

考虑随机时滞的非线性网控系统鲁棒 L_2 - L_∞ 控制

李艳辉[†], 吴迪

(东北石油大学 电气信息工程学院, 黑龙江 大庆 163318)

摘要: 本文考虑了由 Takagi-Sugeno(T-S) 模糊模型描述的一类连续非线性网络控制系统基于观测器的鲁棒 L_2 - L_∞ 控制器设计问题. 假设网络环境下存在的时变时滞具有特定的随机特性, 通过引入一个伯努利随机变量表示时滞在不同区间上分布的概率, 建立一个新的具有概率分布信息的系统模型. 根据平行分布补偿法(parallel distribution compensation, PDC)和Lyapunov稳定性理论, 建立基于T-S模糊模型的使系统均方指数稳定且满足 L_2 - L_∞ 的性能判据. 并利用线性矩阵不等式(linear matrix inequality, LMI)技术同时得到状态观测器增益矩阵和控制器增益矩阵. 最后, 仿真结果验证了该方法的有效性.

关键词: 网络控制系统; T-S模糊模型; 伯努利随机变量; 观测器; 指数稳定

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Robust L_2 - L_∞ control for nonlinear networked control systems with random time delays

LI Yan-hui[†], WU Di

(College of Electrical and Information Engineering, Northeast Petroleum University, Daqing Heilongjiang 163318, China)

Abstract: In this paper, the problem of observer-based robust L_2 - L_∞ controller design for a class of continuous nonlinear networked control systems is considered in Takagi-Sugeno (T-S) fuzzy model. A new system model with probability distribution information is established by introducing a Bernoulli random variable to represent the probability of the delay distribution in different intervals, assuming that the time-varying delay in the network environment has a certain stochastic property. According to the parallel distributed compensation (PDC) and Lyapunov stability theory, the L_2 - L_∞ criterion based on T-S fuzzy model is established to make the system mean square exponential stability. And the state observer gain and the controller gain matrixes are obtained by linear matrix inequality (LMI) technique. Finally, the simulation results verify the effectiveness of the proposed method.

Key words: networked control systems; T-S fuzzy model; Bernoulli random variables; observer; exponential stability

1 引言(Introduction)

近年来, 由于网络控制系统 (networked control systems, NCSs) 具有灵活性高、节约成本、便于实现资源共享等优点被广泛应用于机器人制造、航空航天等领域中. 然而, 由于数据碰撞、节点失败等现象的频繁发生导致数据在传递过程中存在网络诱导时延和数据包丢失^[1-2]等问题, 这大大降低了系统的性能甚至使系统失稳. 目前, 已有很多学者对这一问题进行了研究^[3-5]. 文献[3]研究了一类具有网络诱导时延和数据包丢失的NCSs的鲁棒控制问题, 考虑前向通道和反馈通道双通道丢包情况建模降低系统的保守性; 文献[4]和文献[5]基于时变网络诱导时滞研究了一类NCSs的 H_∞ 控制器设计问题. 而在实际生活中, 时延

和丢包现象的发生通常是随机的^[6-8], 文献[6]通过Bernoulli 0-1分布描述随机时滞, 研究了一类非线性随机系统的鲁棒稳定性问题. 文献[7]和文献[8]分别提出了具有随机时滞的离散NCS的滤波器和动态输出反馈控制器的设计方法. 需要注意的是, 上述对随机时滞的研究大多局限于离散系统, 而对于连续的NCSs, 具有随机时滞的情况还有待进一步研究.

另一方面, 对于实际工程系统, 由于NCSs结构较为复杂, 很多情况下到被控对象的状态信息不能全部测量到, 因而需要引入状态观测器来实现状态反馈. 文献[9]和文献[10]分别针对NCSs和Markov跳变系统研究了一类基于观测器的鲁棒控制器设计问题. 然而, 基于T-S模糊模型的连续非线性NCSs尤其是在具有

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[†]通信作者. E-mail: LY_hui@hotmail.com; Tel.: +86 459-6504797.

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随机时滞的情况下,对状态观测器设计问题的研究成果还相对较少,是一个值得研究的领域.

本文采用T-S模糊模型描述被控对象,研究了基于观测器的NCSs在具有随机时滞的情况下的鲁棒控制问题,当时滞在两个不同的区间上随机分布且每个区间上分布概率已知时,采用Bernoulli 0-1分布,将其转换成具有随机参数的NCSs.然后运用Jensen不等式和LMI技术得到控制器和观测器的设计方法使闭环系统均方指数稳定且满足 L_2-L_∞ 性能,最后通过数值仿真说明分析方法及研究结果的可行性.

2 问题描述(Problem statement)

考虑如下T-S模糊的被控对象:

Rule i :

If $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} and $\dots$ and

$\theta_n(t)$ is M_{in} , then

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + B_{1i}\omega(t), \\ y(t) = C_{1i}x(t), \\ z(t) = C_{2i}x(t), \end{cases} \quad (1)$$

其中: $x(t) \in \mathbb{R}^m$ 是状态变量, $u(t) \in \mathbb{R}^p$ 是输入变量, $y(t) \in \mathbb{R}^q$ 是对象的测量输出, $z(t) \in \mathbb{R}^l$ 是待估计信号, $\omega(t)$ 表示扰动输入, 且 $\omega(t) \in L_2[0, \infty)$, $\theta_1(t), \theta_2(t), \dots, \theta_n(t)$ 是可测的前件变量, $M_{ij}(j=1, 2, \dots, n)$ 是模糊集, r 为模糊规则数; 系统矩阵 A_i, B_i, B_{1i}, C_{1i} 和 C_{2i} 是适维的常数矩阵; $\Delta A_i(t), \Delta B_i(t)$ 是时变参数矩阵, 表示系统模型中存在的 uncertainty. 假设其范数有界且满足

$$[\Delta A_i(t) \ \Delta B_i(t)] = G_i \Lambda_i(t) [H_{1i} \ H_{2i}], \quad (2)$$

其中: G_i, H_{1i}, H_{2i} 适维常数矩阵, $\Lambda_i(t)$ 是具有 Lebesgue 可测元的满足 $\Lambda_i^T(t) \Lambda_i(t) \leq I$ 未知函数矩阵.

应用单点模糊化、乘积推理和加权平均, 可得

$$\begin{cases} \dot{x}(t) = (A(t) + \Delta A(t))x(t) + (B(t) + \Delta B(t))u(t) + B_1(t)\omega(t), \\ y(t) = C_1(t)x(t), \\ z(t) = C_2(t)x(t), \end{cases} \quad (3)$$

其中:

$$h_i(\theta(t)) = \prod_{j=1}^n M_{ij}(\theta_j(t)),$$

$$\mu_i(\theta(t)) = h_i(\theta(t)) / \sum_{i=1}^r h_i(\theta(t)),$$

$$A(t) = \sum_{i=1}^r \mu_i(\theta(t)) A_i, \quad \Delta A(t) = \sum_{i=1}^r \mu_i(\theta(t)) \Delta A_i,$$

$$B(t) = \sum_{i=1}^r \mu_i(\theta(t)) B_i, \quad \Delta B(t) = \sum_{i=1}^r \mu_i(\theta(t)) \Delta B_i,$$

$$C_1(t) = \sum_{i=1}^r \mu_i(\theta(t)) C_{1i}, \quad C_2(t) = \sum_{i=1}^r \mu_i(\theta(t)) C_{2i},$$

$M_{ij}(\cdot)$ 是模糊集 M_{ij} 的隶属函数, $\mu_i(\theta(t))$ 是每条模糊规则的规范化权重. 对任意 t , 满足

$$\mu_i(\theta(t)) \geq 0, \quad \sum_{i=1}^r \mu_i(\theta(t)) = 1.$$

在网络环境下, 考虑控制器-执行器之间存在的时延和丢包, 建立一个新的网控系统, 如图1所示.

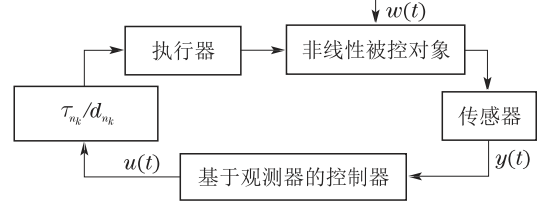


图1 网络控制系统结构图

Fig. 1 Network control system structure

如图1所示, 假设传感器是时间驱动, 信号采样周期为 $h(h > 0)$, 控制器和执行器是事件驱动, 信号经传感器到控制器, 由控制器输出控制信号至执行器并控制被控对象. 控制信号在 $n_k h$ 时刻发出, $n_k h + \tau_{n_k}$ 时刻到达, 其中 τ_{n_k} 表示控制器至执行器间时延. 为了便于数学推导, 本文设计的是只考虑控制器到执行器单侧时滞的理想化模型, 将数据包丢失作为时滞处理, 则系统等价为时变时滞的网络控制系统.

对于 $t \in [n_k h + \tau_{n_k}, n_{k+1} h + \tau_{n_{k+1}})$, 定义 $\eta(t) = t - n_k h$, 则有 $n_k h = t - \eta(t)$, 根据平行分布补偿法 (parallel distributed compensation, PDC) 原理, 因此可考虑如下形式的观测器和控制器:

Rule i :

If $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} and $\dots$ and

$\theta_n(t)$ is M_{in} , then

$$\begin{cases} \dot{\hat{x}}(t) = A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - C_{1i} \hat{x}(t)), \\ u(t) = K_i \hat{x}(t - \eta(t)). \end{cases} \quad (4)$$

对应全局模型如下:

$$\begin{cases} \dot{\hat{x}}(t) = A(t) \hat{x}(t) + B(t) u(t) + L(t)(y(t) - C_1(t) \hat{x}(t)), \\ u(t) = K(t) \hat{x}(t - \eta(t)), \end{cases} \quad (5)$$

其中: $K(t) = \sum_{i=1}^r \mu_i(\theta(t)) K_i$, $L(t) = \sum_{i=1}^r \mu_i(\theta(t)) L_i$ 分别为控制器和观测器增益, $\eta(t) \in [\min_k \tau_{n_k}, (n_{k+1} - n_k)h + \max_k \tau_{n_{k+1}}] = [0, d_2]$, 这里 $\forall k \in \mathbb{N}$.

注1 由于信号传输过程中时延和丢包可能同时发生, 有 $\{n_k\}_{k=1}^\infty \subseteq \mathbb{N}$.

实际上, 在网络环境下, $\eta(t)$ 通常具有随机分布特性, 则存在常数 $d_1(0 \leq d_1 \leq d_2)$, 使得时滞 $\eta(t)$ 以一定的概率在区间 $[0, d_1]$ 和 $(d_1, d_2]$ 上取值. 假定时滞

$\eta(t)$ 的概率分布为

$$P\{\delta(t) = 1\} = P\{0 \leq \eta(t) \leq d_1\} = 1 - \delta_0,$$

$$P\{\delta(t) = 0\} = P\{d_1 \leq \eta(t) \leq d_2\} = \delta_0.$$

引入时变时滞 $\eta_1(t)$ 和 $\eta_2(t)$ 使得 $0 \leq \eta_1(t) \leq d_1$, $d_1 \leq \eta_2(t) \leq d_2$, $\eta_1(t) < \tau_1 < \infty$, $\eta_2(t) < \tau_2 < \infty$. 此时式(5)等价于

$$\begin{cases} \dot{\hat{x}}(t) = A(t)\hat{x}(t) + B(t)u(t) + \\ \quad L(t)(y(t) - C_1(t)\hat{x}(t)), \\ u(t) = \delta(t)K(t)\hat{x}(t - \eta_1(t)) + \\ \quad (1 - \delta(t))K(t)\hat{x}(t - \eta_2(t)). \end{cases} \quad (6)$$

引入估计误差

$$e(t) = x(t) - \hat{x}(t),$$

定义

$$\xi^T(t) = [x^T(t) \ e^T(t)].$$

综合式(3)(6), 应用状态增广思想对模型进行处理, 得到

$$\begin{cases} \dot{\xi}(t) = (\bar{A}(t) + \Delta\bar{A}(t))\xi(t) + \delta_0(\bar{B}(t) + \\ \quad \Delta\bar{B}(t))\xi(t - \eta_1(t)) + \\ \quad (1 - \delta_0)(\bar{B}(t) + \Delta\bar{B}(t))\xi(t - \eta_2(t)) + \\ \quad (\delta(t) - \delta_0)\bar{B}(t)[\xi(t - \eta_1(t)) - \\ \quad \xi(t - \eta_2(t))] + \bar{B}_1(t)\omega(t), \\ z(t) = \bar{C}(t)\xi(t), \end{cases} \quad (7)$$

其中:

$$\begin{aligned} \bar{A}(t) &= \sum_{i=1}^r \mu_i(x(t)) \sum_{j=1}^r \mu_j(x(t)) \begin{bmatrix} A_i & 0 \\ 0 & A_i - L_j C_{1i} \end{bmatrix} = \\ &\begin{bmatrix} A(t) & 0 \\ 0 & A(t) - L(t)C_1(t) \end{bmatrix}, \\ \Delta\bar{A}(t) &= \sum_{i=1}^r \mu_i(x(t)) \begin{bmatrix} \Delta A_i & 0 \\ \Delta A_i & 0 \end{bmatrix} = \begin{bmatrix} \Delta A(t) & 0 \\ \Delta A(t) & 0 \end{bmatrix}, \\ \bar{B}(t) &= \sum_{i=1}^r \mu_i(x(t)) \sum_{j=1}^r \mu_j(x(t)) \begin{bmatrix} B_i K_j & -B_i K_j \\ 0 & 0 \end{bmatrix} = \\ &\begin{bmatrix} B(t)K(t) - B(t)K(t) \\ 0 & 0 \end{bmatrix}, \\ \Delta\bar{B}(t) &= \sum_{i=1}^r \mu_i(x(t)) \sum_{j=1}^r \mu_j(x(t)) \Delta B_i \begin{bmatrix} K_j & -K_j \\ K_j & -K_j \end{bmatrix} = \\ &\begin{bmatrix} \Delta B(t)K(t) - \Delta B(t)K(t) \\ \Delta B(t)K(t) - \Delta B(t)K(t) \end{bmatrix}, \end{aligned}$$

$$\bar{B}_1(t) = \sum_{i=1}^r \mu_i(x(t)) \begin{bmatrix} B_{1i} \\ B_{1i} \end{bmatrix} = \begin{bmatrix} B_1(t) \\ B_1(t) \end{bmatrix},$$

$$\bar{C}(t) = \sum_{i=1}^r \mu_i(x(t)) [C_{2i} \ 0] = [C_2(t) \ 0].$$

针对上述系统矩阵中的不确定项, 定义

$$\begin{aligned} \bar{G} &= \sum_{i=1}^r \mu_i(x(t)) \begin{bmatrix} G_i \\ G_i \end{bmatrix}, \\ \bar{H}_1 &= \sum_{i=1}^r \mu_i(x(t)) [H_{1i} \ 0], \\ \bar{H}_2 &= \sum_{i=1}^r \mu_i(x(t)) \sum_{j=1}^r \mu_j(x(t)) [H_{2i} K_j \ H_{2i} K_j], \end{aligned}$$

则有

$$\Delta\bar{A}(t) = \bar{G}\Lambda(t)\bar{H}_1, \quad \Delta\bar{B}(t) = \bar{G}\Lambda(t)\bar{H}_2.$$

注2 本文考虑的是具有时变时延的非线性网络控制系统, 在时滞随机的分布在不同区间且分布概率已知情况下, 根据Bernoulli 0-1分布的随机性, 建立以上的控制器模型, 使系统(1)转换成具有随机参数的系统(7).

定义1 对于给定常数 $\gamma > 0$, 若满足:

- 1) 闭环增广系统(7)是鲁棒均方指数稳定的;
- 2) 在零初始条件下, 对于所有非零的 $\omega(t)$, 被控输出 $z(t)$ 满足

$$E\{\|z(t)\|_\infty^2\} \leq \gamma^2 \|w(t)\|_2^2,$$

则称该闭环系统具有 L_2 - L_∞ 扰动抑制度 γ .

引理1 对于任意矩阵 $W > 0$, 标量 $b > a$, 向量函数 $\omega : [a, b] \rightarrow \mathbb{R}^n$, 下列积分不等式成立:

$$\begin{aligned} &(\int_a^b \varpi(s) ds)^T W (\int_a^b \varpi(s) ds) \leq \\ &(b-a) \int_a^b \varpi^T(s) W \varpi(s) ds. \end{aligned}$$

3 主要结果(Main results)

本节基于观测器建立使增广系统(7)均方指数稳定且具有 L_2 - L_∞ 性能指标的控制器的充分条件.

定理1 给定标量 $\gamma > 0$, 若存在正定对称矩阵 P, Q_1, Q_2, R_1, R_2 , 满足

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ * & \Omega_{22} & \Omega_{23} \\ * & * & \Omega_{33} \end{bmatrix} < 0, \quad (8)$$

$$\begin{bmatrix} P & \bar{C}_{ij}^T \\ * & \gamma^2 I \end{bmatrix} > 0, \quad (9)$$

其中:

$$\Omega_{11} = \begin{bmatrix} \tilde{\Omega}_{11} & \delta_0 P \bar{B}_{ij} + R_1 & 0 \\ * & -(1 - \tau_1)Q_1 - 2R_1 & R_1 \\ * & * & -R_1 - R_2 \end{bmatrix},$$

$$\begin{aligned}\Omega_{12} &= \begin{bmatrix} (1-\delta_0)P\bar{B}_{ij} & 0 & 0 \\ 0 & 0 & 0 \\ R_2 & 0 & 0 \end{bmatrix}, \\ \Omega_{13} &= \begin{bmatrix} P\bar{B}_{1j} & \varepsilon_1 P G_i & \bar{H}_{1i}^T \\ 0 & 0 & \delta_0 \bar{H}_{2i}^T \\ 0 & 0 & 0 \end{bmatrix}, \\ \Omega_{22} &= \begin{bmatrix} -(1-\tau_2)Q_2 - 2R_2 & R_2 & 0 \\ * & -R_2 & 0 \\ * & * & \tilde{\Omega}_{22} \end{bmatrix}, \\ \Omega_{23} &= \begin{bmatrix} 0 & 0 & (1-\delta_0)\bar{H}_{2i}^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \Omega_{33} &= \text{diag}\{-I, -\varepsilon_1 I, -\varepsilon_1 I\}, \\ \tilde{\Omega}_{11} &= P\bar{A}_{ij} + \bar{A}_{ij}^T P + Q_1 + Q_2 - R_1, \\ \tilde{\Omega}_{22} &= d_1^2 R_1 + (d_2 - d_1)^2 R_2,\end{aligned}$$

则系统(7)均方指数稳定且满足 L_2 - L_∞ 抑制度 γ .

证 构造如下形式的Lyapunov泛函:

$$\begin{aligned}V(t) &= V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \\ V_1(t) &= \xi^T(t)P\xi(t), \\ V_2(t) &= \int_{t-\eta_1(t)}^t \xi^T(s)Q_1\xi(s)ds, \\ V_3(t) &= \int_{t-\eta_2(t)}^t \xi^T(s)Q_2\xi(s)ds, \\ V_4(t) &= \int_{t-d_1}^t \int_s^t \xi^T(\theta)d_1 R_1 \dot{\xi}(\theta)d\theta ds, \\ V_5(t) &= \int_{t-d_2}^{t-d_1} \int_s^t \xi^T(\theta)(d_2 - d_1)R_2 \dot{\xi}(\theta)d\theta ds.\end{aligned}$$

令

$$\begin{aligned}\mathbb{E}\{\delta(t) - \delta_0\} &= 0, \\ \mathbb{E}\{(\delta(t) - \delta_0)^2\} &= \delta_0(1 - \delta_0).\end{aligned}$$

对上述泛函求导, 可得

$$\begin{aligned}\mathbb{E}\{\dot{V}_1(t)\} &= \mathbb{E}\{2\xi^T(t)P\dot{\xi}(t)\}, \\ \mathbb{E}\{\dot{V}_2(t)\} &\leq \mathbb{E}\{\xi^T(t)Q_1\xi(t) - (1-\tau_1)\xi^T(t-\eta_1(t))Q_1\xi(t-\eta_1(t))\}, \\ \mathbb{E}\{\dot{V}_3(t)\} &\leq \mathbb{E}\{\xi^T(t)Q_2\xi(t) - (1-\tau_2)\xi^T(t-\eta_2(t))Q_2\xi(t-\eta_2(t))\}, \\ \mathbb{E}\{\dot{V}_4(t)\} &\leq \mathbb{E}\{\dot{\xi}^T(t)d_1^2\dot{\xi}(t) - \int_{t-d_1}^t \xi^T(s)d_1 R_1 \dot{\xi}(s)ds\}, \\ \mathbb{E}\{\dot{V}_5(t)\} &\leq \mathbb{E}\{\dot{\xi}^T(t)(d_2 - d_1)^2\dot{\xi}(t) - \int_{t-d_2}^{t-d_1} \xi^T(s)(d_2 - d_1)R_2 \dot{\xi}(s)ds\}.\end{aligned}$$

利用引理1对积分项进行处理, 可得

$$\mathbb{E}\{-d_1 \int_{t-d_1}^t \xi^T(s)R_1 \dot{\xi}(s)ds\} \leq$$

$$-\left[\xi(t) - \xi(t - \eta_1(t))\right]^T R_1 [\xi(t) - \xi(t - \eta_1(t))] - \left[\xi(t - \eta_1(t)) - \xi(t - d_1(t))\right]^T R_1 [\xi(t - \eta_1(t)) - \xi(t - d_1(t))].$$

又有

$$\begin{aligned}\mathbb{E}\{(d_2 - d_1) \int_{t-d_2}^{t-d_1} \dot{\xi}^T(s)R_2 \dot{\xi}(s)ds\} &\leq \\ &-\left[\xi(t - d_1) - \xi(t - \eta_2(t))\right]^T R_2 [\xi(t - d_1) - \xi(t - \eta_2(t))] - \left[\xi(t - \eta_2(t)) - \xi(t - d_2)\right]^T R_2 [\xi(t - \eta_2(t)) - \xi(t - d_2)].\end{aligned}$$

定义

$$\begin{aligned}\Phi^T(t) &= \\ &[\xi^T(t) \quad \xi^T(t - \eta_1(t)) \quad \xi^T(t - d_1) \\ &\quad \xi^T(t - \eta_2(t)) \quad \xi^T(t - d_2) \quad \dot{\xi}^T(t) \quad \omega^T(t)].\end{aligned}$$

整理可得 $\mathbb{E}\{\dot{V}(t)\} \leq \Phi^T(t)\bar{\Omega}\Phi(t)$, 其中:

$$\bar{\Omega} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & 0 & \Psi_{14} & 0 & 0 & P\bar{B}_1(t) \\ * & \Psi_{22} & R_1 & 0 & 0 & 0 & 0 \\ * & * & \Psi_{33} & R_2 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & R_2 & 0 & 0 \\ * & * & * & * & -R_2 & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 \\ * & * & * & * & * & * & -I \end{bmatrix},$$

$$\Psi_{11} = P[\bar{A}(t) + \Delta\bar{A}(t)] + [\bar{A}(t) + \Delta\bar{A}(t)]^T P + Q_1 + Q_2 - R_1,$$

$$\Psi_{12} = \delta_0 P[\bar{B}(t) + \Delta\bar{B}(t)] + R_1,$$

$$\Psi_{14} = (1 - \delta_0)P[\bar{B}(t) + \Delta\bar{B}(t)],$$

$$\Psi_{22} = -(1 - \tau_1)Q_1 - 2R_1,$$

$$\Psi_{33} = -R_1 - R_2,$$

$$\Psi_{44} = -(1 - \tau_2)Q_2 - 2R_2,$$

$$\Psi_{66} = d_1^2 R_1 + (d_2 - d_1)^2 R_2.$$

根据文献[9]定义1可知, 当 $\omega(t) \equiv 0$ 时, 根据Schur补引理, 结合不等式(8)可知 $\mathbb{E}\{\dot{V}(t)\} < 0$, 则闭环系统(7)均方指数稳定. 下面讨论闭环系统的 L_2 - L_∞ 性能指标.

定义

$$J = \mathbb{E}\{V(t)\} - \int_0^t \omega^T(s)\omega(s)ds.$$

在零初始条件下, 对于任意非零 $\omega(t) \in L_2[0, \infty)$ 以及 $t > 0$, 有

$$\begin{aligned}J &= \mathbb{E}\left\{\int_0^t [\dot{V}(t) - \omega^T(s)\omega(s)]ds\right\} = \\ &\mathbb{E}\left\{\int_0^t \Phi^T(s)\bar{\Omega}\Phi(s)ds\right\} < 0,\end{aligned}$$

则

$$\begin{aligned} E\{\xi^T(t)P\xi(t)\} &\leq \\ E\{V(t)\} &< E\left\{\int_0^t \omega^T(s)\omega(s)ds\right\} \end{aligned}$$

成立,应用Schur补引理,可知 $\bar{C}^T(t)\bar{C}(t) < \gamma^2 P$,因此

$$\begin{aligned} E\{z^T(t)z(t)\} &\leq \gamma^2 E\{\xi^T(t)P\xi(t)\} < \\ &\gamma^2 \int_0^\infty \omega^T(s)\omega(s)ds, \end{aligned}$$

即 $E\{\|z(t)\|_\infty^2\} \leq \gamma^2 \|\omega(t)\|_2^2$,定理1可证. 证毕.

基于定理1给出的结果,采用变量替换法以及线性矩阵不等式技术,将不等式(8)–(9)转换为LMIs的形式,并得到有效的观测器及控制器的设计方法.

定理2 对给定标量 $\gamma > 0$,如果存在适维矩阵 $W > 0$, $\bar{R}_i > 0$, $\bar{Q}_i > 0$ 和 $M_j, N_g (j = 1, 2, 3, g = 2, 3)$,满足不等式(10)–(13),则闭环系统(7)均方指数稳定且具有 L_2 - L_∞ 抑制度 γ .

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & 0 \\ * & * & * & \Xi_{44} \end{bmatrix} < 0, \quad (10)$$

$$\begin{bmatrix} W_1 & W_2 & W_1 C_{2i}^T \\ * & W_3 & W_2^T C_{2i}^T \\ * & * & \gamma^2 I \end{bmatrix} > 0, \quad (11)$$

所求控制器和观测器的参数为

$$K_i = M_1 W_1^{-1}, L_i = N_2 W_2^{-1},$$

其中:

$$\begin{aligned} \Xi_{11} &= \begin{bmatrix} \tilde{H}_0 & \delta_0 \tilde{H}_1 + \tilde{R}_1 \\ * & \Lambda_4 \end{bmatrix}, \\ \Xi_{12} &= \begin{bmatrix} 0 & (1 - \delta_0) \tilde{H}_1 \\ \tilde{R}_1 & 0 \end{bmatrix}, \\ \Xi_{14} &= \begin{bmatrix} \tilde{H}_2 & \tilde{H}_3 \\ 0 & \delta_0 \tilde{H}_4 \end{bmatrix}, \Xi_{22} = \begin{bmatrix} -\tilde{R}_1 - \tilde{R}_2 & \tilde{R}_2 \\ * & \Lambda_5 \end{bmatrix}, \\ \Xi_{23} &= \begin{bmatrix} 0 & 0 \\ \tilde{R}_2 & 0 \end{bmatrix}, \Xi_{24} = \begin{bmatrix} 0 & 0 \\ 0 & (1 - \delta_0) \tilde{H}_4 \end{bmatrix}, \\ \Xi_{33} &= \begin{bmatrix} -\tilde{R}_2 & 0 \\ * & d_1^2 \tilde{R}_1 + (d_2 - d_1)^2 \tilde{R}_2 \end{bmatrix}, \\ \Xi_{44} &= \begin{bmatrix} -I & 0 \\ * & \text{diag}\{-\varepsilon_1 I, -\varepsilon_1 I\} \end{bmatrix}, \\ \tilde{R}_1 &= \begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} \\ * & \bar{R}_{13} \end{bmatrix}, \tilde{R}_2 = \begin{bmatrix} \bar{R}_{21} & \bar{R}_{22} \\ * & \bar{R}_{23} \end{bmatrix}, \\ \tilde{Q}_1 &= \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} \\ * & \bar{Q}_{13} \end{bmatrix}, \tilde{Q}_2 = \begin{bmatrix} \bar{Q}_{21} & \bar{Q}_{22} \\ * & \bar{Q}_{23} \end{bmatrix}, \end{aligned}$$

$$\tilde{H}_0 = \begin{bmatrix} A_1 & A_2 \\ * & A_3 \end{bmatrix}, \tilde{H}_1 = \begin{bmatrix} A_6 & A_7 \\ 0 & 0 \end{bmatrix}, \tilde{H}_4 = \begin{bmatrix} A_8^T \\ A_9^T \end{bmatrix},$$

$$\tilde{H}_2 = \begin{bmatrix} B_{1i} & \varepsilon_1 G_i \\ B_{1i} & \varepsilon_1 G_i \end{bmatrix}, \tilde{H}_3 = \begin{bmatrix} W_1 H_{1i}^T \\ W_2^T H_{2i}^T \end{bmatrix},$$

$$\Lambda_1 = A_i W_1 + W_1 A_i^T + \bar{Q}_{11} + \bar{Q}_{21} - \bar{R}_{11},$$

$$\Lambda_2 = A_i W_2 + W_2 A_i^T - C_{1i}^T N_2^T +$$

$$\bar{Q}_{12} + \bar{Q}_{22} - \bar{R}_{12},$$

$$\Lambda_3 = A_i W_3 + W_3 A_i^T - N_3 C_{1i} - C_{1i}^T N_3^T +$$

$$\bar{Q}_{13} + \bar{Q}_{23} - \bar{R}_{13},$$

$$\Lambda_4 = -(1 - \tau_1) \bar{Q}_1 - 2 \bar{R}_1,$$

$$\Lambda_5 = -(1 - \tau_5) \bar{Q}_5 - 2 \bar{R}_5,$$

$$\Lambda_6 = B_i M_1 - B_i M_4,$$

$$\Lambda_7 = B_i M_2 - B_i M_3,$$

$$\Lambda_8 = H_{2i} M_1 + H_{2i} M_4,$$

$$\Lambda_9 = H_{2i} M_2 + H_{2i} M_3.$$

证 定义

$$W = P^{-1} = \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix},$$

令

$$\Delta_1 = \text{diag}\{W, W, W, W, W, W, I\},$$

$$\Delta_2 = \text{diag}\{W, I\},$$

采用 Δ_1, Δ_2 对不等式(8)–(9)进行全等变换,并取

$$\bar{Q}_i = W Q_i W, \bar{R}_i = W R_i W, i = 1, 2,$$

$$M_j = K W_j, M_4 = K W_2^T, W_g C_{1i}^T = C_{1i}^T \tilde{W}_g,$$

$$N_g = \tilde{W} L_i^T, j = 1, 2, 3, g = 2, 3,$$

则可得式(10).

同理可得式(11),定理2得证. 证毕.

注3 解决式 $W_g C_{1i}^T = C_{1i}^T \tilde{W}_g (g = 2, 3)$ 中的等式约束问题,这里采用的是奇异值分解法(singular value decomposition, SVD),对于列满秩矩阵 C_1^T ,有

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} C_1^T V = \begin{bmatrix} \Sigma \\ 0 \end{bmatrix},$$

其中: U, V 是正交矩阵, Σ 为对角元素为正的对角阵.

如果矩阵 W_2 满足 $W_2 = U_1^T W_{21} U_1 + U_2^T W_{22} U_2$,其中: $W_{21} > 0, W_{22} > 0$,存在矩阵 $\tilde{W}_2$ 满足 $W_2 C_1^T = C_1^T \tilde{W}_2$,则 $\tilde{W}_2^{-1} = V \Sigma^{-1} W_{21}^{-1} \Sigma V^T$,上述可得控制器和观测器增益

$$K = M_1 W_1^{-1}, L = N V \Sigma^{-1} W_{21}^{-1} \Sigma V^T.$$

4 仿真实例(Numerical example)

考虑系统(1),相关参数给出如下:

$$A_1 = \begin{bmatrix} -0.88 & 0.1 \\ 0.93 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 0.52 & 0.9 \\ 0.11 & -0.9 \end{bmatrix},$$

$$\begin{aligned}
B_1 &= \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} -0.2 \\ -0.8 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix}, \\
B_{12} &= \begin{bmatrix} 1.7 \\ -0.1 \end{bmatrix}, C_{11} = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}^T, C_{12} = \begin{bmatrix} 1.3 \\ 0.2 \end{bmatrix}^T, \\
C_{21} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T, C_{22} = \begin{bmatrix} 0.1 \\ 0.01 \end{bmatrix}^T, H_1 = \begin{bmatrix} 0 & 3 \\ -3.8 & 1 \end{bmatrix}, \\
d_1 &= 0.5, d_2 = 0.8, \tau_1 = 0.1, \tau_2 = 0.6, \\
G &= 1, H_2 = [0 \ 0.05]^T.
\end{aligned}$$

选取 $A(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \sin t \end{bmatrix}$, 由定理2可同时求得控

制器和观测器参数如下:

$$\begin{aligned}
K_1 &= [8.9677 \ 0.7906], L_1 = \begin{bmatrix} -1.7226 \\ -2.0635 \end{bmatrix}, \\
K_2 &= [6.8797 \ -1.9292], L_2 = \begin{bmatrix} -570.6310 \\ -53.6090 \end{bmatrix}.
\end{aligned}$$

为了验证本文方法的有效性, 分别针对具有确定性时滞与随机时滞两种网络控制系统的最小抑制度 γ 值进行对比. 系统在具有确定性时滞的情况下时滞 d 取 d_1 和 d_2 的最大值, 同理 τ 取 τ_1 和 τ_2 的最大值, 即 $d = \max\{d_1, d_2\}$, $\tau = \max\{\tau_1, \tau_2\}$, 考虑随机时滞时给定 $\delta_0 = 0.74$. 表1给出这两种情况下 γ 值对比, 可知, 在时滞相对较小时, 确定性时滞与随机时滞所得的 γ 值相差无几, 但在时滞较大时, 可以看出考虑时滞随机性所得的 $\gamma_2^* = 0.0194$ 明显小于确定性时滞 $\gamma_1^* = 0.0447$. 所以, 本文的方法能够得到更小的 γ 值, 具有更好的 L_2 - L_∞ 干扰抑制水平, 降低了结果的保守性.

表1 有无随机时滞 γ 值的对比

Table 1 Comparison of γ values with or without random time delay

	$d_1 = 5.5, d_2 = 8$	$d_1 = 16, d_2 = 20$
确定性时滞 γ_1^*	0.0293	0.0447
随机时滞 γ_2^*	0.0279	0.0194

选取隶属度函数如下:

$$\begin{aligned}
M_1(x_1(t)) &= e^{-0.5x_1^2(t)}, \\
M_2(x_1(t)) &= 1 - M_1(x_1(t)).
\end{aligned}$$

给出初始状态 $x^T(0) = [0.1 \ -0.1]$, 扰动输入 $\omega(t) = e^{-t}$, 仿真结果图2-3所示. 图2是未加控制器前的开环系统状态响应, 图3是基于观测器的控制作用下的闭环系统状态响应. 仿真结果表明系统在所设计的控制器作用下稳定性较好, 进而证明了本文所提方法的有效性.

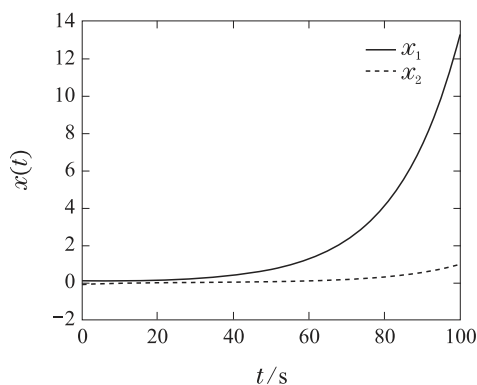


图2 开环状态响应

Fig. 2 Open-loop state response

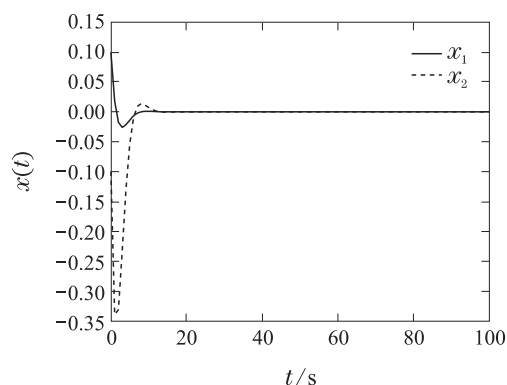


图3 闭环状态响应

Fig. 3 Closed-loop state response

5 结论(Conclusions)

本文基于Lyapunov稳定性理论和线性矩阵不等式技术, 采用Bernoulli 0-1分布描述基于T-S模糊模型的连续非线性网络控制系统中存在的随机时延问题, 建立相应的模型, 设计出基于观测器的使闭环增广系统均方指数稳定且满足 L_2 - L_∞ 性能指标的控制并数值仿真验证了该方法的有效性. 由于传输时滞对前件变量产生影响, 因此将作为下一步研究重点.

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作者简介:

李艳辉 (1970–), 女, 教授, 博士生导师, 从事鲁棒控制及滤波、智能控制等研究, E-mail: LY_hui@hotmail.com;

吴迪 (1994–), 女, 硕士研究生, 从事鲁棒控制及滤波方向研究, E-mail: 8581772@qq.com.