第 34 卷第 5 期
2017 年 5 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 34 No. 5
May 2017

DOI: 10.7641/CTA.2017.60886

# 带有非线性扰动的时变时滞系统的稳定性准则

武　斌[1†], 王长龙[1], 徐锦法[2], 胡永江[1]

(1. 军械工程学院 无人机工程系, 河北 石家庄 050003;

2. 南京航空航天大学 直升机旋翼动力学国家级重点实验室, 江苏 南京 210016)

**摘要**: 研究了带有非线性扰动的时变时滞系统的稳定性问题. 基于时滞分割方法, 提出了保守性更小的系统稳定性分析准则. 利用一个自由参数将时滞区间分割为2个子区间, 进而构造了含有时滞分割点状态信息和3重积分项的Lyapunov-Krasovskii泛函, 并采用自由矩阵积分不等式界定泛函导数中的交叉项. 基于Lyapunov稳定性定理,得到了以线性矩阵不等式描述的时滞相关型稳定性准则. 数值算例表明该稳定性准则能够得到更大的时滞上界, 与已有结果相比具有更小的保守性.

**关键词**: 区间时变时滞; 稳定性准则; Lyapunov-Krasovskii泛函; 时滞分割

**中图分类号**: TP273　　　**文献标识码**: A

# Stability criteria for time-varying delay systems with nonlinear perturbations

WU Bin[1†], WANG Chang-long[1], XU Jin-fa[2], HU Yong-jiang[1]

(1. UAV Engineering Department, Ordnance Engineering College, Shijiazhuang Hebei 050003, China;

2. National Key Laboratory of Rotorcraft Aeromechanics, Nanjing University of Aeronautics and Astronautics, Nanjing Jiangsu 210016, China)

**Abstract:** The stability problem of systems with time-varying delay and nonlinear perturbations is researched. Based on delay decomposition approach, a less conservative stability criterion is proposed. The delay interval is divided into two subintervals by a free parameter. Then, an appropriate Lyapunov-Krasovskii functional (LKF) containing state information of the partition point and some triple-integral terms is constructed. Cross-terms in the time derivative of LKF is dealt with free-matrix integral inequality. According to Lyapunov stability theory, a delay-dependent stability criteria in the framework of linear matrix inequality (LMI) is obtained. Numerical examples demonstrate that the proposed stability criteria can achieve larger upper bounds and has less conservatism than existing results.

**Key words:** interval time-varying delay; stability criterion; Lyapunov-Krasovskii functional; delay decomposition

## 1 引言(Introduction)

时滞现象[1–7]普遍存在于控制系统中, 而实际系统中不可避免存在着非线性干扰[8–11], 两者严重影响着系统的稳定性. 因此, 带有非线性干扰的时滞系统的稳定性分析受到了广泛的关注.

时滞系统稳定性分析的方法有模型转换法、积分不等式方法、自由权矩阵法、时滞分割法等, 得到的稳定性准则可以分为时滞无关和时滞相关两大类, 保守性是评价稳定性准则性能的主要指标. Zhu等[12–13]人采用Jensen不等式分析了时变时滞系统的稳定性. 文献[14]将改进的Jensen不等式和Wirtinger型双重积分不等式相结合, 得到了新的时滞相关稳定性. 为了更多地利用状态信息, 改善系统的保守性, 文献[15]构造了带有4重积分的Lyapunov-Krasovskii泛函(Lyapunov-Krasovskii functional, LKF), 并将状态向量的2重积分项作为扩增状态向量的一项. 现有研究表明, 时滞划分方法是减小保守性的有效方法. 文献[16–17]将时滞区间$N$等分, 不同之处在于文献[16]将每个子区间端点处的状态都作为扩增状态向量的项, 而文献[17]则是对系统时滞属于不同子区间时的情况分别进

收稿日期: 2016−11−22; 录用日期: 2017−03−30.

†通信作者. E-mail: wubin435@163.com; Tel.: +86 15832105279.

本文责任编委: 陈虹.

河北省自然科学基金项目(2015506004)资助.

Supported by National Natural Science Foundation of Hebei Province (2015506004).

行讨论. 文献[18–19]结合积分不等式和时滞分割技术, 把时滞区间分为2个小区间, 允许子区间长度不相等. 文献[20]按几何序列将时滞区间分成$q-1$个子区间, 前一个子区间的长度是后一个的$\alpha$倍.

本文针对带有非线性扰动的时变时滞系统, 研究其稳定性分析方法. 基于时滞划分方法, 利用自由因子将时滞区间划分为2个长度可以不等的子区间. 选取适当的Lyapunov-Krasovskii泛函, 引入与时滞子区间相关的扩增状态变量, 并采用自由矩阵积分不等式对交叉项进行界定, 得到了系统稳定的充分条件. 数值仿真结果表明了本文所提出方法的有效性, 改善了现有方法的保守性.

文中相关符号说明如下: $P>0(\geqslant 0)$表示$P$是正定(半正定)对称矩阵; $I$表示适当维数的单位矩阵; "$*$"表示对称矩阵的对称部分; $\mathrm{sym}\{X\}=X+X^{\mathrm{T}}$.

## 2　问题描述(Problem description)

考虑如下时滞系统:

$$\begin{cases}\dot{x}(t)=Ax(t)+A_{\mathrm{d}}x(t-d(t))+\Delta,\\ x(t)=\varphi(t),\ t\in[-d_{\mathrm{U}},0],\end{cases}\tag{1}$$

其中: $x(t)$为系统状态向量, $\varphi(t)$为连续函数; $d(t)$为时变状态时滞, 满足$0\leqslant d_{\mathrm{L}}\leqslant d(t)\leqslant d_{\mathrm{U}}$, $\dot{d}(t)\leqslant\mu$; $A$, $A_{\mathrm{d}}$为具有适当维数的确定的系数矩阵; $\Delta$描述了系统的非线性扰动, 表达式如下:

$$\Delta=f(t,x(t))+g(t,x(t-d(t))).$$

不产生混淆时, $f(t,x(t))$和$g(t,x(t-d(t)))$分别简写为$f$, $g$, 假设它们满足以下条件:

$$\begin{cases}f^{\mathrm{T}}f\leqslant\alpha^2x^{\mathrm{T}}(t)F^{\mathrm{T}}Fx(t),\\ g^{\mathrm{T}}g\leqslant\beta^2x^{\mathrm{T}}(t-d(t))G^{\mathrm{T}}Gx(t-d(t)),\end{cases}\tag{2}$$

其中: $F$, $G$是已知矩阵, $\alpha$, $\beta$是正常数.

为了得到系统的稳定性准则, 有必要给出如下引理:

**引理 1**　给定矩阵$M>0$, 对任意$x(t)\in\mathbb{R}^n$, 有下式成立:

$$-(\beta-\alpha)\int_\alpha^\beta x^{\mathrm{T}}(s)Mx(s)\mathrm{d}s\leqslant$$
$$-(\int_\alpha^\beta x^{\mathrm{T}}(s)\mathrm{d}s)M(\int_\alpha^\beta x(s)\mathrm{d}s),$$
$$-\frac{(\beta-\alpha)^2}{2}\int_\alpha^\beta\int_\theta^\beta x^{\mathrm{T}}(s)Mx(s)\mathrm{d}s\mathrm{d}\theta\leqslant$$
$$-(\int_\alpha^\beta\int_\theta^\beta x(s)\mathrm{d}s\mathrm{d}\theta)^{\mathrm{T}}M(\int_\alpha^\beta\int_\theta^\beta x(s)\mathrm{d}s\mathrm{d}\theta).$$

**引理 2**　给定常数$r_2\geqslant r_1\geqslant 0$, 对于矩阵$R>0$, 有如下积分不等式成立:

$$-2\zeta^{\mathrm{T}}(t)W\int_{t-r_2}^{t-r_1}\dot{x}(s)\mathrm{d}s\leqslant$$
$$\zeta^{\mathrm{T}}(t)WR^{-1}W^{\mathrm{T}}\zeta(t)+\int_{t-r_2}^{t-r_1}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s.$$

其中: $\zeta(t)$是任意向量, $W$是适当维数的任意矩阵.

**引理 3**　对可导向量函数$x:\ [\alpha,\beta]\to\mathbb{R}^n$, 若存在适当维数的对称矩阵$R$, $Z_1$, $Z_3$和任意矩阵$Z_2$, $N_1$, $N_2$满足

$$\varGamma=\begin{bmatrix}Z_1&Z_2&N_1\\ *&Z_3&N_2\\ *&*&R\end{bmatrix}\geqslant 0,$$

则

$$-\int_\alpha^\beta\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s\leqslant\varpi^{\mathrm{T}}\varPhi\varpi,$$

其中:

$$\varPhi=(\beta-\alpha)(Z_1+\frac{1}{3}Z_3)+$$
$$\mathrm{sym}\{[N_1-N_2\ -N_1-N_2\ 2N_2]\},$$
$$\varpi^{\mathrm{T}}=[x^{\mathrm{T}}(\beta)\ \ x^{\mathrm{T}}(\alpha)\ \ \frac{1}{\beta-\alpha}\int_\alpha^\beta x^{\mathrm{T}}(s)\mathrm{d}s].$$

**引理 4**　给定常数$\gamma_2\geqslant\gamma_1\geqslant 0$, 对任意满足

$$\gamma_1\leqslant\gamma(t)\leqslant\gamma_2$$

的$\gamma(t)$,

$$\varOmega+(\gamma(t)-\gamma_1)\varXi_1+(\gamma_2-\gamma(t))\varXi_2<0,$$

等价于

$$\begin{cases}\varOmega+(\gamma_2-\gamma_1)\varXi_1<0,\\ \varOmega+(\gamma_2-\gamma_1)\varXi_2<0,\end{cases}$$

其中$\varOmega$, $\varXi_1$, $\varXi_2$是适当维数的任意矩阵.

## 3　稳定性准则(Stability criteria)

引入因子$\eta(0<\eta<1)$, 令

$$d_\delta=d_{\mathrm{U}}-d_{\mathrm{L}},\ d_0=\frac{d_{\mathrm{L}}}{2},$$
$$d_1=d_{\mathrm{L}},\ d_2=d_{\mathrm{L}}+\eta d_\delta,\ d_3=d_{\mathrm{U}},$$

将时滞区间分为$[d_1,d_2]$和$[d_2,d_3]$2个子区间.

**定理 1**　对于系统(1), 当$d(t)\in[d_1,d_2]$时, 若存在矩阵$P>0$, 半正定对称矩阵$Q$, $Q_j(j=1,2,3,4)$, $R_i$, $S_i$, $Z_i(i=1,2,3)$, 对称矩阵$M_1$, $M_4$, $N_1$, $N_4$和适当维数的任意矩阵$M_2$, $M_3$, $M_5$, $N_2$, $N_3$, $N_5$, $T_j(j=1,2,3)$, $W_i$, $Y_i(i=1,2,\cdots,12)$以及非负常数$\varepsilon_i\geqslant 0$ $(i=1,2)$, 使得式(3)–(6)的不等式组成立, 则系统是渐近稳定的.

$$\begin{bmatrix}M_1&M_2&M_3\\ *&M_4&M_5\\ *&*&R_1\end{bmatrix}\geqslant 0,\tag{3}$$

$$\begin{bmatrix}N_1&N_2&N_3\\ *&N_4&N_5\\ *&*&R_3\end{bmatrix}\geqslant 0,\tag{4}$$

$$
\begin{bmatrix}
\varXi_{11} & \varXi_{12} & Q_{12} & \varXi_{14} & \varXi_{15} & -P_{14} & \varXi_{17} & \varXi_{18} & \varXi_{19} & \varXi_{1-10} & T_1 & T_1 & d_sW_1 \\
* & \varXi_{22} & \varXi_{23} & \varXi_{24} & \varXi_{25} & \varXi_{26} & \varXi_{27} & \varXi_{28} & \varXi_{29} & \varXi_{2-10} & \varXi_{2-11} & \varXi_{2-12} & d_sW_2 \\
* & * & \varXi_{33} & \varXi_{34} & -Y_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_sW_3 \\
* & * & * & \varXi_{44} & \varXi_{45} & W_6^{\mathrm{T}} & W_7^{\mathrm{T}} & \varXi_{48} & \varXi_{49} & \varXi_{4-10} & W_{11}^{\mathrm{T}} & W_{12}^{\mathrm{T}} & d_sW_4 \\
* & * & * & * & \varXi_{55} & \varXi_{56} & -Y_7^{\mathrm{T}} & \varXi_{58} & \varXi_{59} & \varXi_{5-10} & -Y_{11}^{\mathrm{T}} & -Y_{12}^{\mathrm{T}} & d_sW_5 \\
* & * & * & * & * & \varXi_{66} & 0 & -P_{24}^{\mathrm{T}} & -P_{34}^{\mathrm{T}} & \varXi_{6-10} & 0 & 0 & d_sW_6 \\
* & * & * & * & * & * & \varXi_{77} & P_{12} & P_{13} & P_{14} & T_3 & T_3 & d_sW_7 \\
* & * & * & * & * & * & * & \varXi_{88} & 0 & 0 & 0 & 0 & d_sW_8 \\
* & * & * & * & * & * & * & * & \varXi_{99} & 0 & 0 & 0 & d_sW_9 \\
* & * & * & * & * & * & * & * & * & \varXi_{10-10} & 0 & 0 & d_sW_{10} \\
* & * & * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & d_sW_{11} \\
* & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I & d_sW_{12} \\
* & * & * & * & * & * & * & * & * & * & * & * & -R_2
\end{bmatrix} < 0, \tag{5}
$$

$$
\begin{bmatrix}
\varXi_{11} & \varXi_{12} & Q_{12} & \varXi_{14} & \varXi_{15} & -P_{14} & \varXi_{17} & \varXi_{18} & \varXi_{19} & \varXi_{1-10} & T_1 & T_1 & d_sY_1 \\
* & \varXi_{22} & \varXi_{23} & \varXi_{24} & \varXi_{25} & \varXi_{26} & \varXi_{27} & \varXi_{28} & \varXi_{29} & \varXi_{2-10} & \varXi_{2-11} & \varXi_{2-12} & d_sY_2 \\
 & & \varXi_{33} & \varXi_{34} & -Y_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_sY_3 \\
 & & & \varXi_{44} & \varXi_{45} & W_6^{\mathrm{T}} & W_7^{\mathrm{T}} & \varXi_{48} & \varXi_{49} & \varXi_{4-10} & W_{11}^{\mathrm{T}} & W_{12}^{\mathrm{T}} & d_sY_4 \\
 & & & & \varXi_{55} & \varXi_{56} & -Y_7^{\mathrm{T}} & \varXi_{58} & \varXi_{59} & \varXi_{5-10} & -Y_{11}^{\mathrm{T}} & -Y_{12}^{\mathrm{T}} & d_sY_5 \\
 & & & & & \varXi_{66} & 0 & -P_{24}^{\mathrm{T}} & -P_{34}^{\mathrm{T}} & \varXi_{6-10} & 0 & 0 & d_sY_6 \\
 & & & & & & \varXi_{77} & P_{12} & P_{13} & P_{14} & T_3 & T_3 & d_sY_7 \\
 & & & & & & & \varXi_{88} & 0 & 0 & 0 & 0 & d_sY_8 \\
 & & & & & & & * & \varXi_{99} & 0 & 0 & 0 & d_sY_9 \\
 & & & & & & & * & * & \varXi_{10-10} & 0 & 0 & d_sY_{10} \\
 & & & & & & & * & * & * & -\varepsilon_1 I & 0 & d_sY_{11} \\
 & & & & & & & * & * & * & * & -\varepsilon_2 I & d_sY_{12} \\
 & & & & & & & * & * & * & * & * & -R_2
\end{bmatrix} < 0, \tag{6}
$$

其中:

$$
P=\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ * & P_{22} & P_{23} & P_{24} \\ * & * & P_{33} & P_{34} \\ * & * & * & P_{44} \end{bmatrix},\ Q=\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix},
$$

$$
M_1=\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ * & M_{14} & M_{15} \\ * & * & M_{16} \end{bmatrix},\ M_3=\begin{bmatrix} M_{31} \\ M_{32} \\ M_{33} \end{bmatrix},
$$

$$
M_4=\begin{bmatrix} M_{41} & M_{42} & M_{43} \\ * & M_{44} & M_{45} \\ * & * & M_{46} \end{bmatrix},\ M_5=\begin{bmatrix} M_{51} \\ M_{52} \\ M_{53} \end{bmatrix},
$$

$$
N_1=\begin{bmatrix} N_{11} & N_{12} & N_{13} \\ * & N_{14} & N_{15} \\ * & * & N_{16} \end{bmatrix},\ N_3=\begin{bmatrix} N_{31} \\ N_{32} \\ N_{33} \end{bmatrix},
$$

$$
N_4=\begin{bmatrix} N_{41} & N_{42} & N_{43} \\ * & N_{44} & N_{45} \\ * & * & N_{46} \end{bmatrix},\ N_5=\begin{bmatrix} N_{51} \\ N_{52} \\ N_{53} \end{bmatrix},
$$

$$
\begin{aligned}
\varXi_{11}={}&P_{12}+P_{12}^{\mathrm{T}}+Q_{11}+d_1^2S_1+(d_2-d_1)^2S_2+\\
&(d_3-d_2)^2S_3-d_1^2Z_1-(d_2-d_1)^2Z_2-\\
&(d_3-d_2)^2Z_3+\varepsilon_1\alpha^2F^{\mathrm{T}}F+T_1A+\\
&A^{\mathrm{T}}T_1^{\mathrm{T}}+d_1^3(M_{11}+\frac{1}{3}M_{41})+d_1^2(M_{31}+\\
&M_{31}^{\mathrm{T}}-M_{51}-M_{51}^{\mathrm{T}}),
\end{aligned}
$$

$$
\varXi_{12}=T_1A_{\mathrm{d}}+A^{\mathrm{T}}T_2^{\mathrm{T}}-W_1+Y_1,
$$

$$
\begin{aligned}
\varXi_{14}={}&-P_{12}+P_{13}+d_1^3(M_{12}+\frac{1}{3}M_{42})-\\
&d_1^2(M_{31}-M_{51}+M_{32}^{\mathrm{T}}-M_{52}^{\mathrm{T}})+W_1,
\end{aligned}
$$

$$
\varXi_{15}=-P_{13}+P_{14}-Y_1,
$$

$$
\varXi_{17}=P_{11}-T_1+A^{\mathrm{T}}T_3^{\mathrm{T}},
$$

$$
\begin{aligned}
\varXi_{18}={}&P_{22}^{\mathrm{T}}+d_1Z_1+d_1^2(M_{13}+\frac{1}{3}M_{43})+\\
&d_1(2M_{51}+M_{33}^{\mathrm{T}}-M_{53}^{\mathrm{T}}),
\end{aligned}
$$

$$
\varXi_{19}=P_{23}+(d_2-d_1)Z_2,
$$

$$
\varXi_{1-10}=P_{24}+(d_3-d_2)Z_3,
$$

$$
\begin{aligned}
\varXi_{22}={}&-(1-\mu)Q_4+T_2A_{\mathrm{d}}+A_{\mathrm{d}}^{\mathrm{T}}T_2^{\mathrm{T}}\\
&-W_2-W_2^{\mathrm{T}}+\ Y_2+Y_2^{\mathrm{T}}+\varepsilon_2\beta^2G^{\mathrm{T}}G,
\end{aligned}
$$

$$
\varXi_{23}=-W_3^{\mathrm{T}}+Y_3^{\mathrm{T}},\ \varXi_{24}=W_2-W_4^{\mathrm{T}}+Y_4^{\mathrm{T}},
$$

$$
\varXi_{25}=-Y_2-W_5^{\mathrm{T}}+Y_5^{\mathrm{T}},
$$

$$\Xi_{26} = -W_6^{\rm T} + Y_6^{\rm T},$$
$$\Xi_{27} = -T_2 + A_{\rm d}^{\rm T}T_3^{\rm T} - W_7^{\rm T} + Y_7^{\rm T},$$
$$\Xi_{28} = -W_8^{\rm T} + Y_8^{\rm T},$$
$$\Xi_{29} = -W_9^{\rm T} + Y_9^{\rm T},$$
$$\Xi_{2-10} = -W_{10}^{\rm T} + Y_{10}^{\rm T},$$
$$\Xi_{2-11} = T_2 - W_{11}^{\rm T} + Y_{11}^{\rm T},$$
$$\Xi_{2-12} = T_2 - W_{12}^{\rm T} + Y_{12}^{\rm T},$$
$$\Xi_{33} = Q_{22} - Q_{11} + Q_1,$$
$$\Xi_{34} = -Q_{12} + W_3,$$
$$\Xi_{44} = -Q_{22} - Q_1 + Q_2 + Q_4 - d_1^2(M_{32} - M_{32}^{\rm T} - M_{52} - M_{52}^{\rm T}) + d_1^3(M_{14} + \frac{1}{3}M_{44}) + W_4 + W_4^{\rm T},$$
$$\Xi_{45} = -Y_4 + W_5^{\rm T},$$
$$\Xi_{48} = -P_{22}^{\rm T} + P_{23}^{\rm T} + d_1^2(M_{15} + \frac{1}{3}M_{45}) + d_1(2M_{52} - M_{33}^{\rm T} - M_{53}^{\rm T}) + W_8^{\rm T},$$
$$\Xi_{49} = -P_{23} + P_{33}^{\rm T} + W_9^{\rm T},$$
$$\Xi_{4-10} = -P_{24} + P_{34} + W_{10}^{\rm T},$$
$$\Xi_{55} = -Q_2 + Q_3 + (d_3 - d_2)(N_{11} + \frac{1}{3}N_{41}) + N_{31} + N_{31}^{\rm T} - N_{51} - N_{51}^{\rm T} - Y_5 - Y_5^{\rm T},$$
$$\Xi_{56} = (d_3 - d_2)(N_{12} + \frac{1}{3}N_{42}) - N_{31} - N_{51} + N_{32}^{\rm T} - N_{52}^{\rm T} - Y_6^{\rm T},$$
$$\Xi_{58} = -P_{23}^{\rm T} + P_{24}^{\rm T} - Y_8^{\rm T},$$
$$\Xi_{59} = -P_{33}^{\rm T} + P_{34}^{\rm T} - Y_9^{\rm T},$$
$$\Xi_{5-10} = -P_{34} + P_{44}^{\rm T} + N_{13} + \frac{1}{3}N_{43} + \frac{1}{(d_3 - d_2)}(2N_{51} + N_{33}^{\rm T} - N_{53}^{\rm T}) - Y_{10}^{\rm T},$$
$$\Xi_{66} = -Q_3 + (d_3 - d_2)(N_{14} + \frac{1}{3}N_{44}) - N_{32} - N_{32}^{\rm T} - N_{52} - N_{52}^{\rm T},$$
$$\Xi_{6-10} = -P_{44}^{\rm T} + N_{15} + \frac{1}{3}N_{45} + \frac{1}{(d_3 - d_2)}(2N_{52} - N_{33}^{\rm T} - N_{53}^{\rm T}),$$
$$\Xi_{77} = d_1^3R_1 + (d_2 - d_1)R_2 + (d_3 - d_2)R_3 - T_3 - T_3^{\rm T} + \frac{d_1^4}{4}Z_1 + \frac{(d_2^2 - d_1^2)^2}{4}Z_2 + \frac{(d_3^2 - d_2^2)^2}{4}Z_3,$$
$$\Xi_{88} = -S_1 - Z_1 + d_1(M_{16} + \frac{1}{3}M_{46}) + 2(M_{53} + M_{53}^{\rm T}),$$
$$\Xi_{99} = -S_2 - Z_2,$$
$$\Xi_{10-10} = -S_3 - Z_3 + \frac{1}{(d_3 - d_2)}(N_{16} + \frac{1}{3}N_{46}) + \frac{2}{(d_3 - d_2)^2}(N_{53} + N_{53}^{\rm T}),$$
$$d_s = \sqrt{(d_2 - d_1)}.$$

**证** 选取如下Lyapunov-Krasovskii泛函:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \tag{7}$$

其中:

$$V_1(t) = \xi^{\rm T}(t)P\xi(t), \tag{8}$$

$$V_2(t) = \int_{t-d_0}^{t}\begin{bmatrix} x(s) \\ x(s-d_0) \end{bmatrix}^{\rm T} Q \begin{bmatrix} x(s) \\ x(s-d_0) \end{bmatrix}{\rm d}s + \int_{t-d_1}^{t-d_0} x^{\rm T}(s)Q_1x(s){\rm d}s + \int_{t-d_2}^{t-d_1} x^{\rm T}(s)Q_2x(s){\rm d}s + \int_{t-d_3}^{t-d_2} x^{\rm T}(s)Q_3x(s){\rm d}s + \int_{t-d(t)}^{t-d_1} x^{\rm T}(s)Q_4x(s){\rm d}s, \tag{9}$$

$$V_3(t) = d_1^2\int_{t-d_1}^{t}\int_{\theta}^{t}\dot{x}^{\rm T}(s)R_1\dot{x}(s){\rm d}s{\rm d}\theta + \int_{t-d_2}^{t-d_1}\int_{\theta}^{t}\dot{x}^{\rm T}(s)R_2\dot{x}(s){\rm d}s{\rm d}\theta + \int_{t-d_3}^{t-d_2}\int_{\theta}^{t}\dot{x}^{\rm T}(s)R_3\dot{x}(s){\rm d}s{\rm d}\theta, \tag{10}$$

$$V_4(t) = d_1\int_{t-d_1}^{t}\int_{\theta}^{t}x^{\rm T}(s)S_1x(s){\rm d}s{\rm d}\theta + (d_2 - d_1)\int_{t-d_2}^{t-d_1}\int_{\theta}^{t}x^{\rm T}(s)S_2x(s){\rm d}s{\rm d}\theta + (d_3 - d_2)\int_{t-d_3}^{t-d_2}\int_{\theta}^{t}x^{\rm T}(s)S_3x(s){\rm d}s{\rm d}\theta, \tag{11}$$

$$V_5(t) = \frac{d_1^2}{2}\int_{-d_1}^{0}\int_{\omega}^{0}\int_{t+\theta}^{t}\dot{x}^{\rm T}(s)Z_1\dot{x}(s){\rm d}s{\rm d}\theta{\rm d}\omega + \frac{d_2^2 - d_1^2}{2}\int_{-d_2}^{-d_1}\int_{\omega}^{0}\int_{t+\theta}^{t}\dot{x}^{\rm T}(s)Z_2\dot{x}(s){\rm d}s{\rm d}\theta{\rm d}\omega + \frac{d_3^2 - d_2^2}{2}\int_{-d_3}^{-d_2}\int_{\omega}^{0}\int_{t+\theta}^{t}\dot{x}^{\rm T}(s)Z_3\dot{x}(s){\rm d}s{\rm d}\theta{\rm d}\omega, \tag{12}$$

$$\xi(t) = [x^{\rm T}(t),\ \int_{t-d_1}^{t}x^{\rm T}(s){\rm d}s,\ \int_{t-d_2}^{t-d_1}x^{\rm T}(s){\rm d}s\ \int_{t-d_3}^{t-d_2}x^{\rm T}(s){\rm d}s]^{\rm T}.$$

将$V(t)$沿着系统(1)对$t$求导, 可得

$$\dot{V}_1(t) = 2\xi^{\rm T}(t)P\dot{\xi}(t) =$$

$$x^{\mathrm{T}}(t)(P_{12}+P_{12}^{\mathrm{T}})x(t)+2x^{\mathrm{T}}(t)P_{11}\dot{x}(t)+$$
$$2x^{\mathrm{T}}(t)(-P_{12}+P_{13})x(t-d_1)+$$
$$2x^{\mathrm{T}}(t)(-P_{13}+P_{14})x(t-d_2)-$$
$$2x^{\mathrm{T}}(t)P_{14}x(t-d_3)+2x^{\mathrm{T}}(t)P_{22}^{\mathrm{T}}\int_{t-d_1}^{t}x(s)\mathrm{d}s+$$
$$2x^{\mathrm{T}}(t-d_1)(-P_{22}^{\mathrm{T}}+P_{23}^{\mathrm{T}})\int_{t-d_1}^{t}x(s)\mathrm{d}s+$$
$$2x^{\mathrm{T}}(t)P_{23}\int_{t-d_2}^{t-d_1}x(s)\mathrm{d}s+$$
$$2x^{\mathrm{T}}(t)P_{24}\int_{t-d_3}^{t-d_2}x(s)\mathrm{d}s+$$
$$2x^{\mathrm{T}}(t-d_1)(-P_{23}+P_{33}^{\mathrm{T}})\int_{t-d_2}^{t-d_1}x(s)\mathrm{d}s+$$
$$2x^{\mathrm{T}}(t-d_1)(-P_{24}+P_{34})\int_{t-d_3}^{t-d_2}x(s)\mathrm{d}s+$$
$$2x^{\mathrm{T}}(t-d_2)(-P_{23}^{\mathrm{T}}+P_{24}^{\mathrm{T}})\int_{t-d_1}^{t}x(s)\mathrm{d}s-$$
$$2x^{\mathrm{T}}(t-d_2)(-P_{33}^{\mathrm{T}}+P_{34}^{\mathrm{T}})\int_{t-d_2}^{t-d_1}x(s)\mathrm{d}s-$$
$$2x^{\mathrm{T}}(t-d_2)(-P_{34}+P_{44}^{\mathrm{T}})\int_{t-d_3}^{t-d_2}x(s)\mathrm{d}s-$$
$$2x^{\mathrm{T}}(t-d_3)P_{24}^{\mathrm{T}}\int_{t-d_1}^{t}x(s)\mathrm{d}s+$$
$$2x^{\mathrm{T}}(t-d_3)P_{34}^{\mathrm{T}}\int_{t-d_2}^{t-d_1}x(s)\mathrm{d}s+$$
$$2x^{\mathrm{T}}(t-d_3)P_{44}^{\mathrm{T}}\int_{t-d_3}^{t-d_2}x(s)\mathrm{d}s+$$
$$2\dot{x}^{\mathrm{T}}(t)P_{12}\int_{t-d_1}^{t}x(s)\mathrm{d}s+$$
$$2\dot{x}^{\mathrm{T}}(t)P_{13}\int_{t-d_2}^{t-d_1}x(s)\mathrm{d}s+$$
$$2\dot{x}^{\mathrm{T}}(t)P_{14}\int_{t-d_3}^{t-d_2}x(s)\mathrm{d}s, \tag{13}$$

$$\dot{V}_2(t)=$$
$$x^{\mathrm{T}}(t)Q_{11}x(t)+x^{\mathrm{T}}(t-d_3)Q_3x(t-d_3)-$$
$$x^{\mathrm{T}}(t-d_0)(Q_{22}-Q_{11}+Q_1)x(t-d_0)+$$
$$x^{\mathrm{T}}(t-d_2)(-Q_2+Q_3)x(t-d_2)-$$
$$(1-\dot{d}(t))x^{\mathrm{T}}(t-d(t))Q_4x(t-d(t))+$$
$$x^{\mathrm{T}}(t-d_1)(-Q_{22}-Q_1+Q_2+Q_4)x(t-d_1)+$$
$$2x^{\mathrm{T}}(t)Q_{12}x(t-d_0)-2x^{\mathrm{T}}(t-d_0)Q_{12}x(t-d_1), \tag{14}$$

$$\dot{V}_3(t)=$$
$$-d_1^2\int_{t-d_1}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\mathrm{d}s+(d_2-d_1)\times$$
$$\dot{x}^{\mathrm{T}}(t)R_2\dot{x}(t)-\int_{t-d_3}^{t-d_2}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)\mathrm{d}s+$$
$$(d_3-d_2)\dot{x}^{\mathrm{T}}(t)R_3\dot{x}(t)+d_1^3\dot{x}^{\mathrm{T}}(t)R_1\dot{x}(t)-$$
$$\int_{t-d_2}^{t-d_1}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\mathrm{d}s, \tag{15}$$

$$\dot{V}_4(t)=-d_1\int_{t-d_1}^{t}x^{\mathrm{T}}(s)S_1x(s)\mathrm{d}s-$$
$$(d_2-d_1)\int_{t-d_2}^{t-d_1}x^{\mathrm{T}}(s)S_2x(s)\mathrm{d}s-$$
$$(d_3-d_2)\int_{t-d_3}^{t-d_2}x^{\mathrm{T}}(s)S_3x(s)\mathrm{d}s+$$
$$(d_2-d_1)^2x^{\mathrm{T}}(t)S_2x(t)+d_1^2x^{\mathrm{T}}(t)\times$$
$$S_1x(t)+(d_3-d_2)^2x^{\mathrm{T}}(t)S_3x(t), \tag{16}$$

$$\dot{V}_5(t)=-\frac{d_1^2}{2}\int_{-d_1}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta-$$
$$\frac{d_2^2-d_1^2}{2}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta-$$
$$\frac{d_3^2-d_2^2}{2}\int_{-d_3}^{-d_2}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_3\dot{x}(s)\mathrm{d}s\mathrm{d}\theta+$$
$$\frac{(d_2^2-d_1^2)^2}{4}\dot{x}^{\mathrm{T}}(t)Z_2\dot{x}(t)+\frac{d_1^4}{4}\dot{x}^{\mathrm{T}}(t)Z_1\dot{x}(t)+$$
$$\frac{(d_3^2-d_2^2)^2}{4}\dot{x}^{\mathrm{T}}(t)Z_3\dot{x}(t). \tag{17}$$

由引理1可得

$$-d_1\int_{t-d_1}^{t}x^{\mathrm{T}}(s)S_1x(s)\mathrm{d}s\leqslant$$
$$-(\int_{t-d_1}^{t}x^{\mathrm{T}}(s)\mathrm{d}s)S_1(\int_{t-d_1}^{t}x(s)\mathrm{d}s), \tag{18}$$

$$-(d_2-d_1)\int_{t-d_2}^{t-d_1}x^{\mathrm{T}}(s)S_2x(s)\mathrm{d}s\leqslant$$
$$-(\int_{t-d_2}^{t-d_1}x^{\mathrm{T}}(s)\mathrm{d}s)S_2(\int_{t-d_2}^{t-d_1}x(s)\mathrm{d}s), \tag{19}$$

$$-(d_3-d_2)\int_{t-d_3}^{t-d_2}x^{\mathrm{T}}(s)S_3x(s)\mathrm{d}s\leqslant$$
$$-(\int_{t-d_3}^{t-d_2}x^{\mathrm{T}}(s)\mathrm{d}s)S_3(\int_{t-d_3}^{t-d_2}x(s)\mathrm{d}s), \tag{20}$$

$$-\frac{d_1^2}{2}\int_{-d_1}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\leqslant$$
$$\begin{bmatrix} d_1x(t) \\ \int_{t-d_1}^{t}x(s)\mathrm{d}s \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -Z_1 & Z_1 \\ Z_1 & -Z_1 \end{bmatrix}\begin{bmatrix} d_1x(t) \\ \int_{t-d_1}^{t}x(s)\mathrm{d}s \end{bmatrix}, \tag{21}$$

$$-\frac{d_2^2-d_1^2}{2}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\leqslant$$
$$\begin{bmatrix} (d_2-d_1)x(t) \\ \int_{t-d_2}^{t-d_1}x(s)\mathrm{d}s \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -Z_2 & Z_2 \\ Z_2 & -Z_2 \end{bmatrix}\begin{bmatrix} (d_2-d_1)x(t) \\ \int_{t-d_2}^{t-d_1}x(s)\mathrm{d}s \end{bmatrix}, \tag{22}$$

$$-\frac{d_3^2-d_2^2}{2}\int_{-d_3}^{-d_2}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_3\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\leqslant$$
$$\begin{bmatrix} (d_3-d_2)x(t) \\ \int_{t-d_3}^{t-d_2}x(s)\mathrm{d}s \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -Z_3 & Z_3 \\ Z_3 & -Z_3 \end{bmatrix}\begin{bmatrix} (d_3-d_2)x(t) \\ \int_{t-d_3}^{t-d_2}x(s)\mathrm{d}s \end{bmatrix}. \tag{23}$$

由引理2可得

$$
\begin{aligned}
&-\int_{t-d_2}^{t-d_1}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\mathrm{d}s=\\
&-\int_{t-d(t)}^{t-d_1}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\mathrm{d}s-\\
&\int_{t-d_2}^{t-d(t)}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\mathrm{d}s-\\
&2\phi^{\mathrm{T}}(t)W\int_{t-d(t)}^{t-d_1}\dot{x}(s)\mathrm{d}s-\\
&2\phi^{\mathrm{T}}(t)Y\int_{t-d_2}^{t-d(t)}\dot{x}(s)\mathrm{d}s+\\
&2\phi^{\mathrm{T}}(t)W[x(t-d_1)-x(t-d(t))]+\\
&2\phi^{\mathrm{T}}(t)Y[x(t-d(t))-x(t-d_2)]\leqslant\\
&2\phi^{\mathrm{T}}(t)W\,[x(t-d_1)-x(t-d(t))]+\\
&2\phi^{\mathrm{T}}(t)Y\,[x(t-d(t))-x(t-d_2)]+\\
&(d(t)-d_1)\phi^{\mathrm{T}}(t)WR_2^{-1}W^{\mathrm{T}}\phi(t)+\\
&(d_2-d(t))\phi^{\mathrm{T}}(t)YR_2^{-1}Y^{\mathrm{T}}\phi(t),
\end{aligned}
\tag{24}
$$

其中:

$$
\begin{aligned}
W^{\mathrm{T}}=&[W_1^{\mathrm{T}}\ W_2^{\mathrm{T}}\ W_3^{\mathrm{T}}\ W_4^{\mathrm{T}}\ W_5^{\mathrm{T}}\ W_6^{\mathrm{T}}\\
&W_7^{\mathrm{T}}\ W_8^{\mathrm{T}}\ W_9^{\mathrm{T}}\ W_{10}^{\mathrm{T}}\ W_{11}^{\mathrm{T}}\ W_{12}^{\mathrm{T}}],\\
Y^{\mathrm{T}}=&[Y_1^{\mathrm{T}}\ Y_2^{\mathrm{T}}\ Y_3^{\mathrm{T}}\ Y_4^{\mathrm{T}}\ Y_5^{\mathrm{T}}\ Y_6^{\mathrm{T}}\\
&Y_7^{\mathrm{T}}\ Y_8^{\mathrm{T}}\ Y_9^{\mathrm{T}}\ Y_{10}^{\mathrm{T}}\ Y_{11}^{\mathrm{T}}\ Y_{12}^{\mathrm{T}}],\\
\phi^{\mathrm{T}}(t)=&[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-d(t))\ \ x^{\mathrm{T}}(t-d_0)\\
&x^{\mathrm{T}}(t-d_1)\ \ x^{\mathrm{T}}(t-d_2)\ \ x^{\mathrm{T}}(t-d_3)\\
&\dot{x}^{\mathrm{T}}(t)\ \ \int_{t-d_1}^{t}x^{\mathrm{T}}(s)\mathrm{d}s\ \ \int_{t-d_2}^{t-d_1}x^{\mathrm{T}}(s)\mathrm{d}s\\
&\int_{t-d_3}^{t-d_2}x^{\mathrm{T}}(s)\mathrm{d}s\ \ f^{\mathrm{T}}\ \ g^{\mathrm{T}}].
\end{aligned}
$$

当式(3)和式(4)满足时, 由引理3可得

$$
-\int_{t-d_1}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\mathrm{d}s\leqslant\xi_1^{\mathrm{T}}(t)\varPhi_1\xi_1(t),
\tag{25}
$$

$$
-\int_{t-d_3}^{t-d_2}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)\mathrm{d}s\leqslant\xi_3^{\mathrm{T}}(t)\varPhi_3\xi_3(t),
\tag{26}
$$

其中:

$$
\begin{aligned}
\varPhi_1=&d_1(M_1+\frac{1}{3}M_4)+\\
&\mathrm{sym}\{[M_3-M_5,-M_3-M_5,2M_5]\},\\
\varPhi_3=&(d_3-d_2)(N_1+\frac{1}{3}N_4)+\\
&\mathrm{sym}\{[N_3-N_5,-N_3-N_5,2N_5]\},\\
\xi_1^{\mathrm{T}}(t)=&[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-d_1)\ \ \frac{1}{d_1}\int_{t-d_1}^{t}x^{\mathrm{T}}(s)\mathrm{d}s],\\
\xi_3^{\mathrm{T}}(t)=&[x^{\mathrm{T}}(t-d_2)\ \ x^{\mathrm{T}}(t-d_3)\\
&\frac{1}{d_3-d_2}\int_{t-d_3}^{t-d_2}x^{\mathrm{T}}(s)\mathrm{d}s].
\end{aligned}
$$

由系统方程(1)可得

$$
\begin{aligned}
&2[x^{\mathrm{T}}(t)T_1+x^{\mathrm{T}}(t-d(t))T_2+\dot{x}^{\mathrm{T}}(t)T_3]\times\\
&[Ax(t)+A_{\mathrm{d}}x(t-d(t))+f+g-\dot{x}(t)]=0.
\end{aligned}
\tag{27}
$$

$T_i(i=1,2,3)$是适当维数的任意矩阵.

由式(2)可知, 对$\varepsilon_i\geqslant0(i=1,2)$, 有

$$
\begin{cases}
\varepsilon_1[\alpha^2x^{\mathrm{T}}(t)F^{\mathrm{T}}Fx(t)-f^{\mathrm{T}}f]\geqslant0,\\
\varepsilon_2[\beta^2x^{\mathrm{T}}(t-d(t))G^{\mathrm{T}}Gx(t-d(t))-g^{\mathrm{T}}g]\geqslant0.
\end{cases}
\tag{28}
$$

由式(7)–(28)可得

$$
\begin{aligned}
\dot{V}(t)\leqslant&\phi^{\mathrm{T}}(t)[(d(t)-d_1)WR_2^{-1}W^{\mathrm{T}}+\\
&(d_2-d(t))YR_2^{-1}Y^{\mathrm{T}}+\varXi]\phi(t),
\end{aligned}
\tag{29}
$$

所以当

$$
\begin{aligned}
&\varXi+(d(t)-d_1)WR_2^{-1}W^{\mathrm{T}}+\\
&(d_2-d(t))YR_2^{-1}Y^{\mathrm{T}}<0
\end{aligned}
\tag{30}
$$

时, $\dot{V}(t)<0$, 系统稳定.

由引理4, 式(30)等价于

$$
\begin{cases}
(d_2-d_1)WR_2^{-1}W^{\mathrm{T}}+\varXi<0,\\
(d_2-d_1)YR_2^{-1}Y^{\mathrm{T}}+\varXi<0.
\end{cases}
\tag{31}
$$

由Schur补引理, 式(31)等价于式(5)和式(6).

证毕.

当$d(t)\in[d_2,d_3]$时, 有如下稳定性准则:

**定理 2** 对于系统(1), 当$d(t)\in[d_2,d_3]$时, 若存在矩阵$P>0$, 半正定对称矩阵$Q$, $Q_j(j=1,2,3,4)$, $R_i$, $S_i$, $Z_i(i=1,2,3)$, 对称矩阵$M_1$, $M_4$, $N_1$, $N_4$和适当维数的任意矩阵$M_2$, $M_3$, $M_5$, $N_2$, $N_3$, $N_5$, $T_j(j=1,2,3)$, $W_i$, $Y_i(i=1,2,\cdots,12)$以及非负常数$\varepsilon_i\geqslant0(i=1,2)$, 使得式(32)–(35)的不等式组成立, 则系统是渐近稳定的.

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & Q_{12} & \widetilde{\Xi}_{14} & \widetilde{\Xi}_{15} & \widetilde{\Xi}_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19} & \Xi_{1-10} & T_1 & T_1 & \widetilde{d}_s W_1 \\ * & \Xi_{22} & \Xi_{23} & \widetilde{\Xi}_{24} & \widetilde{\Xi}_{25} & \widetilde{\Xi}_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} & \Xi_{2-10} & \Xi_{2-11} & \Xi_{2-12} & \widetilde{d}_s W_2 \\ * & * & \Xi_{33} & -Q_{12} & W_3 & -Y_3 & 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{d}_s W_3 \\ * & * & * & \widetilde{\Xi}_{44} & \widetilde{\Xi}_{45} & -Y_4 & 0 & \widetilde{\Xi}_{48} & \widetilde{\Xi}_{49} & \widetilde{\Xi}_{4-10} & 0 & 0 & \widetilde{d}_s W_4 \\ * & * & * & * & \widetilde{\Xi}_{55} & \widetilde{\Xi}_{56} & W_7^{\mathrm{T}} & \widetilde{\Xi}_{58} & \widetilde{\Xi}_{59} & \widetilde{\Xi}_{5-10} & W_{11}^{\mathrm{T}} & W_{12}^{\mathrm{T}} & \widetilde{d}_s W_5 \\ * & * & * & * & * & \widetilde{\Xi}_{66} & -Y_7^{\mathrm{T}} & \widetilde{\Xi}_{68} & \widetilde{\Xi}_{69} & \widetilde{\Xi}_{6-10} & -Y_{11}^{\mathrm{T}} & -Y_{12}^{\mathrm{T}} & \widetilde{d}_s W_6 \\ * & * & * & * & * & * & \Xi_{77} & P_{12} & P_{13} & P_{14} & T_3 & T_3 & \widetilde{d}_s W_7 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 & 0 & 0 & 0 & \widetilde{d}_s W_8 \\ * & * & * & * & * & * & * & * & \widetilde{\Xi}_{99} & 0 & 0 & 0 & \widetilde{d}_s W_9 \\ * & * & * & * & * & * & * & * & * & \widetilde{\Xi}_{10-10} & 0 & 0 & \widetilde{d}_s W_{10} \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & \widetilde{d}_s W_{11} \\ * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I & \widetilde{d}_s W_{12} \\ * & * & * & * & * & * & * & * & * & * & * & * & -R_3 \end{bmatrix} < 0, \tag{32}$$

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & Q_{12} & \widetilde{\Xi}_{14} & \widetilde{\Xi}_{15} & \widetilde{\Xi}_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19} & \Xi_{1-10} & T_1 & T_1 & \widetilde{d}_s Y_1 \\ * & \Xi_{22} & \Xi_{23} & \widetilde{\Xi}_{24} & \widetilde{\Xi}_{25} & \widetilde{\Xi}_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} & \Xi_{2-10} & \Xi_{2-11} & \Xi_{2-12} & \widetilde{d}_s Y_2 \\ * & * & \Xi_{33} & -Q_{12} & W_3 & -Y_3 & 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{d}_s Y_3 \\ * & * & * & \widetilde{\Xi}_{44} & \widetilde{\Xi}_{45} & -Y_4 & 0 & \widetilde{\Xi}_{48} & \widetilde{\Xi}_{49} & \widetilde{\Xi}_{4-10} & 0 & 0 & \widetilde{d}_s Y_4 \\ * & * & * & * & \widetilde{\Xi}_{55} & \widetilde{\Xi}_{56} & W_7^{\mathrm{T}} & \widetilde{\Xi}_{58} & \widetilde{\Xi}_{59} & \widetilde{\Xi}_{5-10} & W_{11}^{\mathrm{T}} & W_{12}^{\mathrm{T}} & \widetilde{d}_s Y_5 \\ * & * & * & * & * & \widetilde{\Xi}_{66} & -Y_7^{\mathrm{T}} & \widetilde{\Xi}_{68} & \widetilde{\Xi}_{69} & \widetilde{\Xi}_{6-10} & -Y_{11}^{\mathrm{T}} & -Y_{12}^{\mathrm{T}} & \widetilde{d}_s Y_6 \\ * & * & * & * & * & * & \Xi_{77} & P_{12} & P_{13} & P_{14} & T_3 & T_3 & \widetilde{d}_s Y_7 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 & 0 & 0 & 0 & \widetilde{d}_s Y_8 \\ * & * & * & * & * & * & * & * & \widetilde{\Xi}_{99} & 0 & 0 & 0 & \widetilde{d}_s Y_9 \\ * & * & * & * & * & * & * & * & * & \widetilde{\Xi}_{10-10} & 0 & 0 & \widetilde{d}_s Y_{10} \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & \widetilde{d}_s Y_{11} \\ * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I & \widetilde{d}_s Y_{12} \\ * & * & * & * & * & * & * & * & * & * & * & * & -R_3 \end{bmatrix} < 0, \tag{33}$$

$$\begin{bmatrix} M_1 & M_2 & M_3 \\ * & M_4 & M_5 \\ * & * & R_1 \end{bmatrix} \geqslant 0, \tag{34}$$

$$\begin{bmatrix} N_1 & N_2 & N_3 \\ * & N_4 & N_5 \\ * & * & R_2 \end{bmatrix} \geqslant 0, \tag{35}$$

其中:

$$\begin{aligned} \widetilde{\Xi}_{14} = {} & -P_{12} + P_{13} + d_1^3(M_{12} + \frac{1}{3}M_{42}) - \\ & d_1^2(M_{31} - M_{51} + M_{32}^{\mathrm{T}} - M_{52}^{\mathrm{T}}), \end{aligned}$$

$\widetilde{\Xi}_{15} = -P_{13} + P_{14} + W_1,\ \widetilde{\Xi}_{16} = -P_{14} - Y_1,$

$\widetilde{\Xi}_{24} = -W_4^{\mathrm{T}} + Y_4^{\mathrm{T}},\ \widetilde{\Xi}_{25} = W_2 - W_5^{\mathrm{T}} + Y_5^{\mathrm{T}},$

$\widetilde{\Xi}_{26} = -Y_2 - W_6^{\mathrm{T}} + Y_6^{\mathrm{T}},$

$$\begin{aligned} \widetilde{\Xi}_{44} = {} & -Q_{22} - Q_1 + Q_2 + Q_4 + N_{31} + \\ & N_{31}^{\mathrm{T}} - N_{51} - N_{51}^{\mathrm{T}} - d_1^3(M_{32} - M_{32}^{\mathrm{T}} - \\ & M_{52} - M_{52}^{\mathrm{T}}) + d_1^3(M_{14} + \frac{1}{3}M_{44}) + \\ & (d_2 - d_1)(N_{11} + \frac{1}{3}N_{41}), \end{aligned}$$

$$\begin{aligned} \widetilde{\Xi}_{45} = {} & W_4 + (d_2 - d_1)(N_{12} + \frac{1}{3}N_{42}) - \\ & N_{31} - N_{51} + N_{32}^{\mathrm{T}} - N_{52}^{\mathrm{T}}, \end{aligned}$$

$$\begin{aligned} \widetilde{\Xi}_{48} = {} & -P_{22}^{\mathrm{T}} + P_{23}^{\mathrm{T}} + d_1^2(M_{15} + \frac{1}{3}M_{45}) + \\ & d_1(2M_{52} - M_{33}^{\mathrm{T}} - M_{53}^{\mathrm{T}}), \end{aligned}$$

$$\begin{aligned} \widetilde{\Xi}_{49} = {} & -P_{23} + P_{33}^{\mathrm{T}} + N_{13} + \frac{1}{3}N_{43} + \\ & \frac{1}{d_2 - d_1}(2N_{51} + N_{33}^{\mathrm{T}} - N_{53}^{\mathrm{T}}), \end{aligned}$$

$\widetilde{\Xi}_{4-10} = -P_{24} + P_{34},$

$$\begin{aligned} \widetilde{\Xi}_{55} = {} & -Q_2 + Q_3 + W_5 + W_5^{\mathrm{T}} + \\ & (d_2 - d_1)(N_{14} + \frac{1}{3}N_{44}) - \\ & N_{32} - N_{32}^{\mathrm{T}} - N_{52} - N_{52}^{\mathrm{T}}, \end{aligned}$$

$\widetilde{\Xi}_{56} = -Y_5 + W_6^{\mathrm{T}},\ \widetilde{\Xi}_{58} = -P_{23}^{\mathrm{T}} + P_{24}^{\mathrm{T}} + W_8^{\mathrm{T}},$

$$\begin{aligned} \widetilde{\Xi}_{59} = {} & -P_{33}^{\mathrm{T}} + P_{34}^{\mathrm{T}} + W_9^{\mathrm{T}} + N_{15} + \frac{1}{3}N_{45} + \\ & \frac{1}{d_2 - d_1}(2N_{52} - N_{33}^{\mathrm{T}} - N_{53}^{\mathrm{T}}), \end{aligned}$$

$\widetilde{\Xi}_{5-10} = -P_{34} + P_{44}^{\mathrm{T}} + W_{10}^{\mathrm{T}},$

$$\tilde{\tilde{\Xi}}_{66} = -Q_3 - Y_6 - Y_6^{\mathrm{T}},\ \tilde{\tilde{\Xi}}_{68} = -Y_8^{\mathrm{T}} - P_{24}^{\mathrm{T}},$$
$$\tilde{\tilde{\Xi}}_{69} = -Y_9^{\mathrm{T}} - P_{34}^{\mathrm{T}},\ \tilde{\tilde{\Xi}}_{6-10} = -P_{44}^{\mathrm{T}} - Y_{10}^{\mathrm{T}},$$
$$\tilde{\tilde{\Xi}}_{99} = -S_2 - Z_2 + \frac{1}{d_2 - d_1}(N_{16} + \frac{1}{3}N_{46}) + \frac{2}{(d_2 - d_1)^2}(N_{53} + N_{53}^{\mathrm{T}}),$$
$$\tilde{\tilde{\Xi}}_{10-10} = -S_3 - Z_3,\ \tilde{d}_s = \sqrt{(d_3 - d_2)}.$$

定理2的证明方法与定理1类似, 此处省略.

**注 1**　结合定理1和定理2, 当$d(t) \in [d_1, d_3]$时, 不等式组式(3)–(6)和式(32)–(35)均有解时, 系统渐近稳定.

**注 2**　当$\dot{d}(t)$未知时, 即$d(t)$可导但$\mu$未知或者$d(t)$不可导时，只需去掉定理1和定理2中与$Q_4$相关的项即可得到系统稳定性准则.

**注 3**　若不考虑非线性扰动, 即$f = g = 0$时, 采用与定理1类似的方法也可以得到相应的稳定性准则, 将与定理1和定理2对应的结论分别记为推论1和推论2.

## 4　数值仿真(Numerical simulation)

通过典型的算例对比分析本文提出方法的保守性.

**例 1**　对系统(1), 选取系统参数如下:

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1.0 \end{bmatrix},\ A_{\mathrm{d}} = \begin{bmatrix} -0.6 & 0.7 \\ -1.0 & -0.8 \end{bmatrix},$$
$$F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

当$d_{\mathrm{L}} = 1$时, 取$\alpha, \beta, \mu$为相应值时, 利用本文定理1和定理2可以分别计算出系统稳定的最大允许时滞, 结果如表1所示. 当$d(t)$为常数时, $d_{\mathrm{L}} = 0$时的结果见表2.

表 1　最大允许时滞$d_{\mathrm{U}}(d_{\mathrm{L}} = 1)$
Table 1　Maximum allowable delay bound (MADB) for $d_{\mathrm{U}}(d_{\mathrm{L}} = 1)$

| | $\alpha$和$\beta$ | $\alpha = 0$, $\beta = 0.1$ | $\alpha = 0.1$, $\beta = 0.1$ |
|---|---|---|---|
| $\mu = 0.5$ | 文献[17]($N = 4$) | 1.9930 | 1.6380 |
| | 文献[14] | 1.8316 | 1.6541 |
| | 定理1($\eta = 0.3$) | 2.9242 | 2.4929 |
| | 定理2($\eta = 0.3$) | 2.2454 | 2.0007 |
| $\mu = 0.9$ | 文献[17]($N = 4$) | 1.9930 | 1.6380 |
| | 文献[14] | 1.8316 | 1.6541 |
| | 定理1($\eta = 0.3$) | 2.9242 | 2.4929 |
| | 定理2($\eta = 0.3$) | 2.2454 | 2.0007 |
| $\mu = 1.1$ | 文献[17]($N = 4$) | 1.9930 | 1.6380 |
| | 文献[14] | 1.8316 | 1.6541 |
| | 定理1($\eta = 0.3$) | 2.9242 | 2.4929 |
| | 定理2($\eta = 0.3$) | 2.2454 | 2.0007 |

表 2　定常时滞时的最大允许时滞$d_{\mathrm{U}}(d_{\mathrm{L}} = 0)$
Table 2　MADB with constant delay for $d_{\mathrm{U}}(d_{\mathrm{L}} = 0)$

| $\alpha$和$\beta$ | $\alpha = 0,\ \beta = 0.1$ | $\alpha = 0.1,\ \beta = 0.1$ |
|---|---|---|
| 文献[14] | 2.9081 | 1.9672 |
| 定理1($\eta = 0.5$) | 9.1393 | 6.3774 |
| 定理2($\eta = 0.5$) | 4.0465 | 2.9389 |

由表1–2可知, 利用本文提出的方法可以得到更大的允许时滞上界, 说明本文的稳定性准则具有更小的保守性.

当$\mu$未知时, 将本文的计算结果与文献[15,19]对比, 保证系统稳定的最大允许时滞如表3所示. 由表3可知, 本文推论计算出的最大允许时滞比文献[15,19]的结果都大, 因此具有更小的保守性.

表 3　最大允许时滞$d_{\mathrm{U}}$($\mu$未知)
Table 3　MADB for $d_{\mathrm{U}}$ (unknown $\mu$)

| $d_{\mathrm{L}}$ | 0.3 | 0.5 | 0.8 | 1.0 | 2.0 |
|---|---|---|---|---|---|
| 文献[15] | 1.4210 | 1.5240 | 1.7078 | 1.8446 | 2.6344 |
| 文献[19]($\eta = 0.55$) | 1.6350 | 1.7751 | 1.9978 | 2.1519 | 2.9669 |
| 推论1($\eta = 0.55$) | 2.0188 | 2.1042 | 2.2640 | 2.3858 | 3.1034 |
| 推论2($\eta = 0.55$) | 2.3058 | 2.4267 | 2.6362 | 2.8031 | 3.7143 |

**例 2**　考虑如下系统:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} x(t - d(t)).$$

**例 3**　考虑如下3阶系统:

$$\dot{x}(t) = \begin{bmatrix} -1.0 & 0.0 & 0.0 \\ 0.0 & -2.0 & 0.0 \\ 0.0 & 0.0 & -0.9 \end{bmatrix} x(t) + \begin{bmatrix} -1.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.1 \\ 0.0 & -1.0 & -1.0 \end{bmatrix} x(t - d(t)).$$

当$\mu = 0.3$时, 给定$d_{\mathrm{L}}$的值, 计算保证系统稳定的最大允许时滞, 本文和文献 [3] 的对比结果如表4所示. 由表4可知, 本文方法能够获得更大的允许时滞, 保守性更小.

表 4　最大允许时滞$d_{\mathrm{U}}(\mu = 0.3)$
Table 4　MADB for $d_{\mathrm{U}}$ ($\mu = 0.3$)

| $d_{\mathrm{L}}$ | 0 | 0.1 | 0.4 | 0.7 | 1.0 |
|---|---|---|---|---|---|
| 文献[3] | 3.4724 | 3.5303 | 3.6780 | 3.7870 | 3.8565 |
| 推论1($\eta = 0.4$) | 6.3803 | 7.0271 | 7.5342 | 8.1751 | 8.8393 |
| 推论2($\eta = 0.4$) | 3.8556 | 3.9115 | 4.0253 | 4.0537 | 4.1213 |

## 5　结语(Conclusions)

本文讨论了带有非线性扰动的时变时滞的系统的稳定性问题. 采用时滞分割方法, 把时滞区间拆分为

2个子区间, 构造了适当的Lyapunov-Krasovskii泛函, 采用自由矩阵积分不等式对Lyapunov-Krasovskii泛函的导数进行放缩, 以线性矩阵不等式形式给出了时滞依赖型系统稳定性准则. 最后, 通过数值仿真实例说明了本文提出的方法可以获得更大的允许时滞, 改善了现有方法的保守性, 验证了本文方法的优越性.

## 参考文献(References):


[1] JIAO J M. Robust stability and stabilization of discrete singular systems with interval time-varying delay and linear fractional uncertainty [J]. *International Journal of Automation and Computing*, 2012, 9(1): 8 – 15.

[2] YANG Yuanhua, HAN Chunyan, LIU Xiaohua. Optimal estimation for networked control systems with bounded random measurement delays [J]. *Control Theory & Applications*, 2014, 31(2): 181 – 187.
(杨园华, 韩春艳, 刘晓华. 有界随机测量时滞的网络控制系统的最优估计 [J]. 控制理论与应用, 2014, 31(2): 181 – 187.)

[3] HUI J J, ZHANG H X, KONG X Y. Delay-dependent non-fragile $H_\infty$ control for linear systems with interval time-varying delay [J]. *International Journal of Automation and Computing*, 2015, 12(1): 109 – 116.

[4] YANG J, LUO W P, WANG Y H, et al. Improved stability criteria for T-S fuzzy systems with time-varying delay by delay-partitioning approach [J]. *International Journal of Control, Automation, and Systems*, 2015, 13(6): 1521 – 1529.

[5] ZENG H B, HE Y, WU M, et al. Free-matrix-based integral inequality for stability analysis of systems with time-varying delay [J]. *IEEE Transactions on Automatic Control*, 2015, 60(10): 2768 – 2772.

[6] BOUKAL Y, ZASADZINSKI M, DAROUACH M, et al. Stability and stabilizability analysis of fractional-order time-varying delay systems via diffusive representation [C] //*Proceedings of the 2016 5th International Conference on Systems and Control*. Marrakesh, Morocco: IEEE, 2016: 262 – 266.

[7] LIU Yonghua, HUANG Liangpei, XIAO Dongming, et al. Dynamic state feedback stabilization for a class of nonaffine nonlinear systems with unknown time delays [J]. *Control Theory & Applications*, 2016, 33(7): 923 – 928.
(刘勇华, 黄良沛, 肖冬明, 等. 一类非仿射非线性时滞系统的动态状态反馈镇定 [J]. 控制理论与应用, 2016, 33(7): 923 – 928.)

[8] LIU Y J, LEE S M, KWON O M, et al. Delay-dependent exponential stability criteria for neutral systems with interval time-varying delays and nonlinear perturbations [J]. *Journal of the Franklin Institute*, 2013, 350(10): 3313 – 3327.

[9] LIU P L. New Results on stability analysis for time-varying delay systems with non-linear perturbations [J]. *ISA Transactions*, 2013, 52(10): 318 – 325.

[10] WANG W Q, NGUANG S K, ZHONG S M, et al. Novel delay-dependent stability criterion for time-varying delay systems with parameter uncertainties and nonlinear perturbations [J]. *Information Sciences*, 2014, 281: 321 – 333.

[11] YUAN C A, DONG W H, QU P S. Robust $H_\infty$ guaranteed cost control for a class of uncertain nonlinear time-delay systems [J]. *International Journal of Control and Automation*, 2016, 9(2): 191 – 198.

[12] ZHU X L, YANG G H. Jensen integral inequality approach to stability analysis of continuous-time systems with time varying delay [J]. *IET Control Theory and Applications*, 2008, 2(6): 524 – 534.

[13] ZHU X L, YANG G H. New results of stability analysis for systems with time-varying delay [J]. *International Journal of Robust and Nonlinear Control*, 2010, 20(5): 596 – 606.

[14] ZHAO Zhanshan, LI Xiaomeng, ZHANG Jing, et al. Stability criteria for a class of time-varying delay systems [J]. *Control and Decision*, 2016, 31(11): 2090 - 2094.
(赵占山, 李晓蒙, 张静, 等. 一类时变时滞系统的稳定性分析 [J]. 控制与决策, 2016, 31(11): 2090 – 2094.)

[15] KWON O M, PARK M J, PARK J H, et al. Analysis on robust $H_\infty$ performance and stability for linear systems with interval time-varying state delays via some new augmented Lyapunov-Krasovskii functional [J]. *Applied Mathematics and Computation*, 2013, 224: 108 – 122.

[16] RAMAKRISHNAN K, RAY G. Robust stability criteria for uncertain linear systems with interval time-varying delay [J]. *Journal of Control Theory and Applications*, 2011, 9(4): 559 – 566.

[17] HUI J J, KONG X Y, ZHANG H X, et al. Delay-partitioning approach for systems with interval time-varying delay and nonlinear perturbations [J]. *Journal of Computational and Applied Mathematics*, 2015, 281(C): 74 – 81.

[18] LIU P L. New results on delay-range-dependent stability analysis for interval time-varying delay systems with non-linear perturbations [J]. *ISA Transactions*, 2015, 57: 93 – 100.

[19] AN J Y, LI Z Y, WANG X M. A novel approach to delay-fractional-dependent stability criterion for linear systems with interval delay [J]. *ISA Transactions*, 2014, 53(2): 210 – 219.

[20] CHEN H, ZHONG S M, LI M, et al. Stability criteria for T-S fuzzy systems with interval time-varying delays and nonlinear perturbations based on geometric progression delay partitioning method [J]. *ISA Transactions*, 2016, 63: 69 – 77.

[21] TIAN E G, YUE D, ZHANG Y J. On improved delay-dependent robust $H_\infty$ control for systems with interval time-varying delay [J]. *Journal of the Franklin Institute*, 2011, 348(4): 555 – 567.


## 作者简介:


**武 斌** (1990–), 男, 博士研究生, 研究方向为无人机鲁棒控制, E-mail: wubin435@163.com;

**王长龙** (1965–), 男, 教授, 研究方向为无人机测控技术, E-mail: wang-oec@126.com;

**徐锦法** (1963–), 男, 教授, 研究方向为直升机飞行力学及控制, E-mail: xjfae@nuaa.edu.cn;

**胡永江** (1977–), 男, 博士, 研究方向为无人机测控技术, E-mail: hyjbwcx@163.com.