定义

$$l_{i,1} = \max_{1 \leqslant j \leqslant n_i} l_{i,j,1},$$

于是有

$$|p_{i,j}| \leq ||y_{\rm d} - y_{\rm rd}||l_{i,1},$$
(7)

对于式(6), 由假设1可知,  $||y_{rd} - y_r||$ 有界, 故代换误差  $q_{i,j}$ 有界, 因而存在未知常数 $\psi_{i,j,1}$ 满足

$$|q_{i,j}| \leqslant \psi_{i,j,1}.\tag{8}$$

由式(3)可知,系统(1)可写为

$$\begin{cases} \dot{x}_{i,j} = x_{i,j+1} + f_{i,j}(\bar{x}_{i,j}) + h_{i,j}(y_{\rm r}) + \\ p_{i,j} + q_{i,j}, \\ j = 1, \cdots, n_i - 1, \ i = 1, \cdots, N, \\ \dot{x}_{i,n_i} = \omega_i^{\rm T} u_i + f_{i,n_i}(x_i) + h_{i,n_i}(y_{\rm r}) + \\ p_{i,n_i} + q_{i,n_i}. \end{cases}$$
(9)

采用模糊逻辑系统作为系统(9)中未知函数的逼近 器. 若采用单点模糊化、乘积运算的模糊蕴含规则、 重心法解模糊和高斯函数的隶属度函数,则该模糊逼 近器可表示为

$$f(x|\theta) = \sum_{l=1}^{M} \bar{y}_l \phi_l(x) = \theta^{\mathrm{T}} \phi(x), \qquad (10)$$

其中:  $x = [x_1 \cdots x_n]^T$ 为逼近器的输入;  $f(x|\theta)$ 为 逼近器的输出;  $\theta = (\bar{y}_1, \cdots, \bar{y}_M)$ 为未知参数向量,  $\bar{y}_l$  $=\max_{y\in\mathbb{R}}G^{l}(y);\phi(x)=[\phi_{1}(x)\quad\cdots\quad\phi_{M}(x)]^{\mathrm{T}}\,\,$ 基函数向量, M为模糊规则集合中的规则数目. 根据 模糊逻辑系统的逼近定理<sup>[18]</sup>,对于紧集 $\Omega_{Fuzzy} \in \mathbb{R}^n$ 中的连续非线性函数F(x),存在式(10)所示的模糊逻 辑系统,使得

$$F(x) = \theta^{\mathrm{T}} \phi(x) + \varepsilon(x), \qquad (11)$$

且存在未知常数 $\psi > 0$ ,使得逼近误差

$$\varepsilon(x) < \psi.$$

对于式(9)中的关联项
$$h_{i,j}(y_r), j = 1, \cdots, n_i, 有$$

$$h_{i,j}(y_{\mathbf{r}}) = \theta_{i,j}^{\mathrm{T}} \phi_{i,j}(y_{\mathbf{r}}) + \varepsilon_{i,j}(y_{\mathbf{r}}).$$
(12)

由式(11),存在未知常数 $\psi_{i,j,2} > 0$ ,使得 $|\varepsilon_{i,j}| < \psi_{i,j,2}$ . 令 $e_{i,i} = q_{i,i} + \varepsilon_{i,i}$ ,结合式(8),则有

$$e_{i,j} \leq |q_{i,j}| + |\varepsilon_{i,j}| \leq \psi_{i,j,1} + \psi_{i,j,2}.$$
(13)

令
$$\psi_i = \max_{1 \leq j \leq n_i} \{ \psi_{i,j,1} + \psi_{i,j,2} \}, 则$$
  
 $e_{i,j} \leq \psi_i.$ 

$$y_j \leqslant \psi_i.$$
 (14)

由式(14)可知, 未知常数 $\psi_i$ 中包含了逼近误差和代换 误差的一部分,它将在下文中采用自适应的方法进行 处理. 将式(12)和ei,i代入式(9)有

$$\begin{cases} \dot{x}_{i,j} = x_{i,j+1} + f_{i,j}(\bar{x}_{i,j}) + \theta_{i,j}^{\mathrm{T}}\phi_{i,j}(y_{\mathrm{r}}) + \\ p_{i,j} + e_{i,j}, \\ j = 1, \cdots, n_{i} - 1, \ i = 1, \cdots, N, \\ \dot{x}_{i,n_{i}} = \omega_{i}^{\mathrm{T}}u_{i} + f_{i,n_{i}}(x_{i}) + \theta_{i,n_{i}}^{\mathrm{T}}\phi_{i,n_{i}}(y_{\mathrm{r}}) + \\ p_{i,n_{i}} + e_{i,n_{i}}. \end{cases}$$
(15)

考虑如下4种执行器失效模型, 即损伤(loss of effectiveness, LOE)、卡死(lock in place, LIP)、飞车或饱和

(hard over fault, HOF)、松浮(float), 如下所示:

$$u_{i}(t) = \begin{cases} u_{i}^{c}(t), & \forall t > 0, \text{ no-failure}, \\ k_{i}(t)u_{i}^{c}(t), & \forall t \ge t_{i}^{\mathrm{F}}, \text{ LOE}, \\ u_{i}^{c}(t_{i}^{\mathrm{F}}), & \forall t \ge t_{i}^{\mathrm{F}}, \text{ LIP}, \\ \underline{u}_{i} \vec{\boxtimes} \bar{u}_{i}, & \forall t \ge t_{i}^{\mathrm{F}}, \text{ HOF}, \\ 0, & \forall t \ge t_{i}^{\mathrm{F}}, \text{ float}, \end{cases}$$
(16)

其中:  $0 < k_i(t) < 1$ ;  $u_i^c(t)$ 为系统控制器的输出, 同时 也是执行器的输入;  $u_i(t)$ 为执行器的输出;  $u_i^c(t_i^F)$ 为 执行器卡死时的输出值;  $\bar{u}_i$ 和 $\underline{u}_i$ 为 $u_i^c(t)$ 的上下限;  $t_i^F$ 为执行器发生损伤的时间; 下标i表示第i个执行器. 针对本文系统(1),这些模型可以用公式表示为

$$\begin{cases} u_{i,k}(t) = k_{i,k}(t)u_{i,k}^{c}(t) + u_{i,k}^{F}, \ \forall t \ge t_{i,k}^{F}, \\ k_{i,k}(t)u_{i,k}^{F} = 0, \end{cases}$$
(17)

其中:下标i表示系统(1)中第i个子系统; 下标k表示该 子系统中第k个执行器;  $k_{i,k}(t) \in [0,1]$ 为常数;  $1 \leq k \leq$  $m_i$ . 当 $k_{i,k}(t) = 1, u_{i,k}^{\mathrm{F}} = 0$ 时, 无故障; 当 $0 < k_{i,k}(t) <$ 1,  $u_{i,k}^{\rm F} = 0$ 时, LOE; 当 $k_{i,k}(t) = 0$ ,  $u_{i,k}^{\rm F} = u_{i,k}^{\rm c}(t_{i,k}^{\rm F})$ 时, LIP; 当 $k_{i,k}(t) = 0$ ,  $u_{i,k}^{\mathrm{F}} = \overline{u}_{i,k}$  或 $u_{i,k}^{\mathrm{F}} = \underline{u}_{i,k}$ 时, HOF; 当 $k_{i,k}(t) = 0, u_{i,k}^{F} = 0$ 时, float. 由式(17)可知

$$\sum_{k=1}^{m_{i}} u_{i,k} = \sum_{k=k_{1},\cdots,k_{p}} k_{i,k} u_{i,k}^{c} + \sum_{k \neq k_{1},\cdots,k_{p}} u_{i,k}^{F}.$$
(18)

$$\underbrace{ \underbrace{ \sum_{k=k_1,\cdots,k_p} \mathfrak{H} \sum_{1}}_{k=k_1,\cdots,k_p} }_{k_{j+1},\cdots,k_p} \underbrace{ \underbrace{ \sum_{k=1}^{m_i} u_{i,k} = \sum_{1} k_{i,k} u_{i,k}^{c} + \sum_{2} u_{i,k}^{F}, }_{2} }_{k_{i,k}}$$

于是有

$$\omega_i^{\mathrm{T}} u_i = \sum_{k=1}^{m_i} \omega_{i,k} u_{i,k} =$$
$$\sum_1 \omega_{i,k} k_{i,k} u_{i,k}^{\mathrm{c}} + \sum_2 \omega_{i,k} u_{i,k}^{\mathrm{F}}.$$
(20)

为了构造合适的控制器,本文采用下式来构成控制器 输出*u*<sup>c</sup><sub>*i*,*k*</sub><sup>[14, 19–20]</sup>:

$$u_{i,k}^{c} = b_{i,k}(x_{i,n_{i}})u_{i0}, \qquad (21)$$

和 $b_{i,k}$ 分别是 $b_{i,k}(x_{i,n_i})$ 取值的上下界.  $u_{i0}$ 是下一小节 将要基于反推控制方法设计的自适应控制器.将式 (21)代入式(20),有

$$\omega_{i}^{\mathrm{T}}u_{i} = \sum_{1} \omega_{i,k} k_{i,k} b_{i,k} u_{i0} + \sum_{2} \omega_{i,k} u_{i,k}^{\mathrm{F}}.$$
 (22)

假设2 对于关联大系统(1)的任何一个子系统 来说,若其中有任意不大于 $m_i - 1$ 个执行器发生LIP, HOF或float,剩余的执行器仍可驱使闭环系统达到上 述控制目标.这也是研究容错控制问题的基本假设.

**注** 2 在式(3)中,本文提出了时滞代换的方法来处理 系统中的时滞项,可以使得控制器的设计和应用不再依赖于 时滞假设条件,进而增加系统控制器设计的便利性.这样的 时滞代换处理方法,使得式(12)中模糊逼近器 $\theta_{i,j}^{T}\phi_{i,j}(y_r) + \varepsilon_{i,j}(y_r)$ 的输入为有界的系统参考信号 $y_r$ ,故该模糊逼近器与 系统变量 $x_{i,j}$ 无关,所以 $x_{i,j}$ 可在全局范围内取值,即本文的 稳定性结果将是全局性的.

在容错控制方面,现有一些研究结果如[13-16]等仅考虑 了LOE和LIP模型.由于松浮(操纵面脱离控制)、飞车或饱 和(执行器处于极限位置)这两种故障类型也可导致严重的故 障(如1985年日本的flight123,2002年阿拉斯加的flight85),故 在设计容错控制器时考虑到这两种执行器故障的影响,具有 重要的现实意义.

#### 4 控制器设计(Controller design)

#### 4.1 容错反推控制器设计(FTC design)

对于系统(15),为了避免子系统每一阶均需引入未 知参数 $\theta_{i,j}$ 的参数自适应律,本文定义

$$\theta_i = \begin{bmatrix} \theta_{i,1}^{\mathrm{T}} & \cdots & \theta_{i,n_i}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$
(23)

$$记\phi_{i,j} 维零向量为0_{(\phi_{i,j})} = [0 \cdots 0]^{\mathrm{T}}, 则可定义$$

$$\varphi_{i,j} = \begin{bmatrix} 0_{\phi_{i,1}} & \cdots & 0_{\phi_{i,j-1}} & \phi_{i,j} & 0_{\phi_{i,j+1}} & \cdots & 0_{\phi_{i,n_i}} \end{bmatrix}^{\mathrm{T}}.$$
(24)

于是有

$$\begin{cases} \dot{x}_{i,j} = x_{i,j+1} + f_{i,j} + \theta_i^{\mathrm{T}} \varphi_{i,j} + \\ p_{i,j} + e_{i,j}, \\ j = 1, \cdots, n_i - 1, \ i = 1, \cdots, N, \\ \dot{x}_{i,n_i} = \omega_i^{\mathrm{T}} u_i + f_{i,n_i} + \theta_i^{\mathrm{T}} \varphi_{i,n_i} + \\ p_{i,n_i} + e_{i,n_i}. \end{cases}$$
(25)

定义坐标变换

$$z_{i,j} = x_{i,j} - y_{i,r}^{(j-1)} - \alpha_{i,j-1},$$
(26)

并约定

$$z_{i,0} \triangleq 0, \ \alpha_{i,0} \triangleq 0. \tag{27}$$

下面的设计过程基于文献[21]中的调节函数方法, 为此作者略去了有关设计的具体步骤.在下面的式子 中,作者添加了一些关键项,这些项使得作者能在不 依赖于时滞假设条件和可能发生执行器失效的情况 下,消除未知时滞和未知关联项对闭环系统稳定性的 影响,并保证对系统(1)的容错控制.

考虑正定函数

$$\begin{cases} V_{i,1} = \frac{1}{2}z_{i,1}^{2} + \frac{1}{2}\tilde{\theta}_{i}^{\mathrm{T}}\Gamma_{i}^{-1}\tilde{\theta}_{i} + \frac{1}{2}\gamma_{i,1}^{-1}\tilde{\psi}_{i}^{2} + \frac{1}{2}\gamma_{i,2}^{-1}\tilde{\rho}_{i}^{2}, \\ V_{i,l} = \frac{1}{2}z_{i,l}^{2} + V_{i,l-1}, \ 2 \leqslant l \leqslant n_{i}, \end{cases}$$

$$(28)$$

其中:  $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0, \gamma_{i,1} > 0$ 和 $\gamma_{i,2} > 0$ 为自适应增

益. 自适应模糊容错控制器可设计如下:

$$\alpha_{i,1} = -c_{i,1}z_{i,1} - (\xi_{i,1} + \hat{\rho}_i)z_{i,1} - f_{i,1} - \hat{\theta}_i^{\mathrm{T}} w_{\theta_i,1} - \hat{\psi}_i \beta_{i,1} \tanh \frac{z_{i,1}\beta_{i,1}}{\delta_i}, \qquad (29)$$

$$\alpha_{i,l} = -z_{i,l-1} - c_{i,l} z_{i,l} - \xi_{i,l} z_{i,l} - f_{i,l} - \hat{\theta}_i^{\mathrm{T}} w_{\theta_i,l} - \hat{\psi}_i \beta_{i,l} \tanh \frac{z_{i,l} \beta_{i,l}}{\delta_i} + \Delta_{i,l-1},$$
(30)

$$\omega_{i}^{\mathrm{T}}u_{i} = -z_{i,n_{i}-1} - c_{i,n_{i}}z_{i,n_{i}} - \xi_{i,n_{i}}z_{i,n_{i}} - f_{i,n_{i}} - \hat{\theta}_{i}^{\mathrm{T}}w_{\theta_{i},n_{i}} - \hat{\psi}_{i}\beta_{i,n_{i}} \tanh\frac{z_{i,n_{i}}\beta_{i,n_{i}}}{\delta_{i}} + y_{i,r}^{(n_{i})} + \Delta_{i,n_{i}-1},$$
(31)

$$w_{\theta_{i,l}} = \varphi_{i,l} - \sum_{j=1}^{l-1} \frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}} \varphi_{i,j}, \qquad (32)$$

$$w_{p_i,l} = p_{i,l} - \sum_{j=1}^{l-1} \frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}} p_{i,j}, \qquad (33)$$

$$w_{e_{i},l} = e_{i,l} - \sum_{j=1}^{l-1} \frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}} e_{i,j},$$
(34)

$$\tau_{\theta_i,l} = \tau_{\theta_i,l-1} + z_{i,l} w_{\theta_i,l}, \tag{35}$$

$$\tau_{e_i,l} = \tau_{e_i,l-1} + z_{i,l}\beta_{i,l}\tanh\frac{z_{i,l}\beta_{i,l}}{\delta_i},\tag{36}$$

$$\Delta_{i,l} = \sum_{j=1}^{l} \frac{\partial \alpha_{i,l}}{\partial x_{i,j}} (x_{i,j+1} + f_{i,j}) + \frac{\partial \alpha_{i,l}}{\partial \hat{\theta}_{i}} \dot{\hat{\theta}}_{i} + \frac{\partial \alpha_{i,l}}{\partial \hat{\rho}_{i}} \dot{\hat{\rho}}_{i} + \frac{\partial \alpha_{i,l}}{\partial \hat{\psi}_{i}} \dot{\hat{\psi}}_{i} + \sum_{j=1}^{l} \sum_{k=1}^{N} \frac{\partial \alpha_{i,l}}{\partial y_{k,r}^{(j-1)}} y_{k,r}^{(j)},$$
(37)

其中:  $c_{i,l} > 0$ 为常数,  $\alpha_{i,l}$ 为稳定化函数,  $\tau_{e_i,l}$ 为调节 函数,  $\delta_i$ 为常数,  $1 \leq l \leq n_i$ . 引入的变量 $\xi_{i,l}$ 和 $\beta_{i,l}$ 定义 为

$$\xi_{i,l} = 1 + \sum_{j=1}^{l-1} \left(\frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}}\right)^2,$$
  
$$\beta_{i,l} = l + \sum_{j=1}^{l-1} \left(\frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}}\right)^2.$$
 (38)

为了消除未知时变时滞对闭环系统稳定性的影响,本 文引入量 $\rho_i$ ,其定义为

$$\rho_i = \frac{1}{2} \sum_{k=1}^{N} n_k (n_k + 1) l_{k,1}^2.$$
(39)

根据式(31), 控制率ui0可选为

$$u_{i0} = \frac{1}{\sum_{1} w_{i,k} k_{i,k} b_{i,k}} [-z_{i,n_i-1} - c_{i,n_i} z_{i,n_i} - \xi_{i,n_i} z_{i,n_i} - f_{i,n_i} - \hat{\theta}_i^{\mathrm{T}} w_{\theta_i,n_i} - \hat{\psi}_i \beta_{i,n_i} \tanh \frac{z_{i,n_i} \beta_{i,n_i}}{\delta_i} + y_{i,r}^{(n_i)} + \Delta_{i,n_i-1} - \sum_{2} w_{i,k} u_{i,k}^{\mathrm{F}}].$$
(40)

参数自适应律可选择为

$$\begin{cases} \dot{\hat{\theta}}_{i} = \Gamma_{i}(\tau_{\theta_{i},n_{i}} - r_{i}\hat{\theta}_{i}), \\ \dot{\hat{\psi}}_{i} = \gamma_{i,1}(\tau_{e_{i},n_{i}} - r_{i}\hat{\psi}_{i}), \\ \dot{\hat{\rho}}_{i} = \gamma_{i,2}(z_{i,1}^{2} - r_{i}\hat{\rho}_{i}), \end{cases}$$
(41)

其中*r<sub>i</sub>* > 0为设计参数. 由式(29)-(31), 可得闭环系 统如下:

$$\begin{aligned}
\dot{z}_{i,1} &= -c_{i,1}z_{i,1} + z_{i,2} + \tilde{\theta}_{i}^{\mathrm{T}}w_{\theta_{i},1} + \\ & [w_{p_{i},1} - (\xi_{i,1} + \hat{\rho}_{i})z_{i,1}] + \\ & [w_{e_{i},1} - \hat{\psi}_{i}\beta_{i,1}\tanh\frac{z_{i,1}\beta_{i,1}}{\delta_{i}}], \\ \dot{z}_{i,l} &= -z_{i,l-1} - c_{i,l}z_{i,l} + z_{i,l+1} + \tilde{\theta}_{i}^{\mathrm{T}}w_{\theta_{i},l} + \\ & (w_{p_{i},l} - \xi_{i,l}z_{i,l}) + \\ & [w_{e_{i},l} - \hat{\psi}_{i}\beta_{i,l}\tanh\frac{z_{i,l}\beta_{i,l}}{\delta_{i}}], \\ \dot{z}_{i,n_{i}} &= -z_{i,n_{i}-1} - c_{i,n_{i}}z_{i,n_{i}} + \tilde{\theta}_{i}^{\mathrm{T}}w_{\theta_{i},n_{i}} + \\ & (w_{p_{i},n_{i}} - \xi_{i,n_{i}}z_{i,n_{i}}) + \\ & [w_{e_{i},n_{i}} - \hat{\psi}_{i}\beta_{i,n_{i}}\tanh\frac{z_{i,n_{i}}\beta_{i,n_{i}}}{\delta_{i}}].
\end{aligned}$$
(42)

对于
$$z_{i,l}w_{p_i,l}$$
,根据式(7),有下式成立:  
 $z_{i,l}w_{p_i,l} = z_{i,l}(p_{i,l} - \sum_{j=1}^{l-1} \frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}} p_{i,j}) \leqslant$   
 $\frac{1}{4}z_{i,l}^2 + p_{i,l}^2 + \frac{1}{4}z_{i,l}^2 \sum_{j=1}^{l-1} (\frac{\partial \alpha_{i,l-1}}{\partial x_{i,l}})^2 + \sum_{j=1}^{l-1} p_{i,j}^2 \leqslant$   
 $z_{i,l}^2 [1 + \sum_{j=1}^{l-1} (\frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}})^2] + \sum_{j=1}^{l} p_{i,j}^2 =$   
 $z_{i,l}^2 \xi_{i,l} + ll_{i,1}^2 \sum_{k=1}^{N} z_{i,k}^2 (t - d_k(t)),$  (43)

其中
$$\xi_{i,l} = 1 + \sum_{j=1}^{r-1} \left( \frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}} \right)^2.$$
  
对于 $z_{i,l} w_{e_i,l},$ 基于  
 $|\eta| \leq \eta \tanh(\frac{\eta}{\varepsilon}) + \kappa \varepsilon, \ \kappa = 0.2875^{[22]},$ 

有

$$z_{i,l}w_{e_i,l} = z_{i,l}(e_{i,l} - \sum_{j=1}^{l-1} \frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}} e_{i,j}) \leqslant$$

$$|z_{i,l}|\psi_i(1 + \sum_{j=1}^{l-1} |\frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}}|) \leqslant$$

$$|z_{i,l}|\psi_i(1 + \sum_{j=1}^{l-1} [1 + (\frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}})^2] =$$

$$z_{i,l}\psi_i\beta_{i,l} \leqslant$$

$$\psi_i z_{i,l}\beta_{i,l} \tanh(\frac{z_{i,l}\beta_{i,l}}{\delta_i}) + \psi_i\kappa\delta_i, \quad (44)$$

$$\Downarrow \oplus \beta_{i,l} = l + \sum_{j=1}^{l-1} (\frac{\partial \alpha_{i,l-1}}{\partial x_{i,j}})^2.$$

## 4.2 稳定性分析(Stability analysis)

定义Lyapunov函数为

$$V = \sum_{i=1}^{N} V_{i,n_i} + \frac{1}{2} \sum_{i=1}^{N} n_i (n_i + 1) l_{i,1}^2 \cdot \sum_{k=1}^{N} \int t_{t-d_k}^t z_{k,1}^2(\sigma) \mathrm{d}\sigma.$$
(45)

对V求导可得

$$\dot{V} \leqslant \sum_{i=1}^{N} \left[ -\sum_{j=1}^{n_{i}} c_{i,j} z_{i,j}^{2} + r_{i} \hat{\theta}_{i} \tilde{\theta}_{i} + r_{i} \hat{\psi}_{i} \tilde{\psi}_{i} + r_{i} \hat{\rho}_{i} \tilde{\rho}_{i} + n_{i} \psi_{i} \kappa \delta_{i} \right] - \sum_{i=1}^{N} \rho_{i} z_{i,1}^{2} + \frac{1}{2} \sum_{i=1}^{N} n_{i} (n_{i} + 1) l_{i,1}^{2} \sum_{k=1}^{N} z_{k,1}^{2} (t).$$

$$(46)$$

考虑到式(39)中 $\rho_i$ 的定义,有

$$-\sum_{i=1}^{N} \rho_{i} z_{i,1}^{2} + \frac{1}{2} \sum_{i=1}^{N} n_{i} (n_{i} + 1) l_{i,1}^{2} \sum_{k=1}^{N} z_{k,1}^{2} (t) = -\sum_{i=1}^{N} [\frac{1}{2} \sum_{k=1}^{N} n_{k} (n_{k} + 1) l_{k,1}^{2}] z_{i,1}^{2} + \frac{1}{2} \sum_{i=1}^{N} n_{i} (n_{i} + 1) l_{i,1}^{2} \sum_{k=1}^{N} z_{k,1}^{2} (t) = -\frac{1}{2} \sum_{k=1}^{N} n_{k} (n_{k} + 1) l_{k,1}^{2} \sum_{i=1}^{N} z_{i,1}^{2} + \frac{1}{2} \sum_{i=1}^{N} n_{i} (n_{i} + 1) l_{i,1}^{2} \sum_{k=1}^{N} z_{k,1}^{2} (t) = 0,$$
(47)

于是可得

$$\dot{V} \leqslant \sum_{i=1}^{N} \left[ -\sum_{j=1}^{n_i} c_{i,j} z_{i,j}^2 + r_i \hat{\theta}_i \tilde{\theta}_i + r_i \hat{\psi}_i \tilde{\psi}_i + r_i \hat{\rho}_i \tilde{\rho}_i + n_i \psi_i \kappa \delta_i \right].$$
(48)

对于未知参数a,其估计值â和估计误差ã,有

$$\hat{a}\tilde{a} = \leqslant \frac{1}{2}a^2 + \frac{1}{2}\tilde{a}^2 - \tilde{a}^2 = \frac{1}{2}a^2 - \frac{1}{2}\tilde{a}^2.$$
(49)

根据式(49),式(48)可进一步写为

$$\dot{V} \leqslant \sum_{i=1}^{N} \left( -\sum_{j=1}^{n_{i}} c_{i,j} z_{i,j}^{2} - \frac{1}{2} r_{i} \tilde{\theta}_{i}^{\mathrm{T}} \tilde{\theta}_{i} - \frac{1}{2} r_{i} \tilde{\psi}_{i}^{2} - \frac{1}{2} r_{i} \tilde{\rho}_{i}^{2} \right) + \sum_{i=1}^{N} \left( \frac{1}{2} r_{i} \theta_{i}^{\mathrm{T}} \theta_{i} + \frac{1}{2} r_{i} \psi_{i}^{2} + \frac{1}{2} r_{i} \rho_{i}^{2} + n_{i} \psi_{i} \kappa \delta_{i} \right) \leqslant - a_{0} \sum_{i=1}^{N} \left( \sum_{j=1}^{n_{i}} z_{i,j}^{2} + \tilde{\theta}_{i}^{\mathrm{T}} \Gamma_{i}^{-1} \tilde{\theta}_{i} + \gamma_{i,1}^{-1} \tilde{\psi}_{i}^{2} + \gamma_{i,2}^{-1} \tilde{\rho}_{i}^{2} \right) + b_{0},$$
(50)

其中:

$$0 < a_0 < \min_{\substack{1 \le j \le n_i, 1 \le i \le N}} \{c_{i,j}, \frac{r_i}{2\lambda_{\max}(\Gamma_i^{-1})}, \frac{1}{2}r_i\gamma_{i,1}, \frac{1}{2}r_i\gamma_{i,2}\},\$$

$$b_{0} = \sum_{i=1}^{N} \left( \frac{1}{2} r_{i} \theta_{i}^{\mathrm{T}} \theta_{i} + \frac{1}{2} r_{i} \psi_{i}^{2} + \frac{1}{2} r_{i} \rho_{i}^{2} + n_{i} \psi_{i} \kappa \delta_{i} \right).$$
(51)

定义集合

$$\Omega = \{ (z_{i,j}, \theta_i, \psi_i, \rho_i) : a_0 \sum_{i=1}^{N} (\sum_{j=1}^{n_i} z_{i,j}^2 + \tilde{\theta}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{\theta}_i + \gamma_{i,1}^{-1} \tilde{\psi}_i^2 + \gamma_{i,2}^{-1} \tilde{\rho}_i^2) \leq b_0 \},$$

$$\mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq \Omega \& \mathrm{Arbit}, \, \mathrm{Kpt} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq \Omega \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \theta_i, \psi_i, \rho_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \psi_i, \varphi_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \psi_i, \varphi_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \psi_i, \varphi_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \psi_i, \varphi_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{Ext} \leq (z_{i,j}, \psi_i, \varphi_i) \& \mathrm{Ext} \leq 0 \\ \mathrm{E$$

$$a_{0}\sum_{i=1}^{N} \left(\sum_{j=1}^{n_{i}} z_{i,j}^{2} + \tilde{\theta}_{i}^{\mathrm{T}} \Gamma_{i}^{-1} \tilde{\theta}_{i} + \gamma_{i,1}^{-1} \tilde{\psi}_{i}^{2} + \gamma_{i,2}^{-1} \tilde{\rho}_{i}^{2}\right) > b_{0}$$
(53)

时, 有 $\dot{V}$  < 0. 给定系统任意有界的初始状态, 即V(0)有界, 则V(t)有界, 且闭环系统(42)的所有解最终都 将一致收敛于紧集 $\Omega$ 内, 这也就是说, 闭环系统的信 号 $z_{i,j}$ ,  $\theta_i$ ,  $\psi_i$ ,  $\rho_i$ 均有界. 根据式(29)和式(30)–(40), 可 知 $\alpha_{i,j}$ ,  $u_{i0}$ 有界. 再由式(26)知 $x_{i,j}$ 均有界. 因此, 闭环 系统(42)为GUUB.

更进一步,由式(50)可得  

$$a_0 z_{i,1}^2 \leqslant$$
  
 $a_0 \sum_{i=1}^N (\sum_{j=1}^{n_i} c_{i,j} z_{i,j}^2) \leqslant$   
 $-\dot{V} - a_0 \sum_{i=1}^N (\tilde{\theta}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{\theta}_i + \gamma_{i,1}^{-1} \tilde{\psi}_i^2 + \gamma_{i,2}^{-1} \tilde{\rho}_i^2) + b_0 \leqslant$   
 $-\dot{V} + b_0,$  (54)

于是可得

$$a_0 \int_0^t z_{i,1}^2(\delta) \mathrm{d}\delta \leqslant -\int_0^t \dot{V}(\delta) \mathrm{d}\delta + \int_0^t b_0 \mathrm{d}\delta \leqslant V(0) - V(t) + b_0 t, \tag{55}$$

根据V(0)和V(t)的有界性,可得

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t z_{i,1}^2(\delta) \mathrm{d}\delta \leqslant \frac{b_0}{a_0},\tag{56}$$

即跟踪误差 $z_{i,1}$ 最终将一致收敛到包含原点在内的小 邻域内. 由式(51)和式(56)可知, 增大 $a_0$ , 减小 $b_0$ , 使得 这一邻域变小, 即系统的跟踪性能得到增强. 也就是 说, 通过增大参数 $c_{i,j}$ ,  $\frac{r_i}{2\lambda_{\max}(\Gamma_i^{-1})}$ ,  $\frac{1}{2}r_i\gamma_{i,1}$ ,  $\frac{1}{2}r_i\gamma_{i,2}$  的值, 减小参数 $r_i$ 的值, 可以改善闭环系统的跟踪性 能.

上述讨论可总结出本文的稳定性定理.

**定理1** 考虑由时滞关联大系统(1)、控制器(40) 和参数自适应律(41)构成的闭环系统. 在假设1–2的条件下,给定系统任意有界的初始状态,则下述特性成立:

1) 式(16)所示的所有类型的执行器故障均可在线 补偿, 且闭环系统为GUUB;

2) 跟踪误差 $z_{i,1} = y_{i,1} - y_{i,r}$ 一致收敛到包含原

点在内的小邻域内.

#### 5 仿真实例(Simulatin examples)

为验证本文所提出的控制方案的有效性,现将本 文方法应用到下述关联时滞大系统中. 该模型两个子 系统分别由[*u*<sub>1,1</sub> *u*<sub>1,2</sub> *u*<sub>1,3</sub>]和[*u*<sub>2,1</sub> *u*<sub>2,2</sub> *u*<sub>2,3</sub>]驱动,执 行器的输入为系统控制器的输出*u*<sub>i0</sub>.

$$\begin{cases} \dot{x}_{1,1} = x_{1,2} + \sin x_{1,1} + h_{1,1}(y_{\rm d}), & (57) \\ \dot{x}_{1,2} = \omega_1^{\rm T} u_1 + x_{1,1} + x_{1,2} + h_{1,2}(y_{\rm d}), & \\ \dot{x}_{2,1} = x_{2,2} + 0.1 x_{2,1}^3 + h_{2,1}(y_{\rm d}), & \\ \dot{x}_{2,2} = \omega_2^{\rm T} u_2 + x_{2,1} x_{2,2} + h_{2,2}(y_{\rm d}), & \end{cases}$$

系统输出为 $y_1 = x_{1,1}, y_2 = x_{2,1}$ . 参考信号的选择为  $y_{1,r}(t) = \sin t + \sin(0.5t) \pi y_{2,r}(t) = \sin(0.5t)$ ,系 统时滞选为 $d_1(t) = 1.6(1 + \sin t) \pi d_2(t) = 1.6(1 - \cos t)$ . 未知关联函数选择为

$$\begin{split} h_{1,1} &= y_1(t - d_1(t)) + y_2(t - d_2(t)), \\ h_{1,2} &= y_1(t - d_1(t)) + \sin(y_2(t - d_2(t))), \\ h_{2,1} &= y_1^3(t - d_1(t)) + \sin(y_2(t - d_2(t))), \\ h_{2,2} &= \tanh(y_1(t - d_1(t))) + \sin(y_2(t - d_2(t))), \\ \omega_1 &= [6 \ 6 \ 6]^{\mathrm{T}}, \ u_1 &= [u_{1,1} \ u_{1,2} \ u_{1,3}], \\ \omega_2 &= [5 \ 5 \ 5]^{\mathrm{T}}, \ u_2 &= [u_{2,1} \ u_{2,2} \ u_{2,3}]. \end{split}$$

模糊隶属度函数选择为

$$\mu_{h_{i,j}^l}(y_{k,r}) = e^{-10(y_{k,r}+0.2l-1)^2}$$

$$l = 0, \cdots, 9, \ i = 1, 2, j = 1, 2, k = 1, 2.$$

模糊基函数选择为

$$\phi_{i,j,l} = \prod_{k=1}^{2} \mu_{h_{i,j}^{l}}(y_{k,r}) / \sum_{l=1}^{10} (\prod_{k=1}^{2} \mu_{h_{i,j}^{l}}(y_{k,r})),$$
  
$$\phi_{i,j} = [\phi_{i,j,1} \cdots \phi_{i,j,M}]^{\mathrm{T}}.$$

根据式(12), 可构造模糊逼近器 $\hat{h}_{i,j}(y_r)$ .

分散控制器的设计参数选择如下: 对于子系统1, 有 $c_{1,1} = 25, c_{1,2} = 25, \gamma_{1,1} = 10, \gamma_{1,2} = 10, \Gamma_1 = 10I,$  $r_1 = 5, \delta_{1,2} = 0.1;$  对于子系统2,有 $c_{2,1} = 25, c_{2,2} = 25, \gamma_{2,1} = 10, \gamma_{2,2} = 10, \Gamma_2 = 10I, r_2 = 5, \delta_{2,2} = 0.1.$ 式(21)中的比例函数选为 $b_{1,k}(x_1) = b_{2,k}(x_2) = 0.5, k = 1, 2, 3.$  仿真结果如下:

1) 在仿真过程中,执行器故障模型选择如下. 对 于子系统1来说,当t > 10时发生LOE, $u_{1,1} = 0.8u_{1,1}^c$ ; 当t > 8时发生HOF, $u_{1,2} = -100$ ;当t > 5时发生 LIP, $u_{1,3} = 13$ .对于子系统2来说,当t > 6时发生 LOE, $u_{2,1} = 0.6u_{2,1}^c$ ;当t > 12时发生LIP, $u_{2,2} = 5$ ;当 t > 9时发生float, $u_{2,3} = 0$ .上述 $u_{i,k}^c$ 由式(21)确定. 系统初始状态选择为 $x_{1,1}(0) = 1, x_{2,1}(0) = -1$ .仿 真结果如图1-4所示,其中:图1和图3为子系统输出跟 踪参考信号仿真曲线,图2和图4为执行器输出曲线. 从这些曲线图上可以看出,在发生所有4类执行器故 障的情况下,本文方法可以保证闭环系统的所有信号 有界,同时跟踪误差可以收敛到原点附近的小邻域内.









图 2 执行器输出 $u_{1,1}, u_{1,2}$ 和 $u_{1,3}$ Fig. 2 Outputs of actuators  $u_{1,1}, u_{1,2}$  and  $u_{1,3}$ 



图 3 系统输出y2和参考信号y2,r









2) 参数选择对仿真结果的影响. 仿真结果如图 5-6所示. 由于在式(56)中,参数ri的增大或减小会导  $r_i$  $\overline{r_1}, \frac{1}{2}r_i\gamma_{i,1}, \frac{1}{2}r_i\gamma_{i,2}$ 的值相应增大或 致参数  $2\lambda_{\max}(\overline{\Gamma_i})$ 减小,所以作者在本实例中采用固定 $r_i$ 而改变 $\Gamma_i$ ,  $\gamma_{i,1}$ 和 $\gamma_{i,2}$ 的方式来考证这些参数对控制效果的影响. 图5-6中,保持 $r_i = 5$ 和 $\delta_{i,2} = 0.1$ 不变,第1组参数选 择为 $c_{i,j} = 5, \gamma_{i,j} = 2, \Gamma_i = 2I, i = 1, 2, j = 1, 2, 所$ 得仿真结果如y1(1)和y2(1)所示; 第2组参数选择 为 $c_{i,j} = 15, \gamma_{i,j} = 6, \Gamma_i = 6I,$ 所得仿真结果如 $y_1(2)$ 和 $y_2(2)$ 所示; 第3组参数选择为 $c_{i,j} = 25, \gamma_{i,j} = 10,$  $\Gamma_i = 10I$ ,所得仿真结果如 $y_1(3)$ 和 $y_2(3)$ 所示.从图 中可以看出, 增大设计参数 $c_{i,j}$ ,  $\gamma_{i,j}$ 和 $\Gamma_i$ 的值, 即增大 式(56)中a0的值,可以显著改善闭环系统的跟踪性能.



## 结论(Conclusions)

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本文讨论了一类关联时滞大系统的自适应模糊容 错控制问题.所提出的控制方案能够在线补偿所有常 见的4种执行器故障.通过代换的方法,使得模糊逼近 器的输入信号为有界的参考信号,从而保证了闭环系 统所有信号的全局稳定性.这种代换还使得控制器的 设计不再依赖于对时滞信号所做的假设,大大增加了 控制器设计的便易性.本文方法可以直接推广到系统 输出含有时滞的其他不确定系统中,如输出反馈系 统、控制增益不为1的系统等.对含有时变或随机执行 器故障的非线性时滞系统的容错控制则是作者下一 步研究的方向.

#### 参考文献(References):

- NGUANG S K. Robust stabilization of a class of time-delay nonlinear systems [J]. *IEEE Transactions on Automatic Control*, 2000, 45(4): 756 – 762.
- [2] GUO T, LIU G Y. Adaptive fuzzy control for unknown nonlinear time-delay systems with virtual control functions [J]. *International Journal of Control, Automation and Systems*, 2011, 9(6): 1227 – 1234.
- [3] GUO T, WANG A M. Simplified output feedback stabilization for time-delay interconnected systems based on dynamic surface control
   [J]. *International Review on Computers and Software*, 2012, 7(1): 275 – 282.
- [4] WU J, CHENW S, LI J. Fuzzy-approximation-based global adaptive control for uncertain strict-feedback systems with a priori known tracking accuracy [J]. *Fuzzy Sets and Systems*, 2015, 273(1): 1 – 25.
- [5] LIU Yonghua, HUANG Liangpei, XIAO Dongming, et al. Dynamic state feedback stabilization for a class of nonaffine nonlinear sstems with unknown time delays [J]. *Control Theory & Applications*, 2016, 33(7): 923 928.
  (刘勇华, 黄良沛, 肖东明, 等. 一类非仿射非线性时滞系统的动态状

态反馈镇定 [J]. 控制理论与应用, 2016, 33(7): 923 – 928.)

- [6] HUANG Dong, SUN Guofa. Active disturbance rejection dynamic surface control of time-delay system [J]. *Control Theory & Applications*, 2016, 33(11): 1501 1507.
  (黄东, 孙国法. 时延系统的自抗扰动态面控制 [J]. 控制理论与应用, 2016, 33(11): 1501 1507.)
- [7] ZHANG X, LIN Y. Adaptive control of nonlinear time-delay systems with application to a two-stage chemical reactor [J]. *IEEE Transactions on Automatic Control*, 2015, 60(4): 1074 – 1079.
- [8] CHEN W S, ZHANG Z Q. Globally stable adaptive backstepping fuzzy control for output-feedback systems with unknown highfrequency gain sign [J]. *Fuzzy Sets and Systems*, 2010, 161(6): 821 – 836.
- [9] CHEN W S, JIAO L C, WU J S. Globally stable adaptive robust tracking control using rbf neural networks as feedforward compensators [J]. *Neural Computing & Applications*, 2012, 21(2): 351 – 363.
- [10] HUANG J T. Global tracking control of strict-feedback systems using neural networks [J]. *IEEE Transactions on Neural Networks and Learning Systems*, 2012, 23(11): 1714 – 1725.
- [11] HUANG J T. Global adaptive neural dynamic surface control of strictfeedback systems [J]. *Neurocomputing*, 2015, 165(1): 403 – 413.
- [12] FU J, MA R C, CHAI T Y. Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers [J]. *Automatica*, 2015, 54(1): 360 – 373.

- [13] LI P, YANG G H. Backstepping adaptive fuzzy control of uncertain nonlinear systems against actuator faults [J]. *Journal of Control Theory and Applications*, 2009, 7(3): 248 – 256.
- WANG W, WEN C Y. Adaptive actuator failure compensation control of uncertain nonlinear systems with guaranteed transient performance
   [J]. Automatica, 2010, 46(12): 2082 – 2091.
- [15] HASHEMI M, ASKARI J, GHAISARI J, et al. Adaptive compensation for actuator failure in a class of non-linear time-delay systems [J]. *IET Control Theory and Applications*, 2015, 9(5): 710 – 722.
- [16] TONG S C, HUO B Y, LI Y M. Observer-based adaptive decentralized fuzzy fault-tolerant control of nonlinear large-scale systems with actuator failures [J]. *IEEE Transactions on Fuzzy Systems*, 2014, 22(1): 1 – 15.
- [17] CHEN Longsheng, WANG Qi. Adaptive dynamic surface faulttolerant control for uncertain non-affine pure feedback systems with input constraint [J]. Control Theory & Applications, 2016, 33(2): 221 - 227. (陈龙胜, 王琦. 输入受限的非仿射纯反馈不确定系统自适应动态面)

(陈龙胜, 土琦, 输入受限的非伤射纯反顷不哺定系统自适应动态॥ 容错控制 [J]. 控制理论与应用, 2016, 33(2): 221 – 227.)

- [18] WANG L X. Adaptive Fuzzy Systems and Control: Design and Stability Analysis [M]. New Jersey: Prentice-Hall, 1994.
- [19] TANG X D, TAO G, JOSHI S M. Adaptive actuator failure compensation for nonlinear MIMO systems with an aircraft control application [J]. Automatica, 2007, 43(11): 1869 – 1883.
- [20] TONG S C, LI Y, LI Y M, et al. Observer-based adaptive fuzzy backstepping control for a class of stochastic nonlinear strict-feedback systems [J]. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 2011, 41(6): 1693 – 1704.
- [21] KRSTIC M, KOKOTOVIC P V, KANELLAKOPOULOS I. Nonlinear and Adaptive Control Design [M]. New York: John Wiley & Sons, 1995.
- [22] POLYCARPOU M. Stable adaptive neural control scheme for nonlinear systems [J]. *IEEE Transactions on Automatic Control*, 1996, 41(3): 447 – 451.

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