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一类3阶非线性系统的非奇异终端滑模控制

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摘要: 针对传统非奇异终端滑模控制方法不适用于3阶系统的问题, 提出一类具有不确定和外干扰的3阶非线性系统的新型非奇异终端滑模控制方法. 该方案首先结合backstepping控制中的动态面方法和传统2阶非奇异终端滑模控制构造非奇异3阶终端滑模面, 首次提出采用高阶滑模微分器估计值代替控制器中的负指数项. 采用非线性干扰观测器任意精度地估计不确定和干扰, 设计控制器中的补偿项. 采用终端吸引子函数做趋近律避免抖振的同时能保证有限时间趋近滑模面. 基于有限时间稳定李雅普诺夫定理证明了被控状态将在有限时间内收敛到任意小的闭球内. 所提出方案快于传统的递阶线性滑模控制和其他非奇异终端滑模控制. 仿真中与其他滑模控制方案对比, 总误差减小18%以上, 超调及收敛时间也显著下降.

关键词: 非奇异; 滑模控制; 微分器; 非线性系统; 动态面

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Nonsingular terminal sliding mode control for a class of 3-order nonlinear systems

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Abstract: Traditional nonsingular terminal sliding mode control cannot be used for 3-order systems. To solve this problem, a novel nonsingular terminal sliding mode control for a class of 3-order nonlinear systems with uncertainties and disturbances is proposed. Firstly, the dynamic surface of Backstepping control is combined with 2-order nonsingular terminal sliding mode control (TSMC) to construct the 3-order nonsingular terminal sliding modes. Then, the approximations of negative fractional exponential terms are obtained by higher-order sliding mode differentiator (HOSMD) to eliminate the singularity. Then, nonlinear disturbance observer (NDO) is used to approximate unknown uncertainties and disturbance. Terminal attractor is used as reaching law to avoid controller chattering. Based on finite time stability Lyapunov theorem, it is proved that the proposed scheme will force system states into an arbitrary small neighborhood including the origin in finite time. The proposed scheme has faster convergence speed than recursive linear sliding mode control (RLSMC) and other nonsingular TSMC (NSTSMC). In simulation, the total error of the proposed method decreased at least 18% compared with other sliding mode controllers. Overshoot and convergent time are also decreased significantly.

Key words: nonsingular; sliding mode control; differentiator; nonlinear systems; dynamic surface

1 引言(Introduction)

相比其他现代非线性控制方法, 异端滑模控制器(terminal sliding mode control, TSMC)通过将滑模面和终端吸引子函数结合实现有限时间稳定控制, 这是从渐近稳定控制、指数稳定控制到有限时间稳定控制的飞跃[1–2]. 而TSMC易于设计、运算量小的优点使其被广泛应用于磁轴承系统[3]、自动船舶系统[4]、电动汽车[5]等, 并取得理想的控制效果. 后续研究中进一步将线性项结合到TSMC中以加快远离原点阶段的收敛速度, 得到快速TSMC(fast TSMC, FTSMC)[6–7]. FTSMC在全论域内均有较快的收敛速度. 但不论TSMC还是FTSMC用于2阶系统及更高阶系统时均存在奇异性问题[8]. 该问题的本质在于状态或滑模面接近0点时, 控制器中含有的负指数项使得控制器趋向于无穷大. 相比之下, TSMC的抖振问题、非匹配不确定问题、抗干扰问题等均已有较理想的解决方案, 而奇异性问题则仍然是TSMC最难以解决的现存问题, 严重阻碍了TSMC的发展和应用. 其难点在于既要避

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免控制器出现负指数项, 又要保证TSMC有限时间控制的能力. 因此吸引了很多学者开展相关研究. 其中最具标志性和开创性的是冯勇教授在文献[9]提出的2阶系统非奇异TSMC方案. 他将终端吸引子指数倒置并置于1阶微分项中, 使得滑模面一次微分后不会出现负指数项, 而微分方程的动力学特性仍然具有有限时间收敛的能力, 并在理论上证明了二维平面中仅在过原点的一条直线上不满足非奇异性, 但进一步证明了该直线并非空间中的吸引区, 因此并不影响整体的有限时间控制性能. 文献[6, 10]尝试将文献[9]的方案扩展到n阶系统, 但控制器中的负指数项仍然存在. 文献[11]虽然去除了负指数项避免了奇异性, 但是控制器也失去了有限时间收敛的能力. 文献[12]较为理想地解决了滑模面的奇异性, 但控制器的奇异性仍然有待解决. 文献[13–14]采用饱和函数限制控制器输出, 是一种非常实用且工程化的方案, 但是饱和函数会削弱系统的鲁棒性和使系统动态性能变差. 因此用于高阶系统时, 饱和函数层层限制控制器(含虚拟控制器)输出, 也就层层削弱了控制能力. 另外, 该方法中被控系统状态有界性的理论证明也极其繁杂. 所以3阶以及更高阶系统的非奇异TSMC仍然是极具挑战的问题.

考虑到2阶系统的TSMC含滑模面1次微分, 3阶系统TSMC含滑模面2次微分, 所以3阶系统才是非奇异TSMC问题的分界点, 是更高阶系统非奇异TSMC的代表. 因此本文设计一类带有不确定和外扰动的3阶非线性系统的非奇异TSMC. 第2节中首先给出TSMC的奇异性问题, 并分析文献[9]的方法不能推广到3阶系统的原因. 第3节受backstepping控制动态面方法的启发, 采用文献[15]的高阶滑模微分器(higher-order sliding mode differentiator, HOSMD)设计了一类3阶单输入单输出(single input single output, SISO)非线性系统的新型非奇异TSMC. 第4 节将该方法推广到具有不确定和外干扰的3阶多输入多输出(multiple input multiple output, MIMO)非线性系统, 并采用非线性干扰观测器(nonlinear disturbance obsever, NDO)估计未知复合干扰. 采用终端吸引子函数做趋近律, 保证滑模面快速到达并避免抖振. 使得提出方法广泛适用于一般3阶非线性系统. 第5节仿真中定性及定量地证明了相对已有非奇异TSMC 以及其他一些递阶滑模方法, 本文提出方案在控制精度、收敛快速性、减小超调方面均有显著提高. 第6节对全文所做工作和未来可能工作方向提出总结.

本文的出发点是解决3阶系统的TSMC奇异性问题, 但最重要贡献在于研究有限时间控制方法的融合. 融合的意义和目的: 一是改进各种有限时间控制的缺点, 改善控制性能; 二是尽量使得所得结论普适和可推广于各分支方法, 如TSMC、有限时间backstepping、齐次控制、加幂积分控制等. 这些方法之间是有共性和融合基础的, 但当前又缺乏系统性的研究. 以本文所涉及的两种方法为例, 对于高阶非线性系统, 常规的递阶TSMC在滑模面选取中已经暗含了步步反推的步骤, 虚拟控制器集中体现在最后一层子系统的实际控制器之中. 反之, 常规backstepping控制每一层虚拟控制器的选择, 则可看作基于线性滑模面而得到的. 有限时间backstepping控制则对应终端滑模面. 但backstepping与滑模控制结合的当前文献中, 几乎所有方案均采用前$n-1$阶子系统用backstepping设计虚拟控制器, 最后1阶系统用TSMC设计真实控制器, 没有最大程度体现两种方法的优点. 本文所设计的控制器, 是反馈递推地设计每一步非奇异终端滑模面, 这可看作是有限时间backstepping的发展, 因为滑模面是层层反馈设计的, 也可看成递阶非奇异TSMC的改进, 因为既有terminal滑模面, 也有趋近律, 且控制性能优于这两种传统的方法. 实际上, 提出控制方案是直接基于有限时间李氏稳定理论的. 相对传统控制器设计思路, 本文更致力于仅以某一控制指标(如有限时间收敛)作为唯一设计原则, 而不拘泥或限制于某一种具体控制方法. 因为越僵化的控制器设计步骤越不利于复杂高阶系统高性能控制器的灵活设计.

2 问题陈述(Problem statement)

终端吸引子函数$|x|^{\frac{q}{p}}\operatorname{sgn}x$在下文中简化为[16]

$$x^{\frac{q}{p}} \triangleq |x|^{\frac{q}{p}}\operatorname{sgn}x. \tag{1}$$

基于有限时间稳定李雅普诺夫定理可知q和p均为正数且满足$0<q/p<1$.

为将问题陈述清晰, 先考虑简单的3阶SISO系统:

$$\begin{cases}\dot{x}_1 = x_2,\\ \dot{x}_2 = x_3,\\ \dot{x}_3 = u.\end{cases} \tag{2}$$

控制目标是有限时间镇定x_1. 传统的递阶(Recursive TSMC, RTSMC)选择为

$$\begin{aligned}
&s_1 = \dot{x}_1 + k_1 x_1^{\frac{q_1}{p_1}} = x_2 + k_1 x_1^{\frac{q_1}{p_1}},\\
&s_2 = \dot{s}_1 + k_2 s_1^{\frac{q_2}{p_2}} = x_3 + k_1\frac{q_1}{p_1}x_1^{\frac{q_1-p_1}{p_1}}x_2 + k_2 s_1^{\frac{q_2}{p_2}},\\
&\dot{s}_2 = u + k_1\frac{q_1}{p_1}\frac{q_1-p_1}{p_1}x_1^{\frac{q_1-2p_1}{p_1}}x_2^2 +\\
&\qquad k_1\frac{q_1}{p_1}x_1^{\frac{q_1-p_1}{p_1}}x_3 + k_2\frac{q_2}{p_2}s_1^{\frac{q_2-p_2}{p_2}}\times\\
&\qquad (x_3 + k_1\frac{q_1}{p_1}x_1^{\frac{q_1-p_1}{p_1}}x_2),
\end{aligned}$$

其中: $0<q_1<p_1,\ k_1>0,\ 0<q_2<p_2, k_2>0$. 可得控制器如式(3)所示:

$$\begin{aligned}
u = &-k_1\frac{q_1}{p_1}\frac{q_1-p_1}{p_1}x_1^{\frac{q_1-2p_1}{p_1}}x_2^2 -\\
&k_1\frac{q_1}{p_1}x_1^{\frac{q_1-p_1}{p_1}}x_3 - k_2\frac{q_2}{p_2}s_1^{\frac{q_2-p_2}{p_2}}\times
\end{aligned}$$

$$(x_3 + k_1\frac{q_1}{p_1}x_1^{\frac{q_1-p_1}{p_1}}x_2) + f_{\rm r}(s_2),\tag{3}$$

上式: $f_{\rm r}(s_2)$ 为TSMC的趋近律, 可以选择为符号函数[17]、饱和函数[18–19]、终端吸引子函数[20]、或基于智能方法的函数[21–22]. 不论$f_{\rm r}(s_2)$如何选择, 当$x_1=0$或$s_1=0$时总会因为$x_1^{\frac{q_1-p_1}{p_1}}$, $x_1^{\frac{q_1-2p_1}{p_1}}$, $s_1^{\frac{q_2-p_2}{p_2}}$是负指数项使控制器无限大而出现奇异性. 这就是TSMC控制的奇异性问题.

即使将文献[9]提出的方案用于3阶系统(2)也无法避免该问题. 此时滑模面如下:

$$\begin{aligned}s_1 &= \dot x_1^{\frac{p_1}{q_1}} + k_1x_1 = x_2^{\frac{p_1}{q_1}} + k_1x_1,\\ s_2 &= \dot s_1^{\frac{p_2}{q_2}} + k_2s_1 = \\ &(\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}x_3 + k_1x_2)^{\frac{p_2}{q_2}} + k_2s_1.\end{aligned}$$

在上述滑模面中不存在奇异性, 因此文献[9]提出的(nonsingular terminal sliding mode control, NSTSMC)可以避免2阶系统的奇异性. 但对于3阶系统, 奇异性将出现在$\dot s_2$中:

$$\begin{aligned}\dot s_2 =&\frac{p_2}{q_2}(\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}x_3 + k_1x_2)^{\frac{p_2-q_2}{q_2}}\times\\ &(\frac{p_1}{q_1}\frac{p_1-q_1}{q_1}x_2^{\frac{p_1-2q_1}{q_1}}x_3^2 + \frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}u + k_1x_3)+\\ &k_2(\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}x_3 + k_1x_2).\end{aligned}\tag{4}$$

导出的控制器为

$$\begin{aligned}u =&\frac{q_1}{p_1}\frac{q_2}{p_2}x_2^{\frac{q_1-p_1}{q_1}}(\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}x_3 + k_1x_2)^{\frac{q_2-p_2}{q_2}}\times\\ &(-\frac{p_2}{q_2}(\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}x_3 + k_1x_2)^{\frac{p_2-q_2}{q_2}}\times\\ &(\frac{p_1}{q_1}\frac{p_1-q_1}{q_1}x_2^{\frac{p_1-2q_1}{q_1}}x_3^2 + k_1x_3)-\\ &k_2(\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}x_3 + k_1x_2) + f_{\rm r}(s_2)),\end{aligned}\tag{5}$$

其中$x_2^{\frac{q_1-p_1}{q_1}}$和$(\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}x_3 + k_1x_2)^{\frac{q_2-p_2}{q_2}}$是负指数项, 控制器的奇异性仍然存在.

3 3阶SISO系统NSTSMC设计(NSTSMC design for 3-order SISO systems)

基于以上分析可知消除控制器奇异性的关键是避免控制器出现负指数项. 受backstepping控制动态面方法的启示[23–24], 本节将HOSMD作为一种特殊的滤波器用于3阶系统(2)中的NSTSMC设计. 首先给出几个引理.

引理 1[15] 对于n阶滑模微分器(6), 若$f^{(n)}(x)$具有有限的Lipschitz常数$L_{\rm z}$, 则总可以选择恰当参数$h_i>0(i=1,2,\cdots,n)$, $0<q_{\rm z}<p_{\rm z}$使得微分器估计误差$e_{{\rm z}i}=z_i-f^{(i)}(x)(i=0,1,2,\cdots,n)$ 在有限时间内收敛到任意小闭球内且其半径满足式(7).

$$\begin{cases}\dot z_i = v_i,\\ v_i=-h_i|z_i-v_{i-1}|^{\frac{n-i}{n-i+1}}{\rm sgn}(z_i-v_{i-1})+z_{i+1},\\ i=0,1,\cdots,n,\\ v_-1=f(x),\ z_{n+1}=0,\end{cases}\tag{6}$$

$$|e_{{\rm z}i}|\leqslant\frac{L_{\rm z}^{\frac{(n-i+1)p_{\rm z}}{q_{\rm z}}}}{h_n^{\frac{(n-i+1)p_{\rm z}}{q_{\rm z}}}h_{n-1}^{n-i+1}h_{n-2}^{\frac{n-i+1}{2}}\cdots h_i^{\frac{n-i+1}{n-i}}}.\tag{7}$$

引理 2[12, 25–26] 若V满足$\dot V+\alpha V^\beta\leqslant 0(V\neq 0)$, $\alpha>0$, $0<\beta<1$, 则V从初始点$V(0)$收敛到原点的时间是$t\leqslant\frac{1}{\alpha(1-\beta)}|V(0)|^{1-\beta}$, 从初始点收敛到$V_t\in(0,V(0))$的时间是

$$t\leqslant\frac{1}{\alpha(1-\beta)}(|V(0)|^{1-\beta}-|V_t|^{1-\beta}),$$

其中的等号在$\dot V+\alpha V^\beta=0(V\neq 0)$时成立.

引理 3[27] 对于任意正实数$y_i(i=1,2,\cdots,n)$以及$0<b\leqslant 1$, 如下不等式成立:

$$(|y_1|+|y_2|+\cdots+|y_n|)^{\rm b}\leqslant|y_1|^{\rm b}+|y_2|^{\rm b}+\cdots+|y_n|^{\rm b}.$$

在以上准备工作基础上, 设计1阶子系统的滑模面如下:

$$s_1=\dot x_1^{\frac{p_1}{q_1}}+k_1x_1.\tag{8}$$

当$s_1=0$时有$\dot x_1^{\frac{p_1}{q_1}}=-k_1x_1$. 根据引理2可知$x_1$将在有限时间内收敛到0. 设$x_{2{\rm d}}$为$x_2$期望值, 因此$x_{2{\rm d}}$应满足:

$$x_{2{\rm d}}^{\frac{p_1}{q_1}}=-k_1x_1.\tag{9}$$

下一步是设计合适的虚拟控制器使得x_2跟踪上$x_{2{\rm d}}$. 设跟踪误差为

$$e_2=x_2^{\frac{p_1}{q_1}}-x_{2{\rm d}}^{\frac{p_1}{q_1}}.\tag{10}$$

将式(9)代入上式有

$$e_2=x_2^{\frac{p_1}{q_1}}+k_1x_1.\tag{11}$$

上式微分得到

$$\dot e_2=\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}x_3+k_1x_2.\tag{12}$$

选择终端滑模面为$s_2=\dot e_2+k_2e_2^{\frac{q_2}{p_2}}$, $0<\frac{q_2}{p_2}<1$并代入式(12)有$s_2=\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}x_3+k_1x_2+k_2e_2^{\frac{q_2}{p_2}}$. 设$x_3$的希望值为

$$x_{3{\rm d}}=\frac{q_1}{p_1}x_2^{1-\frac{p_1-q_1}{q_1}}(-k_1-k_2x_2(x_2^{\frac{p_1}{q_1}}+k_1x_1)^{\frac{q_2}{p_2}}).\tag{13}$$

式(13)是基于使e_2有限时间内收敛到原点足够小邻域

内这一目标而设计的, 详见定理2证明. 当$q_1/p_1 > 1/2$时, $x_{3\mathrm{d}}$不含负指数项, 因此$x_{3\mathrm{d}}$有界. 再设

$$e_3 = x_3 - x_{3\mathrm{d}}. \tag{14}$$

e_3的导数为

$$\dot{e}_3 = \dot{x}_3 - \dot{x}_{3\mathrm{d}} = u - \dot{x}_{3\mathrm{d}}. \tag{15}$$

若控制器u直接含有$\dot{x}_{3\mathrm{d}}$, 控制器中会出现负指数项. 受动态面方法启示, 采用1阶滑模微分器(16)估计$\dot{x}_{3\mathrm{d}}$, 再将估计值取代原控制器中的$\dot{x}_{3\mathrm{d}}$项.

$$\begin{cases} \dot{z}_0 = -h_0(z_0 - x_{3\mathrm{d}})^{\frac{q_{\mathrm{z}0}}{p_{\mathrm{z}0}}} + z_1, \\ \dot{z}_1 = -h_1(z_1 - \dot{z}_0)^{\frac{q_{\mathrm{z}1}}{p_{\mathrm{z}1}}}, \end{cases} \tag{16}$$

其中z_0和z_1是微分器内部状态. 参数选择满足

$$h_0 > 0,\ h_1 > 0,\ 0 < q_{\mathrm{z}0} < p_{\mathrm{z}0},\ 0 < q_{\mathrm{z}1} < p_{\mathrm{z}1}.$$

基于式(16)有

$$z_1 - \dot{z}_0 = h_0(z_0 - x_{3\mathrm{d}})^{\frac{q_{\mathrm{z}0}}{p_{\mathrm{z}0}}}. \tag{17}$$

再基于式(13)可得$\dot{x}_{3\mathrm{d}}$为

$$\begin{aligned} \dot{x}_{3\mathrm{d}} =& \frac{q_1}{p_1}(1 - \frac{p_1 - q_1}{q_1})x_2^{-\frac{p_1-q_1}{q_1}} x_3 \times \\ & (-k_1 - k_2 x_2 (x_2^{\frac{p_1}{q_1}} + k_1 x_1)^{\frac{q_2}{p_2}}) + \\ & \frac{q_1}{p_1} x_2^{1-\frac{p_1-q_1}{q_1}} (-k_2 x_3 (x_2^{\frac{p_1}{q_1}} + k_1 x_1)^{\frac{q_2}{p_2}} - \\ & k_2 x_2 \frac{q_2}{p_2} (x_2^{\frac{p_1}{q_1}} + k_1 x_1)^{\frac{q_2-p_2}{p_2}} \times \\ & (\frac{p_1}{q_1} x_2^{\frac{p_1-q_1}{q_1}} x_3 + k_1 x_2)). \end{aligned} \tag{18}$$

$\dot{x}_{3\mathrm{d}}$除$x_2 = 0$和$x_2^{\frac{p_1}{q_1}} + k_1 x_1 = 0$外, 在整个空间Lipschitz连续. 基于引理1和式(18)有定理1如下.

定理 1 对于任意给定误差上界, 总存在恰当设计参数$h_i > 0,\ 0 < q_{\mathrm{z}i} < p_{\mathrm{z}i}(i = 1, 2)$使得1阶微分器(16)在论域

$$\Omega_{\mathrm{x}} = \{x_1, x_2, x_3 | x_2^{\frac{p_1}{q_1}} + k_1 x_1 \neq 0, x_2 \neq 0\}$$

内的估计误差$e_{\mathrm{z}0} = z_0 - x_{3\mathrm{d}}, e_{\mathrm{z}1} = z_1 - \dot{x}_{3\mathrm{d}}$在有限时间内收敛到该给定界以内.

证 设$\Omega_{\mathrm{x}} = \Omega_{\mathrm{x}1} \cup \Omega_{\mathrm{x}2} \cup \Omega_{\mathrm{x}3} \cup \Omega_{\mathrm{x}4}$, 其中$\Omega_{\mathrm{x}1}, \Omega_{\mathrm{x}2}, \Omega_{\mathrm{x}3}, \Omega_{\mathrm{x}4}$有如下表达式:

$$\begin{cases} \Omega_{\mathrm{x}1} = \{x_1, x_2, x_3 | x_2^{\frac{p_1}{q_1}} + k_1 x_1 > 0, x_2 > 0\}, \\ \Omega_{\mathrm{x}2} = \{x_1, x_2, x_3 | x_2^{\frac{p_1}{q_1}} + k_1 x_1 > 0, x_2 < 0\}, \\ \Omega_{\mathrm{x}3} = \{x_1, x_2, x_3 | x_2^{\frac{p_1}{q_1}} + k_1 x_1 < 0, x_2 > 0\}, \\ \Omega_{\mathrm{x}4} = \{x_1, x_2, x_3 | x_2^{\frac{p_1}{q_1}} + k_1 x_1 < 0, x_2 < 0\}. \end{cases}$$

考虑式(18), $x_{3\mathrm{d}}, \dot{x}_{3\mathrm{d}}$在$\Omega_{xi}(i = 1, 2, 3, 4)$中均满足引理1的条件因此定理1成立. 证毕.

在有限时间$t_{\mathrm{z}0}, t_{\mathrm{z}1}$内, 存在参数$h_0, h_1, q_{\mathrm{z}0}, p_{\mathrm{z}0}, q_{\mathrm{z}1}, p_{\mathrm{z}1}$使得在论域$\Omega_{\mathrm{x}}$内下式成立:

$$\begin{cases} |e_{\mathrm{z}0}| \leqslant \varepsilon_{\mathrm{z}0}(h_0, h_1, q_{\mathrm{z}0}, p_{\mathrm{z}0}, q_{\mathrm{z}1}, p_{\mathrm{z}1}), \\ |e_{\mathrm{z}1}| \leqslant \varepsilon_{\mathrm{z}1}(h_1, q_{\mathrm{z}1}, p_{\mathrm{z}1}), \end{cases} \tag{19}$$

$\varepsilon_{\mathrm{z}0}, t_{\mathrm{z}0}, \varepsilon_{\mathrm{z}1}, t_{\mathrm{z}1}$是与参数$h_0, h_1, q_{\mathrm{z}0}, p_{\mathrm{z}0}, q_{\mathrm{z}1}, p_{\mathrm{z}1}$有关的任意小常数.

根据式(16)–(17)有

$$\begin{aligned} \dot{z}_0 =& -h_0(z_0 - x_{3\mathrm{d}})^{\frac{q_{\mathrm{z}0}}{p_{\mathrm{z}0}}} - \\ & h_0^{\frac{q_{\mathrm{z}1}}{p_{\mathrm{z}1}}} h_1 \int_0^t (z_0 - x_{3\mathrm{d}})^{\frac{q_{\mathrm{z}0} q_{\mathrm{z}1}}{p_{\mathrm{z}0} p_{\mathrm{z}1}}} \mathrm{d}t. \end{aligned} \tag{20}$$

由于$z_0, x_{3\mathrm{d}}$均已知, 所以$\dot{z}_0$是可得的. 所以进一步可得如下定理:

定理 2 基于1阶滑模微分器(16), 若参数满足$1/2 < q_1/p_1 < 1$, 则3阶系统(2)的非奇异TSMC控制器(21)可在任意有限时间内使x_1收敛到含原点的任意小闭球内:

$$\begin{aligned} u =& \dot{z}_0 - k_3 e_3^{\frac{q_3}{p_3}} = \\ & -h_0(z_0 - x_{3\mathrm{d}})^{\frac{q_{\mathrm{z}0}}{p_{\mathrm{z}0}}} - \\ & h_0^{\frac{q_{\mathrm{z}1}}{p_{\mathrm{z}1}}} h_1 \int_0^t (z_0 - x_{3\mathrm{d}})^{\frac{q_{\mathrm{z}0} q_{\mathrm{z}1}}{p_{\mathrm{z}0} p_{\mathrm{z}1}}} \mathrm{d}t - k_3 e_3^{\frac{q_3}{p_3}}. \end{aligned} \tag{21}$$

证 下述的推导证明过程中, 用e_3, ε_3分别代表$e_3(t), \varepsilon_3(h_0, h_1, q_{\mathrm{z}0}, p_{\mathrm{z}0}, q_{\mathrm{z}1}, p_{\mathrm{z}1}, k_3, q_3, p_3)$以简化过程. $e_{\mathrm{z}1}, e_2, e_1, \varepsilon_{\mathrm{x}2}, \varepsilon_2, \varepsilon_1$有类似用法.

由于式(21)中每一项都无负指数项且可得, 所以控制器非奇异且可得. 将式(21)代入$\dot{e}_3$有

$$\dot{e}_3 = \dot{z}_0 - k_3 e_3^{\frac{q_3}{p_3}} - \dot{x}_{3\mathrm{d}} = e_{\mathrm{z}1} - k_3 e_3^{\frac{q_3}{p_3}}.$$

当$\boldsymbol{x} = [x_1\ \ x_2\ \ x_3]^{\mathrm{T}} \in \Omega_{\mathrm{x}}$时, 由引理2可知在有限时间段

$$t_3 = \frac{p_3}{k_3(p_3 - q_3)}(|e_3(0)|^{\frac{p_3-q_3}{p_3}} - \varepsilon_3^{\frac{p_3-q_3}{p_3}})$$

之后, e_3将收敛到包含原点的某一闭球内, 其半径取决于参数$h_0, h_1, q_{\mathrm{z}0}, p_{\mathrm{z}0}, q_{\mathrm{z}1}, p_{\mathrm{z}1}, k_3, q_3, q_3, p_3$, 即$\exists \varepsilon_3 > 0$满足

$$|e_3| = \varepsilon_3 = (\frac{|e_{\mathrm{z}1}|}{k_3})^{\frac{p_3}{q_3}}. \tag{22}$$

因$|e_{\mathrm{z}1}|$任意小且k_3可任意大, 所以闭球的半径ε_3可任意小. 考虑式(11)–(14)(22)有

$$\begin{aligned} \dot{e}_2 =& \frac{p_1}{q_1} x_2^{\frac{p_1-q_1}{q_1}} (x_{3\mathrm{d}} + e_3) + k_1 x_2 = \\ & -k_2 x_2^2 e_2^{\frac{q_2}{p_2}} + \frac{p_1}{q_1} x_2^{\frac{p_1-q_1}{q_1}} e_3 \leqslant \\ & -k_2 x_2^2 (x_2^{\frac{p_1}{q_1}} + k_1 x_1)^{\frac{q_2}{p_2}} + \frac{p_1}{q_1} x_2^{\frac{p_1-q_1}{q_1}} \varepsilon_3. \end{aligned}$$

当$x_2 = 0$时, 由式(22)有$|\dot{x}_2| = |x_3| = |e_3 + x_{3\mathrm{d}}| = |e_3| \geqslant \varepsilon_3$. 这说明$x_2 = 0$不是吸引域. 也即在稳态阶段满足$|x_2| \geqslant \varepsilon_{2\mathrm{x}}$, $\varepsilon_{2\mathrm{x}}$是任意小的正整数. 因此$e_2$持续

收敛直至

$$k_2x_2^2(x_2^{\frac{p_1}{q_1}}+k_1x_1)^{\frac{q_2}{p_2}}\leqslant\frac{p_1}{q_1}x_2^{\frac{p_1-q_1}{q_1}}\varepsilon_3,$$

即满足

$$|e_2|=\varepsilon_2\triangleq|x_2^{\frac{p_1}{q_1}}+k_1x_1|\leqslant|(\frac{p_1\varepsilon_3}{q_1k_2\varepsilon_{2\mathrm{x}}^{\frac{3q_1-p_1}{q_1}}})^{\frac{p_2}{q_2}}|. \tag{23}$$

e_2的收敛时间则为

$$t_2\leqslant\frac{p_2}{k_2\varepsilon_{2\mathrm{x}}^{\frac{3q_1-p_1}{q_1}}(p_2-q_2)}(|e_2(0)|^{\frac{p_2-q_2}{p_2}}-\varepsilon_2^{\frac{p_2-q_2}{p_2}}).$$

由于ε_3任意小以及原点是唯一的平衡点, 则对于任意给定的ε_3总可以选择足够大的k_2使$x_2^{\frac{p_1}{q_1}}+k_1x_1$足够小. 基于式(23), x_1可以滑动到以$\varepsilon_1=\varepsilon_2/k_1$为半径的任意小闭球内. 其收敛时间可以表示为

$$t_1=\frac{p_1}{k_1(p_1-q_1)}(|x_1(0)|^{\frac{p_1-q_1}{p_1}}-\varepsilon_1^{\frac{p_1-q_1}{p_1}}).$$

综上所述, 经过有限时间$t=t_1+t_2+t_3$, x_1在非奇异TSMC(21)作用下收敛到以ε_1为半径的闭球内.

证毕.

注 1　引理1是充分非必要结论. 因此尽管在

$$\bar{\Omega}_{\mathrm{x}}=\{x_1,x_2,x_3|x_2^{\frac{p_1}{q_1}}+k_1x_1=0,\ x_2=0\}$$

内Lipschitz条件并不满足, 但滑模微分器的跟踪误差仍然是有界的. 因为当$x\in\bar{\Omega}_{\mathrm{x}}$时, 存在正常数$\varepsilon_{\mathrm{H}}$满足$|z_0-x_{3\mathrm{d}}|=\varepsilon_{\mathrm{H}}$. 该误差将使得控制器偏离式(21)并驱使$\boldsymbol{x}$在无限小的时间段后从$\bar{\Omega}_{\mathrm{x}}$进入$\Omega_{\mathrm{x}}$. 所以HOSMD输出始终有界.

注 2　在式(11)中, e_2被设计为$e_2=x_2^{\frac{p_1}{q_1}}-x_{2\mathrm{d}}^{\frac{p_1}{q_1}}$而不是$e_2=x_2-x_{2\mathrm{d}}$, 否则

$$\begin{aligned}&\dot{e}_2=\dot{x}_2-\dot{x}_{2\mathrm{d}}=x_3+((k_1x_1)^{\frac{q_1}{p_1}})'=\\&x_3+k_1^{\frac{q_1}{p_1}}\frac{q_1}{p_1}x_1^{\frac{q_1-p_1}{p_1}}x_2\end{aligned}$$

存在负指数项$x_1^{\frac{q_1-p_1}{p_1}}$, 则控制器设计中需要额外的HOSMD, 增大了总体的运算量.

注 3　一个控制算法是否属于TSMC应当根据以下3个指标判断: 1) 含有终端吸引子函数; 2) 存在收敛的滑模面; 3) 有限时间稳定或收敛. 本节所提出的NSTSMC满足所有要求.

4　具有不确定和外干扰的3n阶MIMO非线性系统NSTSMC设计(NSTSMC design for 3n-order MIMO nonlinear systems with uncertainties and external disturbance)

将第3节提出的控制方案用于具有不确定和外干扰的3n阶MIMO非线性系统(24):

$$\begin{cases}\dot{\boldsymbol{x}}_1=\boldsymbol{x}_2,\\\dot{\boldsymbol{x}}_2=\boldsymbol{x}_3,\\\dot{\boldsymbol{x}}_3=\boldsymbol{f}(\boldsymbol{x})+\boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}+\boldsymbol{d}(\boldsymbol{x},t),\end{cases} \tag{24}$$

其中: $\boldsymbol{x}_i\in\mathbb{R}^n,\ i=1,2,3;\ \boldsymbol{x}=[\boldsymbol{x}_1^{\mathrm{T}}\ \boldsymbol{x}_2^{\mathrm{T}}\ \boldsymbol{x}_3^{\mathrm{T}}]^{\mathrm{T}};\boldsymbol{u}\in\mathbb{R}^n$; $\boldsymbol{f}(\boldsymbol{x}):\mathbb{R}^{3n}\to\mathbb{R}^n$和$\boldsymbol{g}(\boldsymbol{x}):\mathbb{R}^{3n}\to\mathbb{R}^{n\times n}$是$\boldsymbol{x}$的已知函数; 式(24)的输出是$\boldsymbol{x}_1$; $\boldsymbol{d}(\boldsymbol{x},t)\in\mathbb{R}^n$包含系统不确定和外部干扰; $\|\boldsymbol{d}(\boldsymbol{x},t)\|$和$\|\dot{\boldsymbol{d}}(\boldsymbol{x},t)\|$上界存在且未知. 设

$$\begin{aligned}&(\boldsymbol{x}_i^{\frac{q}{p}})^{(k)}=[(x_{i,1}^{\frac{q}{p}})^{(k)}\ \ (x_{i,2}^{\frac{q}{p}})^{(k)}\ \ \cdots\ \ (x_{i,n}^{\frac{q}{p}})^{(k)}]^{\mathrm{T}},\\&\qquad i=1,2,3;\ k=1,2,\cdots,\end{aligned} \tag{25}$$

$$\boldsymbol{k}_i^{\frac{p}{q}}=\mathrm{diag}\{k_{i,1}^{\frac{p}{q}},k_{i,2}^{\frac{p}{q}},\cdots,k_{i,n}^{\frac{p}{q}}\}. \tag{26}$$

首先给出用于估计未知复合干扰项的NDO的相关结论如下:

引理 4[28]　若满足$\dot{\boldsymbol{d}}\approx\boldsymbol{0}$, 则NDO(27)的估计误差$\boldsymbol{e}_{\mathrm{NDO}}=\boldsymbol{d}-\hat{\boldsymbol{d}}$渐近稳定. 其中$\boldsymbol{z}_{\mathrm{D}}$是NDO的内部状态; $\boldsymbol{P}(\boldsymbol{x})$为待设计矩阵; $\boldsymbol{L}(\boldsymbol{x})=\dfrac{\partial\boldsymbol{P}(\boldsymbol{x})}{\partial\boldsymbol{x}}$通常设计为正定对称矩阵.

$$\begin{cases}\hat{\boldsymbol{d}}=\boldsymbol{z}_{\mathrm{D}}+\boldsymbol{P}(\boldsymbol{x}),\\\dot{\boldsymbol{z}}_{\mathrm{D}}=-\boldsymbol{L}(\boldsymbol{x})\boldsymbol{z}_{\mathrm{D}}-\boldsymbol{L}(\boldsymbol{x})(\boldsymbol{P}(\boldsymbol{x})+\boldsymbol{f}(\boldsymbol{x})+\boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}).\end{cases} \tag{27}$$

在引理1–4和定理2基础上, 对于3n阶MIMO非线性系统(24)有如下定理.

定理 3　基于一组1阶滑模微分器(16), 以及采用NDO(27)得到未知项$\boldsymbol{d}$的估计值$\hat{\boldsymbol{d}}$, 则总存在恰当参数$k_{ij}>0,\ i=1,2;\ j=1,2,\cdots,n$,

$$\frac{1}{2}<\frac{q_1}{p_1}<1,\ 0<\frac{q_2}{p_2}<1,\ 0<\frac{q_3}{p_3}<1,$$

使得$\boldsymbol{g}(\boldsymbol{x})$可逆的3$n$阶MIMO非线性系统在非奇异TSMC控制器(28)作用下, 系统输出$\boldsymbol{x}_1$在有限时间内收敛到包含原点的任意小半径闭球内.

$$\begin{cases}\boldsymbol{u}=\boldsymbol{g}^{-1}(\boldsymbol{x})(-\boldsymbol{f}(\boldsymbol{x})-\hat{\boldsymbol{d}}+\dot{\boldsymbol{z}}_0-\boldsymbol{k}_3\boldsymbol{e}_3^{\frac{q_3}{p_3}}),\\\boldsymbol{e}_3=\boldsymbol{x}_3-\boldsymbol{x}_{3\mathrm{d}},\\\boldsymbol{x}_{3\mathrm{d}}=-\frac{q_1}{p_1}\boldsymbol{k}_1\boldsymbol{x}_2^{1-\frac{p_1-q_1}{q_1}}-\frac{q_1}{p_1}\boldsymbol{k}_2\bar{\boldsymbol{x}}_2\bar{\boldsymbol{x}}_2^{1-\frac{p_1-q_1}{q_1}}\boldsymbol{e}_2^{\frac{q_2}{p_2}},\\\boldsymbol{e}_2=\boldsymbol{x}_2^{\frac{p_1}{q_1}}+\boldsymbol{k}_1\boldsymbol{x}_1,\\\bar{\boldsymbol{x}}_2=\mathrm{diag}\{x_{2,1},x_{2,2},\cdots,x_{2,n}\},\\\bar{\boldsymbol{x}}_2^{1-\frac{p_1-q_1}{q_1}}=\\\mathrm{diag}\{x_{2,1}^{1-\frac{p_1-q_1}{q_1}},x_{2,2}^{1-\frac{p_1-q_1}{q_1}},\cdots,x_{2,n}^{1-\frac{p_1-q_1}{q_1}}\},\\\bar{\boldsymbol{x}}_2^{\frac{p_1-q_1}{q_1}}=\mathrm{diag}\{x_{2,1}^{\frac{p_1-q_1}{q_1}},x_{2,2}^{\frac{p_1-q_1}{q_1}},\cdots,x_{2,n}^{\frac{p_1-q_1}{q_1}}\}.\end{cases} \tag{28}$$

证

步骤 1　选择Lyapunov函数为$V_1=0.5\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{x}_1$. 由滑模面$\dot{\boldsymbol{x}}_1+\boldsymbol{k}_1^{\frac{q_1}{p_1}}\boldsymbol{x}_1^{\frac{q_1}{p_1}}=\boldsymbol{0}$得到$\boldsymbol{x}_{2\mathrm{d}}^{\frac{p_1}{q_1}}=-\boldsymbol{k}_1\boldsymbol{x}_1$. 如果$\boldsymbol{x}_2=\boldsymbol{x}_{2\mathrm{d}}$, 考虑式(1)和式(25)有

$$\begin{aligned}&\dot{V}_1=\boldsymbol{x}_1^{\mathrm{T}}\dot{\boldsymbol{x}}_1=\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{x}_{2\mathrm{d}}=-\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{k}_1^{\frac{q_1}{p_1}}\boldsymbol{x}_1^{\frac{q_1}{p_1}}\leqslant\\&-\underline{k}_1(|x_{1,1}|^{2\frac{p_1+q_1}{2p_1}}+|x_{1,2}|^{2\frac{p_1+q_1}{2p_1}}+\cdots+|x_{1,n}|^{2\frac{p_1+q_1}{2p_1}}),\end{aligned}$$

其中$\underline{k}_1=\min\{k_{1,1}^{\frac{q_1}{p_1}}, k_{1,2}^{\frac{q_1}{p_1}}, \cdots, k_{1,n}^{\frac{q_1}{p_1}}\}$. 基于引理3, 对于$0 < p_1+q_1/2p_1 < 1$, $\dot V_1$可化为

$$\begin{aligned}\dot V_1 \leqslant & -\underline{k}_1(|x_{1,1}|^2+|x_{1,2}|^2+\cdots+|x_{1,n}|^2)^{\frac{p_1+q_1}{2p_1}} = \\ & -\underline{k}_1 2^{\frac{p_1+q_1}{2p_1}}\times 0.5^{\frac{p_1+q_1}{2p_1}}\|\boldsymbol{x}_1\|^{2^{\frac{p_1+q_1}{2p_1}}} = \\ & -\underline{k}_1 2^{\frac{p_1+q_1}{2p_1}} V_1^{\frac{p_1+q_1}{2p_1}} \triangleq -\alpha_1 V_1^{\beta_1},\end{aligned}\tag{29}$$

其中: $\alpha_1=\underline{k}_1 2^{\frac{p_1+q_1}{2p_1}}$, $\beta_1=(p_1+q_1)/2p_1$, 所以只要满足$\boldsymbol{x}_{2\mathrm{d}}^{\frac{p_1}{q_1}}=-\boldsymbol{k}_1\boldsymbol{x}_1$, V_1将在有限时间内收敛到原点.

步骤 2 设误差$\boldsymbol{e}_2=\boldsymbol{x}_2^{\frac{p_1}{q_1}}-\boldsymbol{x}_{2\mathrm{d}}^{\frac{p_1}{q_1}}=\boldsymbol{x}_2^{\frac{p_1}{q_1}}+\boldsymbol{k}_1\boldsymbol{x}_1$. 选择李氏函数为$V_2=V_1+0.5\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{e}_2$, 从而有

$$\dot V_2=\dot V_1+\boldsymbol{e}_2^{\mathrm{T}}\dot{\boldsymbol{e}}_2=\dot V_1+\boldsymbol{e}_2^{\mathrm{T}}(\frac{p_1}{q_1}\bar{\boldsymbol{x}}_2^{\frac{p_1-q_1}{q_1}}\boldsymbol{x}_3+\boldsymbol{k}_1\boldsymbol{x}_2).$$

第2层的终端滑模面选择为$\dot{\boldsymbol{e}}_2+\boldsymbol{k}_2\boldsymbol{e}_2^{\frac{q_2}{p_2}}=\boldsymbol{0}$, 由其导出的虚拟控制器

$$\boldsymbol{x}_{3\mathrm{d}}=-\frac{q_1}{p_1}\boldsymbol{k}_1\boldsymbol{x}_2^{1-\frac{p_1-q_1}{q_1}}-\frac{q_1}{p_1}\boldsymbol{k}_2\bar{\boldsymbol{x}}_2\bar{\boldsymbol{x}}_2^{1-\frac{p_1-q_1}{q_1}}\boldsymbol{e}_2^{\frac{q_2}{p_2}}.$$

当$\boldsymbol{x}_3=\boldsymbol{x}_{3\mathrm{d}}$时有

$$\begin{aligned}\dot V_2 =& \dot V_1+\boldsymbol{e}_2^{\mathrm{T}}\times \\ & (-\begin{pmatrix} k_{1,1}x_{2,1} & 0 & \cdots & 0 \\ 0 & k_{1,2}x_{2,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & k_{1,n}x_{2,n}\end{pmatrix} - \\ & \begin{pmatrix} k_{2,1}x_{2,1}^2 & 0 & \cdots & 0 \\ 0 & k_{2,2}x_{2,2}^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & k_{2,n}x_{2,n}^2\end{pmatrix}\times \\ & \boldsymbol{e}_2^{\frac{q_2}{p_2}}+\boldsymbol{k}_1\boldsymbol{x}_2) = \\ & \dot V_1-\sum_{i=1}^{n}|e_{2,i}|^{1+\frac{q_2}{p_2}}k_{2,i}x_{2,i}^2 \leqslant \\ & \dot V_1<0\,(\boldsymbol{x}_1\neq\boldsymbol{0}).\end{aligned}\tag{30}$$

所以$\boldsymbol{x}_{3\mathrm{d}}$作用下的$\boldsymbol{x}_1$和$\boldsymbol{e}_2$的稳定性得证. 下面证明$V_2$是有限时间稳定. 类似第3节中的证明, 可知$\bar\Omega_2\triangleq\{\boldsymbol{x}_2|x_{2,i}=0,\exists i\in(1,n)\}$并非吸引域. 再结合式(29)和式(30)有

$$\dot V_2\leqslant-\alpha_1V_1^{\beta_1}-\underline{k}_2\underline{x}_{2,i}^2\sum_{i=1}^{n}|e_{2,i}|^{1+\frac{q_2}{p_2}},$$

其中:

$$\begin{aligned}&\underline{k}_2=\min\{k_{2,1},k_{2,2},\cdots,k_{2,n}\},\\ &\underline{x}_{2,i}^2=\min\{x_{2,1}^2,x_{2,2}^2,\cdots,x_{2,n}^2\},\\ &\dot V_2\leqslant-\alpha_1V_1^{\beta_1}-\underline{k}_2\underline{x}_{2,i}^2\sum_{i=1}^{n}|e_{2,i}|^{2^{\frac{q_2+p_2}{2p_2}}}\leqslant\\ &-\alpha_1V_1^{\beta_1}-\underline{k}_2\underline{x}_{2,i}^2(\sum_{i=1}^{n}|e_{2,i}|^2)^{\frac{q_2+p_2}{2p_2}}.\end{aligned}$$

取$\bar V_2=0.5\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{e}_2$, 有$V_2=V_1+\bar V_2$及

$$\begin{aligned}\dot V_2\leqslant &-\alpha_1V_1^{\beta_1}-\underline{k}_2\underline{x}_{2,i}^2(2\bar V_2)^{\frac{q_2+p_2}{2p_2}}\triangleq\\ &-\alpha_1V_1^{\beta_1}-\alpha_2\bar V_2^{\beta_2},\end{aligned}$$

其中: $\alpha_2=\underline{k}_2\underline{x}_{2,i}^2 2^{\frac{q_2+p_2}{2p_2}}$, $\beta_2=(q_2+p_2)/2p_2$. 设$\alpha_{12}=\min\{\alpha_1,\alpha_2\}$, 有$\dot V_2\leqslant-\alpha_{12}V_1^{\beta_1}-\alpha_{12}\bar V_2^{\beta_2}$.

下面按$V_1,\bar V_2$以及是否$\beta_1\geqslant\beta_2$分6类情况证明总是存在常数$\beta_2^\circ\in(0,1)$使得$\dot V_2\leqslant-\alpha_{12}V_2^{\beta_2^\circ}$成立, 即$V_2$是有限时间稳定的.

情况 1 当$V_1\geqslant1$, $\bar V_2\geqslant1$时, 设$\bar\beta_{12}=\max\{\beta_1,\beta_2\}$, $\underline{\beta}_{12}=\min\{\beta_1,\beta_2\}$, 显然$\bar\beta_{12}\in(0,1)$, $\underline{\beta}_{12}\in(0,1)$且有$-\alpha_{12}V_1^{\beta_1}\leqslant-\alpha_{12}V_1^{\underline{\beta}_{12}}$, $-\alpha_{12}\bar V_2^{\beta_2}\leqslant-\alpha_{12}\bar V_2^{\underline{\beta}_{12}}$成立. 再根据引理3有下式:

$$\begin{aligned}\dot V_2\leqslant&-\alpha_{12}V_1^{\underline{\beta}_{12}}-\alpha_{12}\bar V_2^{\underline{\beta}_{12}}\leqslant\\ &-\alpha_{12}(V_1+\bar V_2)^{\underline{\beta}_{12}}=-\alpha_{12}V_2^{\underline{\beta}_{12}}.\end{aligned}\tag{31}$$

情况 2 当$V_1\geqslant1$, $\bar V_2<1$, $\beta_1\geqslant\beta_2$时, 因为$\bar V_2^{\beta_1}\leqslant\bar V_2^{\beta_2}$, 根据引理3有下式:

$$\begin{aligned}\dot V_2\leqslant&-\alpha_{12}V_1^{\beta_1}-\alpha_{12}\bar V_2^{\beta_1}\leqslant\\ &-\alpha_{12}(V_1+\bar V_2)^{\beta_1}=-\alpha_{12}V_2^{\beta_1}.\end{aligned}\tag{32}$$

情况 3 当$V_1\geqslant1$, $\bar V_2<1$, $\beta_1<\beta_2$时, 则$V_1^{\beta_1}\geqslant1$, $\bar V_2^{\beta_2}\in(0,1)$. 令$V_1^{\beta_1}+\bar V_2^{\beta_2}\triangleq(V_1+\bar V_2)^{\beta_{\mathrm{A}}}=V_2^{\beta_{\mathrm{A}}}$, $\beta_{\mathrm{A}}>0$. 下面要证明$\beta_{\mathrm{A}}>\beta_1$. 在该情况下$V_1\geqslant1$, $\bar V_2<1$, 所以$V_2=V_1+\bar V_2>1$, $V_2^{\beta_{\mathrm{A}}}=V_1^{\beta_1}+\bar V_2^{\beta_2}>1$成立. 又$V_1^{\beta_1}$, $V_2^{\beta_{\mathrm{A}}}$是$\beta_1,\beta_{\mathrm{A}}$的增函数而$\bar V_2^{\beta_2}$是$\beta_2$的减函数. 设$U_{v2}$为$V_2^{\beta_{\mathrm{A}}}$的上界, L_{v2}为$V_2^{\beta_{\mathrm{A}}}$的下界, 于是有

$$\begin{cases}V_2^{\beta_{\mathrm{A}}}\in(V_1^{\beta_1},V_1^{\beta_1}+1),\\ V_2\in(V_1,V_1+1),\end{cases}\Rightarrow$$

$$\begin{cases}V_1^{\beta_1}<L_{v2}=V_1^{\beta_{\mathrm{A}}}<V_1^{\beta_1}+1,\\ V_1^{\beta_1}<U_{v2}=(V_1+1)^{\beta_{\mathrm{A}}}<V_1^{\beta_1}+1,\end{cases}\Rightarrow$$

$$\begin{cases}\beta_1<\beta_{\mathrm{A}}<\log_{V_1}^{V_1^{\beta_1}+1}\triangleq\beta_{\mathrm{C}},\\ \beta_{\mathrm{D}}\triangleq\log_{V_1+1}^{V_1^{\beta_1}}<\beta_{\mathrm{A}}<\log_{V_1+1}^{V_1^{\beta_1}+1}\triangleq\beta_{\mathrm{E}}.\end{cases}$$

由于$\log_{V_1}^{V_1^{\beta_1}}=\beta_1$和$\log_{V_1}^{x}$是$x$的增函数, 故$\beta_{\mathrm{C}}>\beta_1$对于$V_1$总成立. 类似地有$0<\beta_{\mathrm{D}}<\beta_1$和$\beta_{\mathrm{D}}<\beta_{\mathrm{E}}$. 当$\beta_{\mathrm{A}}>\beta_1$, 上式第2个不等式$V_1^{\beta_1}<(V_1+1)^{\beta_{\mathrm{A}}}<V_1^{\beta_1}+1$也成立. 至此$\beta_1<\beta_{\mathrm{A}}$使以上等式均成立. 再选择$\beta_{12}^{\perp}\in(0,\beta_1]$, 显然$\beta_{12}^{\perp}<\beta_{\mathrm{A}}$, 可使不等式$V_1^{\beta_1}+\bar V_2^{\beta_2}\triangleq V_2^{\beta_{\mathrm{A}}}>V_2^{\beta_1}>V_2^{\beta_{12}^{\perp}}$成立, 也即总是存在$0<\beta_{12}^{\perp}<1$可使$V_1^{\beta_1}+\bar V_2^{\beta_2}\geqslant(V_1+\bar V_2)^{\beta_{12}^{\perp}}$成立, 即

$$\dot V_2\leqslant-\alpha_{12}V_2^{\beta_{12}^{\perp}}.\tag{33}$$

情况 4 当$V_1<1$, $\bar V_2\geqslant1$, $\beta_1\geqslant\beta_2$时, 此时V_1, $\bar V_2$的取值区间刚好分别是Case 3中$\bar V_2$, V_1的取值区间. 证明过程与式(33)证明完全相同, 不再赘述. 必然存在

$0<\beta_{12}^{\angle}<1$使得$V_1^{\beta_1}+\bar V_2^{\beta_2}\geqslant(V_1+\bar V_2)^{\beta_{12}^{\angle}}$成立, 即$\dot V_2\leqslant-\alpha_{12}V_2^{\beta_{12}^{\angle}}$.

情况 5 当$V_1<1,\ \bar V_2\geqslant1,\ \beta_1<\beta_2$时, $V_1^{\beta_2}\leqslant V_1^{\beta_1}$成立, 再根据引理3有

$$\begin{aligned}\dot V_2\leqslant&-\alpha_{12}V_1^{\beta_2}-\alpha_{12}\bar V_2^{\beta_2}\leqslant\\&-\alpha_{12}(V_1+\bar V_2)^{\beta_2}=-\alpha_{12}V_2^{\beta_2}.\end{aligned}\tag{34}$$

情况 6 当$V_1<1,\ \bar V_2<1$时, $V_1^{\beta_1}\geqslant V_1^{\bar\beta_{12}},\ \bar V_2^{\beta_2}\geqslant\bar V_2^{\bar\beta_{12}}$成立, 再根据引理3有

$$\begin{aligned}\dot V_2\leqslant&-\alpha_{12}V_1^{\bar\beta_{12}}-\alpha_{12}\bar V_2^{\bar\beta_{12}}\leqslant\\&-\alpha_{12}(V_1+\bar V_2)^{\bar\beta_{12}}=-\alpha_{12}V_2^{\bar\beta_{12}}.\end{aligned}\tag{35}$$

综合式(31)–(36)的结论可知, 总是存在$\beta_2^\circ\in(0,1)$使$\dot V_2\leqslant-\alpha_{12}V_2^{\beta_2^\circ}$在任意情况下成立. 所以若$\boldsymbol x_3=\boldsymbol x_{3\mathrm d}$, V_2将在有限时间内收敛到0. 指数$\beta_2^\circ\in(0,1)$必然是如下6种可能值之一.

$$\beta_2^\circ\in(0,1)=\begin{cases}\underline\beta_{12},V_1\geqslant1,\ \bar V_2\geqslant1,\\\beta_1,\ \ V_1\geqslant1,\ \bar V_2<1,\ \beta_1\geqslant\beta_2,\\\beta_{12}^{\perp},\ V_1\geqslant1,\ \bar V_2<1,\ \beta_1<\beta_2,\\\beta_{12}^{\angle},\ V_1<1,\ \bar V_2\geqslant1,\ \beta_1\geqslant\beta_2,\\\beta_2,\ \ V_1<1,\ \bar V_2\geqslant1,\ \beta_1<\beta_2,\\\bar\beta_{12},\ V_1<1,\bar V_2<1.\end{cases}$$

步骤 3 MIMO的1OSMD设计为

$$\begin{cases}\dot{\boldsymbol z}_0=-\boldsymbol h_0(\boldsymbol z_0-\boldsymbol x_{3\mathrm d})^{\frac{q_{z0}}{p_{z0}}}+\boldsymbol z_1,\\\dot{\boldsymbol z}_1=-\boldsymbol h_1(\boldsymbol z_1-\dot{\boldsymbol z}_0)^{\frac{q_{z1}}{p_{z1}}},\end{cases}\tag{36}$$

其中: $\dot{\boldsymbol z}_i=[z_{i,1}\ \ z_{i,2}\ \ \cdots\ \ z_{i,n}]^{\mathrm T}$, $\boldsymbol h_i=\mathrm{diag}\{h_{i,1},\ h_{i,2},\cdots,h_{i,n}\},i=0,1$. $z_{0,j}(j=1,2,\cdots,n)$与$z_{1,j}(j=1,2,\cdots,n)$是解耦的. 所以可以选择参数$\boldsymbol h_i$和$0<q_{zi}<p_{zi}(i=0,1)$使1阶滑模微分器(37)的误差$\boldsymbol e_{z0}=\boldsymbol z_0-\boldsymbol x_{3\mathrm d}$在有限时间内收敛到任意小闭球内.

设$\boldsymbol e_3=\boldsymbol x_3-\boldsymbol x_{3\mathrm d}$, $\bar V_3=0.5\boldsymbol e_3^{\mathrm T}\boldsymbol e_3$, $V_3=V_2+\bar V_3$, 有

$$\begin{aligned}&\dot V_3=\dot V_2+\boldsymbol e_3^{\mathrm T}\dot{\boldsymbol e}_3=\\&\dot V_2+\boldsymbol e_3^{\mathrm T}(\dot{\boldsymbol z}_0-\boldsymbol k_3\boldsymbol e_3^{\frac{q_3}{p_3}}-\dot{\boldsymbol x}_{3\mathrm d})\leqslant\\&\dot V_2-\underline k_3(|e_{3,1}|^2+|e_{3,2}|^2+\cdots+|e_{3,n}|^2)^{\frac{p_3+q_3}{2p_3}}+\zeta_1\leqslant\\&-\alpha_{12}V_2^{\beta_2^\circ}-\alpha_3\bar V_3^{\beta_3}+\zeta_1\leqslant-\alpha_{123}V_3^{\beta_3^\circ}+\zeta_1,\end{aligned}\tag{37}$$

其中:

$$\underline k_3=\min\{k_{3,1},k_{3,2},\cdots,k_{3,n}\},$$
$$\alpha_3=2^{\beta_3}\underline k_3,\ \beta_3=\frac{p_3+q_3}{2p_3},$$
$$\zeta_1=\|\boldsymbol e_3^{\mathrm T}\|\|\boldsymbol e_{z0}\|,\ \alpha_{123}>0,\ 0<\beta_3^\circ<1.$$

β_3°的定义类似2中β_2°的定义, 不再重复. 选择恰当参数$k_{i,j}(i=1,2,3;\ j=1,\cdots,n)$可以保证$\alpha_{123}$足够大. 再由于$\boldsymbol e_3(0)$有界, 所以$\boldsymbol e_3$有界. 由引理1可知$\|\boldsymbol e_{z0}\|$任意小, 进一步可得$\zeta_1$任意小. 所以存在任意小的$\zeta_2$使$V_3$在有限时间内满足$V_3<\zeta_2$. $\boldsymbol x_1$在控制器(28)作用下收敛到包含原点的足够小的闭球内.

证毕.

注 4 若$\dot{\boldsymbol d}\neq\boldsymbol 0$, NDO的估计误差也可以通过选择$\boldsymbol L(\boldsymbol x)$使其任意小, 并不改变定理3的结论.

注 5 控制器(28)连续, 所以是无抖振的.

注 6 为论述方便, 本文假设$\boldsymbol u$的维数和状态$\boldsymbol x_3$的维数都是n, $\boldsymbol g(\boldsymbol x)$逆存在. 该假设不成立时, 需使用$\boldsymbol g(\boldsymbol x)$的伪逆来求取控制器$\boldsymbol u$的表达式, 如文献[29].

5 仿真(Simulation)

选择如下具有扰动的MIMO非线性系统作为被控对象:

$$\begin{cases}\dot{\boldsymbol x}_1=\boldsymbol x_2,\\\dot{\boldsymbol x}_2=\boldsymbol x_3,\\\dot{\boldsymbol x}_3=\boldsymbol x_1^2+\boldsymbol x_2^2+\boldsymbol x_3+\begin{pmatrix}1+x_{11}^2&0\\0&0.5+x_{22}^2\end{pmatrix}\boldsymbol u+\\\qquad 2\begin{pmatrix}\sin(0.5t)\\\cos(t+0.5)\end{pmatrix},\end{cases}$$

其中: $\boldsymbol x_i=[x_{i1}\ \ x_{i2}]^{\mathrm T}$, $i=1,2,3$. 4种滑模控制器设计分别如下以验证本文方案的结论和改进效果.

方案 1 本文提出方案. 根据定理3, NTSMC控制器设计为

$$\begin{aligned}\boldsymbol u=&\begin{pmatrix}1+x_{11}^2&0\\0&0.5+x_{22}^2\end{pmatrix}^{-1}(-(\boldsymbol x_1^2+\boldsymbol x_2^2+\boldsymbol x_3)-\\&\hat{\boldsymbol d}+\dot{\boldsymbol z}_0-\boldsymbol k_3(\boldsymbol x_3+\frac{q_1}{p_1}\boldsymbol k_1\boldsymbol x_2^{1-\frac{p_1-q_1}{q_1}}+\\&\frac{q_1}{p_1}\boldsymbol k_2\bar{\boldsymbol x}_2\bar{\boldsymbol x}_2^{1-\frac{p_1-q_1}{q_1}}(\boldsymbol x_2^{\frac{p_1}{q_1}}+\boldsymbol k_1\boldsymbol x_1))^{\frac{q_2}{p_2}})^{\frac{q_3}{p_3}},\end{aligned}$$

其中:

$$q_i=3,\ p_i=5,\ i=1,2,3,$$
$$q_{z_j}=3,\ p_{z_j}=5,\ j=1,2,$$
$$\boldsymbol k_1=\boldsymbol k_2=\mathrm{diag}\{1,1\},\boldsymbol k_3=\mathrm{diag}\{4,4\},$$
$$\boldsymbol h_0=\boldsymbol h_1=\mathrm{diag}\{1,1\},$$
$$\boldsymbol z_0(0)=\dot{\boldsymbol z}_0(0)=\boldsymbol z_1(0)=\dot{\boldsymbol z}_1(0)=$$
$$\boldsymbol z_{\mathrm D}(0)=\dot{\boldsymbol z}_{\mathrm D}(0)=\boldsymbol 0,$$
$$\boldsymbol x_1(0)=\boldsymbol x_2(0)=\boldsymbol x_3(0)=[2,2]^{\mathrm T},$$
$$\boldsymbol P(\boldsymbol x_3)=\begin{pmatrix}50x_{31}+\dfrac{90}{3}x_{31}^3\\50x_{32}+\dfrac{90}{3}x_{32}^3\end{pmatrix},$$
$$\boldsymbol L(\boldsymbol x_3)=\begin{pmatrix}50+90x_{31}^2&0\\0&50+90x_{32}^2\end{pmatrix}.$$

方案 2 RLSMC方案. 线性滑模面设计为$\boldsymbol e_{l1}=\boldsymbol x_2+\boldsymbol k_1\boldsymbol x_1$, $\boldsymbol e_{l2}=\dot{\boldsymbol e}_{l1}+\boldsymbol k_2\boldsymbol e_{l1}$. 对应的控制器为

$$\boldsymbol{u}=\begin{pmatrix}1+x_{11}^2 & 0\\ 0 & 0.5+x_{22}^2\end{pmatrix}^{-1}(-\left(\boldsymbol{x}_1^2+\boldsymbol{x}_2^2+\boldsymbol{x}_3\right)-\hat{\boldsymbol{d}}-\boldsymbol{k}_1\boldsymbol{x}_3-\boldsymbol{k}_2\left(\boldsymbol{x}_3-\boldsymbol{k}_1\boldsymbol{x}_2\right)-\boldsymbol{k}_3\mathrm{sgn}\,\boldsymbol{e}_{l2}).$$

所取参数$\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\hat{\boldsymbol{d}},\boldsymbol{x}_1(0),\boldsymbol{x}_2(0),\boldsymbol{x}_3(0)$值与方案1中对应值完全相同. 否则, 容易出现选择参数大的方案控制效果更优的情况, 从而使得方案间的比较不客观. 为消除RLSMC的抖振, 用双曲正切函数

$$-\boldsymbol{k}_3\frac{\mathrm{e}^{\boldsymbol{e}_{l2}}-\mathrm{e}^{-\boldsymbol{e}_{l2}}}{\mathrm{e}^{\boldsymbol{e}_{l2}}+\mathrm{e}^{-\boldsymbol{e}_{l2}}}$$

代替原来的符号函数sgn $\boldsymbol{e}_{l2}$.

方案 3　RTSMC. 滑模面设计为

$$\boldsymbol{s}_1=\boldsymbol{x}_2+\boldsymbol{k}_1\boldsymbol{x}_1^{\frac{q_1}{p_1}},$$
$$\boldsymbol{s}_2=\dot{\boldsymbol{s}}_1+\boldsymbol{k}_2\boldsymbol{s}_1^{\frac{q_2}{p_2}}=\boldsymbol{x}_3+\boldsymbol{k}_1\frac{q_1}{p_1}\bar{\boldsymbol{x}}_1^{\frac{q_1-p_1}{p_1}}\boldsymbol{x}_2+\boldsymbol{k}_2\boldsymbol{s}_1^{\frac{q_2}{p_2}},$$

从而所得控制器为

$$\boldsymbol{u}=\begin{pmatrix}1+x_{11}^2 & 0\\ 0 & 0.5+x_{22}^2\end{pmatrix}^{-1}(-(\boldsymbol{x}_1^2+\boldsymbol{x}_2^2+\boldsymbol{x}_3)-\hat{\boldsymbol{d}}-\boldsymbol{k}_1\frac{q_1}{p_1}\frac{q_1-p_1}{p_1}\bar{\boldsymbol{x}}_1^{\frac{q_1-2p_1}{p_1}}\boldsymbol{x}_2^2-\boldsymbol{k}_1\frac{q_1}{p_1}\bar{\boldsymbol{x}}_1^{\frac{q_1-p_1}{p_1}}\boldsymbol{x}_3-\boldsymbol{k}_2\frac{q_2}{p_2}\boldsymbol{s}_1^{\frac{q_2-p_2}{p_2}}-\boldsymbol{k}_3s_2^{\frac{q_3}{p_3}}).$$

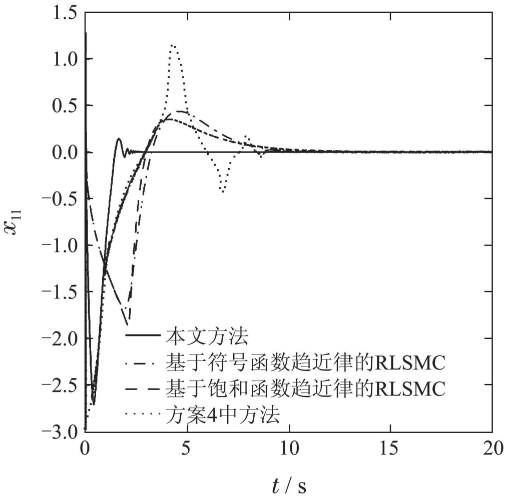
参数选择同前两种方案.

方案 4　文献[14]中的NSTSMC. 滑模面和控制器设计为

$$\boldsymbol{s}=\boldsymbol{x}_3+\boldsymbol{x}_2^{\frac{3}{5}}+\boldsymbol{x}_1^{\frac{3}{7}},$$
$$\boldsymbol{u}_{\mathrm{f}}=-\frac{3}{5}\bar{\boldsymbol{x}}_2^{\frac{3}{5}-1}\boldsymbol{x}_3-\frac{3}{7}\bar{\boldsymbol{x}}_1^{\frac{3}{7}-1}\boldsymbol{x}_2,$$
$$\boldsymbol{u}=-\hat{\boldsymbol{d}}-\boldsymbol{x}_2^3+\mathrm{sat}(\boldsymbol{u}_{\mathrm{f}},3)-20.1\mathrm{sgn}\,\boldsymbol{s}.$$

为了客观地比较, 在该方案中采用NDO以克服干扰的影响.

上述4种方案的仿真结果由图1中给出.



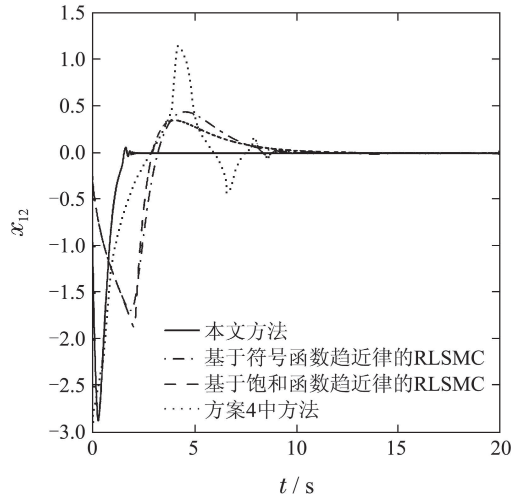


图 1　状态轨迹

Fig. 1　State tracks

本文提出方案NSTSMC相对其他方案具有更快的收敛速度. 这一特性得益于每一层滑模面均具有TSMC的结构. 方案4虽然避免了奇异性, 但是超调时间明显较长. 总误差为$E\triangleq|x_{11}(t)|^2+|x_{12}(t)|^2,\ t\in[0,20]$. 总误差包含了初期误差和稳态误差, 是超调量和控制精度的综合体现. 4种控制方案的总误差由表1给出.本文提出方案具有最小的总误差. 相对其他3种方案减小18%以上.

表 1　总误差分析

Table 1　Total error analysis

	方案 1	方案 2
总误差E	$E_1=9643.4$	$E_2=11422$
比例	—	$r_1=\frac{E_2}{E_1}=118.44\%$
	方案 3	方案 4
总误差E	$E_3=12881$	$E_4=13876$
比例	$r_2=\frac{E_3}{E_1}=133.57\%$	$r_3=\frac{E_4}{E_1}=143.89\%$

6　结论(Conclusions)

现代非线性控制器设计的思路是让控制器某些项精确抵消系统中不稳定或动态性能不够好的项, 再使控制器剩下项驱使被控系统产生期望的运动. 这一思路简洁有效, 但在高阶系统中与有限时间稳定控制设计存在一定的矛盾, 因而造成了TSMC最难解决的奇异性问题. 本文尝试采用融合backstepping控制与TSMC的思路来解决这个问题. 通过结合动态面方法和高阶滑模微分器, 3阶非线性系统TSMC的奇异性问题可以得到较好解决, 同时保留TSMC作用下系统状态有限时间收敛的优点和收敛的快速性. NDO显著增强了系统的鲁棒性, 减小了保守性.基于反步递推的思路, 每一层滑模面均为TSMC滑模面, 保证了系统的

收敛速度, 这比仅设计总的系统为有限时间收敛的方法有更快的收敛速度. 因为后者无法保证每一个子系统的收敛过程, 可能会因为子系统间的耦合而存在反复振荡的现象, 从而影响整体的性能. 仿真证实了理论推导的结论. 本文提出的方案区别于其他现有非奇异TSMC[30], 并有希望推广到更一般的如下系统:

$$\begin{cases} \dot{x}_1 = f_1(\boldsymbol{x}) + g_1(\boldsymbol{x})x_2 + d_1(\boldsymbol{x}, t), \\ \dot{x}_2 = f_2(\boldsymbol{x}) + g_2(\boldsymbol{x})x_3 + d_2(\boldsymbol{x}, t), \\ \qquad\qquad \vdots \\ \dot{x}_n = f_n(\boldsymbol{x}) + g_n(\boldsymbol{x})u + d_n(\boldsymbol{x}, t). \end{cases}$$

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