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具有输入未建模动态的纯反馈非线性系统自适应控制

张天平†, 葛继伟, 夏晓南

(扬州大学 信息工程学院 自动化专业部, 江苏 扬州 225127)

摘要: 对一类具有状态和输入未建模动态且控制增益符号未知的纯反馈非线性系统, 利用非线性变换、改进的动态面控制方法以及Nussbaum函数性质, 提出两种自适应动态面控制方案. 利用正则化信号来约束输入未建模动态, 从而有效地抑制其产生的扰动. 通过引入动态信号, 有效地处理了由状态未建模动态引起的动态不确定性. 通过在总的李雅普诺夫函数中引入非负正则化信号, 并利用稳定性分析中引入的紧集, 证明了闭环控制系统是半全局一致终结有界的. 数值仿真验证了所提方案的有效性.

关键词: 输入未建模动态; 动态面控制; 积分型Lyapunov函数; Nussbaum函数

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Adaptive control of pure-feedback nonlinear systems with input unmodeled dynamics

ZHANG Tian-ping†, GE Ji-wei, XIA Xiao-nan

(Department of Automation, College of Information Engineering, Yangzhou University, Yangzhou Jiangsu 225127, China)

Abstract: Based on dynamic surface control (DSC) method and using Nussbaum function property, two adaptive DSC schemes are developed for a class of pure-feedback nonlinear systems with state and input unmodeled dynamics as well as unknown control gain sign in this paper. Normalization signal is designed to restrict the input unmodeled dynamics, and the disturbance produced by it is effectively suppressed. Dynamic signal is introduced to deal with the dynamic uncertainty caused by unmodeled dynamics. By adding the normalization signal to the whole Lyapunov function and using the defined compact set in stability analysis, all the signals in the closed-loop system are proved to be semi-globally uniformly ultimately bounded (SGUUB). Numerical simulation verifies the effectiveness of the proposed approach.

Key words: input unmodeled dynamics; dynamic surface control; integral Lyapunov function; Nussbaum function

1 引言(Introduction)

自从文献[1]提出后推设计以来, 它已成为非线性系统控制的主要设计工具. 其缺点是在后推的每一步需对虚拟控制反复求导, 随着系统阶次的增加, 控制器的结构越加复杂, 通常称为“微分爆炸”问题. 文献[2]通过在后推的每一步引入一个1 阶滤波器, 用代数运算代替微分运算来消除传统后推设计的不足. 文献[3–4]在文献[2]基础上分别对严格反馈及纯反馈两类非线性系统提出两种自适应动态面控制方案. 进一步, 文献[5]提出一种改进的动面控制策略.

近年来, 带有输入未建模动态的自适应控制受到了人们广泛的关注, 并取得了一些研究成果. 文献[6]首次对输入未建模动态展开了研究, 并分别对具有线性输入未建模动态的严格反馈非线性系统和输出反馈非线性系统, 利用正则化信号、动态非线性阻尼设计和后推技术, 设计了相应的控制律. 该设计保证了对于传递函数描述下的输入未建模增益, 存在一个独立于初始条件的正则化信号, 使得系统所有输入与状态收敛于一个区间内. 文献[7]利用小增益定理拓展了文献[6]关于输入未建模动态的研究思路. 文献[8]在文献[6–7]的基础上得到了进一步的结果, 证明了未建模动态子系统为零相对阶的最小相位系统的有界性. 文献[9–15]关于输入未建模动态展开了不同的讨论. 对于线性输入未建模动态, 相应的约束条件是子系统为最小相位系统, 而对于非线性输入未建模动态, 要求子系统零动态是输入状态稳定的. 在该假设条件下, 根据输入未建模动态李雅普诺夫函数的指数收敛率, 设计正则化信号, 提出自适应后推控制律, 但系统高

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†通信作者. E-mail: tpzhang@yzu.edu.cn; Tel.: +86 514-87978319.

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频增益符号假设是已知的.

众所周知, 当系统的控制方向未知时常常给控制器的设计带来较大困难. 由于具有广阔的应用背景, 控制增益符号未知的非线性系统受到广泛的讨论. 文献[16]为控制方向未知的系统提供了一种通用性控制方法, 即Nussbaum函数增益技术. 文献[17–18]针对存在未知高频增益和时变不确定性的非线性系统, 利用Nussbaum 函数和后推技术, 提出了一种鲁棒控制策略. 文献[19]利用Nussbaum函数性质讨论了一类具有时滞不确定性的严格反馈系统的自适应控制问题, 同时给出了时变控制增益符号未知的闭环系统稳定的判断定理. 文献[20]对一类具有未建模动态的纯反馈非线性系统, 在虚拟控制增益已知和未知的两种情形下, 分别提出了自适应动态面控制方案, 并利用Nussbaum函数解决了虚拟控制增益未知的问题. 文献[21]对一类具有未建模动态及动态不确定性的严格反馈非线性系统, 利用李雅普诺夫函数刻画状态未建模动态, 提出一种新的自适应动态面控制方案. 文献[22–23]对一类带有输入未建模动态的输出反馈非线性系统, 利用正则化信号约束输入未建模动态, 提出两种输出反馈自适应动态面控制策略. 文献[24]对一类具有未建模动态和死区的纯反馈非线性系统, 在假设控制增益符号已知的条件下, 提出一种基于改进动态面控制的自适应神经网络控制方案.

本文在文献[5, 20, 22, 24]的基础上, 对一类纯反馈非线性系统, 提出了两种新的鲁棒自适应动态面控制策略. 主要贡献如下: 1) 对同时具有状态和输入未建模动态的非线性系统, 分别讨论了控制增益$g_n(x)$符号已知和未知两种情况, 提出了两种不同的自适应控制策略, 而文献[22–23]中讨论的系统是一类输出反馈非线性系统. 2) 通过非线性变换将纯反馈系统转化为更容易分析的严格反馈系统形式, 采用改进的动态面控制方法, 避免采用中值定理, 从而移去了虚拟控制增益符号及其上下界已知的假设条件, 并简化了设计. 3) 在后推设计的前$n-1$步仅有一个参数需要在线调节, 减轻了计算量. 4) 通过在总的李雅普诺夫函数中加入非负正则化信号, 并利用动态面控制证明的特点, 有效地处理了控制信号的有界性.

2 问题的描述及基本假设(Problem statement and basic assumptions)

考虑如下一类具有输入未建模动态的纯反馈非线性系统:

$$\begin{cases} \dot{z} = q(t,z,x), \\ \dot{x}_i = f_i(\bar{x}_i, x_{i+1}) + \Delta_i(t,z,x), \\ \qquad 1 \leqslant i \leqslant n-1, \\ \dot{x}_n = f_n(x) + g_n(x)\omega + \Delta_n(t,z,x),\ n \geqslant 2, \\ y = x_1, \end{cases} \tag{1}$$

式中: $\bar{x}_i = [x_1\ x_2\ \cdots\ x_i]^{\mathrm{T}} \in \mathbb{R}^i$, $i = 1, 2\ \cdots, n$; $x = [x_1\ \cdots\ x_n] \in \mathbb{R}^n$是状态向量, $\omega \in \mathbb{R}$是作用在非线性系统上的不可量测信号, $y \in \mathbb{R}$是系统输出, $g_n(x)$, $f_i(\cdot)(i = 1, \cdots, n)$是未知光滑函数, $z \in \mathbb{R}^{n_0}$是不可测量状态, $\Delta_i(t,z,x)(i = 1, \cdots, n)$为未知不确定扰动.

输入未建模子系统描述如下:

$$\dot{p} = A_\Delta(p) + b_\Delta u, \tag{2}$$

$$\omega = c_\Delta(p) + d_\Delta u, \tag{3}$$

式中: $p \in \mathbb{R}^{n_1}$是由输入$u \in \mathbb{R}$所产生的未建模状态, $\omega \in \mathbb{R}$是n_1阶子系统的输出, $A_\Delta(\cdot)$和b_Δ是未知向量, $c_\Delta(\cdot)$是未知函数并且d_Δ未知常数.

控制目标: 设计自适应控制律u, 使得系统的输出y尽可能好地跟踪一个给定的期望信号y_{d}, 并保证闭环系统是半全局一致终结有界的, 且跟踪误差收敛到一个小的残差集内.

定义 1[25] 对于系统$\dot{z} = q(t,z,x)$, 如果存在K_∞类函数$\bar{\alpha}_1$, $\bar{\alpha}_2$和一个Lyapunov函数$V_0(z)$使得

$$\bar{\alpha}_1(\|z\|) \leqslant V_0(z) \leqslant \bar{\alpha}_2(\|z\|), \tag{4}$$

以及存在两个常数$c > 0$, $d \geqslant 0$和一个K_∞类函数$\gamma(\cdot)$使得

$$\frac{\partial V_0(z)}{\partial z} q(t,z,x) \leqslant -cV_0(z) + \gamma(|x_1|) + d, \tag{5}$$

式中: $c > 0$, $d \geqslant 0$是两个已知常数, $\gamma(\cdot)$是一个已知K_∞类函数, 则称未建模动态是指数输入状态实用稳定(exponentially input-state-practically stable, exp-ISpS).

假设 1[25] 未建模动态是指数输入状态实用稳定(exp-ISpS)的.

假设 2 $g_n(x)$的符号是已知的, 且存在常数g_{i0}和g_{n1}, 使得

$$0 < g_{i0} \leqslant |g_i(\bar{x}_{i+1})|,\ 1 \leqslant i \leqslant n-1,$$
$$0 < g_{n0} \leqslant |g_n(x)| \leqslant g_{n1},$$

其中

$$g_i(\bar{x}_{i+1}) = \frac{\partial f_i(\bar{x}_i, x_{i+1})}{\partial x_{i+1}},\ i = 1, \cdots, n-1.$$

不失一般性, 假设$g_n(x) > 0$.

假设 3[3] 期望轨迹向量$x_{\mathrm{d}} = [y_{\mathrm{d}}\ \dot{y}_{\mathrm{d}}\ \ddot{y}_{\mathrm{d}}]^{\mathrm{T}} \in \Omega_{\mathrm{d}}$连续可测, 其中$\Omega_{\mathrm{d}} = \{x_{\mathrm{d}} : y_{\mathrm{d}}^2 + \dot{y}_{\mathrm{d}}^2 + \ddot{y}_{\mathrm{d}}^2 \leqslant B_0\}$是一个紧集, B_0是一个已知正常数.

假设 4[25] 对未知不确定扰动$\Delta_i(t,z,x)$, $i = 1, \cdots, n$, 存在未知非负连续函数$\rho_{i1}(\cdot)$, 未知非负连续单调递增函数$\rho_{i2}(\cdot)$, 使得

$$|\Delta_i(t,z,x)| \leqslant \rho_{i1}(\|\bar{x}_i\|) + \rho_{i2}(\|z\|),\ i = 1, \cdots, n,$$

其中$\|\cdot\|$表示欧氏范数.

假设 5[13] 对于输入未建模动态子系统(2)–(3), 其相对阶数为零, 即$d_\Delta \neq 0$, 且存在一个常数$\bar{c}$, 使得$\|c_\Delta(p)\| \leqslant \bar{c}\|p\|$.

假设 6[13] 对于输入未建模动态子系统(2)–(3), 存在一个Lyapunov函数$W(p)$, 满足

$$\beta_{p1}\|p\|^2 \leqslant W(p) \leqslant \beta_{p2}\|p\|^2, \tag{6}$$

$$\frac{\partial W}{\partial p}A_\Delta(p) \leqslant -2\delta_0 W(p), \tag{7}$$

$$\|\frac{\partial W}{\partial p}\| \leqslant \beta_{p3}\|p\|, \tag{8}$$

式中: $\beta_{p1}, \beta_{p2}, \beta_{p3}$是正常数, δ_0是已知正常数.

引理 1[25] 若$V_0(t)$是系统$\dot{z} = q(t,z,x)$的一个exp-ISpS李雅普诺夫函数,即假设1成立, 则对于任意常数$\bar{c}_\mathrm{f} \in (0, c)$, 任意初始时间$t_0 > 0$, 任意初始状态$z_0 = z(t_0)$, $v_0 > 0$和任意$\bar{\gamma}(|x_1|) \geqslant \gamma(|x_1|)$, 存在有限时间

$$T_0 = V_0(z_0)\mathrm{e}^{(c-\bar{c}_\mathrm{f})t_0}/v_0(c-\bar{c}_\mathrm{f}) \geqslant 0.$$

对于非负函数$D(t_0,t)$, 定义动态信号$\dot{v} = -\bar{c}_\mathrm{f}v + \bar{\gamma}(|x_1|) + d$. 当$t \geqslant t_0 + T_0$时, 存在$D(t_0,t) = 0$, 使得$V_0(z) \leqslant v(t) + D(t_0,t)$. 不失一般性, 取$\bar{\gamma}(|x_1|) = \gamma(|x_1|)$.

引理 2 若假设6成立, $\dot{\bar{m}} = -\delta_0\bar{m} + |u|$, 则存在常数$c_1, c_2 > 0$使得

$$\|p(t)\| \leqslant c_1(\|p(0)\| + |\bar{m}(0)|)\mathrm{e}^{-\delta_0 t} + c_2|\bar{m}(t)|, \tag{9}$$

其中δ_0由式(7)确定. 引理2证明参见文献[13].

注 1 假设1是对未建模动态的要求; 假设2是为了保证所讨论的下三角型系统是能控的而对未知系统函数提出的基本要求; 假设3是对跟踪信号的要求; 假设4是对动态不确定性提出的要求; 假设5–6是对输入未建模动态的刻画. 假设1–6在现有文献中已被广泛使用. 仿真中应该验证状态未建模动态和输入未建模动态满足假设1, 4–6. 此外, 需要构造适当的李雅普诺函数, 如$V_0(z) = \frac{1}{2}z^2$, $W(p) = \frac{1}{2}p^2$来确定设计动态信号、正则化信号用到的设计参数$\bar{c}_\mathrm{f}$及δ_0.

3 控制增益符号已知的控制器设计(Controller design with known gain sign)

本节中, 首先讨论系统控制增益$g_n(x)$及d_Δ符号已知的情形, 不妨假设全为正.

令$F_i(\bar{x}_i, x_{i+1}) = f_i(\bar{x}_i, x_{i+1}) - x_{i+1}$, $i = 1,\cdots,n-1$. 则系统(1)可改写为如下形式:

$$\begin{cases}\dot{z} = q(t,z,x),\\ \dot{x}_i = F_i(\bar{x}_i, x_{i+1}) + x_{i+1} + \varDelta_i(t,z,x),\\ \qquad 1 \leqslant i \leqslant n-1,\\ \dot{x}_n = f_n(x) + g_n(x)\varOmega + \varDelta_n(t,z,x),\ n \geqslant 2,\\ y = x_1.\end{cases} \tag{10}$$

对于未知连续函数$F_i(\bar{x}_i, x_{i+1})$, $1 \leqslant i \leqslant n-1$, 在给定的紧集$\varOmega_{Z_i}$上, 本文将采用径向基函数神经网络进行逼近, 即

$$\begin{aligned}&F_i(Z_i) = F_i(\bar{x}_i, x_{i+1}) =\\ &\theta_i^{*\mathrm{T}}\xi_i(Z_i) + \varepsilon_i(Z_i),\ Z_i \in \varOmega_{Z_i},\end{aligned} \tag{11}$$

式中: $Z_i = \bar{x}_{i+1}, \varepsilon_i(Z_i)$是逼近误差, $i = 1,\cdots,n-1$, $F_n(Z_n)$将在最后一步中给出, $Z_n = [x^\mathrm{T}\ \ s_n\ \ \dot{\beta}_n\ \ v]^\mathrm{T}$. 基向量$\xi_i(Z_i) = [\xi_{i1}(Z_i)\ \ \cdots\ \ \xi_{il_i}(Z_i)]^\mathrm{T} \in \mathbb{R}^{l_i}$, 基函数定义如下:

$$\xi_{ik}(Z_i) = \exp(-\frac{\|Z_i - b_{ik}\|^2}{a_{ik}^2}), \tag{12}$$

其中: b_{ik}和a_{ik}分别为高斯函数的中心和宽度, $k = 1,\cdots,l_i$, 理想权向量θ_i^*定义为

$$\theta_i^* = \arg\min_{\theta_i \in R^{l_i}}\{\sup_{Z_i \in \varOmega_{Z_i}}|F_i(Z_i) - \theta_i^\mathrm{T}\xi_i(Z_i)|\}. \tag{13}$$

控制器设计分为n步, β_i是以α_i为输入的一阶滤波器的输出, $i = 2,\cdots,n$. 最后, 控制律u将在第n步提出.

为了叙述方便, 定义一些如下形式的Lyapunov函数:

$$V_{s_i} = \frac{1}{2}s_i^2,\ i = 1,\cdots,n-1, \tag{14}$$

$$V_1 = V_{s_1} + \frac{v}{\lambda_0}, \tag{15}$$

$$V_i = \sum_{j=1}^{i}V_{s_j},\ i = 2,\cdots,n, \tag{16}$$

式中: $s_1 = x_1 - \beta_1 = y - y_\mathrm{d}$, $s_i = x_i - \beta_i$, $i = 2,\cdots,n$.

第1步 由式(10)可知

$$\dot{x}_1 = F_1(\bar{x}_2) + x_2 + \varDelta_1(t,z,x). \tag{17}$$

对s_1求导得

$$\dot{s}_1 = \theta_1^{*^\mathrm{T}}\xi_1(\bar{x}_2) + \varepsilon_1(\bar{x}_2) + x_2 + \varDelta_1(t,z,x) - \dot{\beta}_1. \tag{18}$$

设计虚拟控制律α_2如下:

$$\alpha_2 = -k_1s_1 - \frac{1}{2a_1^2}\hat{\lambda}s_1\|\xi_1(\bar{x}_2)\|^2 + \dot{\beta}_1, \tag{19}$$

式中: $a_1 > 0, k_1 > 0$是设计常数, $\hat{\lambda}$是λ在t时刻的估计, 而$\lambda = \max\limits_{1\leqslant i\leqslant n}\|\theta_i^*\|^2$.

设计一阶滤波器如下:

$$\tau_2\dot{\beta}_2 + \beta_2 = \alpha_2,\ \beta_2(0) = \alpha_2(0), \tag{20}$$

式中: τ_2为时间常数, α_2为系统输入, β_2为系统状态.

令$y_2 = \beta_2 - \alpha_2$, 可得出$\dot{y}_2 = -\frac{y_2}{\tau_2} - \dot{\alpha}_2$, 因此有

$$\begin{aligned}y_2\dot{y}_2 \leqslant\ & -\frac{y_2^2}{\tau_2} + |y_2|\eta_2(\bar{s}_n, \bar{y}_n, \hat{\lambda}, v, y_\mathrm{d}, \dot{y}_\mathrm{d}, \ddot{y}_\mathrm{d}) \leqslant\\ & -\frac{y_2^2}{\tau_2} + y_2^2 + \frac{\eta_2^2}{4},\end{aligned} \tag{21}$$

式中$\eta_2(\bar{s}_n,\bar{y}_n,\hat{\lambda},v,y_{\rm d},\dot{y}_{\rm d},\ddot{y}_{\rm d})$是一个非负连续函数.

对V_{s1}关于时间t求导, 得

$$\begin{aligned}\dot{V}_{s_1}=&s_1[\theta_1^{*\rm T}\xi_1(\bar{x}_2)+\varepsilon_1(\bar{x}_2)+x_2+\\&\Delta_1(t,z,x)-\dot{y}_{\rm d}]+\frac{\dot{v}}{\lambda_0}\leqslant\\&s_1(s_2+y_2+\alpha_2)+\frac{1}{2a_1^2}\lambda s_1^2\|\xi_1(\bar{x}_2)\|^2+\\&s_1[\rho_{11}(\|x_1\|)+\rho_{12}(\|z\|)]-\\&s_1\dot{y}_{\rm d}+\frac{\gamma(x_1)}{\lambda_0}-\frac{\bar{c}_{\rm f}}{\lambda_0}v+\frac{d}{\lambda_0}+\frac{a_1^2}{2}+s_1\varepsilon_1(\bar{x}_2)\leqslant\\&-(k_1-\frac{1}{2})s_1^2+s_2^2+y_2^2-\frac{1}{2a_1^2}\tilde{\lambda}s_1^2\|\xi_1(\bar{x}_2)\|^2+\\&s_1[\rho_{11}(\|x_1\|)+\rho_{12}(\|z\|)]+\\&\frac{\gamma(x_1)}{\lambda_0}-\frac{\bar{c}_{\rm f}}{\lambda_0}v+\frac{d}{\lambda_0}+\frac{a_1^2}{2}+s_1\varepsilon_1(\bar{x}_2),\end{aligned}\tag{22}$$

式中$\tilde{\lambda}=\hat{\lambda}-\lambda$.

由假设4和引理1可知存在一个正常数D_0, 使得$D(t_0,t)\leqslant D_0$, $\forall t\geqslant 0$, 可得

$$\|z\|\leqslant\bar{\alpha}_1^{-1}(v+D_0).\tag{23}$$

因此

$$\begin{aligned}\dot{V}_{s_1}\leqslant&-(k_1-\frac{1}{2})s_1^2+s_2^2+y_2^2-\\&\frac{1}{2a_1^2}\tilde{\lambda}s_1^2\|\xi_1(\bar{x}_2)\|^2+\frac{a_1^2}{2}+s_1\varepsilon_1(\bar{x}_2)+\\&\frac{\gamma(x_1)}{\lambda_0}-\frac{\bar{c}_{\rm f}}{\lambda_0}v+\frac{d}{\lambda_0}+s_1[\rho_{11}(\|x_1\|)+\\&\rho_{12}\circ\bar{\alpha}_1^{-1}(v+D_0)],\end{aligned}\tag{24}$$

式中: $\rho_{12}\circ\bar{\alpha}_1^{-1}(V_0(v+D_0))$表示$\rho_{12}(\bar{\alpha}_1^{-1}(V_0(v+D_0)))$.

由Young's不等式得

$$s_1\rho_{11}(\|x_1\|)\leqslant\frac{s_1^2}{4}+\rho_{11}^2(\|x_1\|),\tag{25}$$

$$s_1[\rho_{12}\circ\bar{\alpha}_1^{-1}(v+D_0)]\leqslant\frac{s_1^2}{4}+\rho_{12}^2\circ\bar{\alpha}_1^{-1}(v+D_0).\tag{26}$$

将式(25)–(26)代入式(24), 可得

$$\begin{aligned}\dot{V}_{s_1}\leqslant&-(k_1-\frac{5}{4})s_1^2+s_2^2+y_2^2-\frac{1}{2a_1^2}\tilde{\lambda}s_1^2\|\xi_1(\bar{x}_2)\|^2+\\&\frac{a_1^2}{2}+\kappa_1^2+\rho_{11}^2(\|x_1\|)+\rho_{12}^2\circ\bar{\alpha}_1^{-1}(v+D_0)+\\&\frac{\gamma(x_1)}{\lambda_0}-\frac{\bar{c}_{\rm f}}{\lambda_0}v+\frac{d}{\lambda_0},\end{aligned}\tag{27}$$

式中: $|\varepsilon_1(\bar{x}_2)|\leqslant\kappa_1(\bar{s}_n,\bar{y}_n,\hat{\lambda},y_{\rm d},\dot{y}_{\rm d})$, $\kappa_1(\cdot)$是一个未知的非负连续函数.

第i步$(2\leqslant i\leqslant n-1)$ 对s_i求导得

$$\begin{aligned}\dot{s}_i=&\theta_i^{*\rm T}\xi_i(\bar{x}_{i+1})+\varepsilon_i(\bar{x}_{i+1})+x_{i+1}+\\&\Delta_i(t,z,x)-\dot{\beta}_i.\end{aligned}\tag{28}$$

设计虚拟控制律α_{i+1}如下:

$$\alpha_{i+1}=-k_is_i-\frac{1}{2a_i^2}\hat{\lambda}s_i\|\xi_i(\bar{x}_{i+1})\|^2+\dot{\beta}_i,\tag{29}$$

式中: $a_i>0$, $k_i>0$是设计常数.

设计一阶滤波器如下:

$$\tau_{i+1}\dot{\beta}_{i+1}+\beta_{i+1}=\alpha_{i+1},\ \beta_{i+1}(0)=\alpha_{i+1}(0),\tag{30}$$

式中: τ_{i+1}为时间常数. 令$y_{i+1}=\beta_{i+1}-\alpha_{i+1}$, 可得$\dot{y}_{i+1}=-\dfrac{y_{i+1}}{\tau_{i+1}}-\dot{\alpha}_{i+1}$, 进一步有

$$\begin{aligned}&y_{i+1}\dot{y}_{i+1}\leqslant\\&-\frac{y_{i+1}}{\tau_{i+1}}+|y_{i+1}|\,\eta_{i+1}(\bar{s}_n,\bar{y}_n,\hat{\lambda},v,y_{\rm d},\dot{y}_{\rm d},\ddot{y}_{\rm d})\leqslant\\&-\frac{y_{i+1}^2}{\tau_{i+1}}+y_{i+1}^2+\frac{\eta_{i+1}^2}{4}.\end{aligned}\tag{31}$$

类似于第1步的推导, 易得

$$\begin{aligned}\dot{V}_{si}\leqslant&-(k_i-\frac{5}{4})s_i^2+s_{i+1}^2+\rho_{i1}^2(\|\bar{x}_i\|)+\\&y_{i+1}^2-\frac{1}{2a_i^2}\tilde{\lambda}s_i^2\|\xi_i(\bar{x}_{i+1})\|^2+\frac{a_i^2}{2}+\\&\kappa_i^2+\rho_{i2}^2\circ\bar{\alpha}_1^{-1}(v+D_0),\end{aligned}\tag{32}$$

式中: $|\varepsilon_i(\bar{x}_{i+1})|\leqslant\kappa_i(\bar{s}_n,\bar{y}_n,\hat{\lambda},y_{\rm d},\dot{y}_{\rm d})$, $\kappa_i(\cdot)$是一个未知的非负连续函数.

第n步 令$s_n=x_n-\beta_n$, 因此可得

$$\dot{s}_n=f_n(x)+g_n(x)\omega+\Delta_n(t,z,x)-\dot{\beta}_n.\tag{33}$$

令$G_n(x)=d_\Delta g_n(x)$, 定义一个光滑Lyapunov函数如下:

$$V_{s_n}=\int_0^{s_n}\frac{\zeta}{G_n(\bar{x}_{n-1},\zeta+\beta_n)}{\rm d}\zeta.\tag{34}$$

由积分第2中值定理可知

$$V_{s_n}=\frac{s_n^2}{2G_n(\bar{x}_{n-1},\sigma s_n+\beta_n)},$$

其中$\sigma\in(0,1)$. 因此V_{s_n}为正定函数. 将V_{s_n}对时间t求导并利用分部积分可得

$$\begin{aligned}\dot{V}_{s_n}=&\frac{s_n\dot{s}_n}{G_n(x)}+s_n^2\sum_{j=1}^{n-1}\Delta_j(t,z,x)\times\\&\int_0^1\sigma\frac{\partial G_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)}{\partial x_j}{\rm d}\sigma+\frac{\dot{\beta}_ns_n}{G_n(x)}+\\&s_n^2\int_0^1\sigma\sum_{j=1}^{n-1}\frac{\partial G_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)}{\partial x_j}\times\\&(x_{j+1}+F_j(\bar{x}_j,x_{j+1})){\rm d}\sigma-\\&\dot{\beta}_ns_n\int_0^1\frac{1}{G_n(\bar{x}_{n-1},\sigma s_n+\beta_n)}{\rm d}\sigma.\end{aligned}\tag{35}$$

由假设4得

$$\Delta_n(t,z,x)\leqslant\rho_{n1}(\|x\|)+\rho_{n2}(\|z\|).\tag{36}$$

同理, 与第1步类似, 由假设4和引理1可得

$$\rho_{n2}(\|z\|)\leqslant\rho_{n2}\circ\bar{\alpha}_1^{-1}(v+D_0).\tag{37}$$

由Young's不等式可得

$$s_n^2(\sum_{j=1}^{n-1}\int_0^1\sigma\frac{\partial G_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)}{\partial x_j}\mathrm{d}\sigma)\varDelta_j(t,z,x)\leqslant \frac{s_n^2}{2}\sum_{j=1}^{n-1}(\int_0^1\sigma\frac{\partial G_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)}{\partial x_j}\mathrm{d}\sigma)^2+\frac{s_n^2}{2}\sum_{j=1}^{n-1}\varDelta_j^2(t,z,x).\tag{38}$$

由假设4和引理1, 可得

$$\frac{s_n^2}{2}\sum_{j=1}^{n-1}\varDelta_j^2(t,z,x)\leqslant \frac{s_n^2}{2}\sum_{j=1}^{n-1}[\rho_{j1}(\|x\|)+\rho_{j2}(\|z\|)]^2\leqslant s_n^2\sum_{j=1}^{n-1}\rho_{j1}^2(\|x\|)+s_n^2\sum_{j=1}^{n-1}\rho_{j2}^2\circ\bar{\alpha}_1^{-1}(v+D_0).\tag{39}$$

令

$$F_n(Z_n)= s_n\int_0^1\sigma\sum_{j=1}^{n-1}\frac{\partial G_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)}{\partial x_j}\times (x_{j+1}+F_j(\bar{x}_j,x_{j+1}))\mathrm{d}\sigma+\frac{f_n(x)}{G_n(x)}- \dot{\beta}_n\int_0^1\frac{1}{G_n(\bar{x}_{n-1},\sigma s_n+\beta_n)}\mathrm{d}\sigma+\frac{\rho_{n1}(\|x\|)}{G_n(x)}+ \frac{\rho_{n2}\circ\bar{\alpha}_1^{-1}(v+D_0)}{G_n(x)}+ \frac{s_n}{2}\sum_{j=1}^{n-1}(\int_0^1\sigma\frac{\partial G_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)}{\partial x_j}\mathrm{d}\sigma)^2+ s_n\sum_{j=1}^{n-1}\rho_{j1}^2(\|x\|)+s_n\sum_{j=1}^{n-1}\rho_{j2}^2\circ\bar{\alpha}_1^{-1}(v+D_0),\tag{40}$$

式中$Z_n=[x^{\mathrm{T}}\ \ s_n\ \ \dot{\beta}_n\ \ v]^{\mathrm{T}}\in\mathbb{R}^{n+3}$.

对于未知连续函数$F_n(Z_n)$, 在给定的紧集$\varOmega_{Z_n}$上采用径向基函数神经网络进行逼近, 即

$$F_n(Z_n)=\theta_n^{*\mathrm{T}}\xi_n(Z_n)+\varepsilon_n(Z_n).\tag{41}$$

将式(33)(36)–(41)代入式(35), 可得

$$\dot{V}_{sn}\leqslant\frac{s_n\omega}{d_\varDelta}+\frac{1}{2a_n^2}\lambda s_n^2\left\|\xi_n(Z_n)\right\|^2+ s_n\varepsilon_n(Z_n)+\frac{a_n^2}{2}.\tag{42}$$

将式(3)代入式(42), 并利用Young's不等式, 可得

$$\dot{V}_{sn}\leqslant\frac{s_n}{d_\varDelta}\left[c_\varDelta(p)+d_\varDelta u\right]+\frac{1}{2a_n^2}\lambda s_n^2\left\|\xi_n(Z_n)\right\|^2+ s_n\varepsilon_n(Z_n)+\frac{a_n^2}{2}\leqslant \frac{c_\varDelta^2(p)}{d_\varDelta^2}+\frac{s_n^2}{4}+s_nu+\frac{1}{2a_n^2}\lambda s_n^2\left\|\xi_n(Z_n)\right\|^2+ s_n\varepsilon_n(Z_n)+\frac{a_n^2}{2}.\tag{43}$$

为了处理上式中$\dfrac{c_\varDelta^2(p)}{d_\varDelta^2}$项, 由假设5–6及引理2可知

$$|c_\varDelta(p)|\leqslant \bar{c}c_1(\|p(0)\|+|\bar{m}(0)|)\mathrm{e}^{-\delta_0 t}+\bar{c}c_2|\bar{m}(t)|.\tag{44}$$

设$H_{\bar{m}}=\max\{\bar{c}c_1(\|p(0)\|+|\bar{m}(0)|),\ \bar{c}c_2\}$, 则可得

$$\frac{|c_\varDelta(p)|}{(1+|\bar{m}(t)|)}\leqslant H_{\bar{m}}.\tag{45}$$

不妨令$H_c=\dfrac{H_{\bar{m}^2}}{d_\varDelta^2}$, $H=H_c\varepsilon^{*-1}$. 将其代入上式, 可得

$$\frac{|c_\varDelta(p)|^2}{d_\varDelta^2}\leqslant P_{\bar{m}}s_n^2H+(1-\frac{s_n^2}{\varepsilon^*})P_{\bar{m}}H_c,\tag{46}$$

式中$P_{\bar{m}}=(1+|\bar{m}(t)|)^2$.

设计下面的控制律u:

$$u=-(k_ns_n+\frac{1}{2a_n^2}s_n\hat{\lambda}\|\xi_n(Z_n)\|^2+s_nP_{\bar{m}}\hat{H}),\tag{47}$$

式中: $a_n>0$, $k_n>0$是设计常数, $\hat{H}$是H在t时刻的估计.

将式(46)和式(47)代入式(43), 并利用Young's不等式, 可得

$$\dot{V}_{s_n}\leqslant -(k_n-\frac{5}{4})s_n^2-P_{\bar{m}}s_n^2\tilde{H}+(1-\frac{s_n^2}{\varepsilon^*})P_{\bar{m}}H_c- \frac{1}{2a_n^2}s_n^2\tilde{\lambda}_n\|\xi_n(Z_n)\|^2+s_n\varepsilon_n(Z_n)+\frac{a_n^2}{2}\leqslant -(k_n-\frac{3}{2})s_n^2-\frac{1}{2a_n^2}s_n^2\tilde{\lambda}\|\xi_n(Z_n)\|^2-P_{\bar{m}}s_n^2\tilde{H}+ (1-\frac{s_n^2}{\varepsilon^*})P_{\bar{m}}H_c+\kappa_n^2+\frac{a_n^2}{2},\tag{48}$$

式 中: $|\varepsilon_n(Z_n)|\leqslant\kappa_n(\bar{s}_n,\bar{y}_n,\hat{\lambda},v,y_{\mathrm{d}},\dot{y}_{\mathrm{d}})$, $\kappa_n(\cdot)$是 一个未知的非负连续函数, $\tilde{H}=\hat{H}-H$.

设计参数$\hat{\lambda}$, $\hat{H}$的自适应调节律如下:

$$\dot{\hat{\lambda}}=\gamma_1(\sum_{i=1}^{n-1}\frac{\|\xi_i(\bar{x}_{i+1})\|^2s_i^2}{2a_i^2}+\frac{\|\xi_n(Z_n)\|^2s_n^2}{2a_n^2}-\sigma_1\hat{\lambda}),\tag{49}$$

$$\dot{\hat{H}}=\gamma_2(s_n^2P_{\bar{m}}-\sigma_2\hat{H}),\tag{50}$$

式中$\gamma_1,\gamma_2,\sigma_1,\sigma_2>0$是设计常数.

定义紧集

$$\varOmega_n=\{[\bar{s}_n^{\mathrm{T}},\bar{y}_n^{\mathrm{T}},\hat{\lambda},\hat{H},v,\bar{m}]^{\mathrm{T}}:\sum_{i=1}^nV_{s_i}+\sum_{i=2}^n\frac{1}{2}y_i^2+ \frac{1}{2\gamma_1}\tilde{\lambda}^2+\frac{1}{2\gamma_2}\tilde{H}^2+\frac{v}{\lambda_0}+\frac{\bar{m}}{\gamma_3}\leqslant J\}\subset\mathbb{R}^{p_n},\tag{51}$$

式中: $\gamma_3>0$是一个设计常数, J为任给的正常数, $p_n=2n+3$.

令连续函数κ_i在紧集$\varOmega_n\times\varOmega_{\mathrm{d}}$上的最大值为$M_{1i}$

$(i=1,\cdots,n)$, η_i在紧集$\Omega_n\times\Omega_{\rm d}$上的最大值为$M_{2i}$ $(i=2,\cdots,n)$, $|u|$在紧集$\Omega_n\times\Omega_{\rm d}$上的最大值为$M_3$.

定理 1 考虑由系统(1)、控制律(47)、自适应律(49)–(50)组成的闭环系统, 若假设(1)–(6)成立, 对于任意有界初始条件及$V(0)\leqslant J$, 存在常数$k_i,\tau_i,\gamma_1,\gamma_2,\sigma_1,\sigma_2$使得闭环系统半全局一致终结有界, 其中$k_i,1/\tau_i,\alpha_0$满足如下条件:

$$\begin{cases} k_i\geqslant \dfrac{9}{4}+\dfrac{\alpha_0}{2},\ i=1,2,\cdots,n,\\ \dfrac{1}{\tau_i}\geqslant \dfrac{5}{4}+\dfrac{\alpha_0}{2},\ i=1,2,\cdots,n,\\ \alpha_0\leqslant\min\{\bar c_{\rm f},\gamma_1\sigma_1,\gamma_2\sigma_2,\delta_0\}. \end{cases} \tag{52}$$

证 选取如下Lyapunov函数:

$$V=\sum_{i=1}^n V_{s_i}+\sum_{i=2}^n\frac{1}{2}y_i^2+\frac{1}{2\gamma_1}\tilde\lambda^2+\frac{1}{2\gamma_2}\tilde H^2+\frac{v}{\lambda_0}+\frac{\bar m}{\gamma_3}. \tag{53}$$

将V对时间t求导, 可得

$$\dot V=\sum_{i=1}^n\dot V_{s_i}+\sum_{i=2}^n y_i\dot y_i+\frac{1}{\gamma_1}\tilde\lambda\dot{\hat\lambda}+\frac{1}{\gamma_2}\tilde H\dot{\hat H}+\frac{\dot v}{\lambda_0}+\frac{\dot{\bar m}}{\gamma_3}. \tag{54}$$

因为

$$-\hat\lambda\tilde\lambda=-\tilde\lambda(\tilde\lambda+\lambda)\leqslant\frac{1}{2}[-\tilde\lambda^2+\lambda^2],$$
$$-\hat H\tilde H=-\tilde H(\tilde H+H)\leqslant\frac{1}{2}[-\tilde H^2+H^2],$$

所以当$V\leqslant J$时, 易得

$$\dot V\leqslant -\sum_{i=1}^n(k_i-\frac{9}{4})s_i^2-\sum_{i=1}^{n-1}(\frac{1}{\tau_{i+1}}-\frac{5}{4})y_{i+1}^2+\mu_1-\frac{\bar c_{\rm f}v}{\lambda_0}-\frac{\sigma_1\tilde\lambda^2}{2}-\frac{\sigma_2\tilde H^2}{2}-\frac{\Delta_0\bar m}{\gamma_3}+Q(\bar x_{n-1},v), \tag{55}$$

式中:

$$\mu_1=\frac{d}{\lambda_0}+\sum_{i=1}^n\frac{a_i^2}{2}+\sum_{i=1}^n M_{1i}^2+\sum_{i=2}^n M_{2i}^2+\frac{\sigma_1\lambda^2}{2}+\frac{\sigma_2H^2}{2}+\frac{M_3}{\gamma_3},$$
$$Q(\bar x_{n-1},v)=\sum_{i=1}^{n-1}\rho_{i2}^2\circ\bar\alpha_1^{-1}(v+D_0)+\sum_{i=1}^{n-1}\rho_{i1}^2(\|\bar x_i\|)+\frac{\gamma(|x_1|)}{\lambda_0}.$$

将式(52)代入式(55), 可得

$$\dot V\leqslant-\alpha_0V+\mu_1+Q(\bar x_{n-1},v). \tag{56}$$

当$V\leqslant J$, 可得$\bar s_n,\bar y_n,\hat\lambda,\hat H,v,\bar m$有界. 因为$x_1=s_1+y_{\rm d}$, $x_i=s_i+y_i+\alpha_i$, 利用式(20)–(30), 依次可得$x_1,\alpha_2,x_2,\cdots,\alpha_n,x_n$是有界的. 由$\bar m\in L_\infty$, 可得$P_{\bar m}$是有界的. 根据式(47)及$\hat\lambda,\hat H,P_{\bar m}\in L_\infty$, 可得$u\in L_\infty$. 因为$Q(\bar x_{n-1},v)$是一个非负连续函数, $\bar x_{n-1},v$有界, 所以$Q(\bar x_{n-1},v)$有界. 可设$Q(\bar x_{n-1},v)\leqslant\mu_0$, μ_0是正常数. 由上式可得

$$\dot V\leqslant-\alpha_0V+\mu_1+\mu_0. \tag{57}$$

如果$V=J$且$\alpha_0\geqslant(\mu_0+\mu_1)/J$, 那么$\dot V\leqslant0$. 进一步, 如果$V(0)\leqslant J$, 那么$V(t)\leqslant J$, $\forall t>0$. 式(57)两边同乘以${\rm e}^{\alpha_0t}$可得

$$\frac{{\rm d}V((t){\rm e}^{\alpha_0t})}{{\rm d}t}\leqslant{\rm e}^{\alpha_0t}(\mu_0+\mu_1). \tag{58}$$

对式(58)积分, 可得

$$0\leqslant V(t)\leqslant\frac{\mu_0+\mu_1}{\alpha_0}+[V(0)-\frac{\mu_0+\mu_1}{\alpha_0}]{\rm e}^{-\alpha_0t}. \tag{59}$$

因此, 闭环系统的所有信号$\bar s_n,\bar y_n,\hat\theta,v,\bar m$和$\hat H$是一致终结有界的. 进一步有$x_i,y_{i+1}$和$\alpha_i,u$一致终结有界.

4 控制增益符号未知的控制器设计(Controller design with unknown gain sign)

本节中, 将放宽假设条件, 研究含有Nussbaum函数的自适应动态面控制器来处理控制增益符号未知且具有输入未建模动态情形的控制问题.

假设 7 $g_n(x)$的符号是未知的, 且存在常数g_{i0}和g_{n1}, 使得

$$0<g_{i0}\leqslant|g_i(\bar x_{i+1})|,$$
$$1\leqslant i\leqslant n-1,\ 0<g_{n0}\leqslant|g_n(x)|\leqslant g_{n1},$$

其中

$$g_i(\bar x_{i+1})=\frac{\partial f_i(\bar x_i,x_{i+1})}{\partial x_{i+1}},\ i=1.\cdots,n-1.$$

Nussbaum函数性质如下:

$$\text{i)}\quad \lim_{s\to+\infty}\sup\frac{1}{s}\int_0^s N(\zeta_n){\rm d}\zeta_n=+\infty, \tag{60}$$

$$\text{ii)}\quad \lim_{s\to+\infty}\inf\frac{1}{s}\int_0^s N(\zeta_n){\rm d}\zeta_n=-\infty. \tag{61}$$

常用的Nussbaum函数包括: $\zeta_n^2\cos\zeta$, $\zeta_n^2\sin\zeta$和$\exp(\zeta_n^2)\cos((\pi/2)\zeta_n)$, 本文选取

$$N(\zeta_n)=\exp(\zeta_n^2)\cos((\pi/2)\zeta_n).$$

引理 3 已知$V(\cdot),\zeta(\cdot)$都是$[0,t_{\rm f})$上的光滑函数, 且$V(t)\geqslant0,\forall t\in[0,t_{\rm f})$, $N(\cdot)$是一个Nussbaum函数, 如果下列不等式成立

$$V(t)\leqslant c+\int_0^t(g(x(\tau))N(\zeta)+1)\dot\zeta{\rm e}^{-\alpha(t-\tau)}{\rm d}\tau, \tag{62}$$

其中: c为非负常数, $g(x(\tau))$是一个在闭区间$[l^-,l^+]$取值的时变参数, α是一个正常数. 可得$V(t)$, $\zeta(t)$和$\int_0^t g(x(\tau))N(\zeta)\dot\zeta{\rm d}\tau$一定在$[0,t_{\rm f})$上有界.

第i步$(0\leqslant i\leqslant n-1)$ 与第3节讨论相同, 在此

不再赘述.

第n步 令$s_n = x_n - \beta_n$, 因此可得

$$\dot{s}_n = f_n(x) + g_n(x)\omega + \varDelta_n(t,z,x) - \dot{\beta}_n. \tag{63}$$

由假设7, 定义一个光滑Lyapunov函数如下:

$$V_{s_n} = \int_0^{s_n} \frac{\zeta}{|g_n(\bar{x}_{n-1}, \zeta + \beta_n)|}\mathrm{d}\zeta. \tag{64}$$

由积分第2中值定理可知, V_{s_n}可改写为

$$V_{s_n} = \frac{s_n^2}{2}|g_n(\bar{x}_{n-1}, \sigma s_n + \beta_n)|,$$

其中$\sigma \in (0,1)$. 对V_{s_n}在时间t上求导, 可得

$$\begin{aligned}\dot{V}_{s_n} = &\frac{s_n\dot{s}_n}{|g_n(x)|} + s_n^2\sum_{j=1}^{n-1}\varDelta_j(t,z,x)\times\\ &\int_0^1 \sigma\frac{\partial|g_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)|}{\partial x_j}\mathrm{d}\sigma + \frac{\dot{\beta}_n s_n}{|g(x)|} +\\ &s_n^2\int_0^1\sigma\sum_{j=1}^{n-1}\frac{\partial|g_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)|}{\partial x_j}\times\\ &(x_{j+1}+F_j(\bar{x}_j,x_{j+1}))\mathrm{d}\sigma -\\ &\dot{\beta}_n s_n\int_0^1\frac{1}{|g_n(\bar{x}_{n-1},\sigma s_n+\beta_n)|}\mathrm{d}\sigma.\end{aligned} \tag{65}$$

类似于第3节的推导, 易得

$$\begin{aligned}\dot{V}_{s_n} \leqslant &\frac{g_n(x)}{|g_n(x)|}s_n[c_\varDelta(p) + d_\varDelta u] +\\ &\frac{1}{2a_n^2}\lambda s_n^2\|\xi_n(Z_n)\|^2 + s_n\varepsilon_n(Z_n) + \frac{a_n^2}{2} \leqslant\\ &c_\varDelta^2(p) + \frac{s_n^2}{4} + \frac{g_n(x)}{|g_n(x)|}d_\varDelta s_n u +\\ &\frac{1}{2a_n^2}\lambda s_n^2\|\xi_n(Z_n)\|^2 + s_n\varepsilon_n(Z_n) + \frac{a_n^2}{2},\end{aligned} \tag{66}$$

式中:

$$\begin{aligned}&F_n(Z_n) =\\ &\frac{f_n(x)}{|g_n(x)|} + s_n\int_0^1\sigma\sum_{j=1}^{n-1}\frac{\partial|g_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)|}{\partial x_j}\times\\ &(x_{j+1}+F_j(\bar{x}_j,x_{j+1}))\mathrm{d}\sigma + \frac{\rho_{n2}\circ\bar{\alpha}_1^{-1}(v+D_0)}{|g_n(x)|} -\\ &\dot{\beta}_n\int_0^1\frac{1}{|g_n(\bar{x}_{n-1},\sigma s_n+\beta_n)|}\mathrm{d}\sigma + \frac{\rho_{n1}(\|x\|)}{|g_n(x)|} +\\ &\frac{s_n}{2}\sum_{j=1}^{n-1}(\int_0^1\sigma\frac{\partial|g_n^{-1}(\bar{x}_{n-1},\sigma s_n+\beta_n)|}{\partial x_j}\mathrm{d}\sigma)^2 +\\ &s_n\sum_{j=1}^{n-1}\rho_{j1}^2(\|x\|) + s_n\sum_{j=1}^{n-1}\rho_{j2}^2\circ\bar{\alpha}_1^{-1}(v+D_0).\end{aligned} \tag{67}$$

设计控制律如下:

$$u = N(\zeta_n)(k_n s_n + \frac{1}{2a_n^2}s_n\hat{\lambda}\,\|\xi_n(Z)\|^2), \tag{68}$$

$$\dot{\zeta}_n = k_n s_n^2 + \frac{1}{2a_n^2}s_n^2\hat{\lambda}\,\|\xi_n(Z)\|^2. \tag{69}$$

令$H_c = H_{\bar{m}}^2$, 类似于式(44)–(45)的推导, 可得

$$|c_\varDelta(p)|^2 \leqslant P_{\bar{m}}H_c. \tag{70}$$

将式(68)–(70)代入式(68), 并利用Young's不等式得

$$\begin{aligned}&\dot{V}_{s_n} \leqslant\\ &-(k_n - \frac{1}{2})s_n^2 + [\frac{g_n(x)}{|g_n(x)|}d_\varDelta N(\zeta_n) + 1]\dot{\zeta}_n -\\ &\frac{1}{2a_n^2}s_n^2\tilde{\lambda}\|\xi_n(Z_n)\|^2 + P_{\bar{m}}H_c + \kappa_n^2 + \frac{a_n^2}{2}.\end{aligned} \tag{71}$$

定义总的Lyapunov函数如下:

$$V = \sum_{i=1}^n V_{s_i} + \sum_{i=2}^n\frac{1}{2}y_i^2 + \frac{1}{2\gamma_1}\tilde{\lambda}^2 + \frac{v}{\lambda_0} + \frac{\bar{m}}{\gamma_3}, \tag{72}$$

式中$\gamma_3 > 0$是设计常数.

定义紧集

$$\varOmega_n = \{[\bar{s}_n^{\mathrm{T}}\ \ \bar{y}_n^{\mathrm{T}}\ \ \hat{\lambda}\ \ v\ \ \bar{m}]^{\mathrm{T}} : V \leqslant J\} \subset \mathbb{R}^{p_n}, \tag{73}$$

式中: J为任给的正常数, $p_n = 2n+2$.

令连续函数κ_i在紧集$\varOmega_n \times \varOmega_{\mathrm{d}}$上的最大值为$M_{1i}$, $i = 1,\cdots,n$, η_i在紧集$\varOmega_n \times \varOmega_{\mathrm{d}}$上的最大值为$M_{2i}$, $i = 2,\cdots,n$.

定理 2 考虑一类由系统(1)、控制律(68)–(69)、自适应律(48)组成的闭环系统, 若假设1, 3–7成立, 则对于任意有界初始条件及$V(0) \leqslant J$, 存在常数$k_i, \tau_i, \gamma_1, \gamma_2, \sigma_1, \sigma_2$使得闭环系统半全局一致终结有界, 其中$k_i, 1/\tau_i, \alpha_0$满足如下条件:

$$\begin{cases} k_i \geqslant \dfrac{9}{4} + \dfrac{\alpha_0}{2},\ i = 1,2,\cdots,n,\\ \dfrac{1}{\tau_i} \geqslant \dfrac{5}{4} + \dfrac{\alpha_0}{2},\ i = 1,2,\cdots,n,\\ \alpha_0 \leqslant \min\{\bar{c}_{\mathrm{f}}, \gamma_1\sigma_1, \delta_0\}.\end{cases} \tag{74}$$

证 总的Lyapunov函数V由式(72)确定.

当$V \leqslant J$时, 对Lyapunov函数V求导并利用式(68)–(69)可得

$$\begin{aligned}\dot{V} \leqslant &-\sum_{i=1}^n(k_i - \frac{9}{4})s_i^2 - \sum_{i=1}^{n-1}(\frac{1}{\tau_{i+1}} - \frac{5}{4})y_{i+1}^2 + \mu_1 -\\ &\frac{\sigma_1\tilde{\lambda}^2}{2} - \frac{\delta_0\bar{m}}{\gamma_3} + [\frac{g_n(x)}{|g_n(x)|}d_\varDelta N(\zeta_n) + 1]\dot{\zeta}_n -\\ &\frac{\bar{c}_{\mathrm{f}}}{\lambda_0}v + Q(\bar{x}_{n-1}, v),\end{aligned} \tag{75}$$

式中:

$$\begin{aligned}\mu_1 = &\frac{d}{\lambda_0} + \sum_{i=1}^n\frac{a_i^2}{2} + \sum_{i=1}^n M_{1i}^2 + \sum_{i=2}^n M_{2i}^2 +\\ &\frac{\sigma_1\lambda^2}{2} + (1+J)^2H_c,\end{aligned}$$

$$\begin{aligned}Q(\bar{x}_{n-1}, v) = &\sum_{i=1}^{n-1}\rho_{i2}^2\circ\bar{\alpha}_1^{-1}(v+D_0) +\\ &\sum_{i=1}^{n-1}\rho_{i1}^2(\|\bar{x}_i\|) + \frac{\gamma(x_1)}{\lambda_0}.\end{aligned}$$

将式(74)代入式(75), 可得

$$\dot{V} \leqslant -\alpha_0 V + \mu_1 + Q(\bar{x}_{n-1}, v) + [\frac{g_n(x)}{|g_n(x)|} d_\Delta N(\zeta_n) + 1]\dot{\zeta}_n. \tag{76}$$

若$V \leqslant J$, 则有$\bar{s}_n, \bar{y}_n, \hat{\lambda}, \hat{H}, v, \bar{m}$有界, 类似于定理1的分析可得$\bar{x}_n, \alpha_i$有界. 根据$\bar{m} \in L_\infty$, 可知$P_{\bar{m}}$有界. 因为$Q(\bar{x}_{n-1}, v)$是一个非负连续函数, $\bar{x}_{n-1}, v$有界, 所以$Q(\bar{x}_{n-1}, v)$有界. 可设$Q(\bar{x}_{n-1}, v) \leqslant \mu_0$, μ_0是一个未知正常数. 由式(78)得

$$\dot{V} \leqslant -\alpha_0 V + \mu_1 + \mu_0 + [\frac{g_n(x)}{|g_n(x)|} d_\Delta N(\zeta_n) + 1]\dot{\zeta}_n. \tag{77}$$

类似于第2节的讨论, 可得

$$V(t) \leqslant \int_0^t [\frac{g_n(x)}{|g_n(x)|} d_\Delta N(\zeta_n) + 1]\dot{\zeta}_n \mathrm{e}^{-\alpha_0(t-\tau)} \mathrm{d}\tau + \frac{\mu_0 + \mu_1}{\alpha_0} + V(0). \tag{78}$$

由引理3可知, $\int_0^t [\frac{g_n(x)}{|g_n(x)|} d_\Delta N(\zeta_n) + 1]\dot{\zeta}_n \mathrm{e}^{-\alpha_0(t-\tau)} \mathrm{d}\tau$, $V(t)$和$\zeta(t)$在$[0, t_\mathrm{f})$上有界. 由于t_f是任意正常数, 因此, $\int_0^\mathrm{T} [\frac{g_n(x)}{|g_n(x)|} d_\Delta N(\zeta_n) + 1]\dot{\zeta}_n \mathrm{e}^{-\alpha_0(t-\tau)} \mathrm{d}\tau$, $V(t)$和$\zeta(t)$在$[0, \infty)$上有界. 进一步由式(69)可知, 式(77)右边第4项是有界的, 即存在正常数μ_2使得$|[\frac{g_n(x) d_\Delta}{|g_n(x)|} \times N(\zeta_n) + 1]\dot{\zeta}_n| \leqslant \mu_2$. 由式(77)可得

$$0 \leqslant V(t) \leqslant \frac{\mu_0 + \mu_1 + \mu_2}{\alpha_0} + [V(0) - \frac{\mu_0 + \mu_1 + \mu_2}{\alpha_0}]\mathrm{e}^{-\alpha_0 t}. \tag{79}$$

如果$V = J$且$\alpha_0 \geqslant (\mu_0 + \mu_1 + \mu_2)/J$, 那么$\dot{V} \leqslant 0$. 进一步, 如果$V(0) \leqslant J$, 那么$V(t) \leqslant J, \forall t \geqslant 0$.

因此, 闭环系统的所有信号$s_i, y_i, \hat{\lambda}, v, \bar{m}$和$\hat{H}$是一致终结有界的. 进一步, 可得$x_i, y_{i+1}$和$\alpha_i, u$一致终结有界.

注 2 本文利用Nussbaum函数, 设计了控制律(68)和Nussbaum参数自适应律(69). 进一步, 在总的李雅普诺夫函数中加入了正则化信号, 从而证明了闭环系统的稳定性.

5 仿真结果(Simulation results)

例1 考虑如下具有未建模动态的倒立摆系统[23]:

$$\begin{cases} \dot{z} = q(t, z, y), \\ \dot{x}_1 = x_2 + \varDelta_1, \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1)v + \varDelta_2, \\ y = x_1, \\ \dot{p}_1 = -2p_1 - p_1^3 + p_2, \\ \dot{p}_2 = -2p_2 + u, \\ v = p_1 + \dfrac{-p_2 + 10p_2^3}{1 + p_2^2} + u, \end{cases} \tag{80}$$

式中:

$$f_2(x_1, x_2) = \frac{g \sin x_1}{l(\dfrac{4}{3} - \dfrac{m_l \cos^2 x_1}{m_c + m_l})} - \frac{\dfrac{m_l l x_2^2 \cos x_1 \sin x_1}{m_c + m_l}}{l(\dfrac{4}{3} - \dfrac{m_l \cos^2 x_1}{m_c + m_l})},$$

$$g_2(x_1) = \frac{\dfrac{\cos x_1}{m_c + m_l}}{l(\dfrac{4}{3} - \dfrac{m_l \cos^2 x_1}{m_c + m_l})},$$

$q(t, z, y) = -2z + y \sin t + 0.5$, $\varDelta_1 = 0.5z$, $\varDelta_2 = x_1 z$, $\mathrm{g} = 9.8\ \mathrm{m/s^2}$重力加速度, $m_c = 1\ \mathrm{kg}$是小车的质量, $m_l = 0.1\ \mathrm{kg}$是半个杆的质量, $l = 0.5\ \mathrm{m}$是半个杆的长度. 期望的轨迹为$y_\mathrm{d} = (\pi/30) \sin t$.

仿真中, $\dot{\bar{m}} = -\delta_0 \bar{m} + |u|$, $\dot{v} = -v + 2.5y^2 + 0.6$; 设计参数取为$k_1 = 5, k_2 = 10, a_1^2 = a_2^2 = 0.05, \gamma_1 = \gamma_2 = 4, \sigma_1 = \sigma_2 = 0.01, \delta_0 = 1.5, \tau_2 = 0.05$; 初值为$x(0) = [0.05 \quad -0.1]^\mathrm{T}$, $z(0) = 0$, $p(0) = [0 \quad 0]^\mathrm{T}$, $\hat{\lambda}(0) = 1.5$, $\hat{H}(0) = 0.15$, $\bar{m}(0) = 0.2, v(0) = 1.5$. 基向量为

$$\xi_i(Z_i) = [\xi_{i1}(Z_i) \quad \cdots \quad \xi_{il_i}(Z_i)]^\mathrm{T} \in \mathbb{R}^{l_i},\ l_1 = 10,$$

$$\xi_{ij}(Z_i) = \exp[-\frac{(Z_i - b_{ij})^\mathrm{T}(Z_i - b_{ij})}{a_{ij}^2}],\ l_2 = 20,$$

$$Z_1 = \bar{x}_2 = [x_1 \quad x_2]^\mathrm{T},\ Z_2 = [x_1 \quad x_2 \quad s_2 \quad \dot{\beta}_2 \quad v]^\mathrm{T},$$

$$s_1 = y - y_\mathrm{d},\ s_2 = x_2 - \beta_2,\ j = 1, \cdots, l_i, i = 1, 2,$$

$$b_{1jk} = 0.2k(j - \frac{l_1}{2}),\ j = 1, \cdots, l_1, k = 1, 2,$$

$$b_{2jk} = 0.2k(j - \frac{l_2}{2}),\ j = 1, \cdots, l_2,\ k = 1, \cdots, 5,$$

$$a_{ij} = 1, j = 1, \cdots, l_i,\ i = 1, 2,$$

仿真结果如图1–3所示. 从图1, 2可知, 本文所设计的自适应控制能够保证闭环系统具有良好的跟踪性能.

例 2 考虑如下一类具有输入和状态未建模动态的纯反馈非线性系统:

$$\begin{cases} \dot{z} = -z + 0.5x_1^2 \sin(x_1 t), \\ \dot{x}_1 = x_1 + x_2 + \dfrac{x_2^3}{5} + \varDelta_1, \\ \dot{x}_2 = x_3 + \dfrac{x_3^3}{2} + \varDelta_2, \\ \dot{x}_3 = x_1 x_2 x_3 + (1 + 0.1 \sin(0.5 x_1 x_2 x_3))v + \varDelta_3, \end{cases} \tag{81a}$$

$$\begin{cases} y = x_1, \\ \dot{p}_1 = -2p_1 - p_1^3 + p_2, \\ \dot{p}_2 = -2p_2 + u, \\ v = p_1 + \dfrac{-p_2 + 4p_2^3}{1 + p_2^2} + u, \end{cases} \tag{81b}$$

式中:

$$\varDelta_1 = 0.1z \sin t + 0.1 \sin(x_1 x_2 x_3 t),$$

$$\Delta_2 = 0.1z\cos(x_2x_3t),$$

$$\Delta_3 = 0.2z\cos(0.5x_2t) - 0.5x_1.$$

期望的跟踪轨迹$y_{\rm d}(t) = 0.5\sin t + 0.25\sin(0.5t)$.

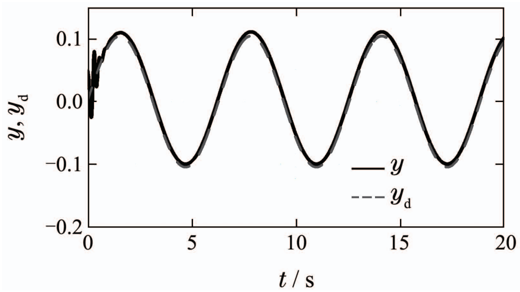

图 1　增益符号已知的倒立摆系统输出y和期望轨迹$y_{\rm d}$

Fig. 1　Output y and desired trajectory $y_{\rm d}$ for inverted pendulum system with known gain sign

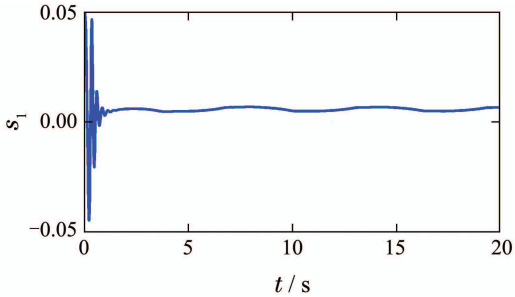

图 2　跟踪误差s_1

Fig. 2　Tracking error s_1

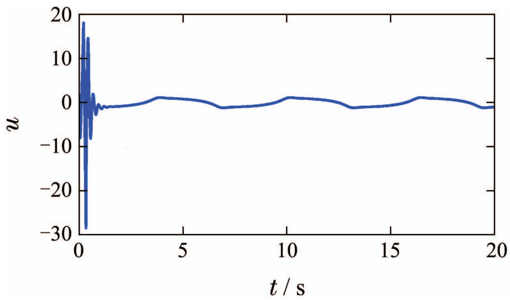

图 3　控制信号u

Fig. 3　Control signal u

对于控制方案1(增益符号已知): 仿真中动态信号为$\dot{v} = -v + 0.5x_1^4 + 0.5$; 设计参数取为

$$\sigma_1 = \sigma_2 = 0.1,\ \tau_2 = \tau_3 = 0.01,$$

$$a_1 = a_2 = a_3 = 10,\ \delta_0 = 1.5,$$

$$k_1 = 7,\ k_2 = k_3 = 5,\ \gamma_1 = \gamma_2 = 5;$$

初值取为

$$z(0) = 0.1, x_1(0) = 0.1,\ x_2(0) = 0.2,\ x_3(0) = 0,$$

$$\hat{\lambda}(0) = 2,\ \beta_2(0) = \beta_3(0) = 0.2,$$

$$p_1(0) = 0.1,\ p_2(0) = 0.1,\ \bar{m}(0) = 0.2,$$

$$\hat{H}(0) = 0.5,\ v(0) = 0.1;$$

神经网络的设计参数为

$$l_1 = 40,\ l_2 = l_3 = 20,$$

$$b_{1jk} = 0.1k(j - \frac{l_1}{2}),\ j = 1, \cdots, l_1,\ k = 1, 2,$$

$$b_{2jk} = 0.1k(j - \frac{l_2}{2}),\ j = 1, \cdots, l_2,\ k = 1, 2, 3,$$

$$b_{3jk} = 0.1k(j - \frac{l_3}{2}),\ j = 1, \cdots, l_3,\ k = 1, \cdots, 6,$$

$$a_{ij} = 1,\ j = 1, \cdots, l_i,\ i = 1, 2, 3.$$

仿真结果如图4–6所示.

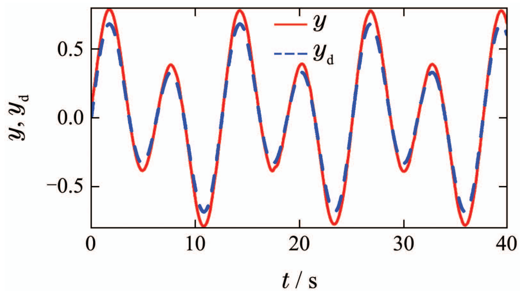

图 4　增益符号已知的纯反馈系统输出y和期望轨迹$y_{\rm d}$

Fig. 4　Output y and desired trajectory $y_{\rm d}$ for pure-feedback system with known gain sign

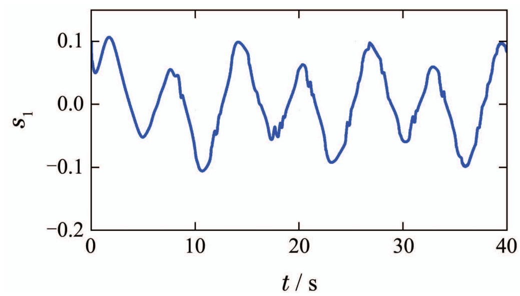

图 5　跟踪误差s_1

Fig. 5　Tracking error s_1

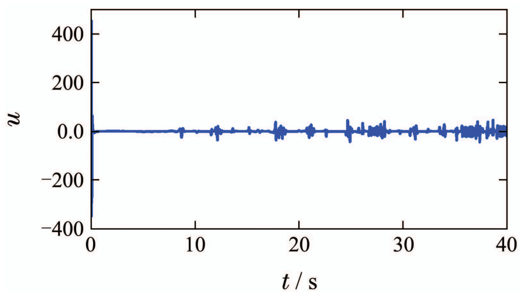

图 6　控制信号u

Fig. 6　Control signal u

对于控制方案2(增益符号未知): 仿真中动态信号为$\dot{v} = -v + 0.5x_1^4 + 0.5$; 设计参数取为

$$\sigma_1 = \sigma_2 = 0.01,\ \tau_2 = \tau_3 = 0.01,$$

$$a_1 = a_2 = a_3 = 15,\ \delta_0 = 1.5,$$

$$k_1 = 7,\ k_2 = 5,\ k_3 = 2.5,\ \gamma_1 = 10.$$

初值取为

$$z(0) = 0.1,\ x_1(0) = 0.1,$$
$$x_2(0) = 0.2,\ x_3(0) = 0,\ \hat{\lambda}(0) = 1.5,$$
$$\beta_2(0) = \beta_3(0) = 0.2,\ p_1(0) = 0.1,\ p_2(0) = 0.1,$$
$$\bar{m}(0) = 0.2,\ v(0) = 0.1,\ \zeta_3(0) = 0.$$

神经网络的设计参数为

$$l_1 = l_2 = l_3 = 10,$$
$$b_{1jk} = 0.2k(j - \frac{l_1}{2}),\ j = 1, \cdots, l_1,\ k = 1, 2,$$
$$b_{2jk} = 0.2k(j - \frac{l_2}{2}),\ j = 1, \cdots, l_2,\ k = 1, 2, 3,$$
$$b_{3jk} = 0.2k(j - \frac{l_3}{2}),\ j = 1, \cdots, l_3,\ k = 1, \cdots, 6,$$
$$a_{ij} = 1,\ j = 1, \cdots, l_i,\ i = 1, 2, 3.$$

仿真结果如图7–9所示.

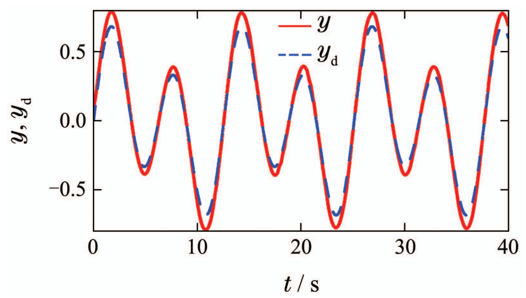

图 7 增益符号未知的纯反馈系统输出y和期望轨迹y_d

Fig. 7 Output y and desired trajectory y_d for pure-feedback system with unknown gain sign

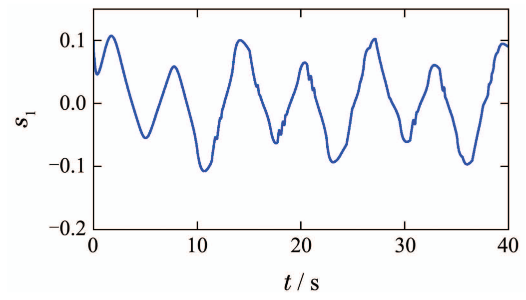

图 8 跟踪误差s_1

Fig. 8 Tracking error s_1

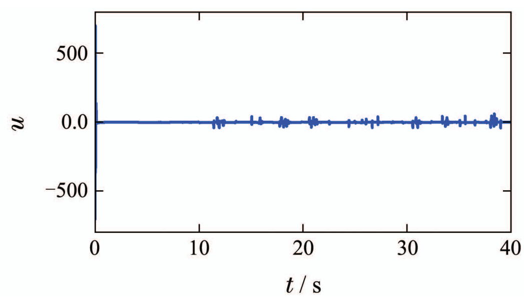

图 9 控制信号u

Fig. 9 Control signal u

6 结论(Conclusions)

本文对一类具有状态和输入未建模动态的纯反馈非线性系统, 利用非线性变换将纯反馈非线性系统转换为形式上的严格反馈非线性系统, 进一步, 利用动态面控制方法, 对控制增益符号已知和未知情况, 提出两种自适应控制方案. 通过引入一阶滤波器, 降低了控制器设计的复杂性. 利用径向基函数神经网络逼近系统中的未知光滑非线性函数. 利用积分型李雅普诺夫函数放宽了控制增益的要求. 利用Young's不等式, 对推导过程中的不确定项进行放缩, 从而减少神经网络在线调节参数的数目. 利用Nussbaum函数的性质, 处理虚拟控制增益符号未知问题. 在未来的研究工作中进一步将其结果推广到具有输出和状态约束的非线性系统.

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作者简介:

张天平 (1964–), 男, 博士, 教授, 博士生导师, 目前主要从事自适应控制、模糊控制理论、智能控制及非线性控制等研究工作, E-mail: tpzhang@yzu.edu.cn;

葛继伟 (1992–), 男, 硕士研究生, 主要研究方向为自适应控制、非线性控制等研究工作, E-mail: 18252738271@139.com;

夏晓南 (1970–), 女, 博士, 讲师, 目前主要从事复杂系统的鲁棒自适应控制、智能控制及非线性系统的控制等研究工作, E-mail: xnxia@yzu.edu.cn.