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组合航天器的姿态控制与结构鲁棒控制分配

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摘要: 针对空间臂捕获未知目标航天器后的控制问题, 本文提出了一种新方案. 基于动量的估计方法和递推最小二乘算法在线估计组合式航天器的惯性参数, 并通过一种基于比例微分反馈的直接参数方法处理组合姿态控制系统, 此方法给出了完整的参数化双反馈增益. 考虑到推力器的配置和配置矩阵的测量误差, 提出了具有多面体和多胞体形式摄动的鲁棒控制分配方法. 最后, 数值仿真结果验证了所提方法的有效性.

关键词: 鲁棒控制分配; 组合航天器; 姿态控制; 直接参数化方法; 参数辨识

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Attitude control and structure robust control allocation for combined spacecraft

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Abstract: This paper presents a new control scheme for the problem of a space manipulator after capturing an unknown target. Since the inertia parameters of the combined spacecraft have been identified online depending on momentum-based estimation method and recursive least squares algorithm, a direct parametric approach via proportional plus derivative feedback is proposed for the combined attitude control system, which gives a complete parametrization of the pair of feedback gains. Considering the thruster's configuration and the measurement error of the configuration matrix, robust control allocations with perturbation both in regular polyhedral and polytopic form are developed. Finally, numerical simulations demonstrate the effectiveness of the proposed approach.

Key words: robust control allocation; combined spacecraft; attitude control; direct parametric approach; parameter identification

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1 Introduction

To capture a fault/failed and possibly uncooperative target satellite by space robotic arm is a precondition for on-line service missions. The capture process mainly includes four phases[1]: observing and planing, final approaching, impact and capture, post-capturing stabilization. This paper concentrates on the post-capturing phase and tries to stabilize the captured system.

After the target has been captured, estimating the inertia parameter of the combined spacecraft accurately in real time is the premise for postcapture stabilization algorithms. Some scholars have made a great contribution to inertia parameter estimation for the combined spacecraft. According to the conservation principle of linear and angular momentum and Newton-Euler equations of motion, Yoshisada, etc[2] identified inertial parameters of the unknown object handled by manipulators on a free-flying space robot. Kazuya, etc[3] developed an identification algorithm which did not require torque or acceleration measurement by using the law of momentum conservation. The mass and mass center of a rigid spacecraft could be determined using only torque-producing actuators such as control-moment gyros or reaction wheels, and commonly available sensors, e.g., rate gyros and accelerometers[4]. Liu, etc[5] identified the mass of target satellite by using the least

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square method and identified mass center of the combination satellite by using one, three and four point three orientations' acceleration together with gyro information. The precise operations and coordinated parameter identification (CPI) allowing the motion of multi-joints for non-cooperative target space operations was investigated in [6]. More importantly, Nguyenhuynh, etc[7] proposed an adaptive reactionless control scheme based on the momentum conservation equation and the recursive least-squares (RLS) procedure for parameter adaptation, which is the main idea of identifying unknown inertia parameters in this paper.

When inertia parameters of the combined spacecraft have been identified, the attitude control problem will turn into a normal case. After some transformation, the attitude system becomes a second-order form and it is easier to find a controller[8–9]. According to Duan's former contribution[10–11] to high-order generalized Sylvester matrix equations, the author proposed a direct parametric control approach for a type of general fully-actuated second-order nonlinear systems[12], and generalized the method to the satellite attitude control system[13]. Some other condition have been also taken in the attitude control[14–15], which will be done in the future.

The combination of target spacecraft and base spacecraft will lead the dynamics of base spacecraft to suffer a great shift, which makes the thrusters' configuration change. One way to meet this challenge is using control reallocation and several methods have been developed to solve this problem, such as pseudo-inverse method, daisy chaining method, direct allocation method, linear programming method[16] and dynamic control allocation method[17–19]. However, the error between the shift of real mass center position vector and the estimated one causes the configuration matrix uncertain, robust control allocation becomes necessary in the control reallocation of the combined spacecraft thrusters. Ghaoui, etc[20] firstly investigated least-squares problems where the coefficient matrices were unknown but bounded in general cases. Ma, etc[21] studied the robust transformation from the ellipsoidal uncertain set to equality and extended this result to the uncertain set represented by a conic quadratic inequality. Under the condition of uncertainty included in the control effectiveness matrix, a robust least-squares scheme was proposed to deal with the problem of distributing the three axis moments to the corresponding control surfaces both in unstructured and structured uncertainties cases[22]. Hu, etc[23–24], distributed the previously designed three-axis moments over the available actuators by minimizing the worst-case residual error using programming algorithms. In the previous studies, the scholars all put their concentration on continuous system, Cui, etc[25] found interest in discrete system and designed a new scheme of robust fault-tolerant control allocation for a discrete-time aerodynamic model in a research environment (ADMIRE) aircraft model. Shen, etc[26] set robust control allocation problem as a min–max optimization problem and dealt with actuator faults directly without reconfiguring the controller and ensures some robustness of system performances. Although both the unstructured and structured robust control allocation have been studied, the structured robust control allocation in polyhedral and polytopic form, and more over, the linear structured form have not been considered before, which will be investigated in this paper.

This paper is organized as follows: in Section 2, after inertia parameters of combined spacecraft are identified according to the momentum conservation equation and the RLS algorithm, the attitude error dynamics of combined spacecraft is established in terms of MRP. Direct parametric control approach for the established combined attitude system is proposed in Section 3. Furthermore, Section 4 introduces the robust control allocation with regular polyhedral and polytopic perturbation. Finally, numerical simulations about robust control allocation for the combined spacecraft attitude system illustrates the effective of the approach in Section 5.

2 Attitude dynamics of combined spacecraft

In order to form the attitude tracking control system of the combined spacecraft, several corresponding frames are presented.

1) The combined spacecraft body frame $\mathcal{F}_{\rm c}(O_{\rm c}x_{\rm c}y_{\rm c}z_{\rm c})$ defines the center of mass of the combined spacecraft as its origin, and three mutually perpendicular axes $O_{\rm c}x_{\rm c}$, $O_{\rm c}y_{\rm c}$ and $O_{\rm c}z_{\rm c}$ coincident with the principle axis of inertia.

2) The inertia principal axis frame $\mathcal{F}_{\rm I}(O_{\rm I}xyz)$, in this frame, the inertia matrix of the combined spacecraft is diagonal matrix.

3) The body frame of the ith link $\mathcal{F}_i(O_ix_iy_iz_i)$ defines the center of mass of the ith link as its origin, and three mutually perpendicular axes O_ix_i, O_iy_i and O_iz_i coincident with the principle axis of inertia.

We also assume that the combined spacecraft system consists of a rigid base spacecraft, a rigid target spacecraft and one rigid space manipulator. The launch vehicle interface ring of target spacecraft is captured by the space manipulator, shown in Fig. 1[7]. In the post-capture phase, the joints of space manipulators will be locked, and the dynamics of the combined spacecraft can be represented by a rigid body. For the simpleness of statement, as the same as [27] and [28], the following assumptions need to be satisfied:

1) There is no attitude control capability in the target spacecraft, whose attitude control is taken over by the attitude control system of the base spacecraft.

2) The base spacecraft is driven by thrusters that are assumed to be continuously controllable, and the locations and directions of the thrusters are known.

3) The space manipulators are locked after capture of target spacecraft, once the joints of the space manipulators are locked after capture of target spacecraft.

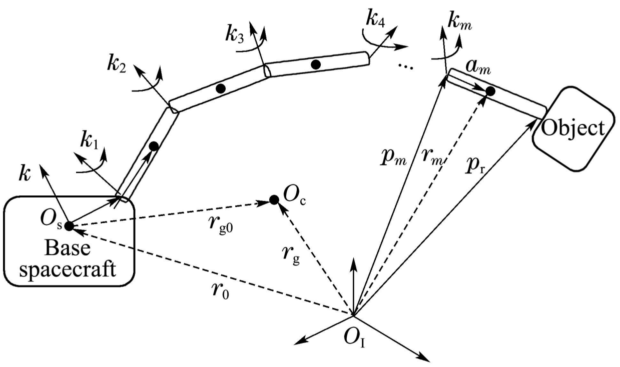


Fig. 1 Model of space manipulators robot

According to the assumption 3), shortly after the manipulator grasps the target, the target will be rigidized relative to the end effector, so that the inertial parameters of the last link are changed after capture. The momentum-based identification method to estimate the inertial parameters of the last link of the system after target capture without the knowledge of its initial angular momentum has been derived in [7]. Then the identification problem can be solved by any of the existing methods. In this paper, We take the RLS algorithm to estimate these parameters. For readers, the main result of [7] is briefly stated in the following, please refer to [7] for more details. The adaptation equation can be transformed into the standard regressor form as

$$y(t) = \Phi(t)\theta(t),$$

where

$$y(t) = \begin{bmatrix} -v_0 - \omega_0 \times (p_m - r_0) - \Lambda \\ \Delta(u \times R_m)a_m + \Delta\Omega_m i_m \end{bmatrix},$$

$$\Phi(t) = \begin{bmatrix} u & \omega_0 \times R_m + \sum\limits_{j=1}^{m} \dot{\phi}_j k_j \times R_m & 0 \\ 0 & \Delta(u \times R_m) & \Delta\Omega_m \end{bmatrix},$$

$$\theta(t) = [1/m_m \quad a_m \quad i_m]^{\mathrm{T}}$$

with

$$\Lambda = \sum_{j=1}^{m} (k_j \times (p_m - p_j))\dot{\phi}_j,\ u = \sum_{i=0}^{m-1} m_i \dot{r}_i,$$

and the symbol Δ denotes the increment between time t_k and t_{k+1}. The matrix $\Delta\Omega_m$ and the vector i_m are defined as follows:

$$\Delta\Omega_m = \Delta\left[R_m \begin{bmatrix} \omega_{mx} & \omega_{mx} & \omega_{mx} & 0 & 0 & 0 \\ 0 & \omega_{mx} & 0 & \omega_{mx} & \omega_{mx} & 0 \\ 0 & 0 & \omega_{mx} & 0 & \omega_{mx} & \omega_{mx} \end{bmatrix}\right],$$

$$i_m = [J_{11} \quad J_{12} \quad J_{13} \quad J_{11} \quad J_{22} \quad J_{33}]^{\mathrm{T}},$$

and the unknown m_m, a_m and I_m denote the mass, the position of mass center and the inertia tensor of the last link, respectively; r_0, m_0, J_0, ω_0 and v_0 are the the position vector of the mass center, the mass, the inertia tensor, the angular velocity and linear velocity of the base spacecraft, respectively; r_i, m_i, J_i and $\omega_i = [\omega_{ix} \quad \omega_{iy} \quad \omega_{iz}]^{\mathrm{T}} (i = 1, 2, \cdots, m)$ are the the position vector of the mass center, the mass, the inertia tensor and the angular velocity of link i, respectively; R_m is the rotation matrix from $\mathcal{F}_m$ frame to the inertia principal axis frame $\mathcal{F}_{\mathrm{I}}$; p_i and $k_i (i = 1, \cdots, m)$ are vectors showed in Fig. 1; $\dot{\phi}$ contains the arm joint rates. Besides, $\times$ represents cross product of two vectors.

Then, the RLS algorithm to compute the updates for $\theta(t)$ is stated in the following form:

$$\theta(t) = \theta(t-1) + K(t)[y(t) - \Phi(t)\theta(t-1)],$$

$$K(t) = P(t-1)\Phi(t)[\lambda I_3 + \Phi(t)P(k-1)\Phi^{\mathrm{T}}(t)]^{-1},$$

$$P(k) = \frac{1}{\lambda}[I_3 - K(t)\Phi(t)]P(k-1),$$

where I_3 is a 3×3 identify matrix.

Remark 1 The initial guess for the adaptation gain matrix P can be chosen as $P(0) = \alpha I_3$ for any $\alpha > 1$. A value of forgetting factor λ very close to 1 is desired to ensure that the ARLC algorithm is stable during the postcapture maneuvering.

Based on the above discussion, we can get the inertia matrix of combined spacecraft in the inertia principal axis frame $\mathcal{F}_{\mathrm{I}}$ as

$$J = J_0 + m_0[(r_0^{\mathrm{T}} r_0)I_3 - r_0 r_0^{\mathrm{T}}] + m_m[(r_m^{\mathrm{T}} r_m)I_3 - r_m r_m^{\mathrm{T}}] + J_m + \sum_{i=1}^{m-1}(J_i + m_i[(r_i^{\mathrm{T}} r_i)I_3 - r_i r_i^{\mathrm{T}}]),$$

then the inertia tensor of combined spacecraft is obtained in the following statement.

In this paper, we use modified rodrigues parameter (MRP) to describe the attitude kinematics of the combined spacecraft. Given a Euler rotation angle $\phi(t) \in [0, 360)$ deg about the Euler principal axis $\eta \in \mathbb{R}^3$, the spacecraft orientation in the combined spacecraft body frame $\mathcal{F}_{\mathrm{c}}(O_{\mathrm{c}}x_{\mathrm{c}}y_{\mathrm{c}}z_{\mathrm{c}})$ with respect to the inertia principal axis frame $\mathcal{F}_{\mathrm{I}}(O_{\mathrm{I}}xyz)$ can be represented by a vector of MRPs $\sigma = \eta \tan\frac{\phi(t)}{4} = [\sigma_1 \quad \sigma_2 \quad \sigma_3]^{\mathrm{T}} \in \mathbb{R}^3$. The direction cosine matrix $A(\sigma)$ can be denoted by

$$A(\sigma) = \frac{(1+\|\sigma\|^2)^2 I_3 + 8[\sigma^{\times}]^2 - 4(1+\|\sigma\|^2)\sigma^{\times}}{(1+\|\sigma\|^2)^2},$$

where I_3 is a 3×3 identify matrix and $\sigma^{\times}$ is a skew-symmetry matrix of σ defined as follows:

$$\sigma^{\times} = \begin{pmatrix} 0 & -\sigma_3 & \sigma_2 \\ \sigma_3 & 0 & -\sigma_1 \\ -\sigma_2 & \sigma_1 & 0 \end{pmatrix}.$$

The kinematics model of the combined spacecraft in the terms of the MRP takes the following form:

$$\dot{\sigma} = F(\sigma)\omega \tag{1}$$

with

$$F(\sigma) = \frac{1}{4}[(1-\sigma^{\mathrm{T}}\sigma)I_3 + 2\sigma^{\times} + 2\sigma\sigma^{\mathrm{T}}],$$

and it is not difficult to get

$$F^{-1}(\sigma) = \frac{16}{(1+\sigma^{\mathrm{T}}\sigma)^2}F^{\mathrm{T}}(\sigma).$$

Also, $\omega \in \mathbb{R}^3$ is the angular velocity of the body frame $\mathcal{F}_{\mathrm{c}}(O_{\mathrm{c}}x_{\mathrm{c}}y_{\mathrm{c}}z_{\mathrm{c}})$ with respect to the inertial frame $\mathcal{F}_{\mathrm{I}}(O_{\mathrm{I}}xyz)$ and expressed in the body frame $\mathcal{F}_{\mathrm{c}}(O_{\mathrm{c}}x_{\mathrm{c}}y_{\mathrm{c}}z_{\mathrm{c}})$.

Now, considering the combination system as a rigid body, the Euler's attitude dynamic equation of the combined spacecraft based on the well-know angular momentum theorem can be given by the following equation:

$$J\dot{\omega} + \omega^{\times}J\omega = T_{\mathrm{c}} + T_{\mathrm{g}}, \tag{2}$$

where J is the inertia tensor of the combined spacecraft written in inertia principle frame, and can be represented as follows:

$$J = \mathrm{diag}\{J_{xx}, J_{yy}, J_{zz}\},$$

and T_{c} is the control torque, T_{g} is the gravity gradient torque.

The gravity gradient torque T_{g} can be easily modeled by integrating the effect of the non-uniform gravity field at each mass point of the combined spacecraft. Such derivation can yield

$$T_{\mathrm{g}} = 3\omega_0^2 A_3(\sigma) \times JA_3(\sigma),$$

where

$$A_3(\sigma) = \frac{1}{1+\sigma^{\mathrm{T}}\sigma}\begin{bmatrix} 8\sigma_1\sigma_3 - 4\sigma_2(1-\sigma^{\mathrm{T}}\sigma) \\ 8\sigma_2\sigma_3 + 4\sigma_1(1-\sigma^{\mathrm{T}}\sigma) \\ 4(\sigma_3^2-\sigma_2^2-\sigma_1^2) + (1-\sigma^{\mathrm{T}}\sigma)^2 \end{bmatrix}.$$

Denote $G(\sigma) = F^{-1}(\sigma)$, $\theta = [J_{xx}\ \ J_{yy}\ \ J_{zz}\ \ 0\ \ 0\ \ 0]$, We take the derivation on the both side of equation (1) and premultiply $G^{\mathrm{T}}(\sigma)JG(\sigma)$, we can get the Lagrange's attitude dynamic equation of the combined spacecraft:

$$H(\theta,\sigma)\ddot{\sigma} + C(\theta,\sigma,\dot{\sigma})\dot{\sigma} + g(\theta,\sigma) = T, \tag{3}$$

where $H(\theta,\sigma)$ is the inertia matrix, $C(\theta,\sigma,\dot{\sigma})$ is the vector of coriolis and centripetal torques, and the T is the control torques. Their expressions are stated as follows:

$$\begin{aligned} &H(\theta,\sigma) = G^{\mathrm{T}}(\sigma)JG(\sigma),\\ &C(\theta,\sigma,\dot{\sigma}) = -G^{\mathrm{T}}(\sigma)JG(\sigma)\dot{F}(\sigma)G(\sigma) + \\ &\qquad\qquad G^{\mathrm{T}}(\sigma)(G(\sigma)\dot{\sigma})^{\times}JG(\sigma),\\ &g(\theta,\sigma) = -3\omega_0^2G^{\mathrm{T}}(\sigma)A_3(\sigma)^{\times}JA_3(\sigma),\\ &T = G^{\mathrm{T}}(\sigma)T_{\mathrm{c}}. \end{aligned}$$

In this paper, we want the combined spacecraft to track the objective MRP position σ_{d} and MRP vector $\dot{\sigma}_{\mathrm{d}}$. The trajectory tracking error ε can be defined as

$$\varepsilon = \sigma - \sigma_{\mathrm{d}}, \tag{4}$$

differentiating (4) with respect to time yields

$$\dot{\varepsilon} = \dot{\sigma} - \dot{\sigma}_{\mathrm{d}}, \tag{5}$$

differentiating (5) twice with respect to time yields

$$\ddot{\varepsilon} = \ddot{\sigma} - \ddot{\sigma}_{\mathrm{d}}. \tag{6}$$

Since the inertia parameter has been identified, θ is known, substituting (5) and (6) into (3) yields

$$\begin{aligned} &H(\varepsilon)\ddot{\varepsilon} + C(\varepsilon,\dot{\varepsilon})\dot{\varepsilon} + H(\varepsilon)\ddot{\sigma}_{\mathrm{d}} + \\ &C(\varepsilon,\dot{\varepsilon})\dot{\sigma}_{\mathrm{d}} + g(\varepsilon) = T. \end{aligned} \tag{7}$$

3 Direct parametric control for combined spacecraft

In this section, the controller is designed based on [13]. To control the system (7), we will design a controller which is composed of two parts:

$$T = u_{\mathrm{c}} + u_{\mathrm{f}},$$

where

$$u_{\mathrm{f}} = H(\sigma)\ddot{\sigma}_{\mathrm{d}} + C(\sigma,\dot{\sigma})\dot{\sigma}_{\mathrm{d}} + g(\sigma),$$

while u_{c} is a proportional plus derivative state feedback in the following form:

$$u_{\mathrm{c}} = K_0(\sigma,\dot{\sigma})\varepsilon + K_1(\sigma,\dot{\sigma})\dot{\varepsilon} + v_{\mathrm{c}},$$

where $K_0(\sigma,\dot{\sigma}) \in \mathbb{R}^{n\times n}$ and $K_1(\sigma,\dot{\sigma}) \in \mathbb{R}^{n\times n}$ are the feedback gains to be designed, they are piece-wisely continuous functions with respect to $\sigma, \dot{\sigma}$, and v_{c} is an external signal. With this controller applied to the fully-actuated system (7), the closed-loop system is obviously obtained as follows:

$$H(\sigma)\ddot{\varepsilon} + (C(\sigma,\dot{\sigma}) - K_1(\sigma,\dot{\sigma}))\dot{\varepsilon} - K_0(\sigma,\dot{\sigma})\varepsilon = v_{\mathrm{c}}. \tag{8}$$

3.1 The problem

The problem to be dealt with can be explained precisely as follows.

Let

$$X = \begin{bmatrix} \varepsilon \\ \dot{\varepsilon} \end{bmatrix},$$

then the closed loop system (8) can be converted into the following first-order form:

$$\dot{X} = A(\sigma,\dot{\sigma})X + B(\sigma,\dot{\sigma})v_{\mathrm{c}},$$

where

$$A(\sigma,\dot{\sigma}) = \begin{bmatrix} 0 & I_3 \\ A_{21} & A_{22} \end{bmatrix},$$

$$B(\sigma,\dot{\sigma}) = \begin{bmatrix} 0 \\ H^{-1}(\sigma) \end{bmatrix}$$

with

$$A_{21} = H^{-1}(\sigma)K_0(\sigma,\dot{\sigma}),$$

$$A_{22} = H^{-1}(\sigma)(K_1(\sigma, \dot{\sigma}) - C(\sigma, \dot{\sigma}))$$

and our design purpose is to let $A(\sigma, \dot{\sigma})$ be similar to an arbitrary given constant negative matrix of the same dimension as stated in the following problem.

Problem A Given an arbitrarily chosen negative matrix $F_0 \in \mathbb{R}^{2n\times 2n}$, find a constant nonsingular matrix $V \in \mathbb{R}^{2n\times 2n}$, and a pair of gain matrices $K_0(\sigma, \dot{\sigma})$ and $K_1(\sigma, \dot{\sigma}) \in \mathbb{R}^{n\times n}$ such that

$$V^{-1}A(\sigma, \dot{\sigma})V = F_0,$$

then the closed-loop system matrix

$$A(\sigma, \dot{\sigma}) = VF_0V^{-1}$$

is a constant one.

3.2 Main result

Define

$$\mathcal{F} = \{F_0| \text{ and } \exists Z \in \mathbb{R}^{n\times 2n} \text{ s.t.} \det \begin{bmatrix} Z \\ ZF_0 \end{bmatrix} \neq 0\},$$

the following result gives a complete answer to Problem A.

Proposition 1 Problem A has solution if and only if $F \in \mathcal{F}$, and in this case all the solutions to Problem A are parameterized as

$$V = V(Z, F_0) = \begin{bmatrix} Z \\ ZF_0 \end{bmatrix} \tag{9}$$

and

$$[K_0(\sigma, \dot{\sigma})\, K_1(\sigma, \dot{\sigma})] = W(\sigma, \dot{\sigma}, Z, F_0)V(Z, F_0)^{-1} \tag{10}$$

with

$$W(\sigma, \dot{\sigma}, Z, F_0) = H(\sigma)ZF_0^2 + C(\sigma, \dot{\sigma})ZF_0, \tag{11}$$

where $Z \in \mathbb{R}^{n\times 2n}$ is an arbitrary parameter matrix satisfying

$$\det \begin{bmatrix} Z \\ ZF_0 \end{bmatrix} \neq 0. \tag{12}$$

Proof According to Theorem 1 in [13], set $B(\theta, x, \dot{x})$ as identity matrix I, $A_2(\theta, x, \dot{x})$ as $H(\sigma)$ and $A_1(\theta, x, \dot{x})$ as $C(\sigma, \dot{\sigma})$, the result is obtained.

QED.

Moreover, the resulted closed-loop system is

$$\dot{X} = (VF_0V^{-1})X + \begin{bmatrix} 0 \\ H^{-1}(\sigma) \end{bmatrix} v_c.$$

4 Structured robust least-squares control allocation

Assuming that the service spacecraft has m actuators, then the resulting control torques of thrusters $T_c \in \mathbb{R}^n$ with respect to the service spacecraft can be denoted as

$$T_c = Au,$$

where $u \in \mathbb{R}^m$ is the force vector of the thrusters, can be denoted as $u = [u_1\ u_2\ \cdots\ u_m]^T$, and $A \in \mathbb{R}^{n\times m}$ is the configuration matrix of the thrusters, can be denoted as

$$A = [d_1 \times e_1\ \ d_2 \times e_2\ \ \cdots\ \ d_m \times e_m]$$

with the position matrix $d = [d_1\ d_2\ \cdots\ d_m]$ and the orientation matrix $e = [e_1\ e_2\ \cdots\ e_m]$. After capturing the target spacecraft, the position matrix has been changed, which can be denoted as

$$d + \Delta d = [d_1 + \Delta d_1\ \ d_2 + \Delta d_2\ \ \cdots\ \ d_m + \Delta d_m]$$

and the resulting control torques of thrusters T with respect to the combined spacecraft's frame can be denoted as

$$T_c = Bu,$$

where

$$B = [(d_1 + \Delta d_1) \times e_1\ \ \cdots\ \ (d_m + \Delta d_m) \times e_m].$$

However, there may be error when measure the configuration matrix B. Without loss of generality, define

$$B = B_0 + \Delta B,$$

where B_0 is the nominal matrix and ΔB is the uncertain part. Our object is to find a set of admissible control effector deflections satisfying the following optimal problem

$$u_{\mathrm{RLSCA}} = \arg \min_{\underline{u}<u<\bar{u}} \max_{\|\Delta B\|_\infty \leqslant \rho} \|(B_0 + \Delta B)u - T_c\|,$$

subject to the following conditions:

1) The uncertainty matrix ΔB is an unknown bounded matrix satisfying

$$\|\Delta B\|_\infty \leqslant \rho.$$

2) The control vector u is between the upper bounded $\bar{u}$ and the lower bound $\underline{u}$.

If the uncertain control effectiveness matrix ΔB is set as

$$\Delta B(\delta) = \sum_{i=1}^{q} \delta_i(t)B_i, \tag{13}$$

where i) $B_i \in \mathbb{R}^{n\times m} (i = 1, 2, \cdots, q)$ are known matrices, which represent the perturbation direction; ii) $\delta_i(t) (i = 1, 2, \cdots, q)$ are arbitrary time functions, which represent the uncertain parameters in the system; iii) $\delta(t) = [\delta_1(t)\ \ \delta_2(t)\ \ \cdots\ \ \delta_q(t)]^T$ is an uncertain parameter vector, which is often assumed to be within a certain compact and convex set Δ.

Remark 2 The formalization of $\Delta B(\delta)$ we used here is reasonable, the reason is stated as follows. The nominal configuration matrix can be written as

$$B_0 = [d_{01} \times e_1\ \ d_{02} \times e_2\ \ \cdots\ \ d_{0m} \times e_m],$$

if the measure error was happened in position vector d_{01}, then the configuration matrix is written as

$$B = [(d_{01} + \Delta d_1) \times e_1\ \ d_{02} \times e_2\ \ \cdots\ \ d_{0m} \times e_m].$$

Without loss of generality, we assume that $\Delta d_1 = [\delta_{11}\ \ \delta_{12}\ \ \delta_{13}]^T$ and $e_1 = [e_{11}\ \ e_{12}\ \ e_{13}]^T$, then

$$\Delta d_1 \times e_1 = \delta_{11}a_1 + \delta_{12}a_2 + \delta_{13}a_3$$

with

$$a_1 = \begin{bmatrix} 0 & -e_{13} & e_{12} \end{bmatrix}^{\mathrm{T}},$$
$$a_2 = \begin{bmatrix} e_{13} & 0 & -e_{11} \end{bmatrix}^{\mathrm{T}},$$
$$a_3 = \begin{bmatrix} -e_{12} & e_{11} & 0 \end{bmatrix}^{\mathrm{T}},$$

if we set

$$B_1 = \begin{bmatrix} a_1 & 0_{3\times(m-1)} \end{bmatrix},$$
$$B_2 = \begin{bmatrix} a_2 & 0_{3\times(m-1)} \end{bmatrix},$$
$$B_3 = \begin{bmatrix} a_3 & 0_{3\times(m-1)} \end{bmatrix},$$

then the configuration matrix can be written as

$$B = B_0 + \Delta B$$

with

$$\Delta B = \delta_{11}B_1 + \delta_{12}B_2 + \delta_{13}B_3,$$

and if the measure errors were happened in some or all position vectors, it is easy to generalize the form of $\Delta B(\delta)$ into (13).

In practical applications, two types of perturbation parameters sets are widely used[29]. One is in the regular polyhedral form

$$\varDelta_I = \{\delta(t) | \delta_i(t) \in [\delta_i^-, \delta_i^+],\ i = 1, 2, \cdots, q\},$$

the other type is in the polytopic form

$$\varDelta_P = \{\delta(t) | \sum_{i=1}^{q} \delta_i(t) = 1,\ \delta_i(t) \geqslant 0,\quad i = 1, 2, \cdots, q\}.$$

The above optimal problem can be transformed to the following form:

$$u_{\mathrm{RLSCA}} = \arg \min_{\underline{u}<u<\bar{u}} \max_{\delta\in\Delta} \|B(\delta)u - T_{\mathrm{c}}\|,$$

where $B(\delta) = B_0 + \Delta B(\delta)$, then the following results can be obtained.

Theorem 1 If $\delta(t) \in \varDelta_{\mathrm{I}}$, the SRLSCA problem has an optimal solution $(\lambda, u_{\mathrm{SRLSCA}})$ if the following is solved for $\forall\delta \in \varDelta_{\mathrm{E}}$:

$$\min_{u,\lambda} \lambda,$$

s.t.

$$\begin{bmatrix} -I & (B_0+\Delta B(\delta))u-T_{\mathrm{c}} \\ ((B_0+\Delta B(\delta))u-T_{\mathrm{c}})^{\mathrm{T}} & -\lambda I \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} b_1^{\mathrm{T}}(u-\bar{u}) & 0 & \cdots & 0 \\ 0 & b_2^{\mathrm{T}}(u-\bar{u}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_m^{\mathrm{T}}(u-\bar{u}) \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} b_1^{\mathrm{T}}(\underline{u}-u) & 0 & \cdots & 0 \\ 0 & b_2^{\mathrm{T}}(\underline{u}-u) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_m^{\mathrm{T}}(\underline{u}-u) \end{bmatrix} < 0 \tag{16}$$

with the variables $u \in \mathbb{R}^m$, and $\lambda > 0$, the control signal is $u^{\mathrm{T}} = [u_1 \ \cdots \ u_m]$, the upper and lower bounds of the control signal are $\bar{u}^{\mathrm{T}} = [\bar{u}_1 \ \cdots \ \bar{u}_m]$ and $\underline{u}^{\mathrm{T}} = [\underline{u}_1 \ \cdots \ \underline{u}_m]$, and $b_i (i = 1, 2, \cdots, m)$ are unit column vectors and satisfy $[b_1 \ b_2 \ \cdots \ b_m] = I_m$, and $\varDelta_{\mathrm{E}} = \{\delta = [\delta_1 \ \delta_2 \ \cdots \ \delta_q]^{\mathrm{T}} | \delta_i = \delta_i^- \text{ or } \delta_i^+,$ $i = 1, 2, \cdots, q\}$.

Proof Since the squared worst-case residual is represented as

$$r^2(u) = \max_{\delta\in\varDelta_{\mathrm{E}}} (B(\delta)u - T_{\mathrm{c}})^{\mathrm{T}}(B(\delta)u - T_{\mathrm{c}}).$$

To ensure $r^2(u) < \lambda$, it holds if

$$(B(\delta)u - T_{\mathrm{c}})^{\mathrm{T}}(B(\delta)u - T_{\mathrm{c}}) - \lambda < 0.$$

Using the Schur complement Lemma[29], we have

$$\begin{bmatrix} -I & (B_0+\Delta B(\delta))u-T_{\mathrm{c}} \\ ((B_0+\Delta B(\delta))u-T_{\mathrm{c}})^{\mathrm{T}} & -\lambda I \end{bmatrix} < 0 \tag{17}$$

for $\delta \in \varDelta_{\mathrm{I}}$, then according to Corollary 4.3.1 in [29], (14) is obtained.

To add the constraints to u, we have

$$\underline{u} < u < \bar{u} \Leftrightarrow \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \vdots \\ \underline{u}_m \end{bmatrix} < \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} < \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \vdots \\ \bar{u}_m \end{bmatrix}, \tag{18}$$

then rewrite (18) as two LMIs:

$$\begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \vdots \\ \underline{u}_m \end{bmatrix} < \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} \Leftrightarrow$$

$$\begin{bmatrix} \underline{u}_1 - u_1 & 0 & \cdots & 0 \\ 0 & \underline{u}_2 - u_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \underline{u}_m - u_m \end{bmatrix} < 0 \Leftrightarrow$$

$$\begin{bmatrix} b_1^{\mathrm{T}}(\underline{u} - u) & 0 & \cdots & 0 \\ 0 & b_2^{\mathrm{T}}(\underline{u} - u) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_m^{\mathrm{T}}(\underline{u} - u_m) \end{bmatrix} < 0.$$

Thus (16) is obtained, by the same way, we can also get (15). QED.

Theorem 2 If $\delta(t) \in \varDelta_{\mathrm{P}}$, the SRLSCA problem has an optimal solution $(\lambda, u_{\mathrm{SRLSCA}})$ if the following is solved for $i = 1, 2, \cdots, q$:

$$\min_{u,\lambda} \lambda,$$

s.t.

$$\begin{bmatrix} -I & (B_0 + B_i)u - T_{\mathrm{c}} \\ ((B_0 + B_i)u - T_{\mathrm{c}})^{\mathrm{T}} & -\lambda I \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} b_1^{\mathrm{T}}(u - \bar{u}) & 0 & \cdots & 0 \\ 0 & b_2^{\mathrm{T}}(u - \bar{u}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_m^{\mathrm{T}}(u - \bar{u}) \end{bmatrix} < 0,$$

$$\begin{bmatrix} b_1^{\mathrm{T}}(\underline{u}-u) & 0 & \cdots & 0 \\ 0 & b_2^{\mathrm{T}}(\underline{u}-u) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_m^{\mathrm{T}}(\underline{u}-u) \end{bmatrix} < 0$$

with the variables $u \in \mathbb{R}^m$, and $\lambda > 0$, the control signal is $u^{\mathrm{T}} = [u_1 \ \cdots \ u_m]$, the upper and lower bounds of the control signal are $\bar{u}^{\mathrm{T}} = [\bar{u}_1 \ \cdots \ \bar{u}_m]$ and $\underline{u}^{\mathrm{T}} = [\underline{u}_1 \ \cdots \ \underline{u}_m]$, and $b_i (i = 1, 2, \cdots, m)$ are unit column vectors and satisfy $[b_1 \ b_2 \ \cdots \ b_m] = I_m$.

Proof The proof is similar to the proof of Theorem 1. After (17) is obtained, using Corollary 4.3.2 in [29], we can get (19). The proof of the constraints on u is similar in Theorem 1; thus, the conclusion is obtained.

In the following part, we will investigate one particular linear structure robust control allocation. Let us introduce a lemma first. QED.

Lemma 1[29] Let $X \in \mathbb{R}^{m\times n}$, $Y \in \mathbb{R}^{n\times m}$, then for $\forall\delta > 0$, there holds

$$XFY + Y^{\mathrm{T}}F^{\mathrm{T}}X^{\mathrm{T}} \leqslant \delta XX^{\mathrm{T}} + \delta^{-1}Y^{\mathrm{T}}Y,$$

if $F \in \mathbb{F} = \{F | F \in \mathbb{R}^{n\times n}, \ F^{\mathrm{T}}F \leqslant I\}$.

If ΔB satisfies $\Delta B = EFH$ with $F^{\mathrm{T}}F \leqslant I$, then we can get the following result.

Theorem 3 If $\Delta B = EFH$ with $F^{\mathrm{T}}F \leqslant I$, the SRLSCA problem has an optimal solution $(\lambda, \delta, u_{\mathrm{SRLSCA}})$ if the following problem is solved:

$$\min_{u,\delta,\lambda} \lambda,$$

s.t.

$$\begin{bmatrix} -I + \delta EE^{\mathrm{T}} & B_0u - T_{\mathrm{c}} & 0 \\ (B_0u - T_{\mathrm{c}})^{\mathrm{T}} & -\lambda I & (Hu)^{\mathrm{T}} \\ 0 & Hu & -\delta I \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} b_1^{\mathrm{T}}(u-\bar{u}) & 0 & \cdots & 0 \\ 0 & b_2^{\mathrm{T}}(u-\bar{u}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_m^{\mathrm{T}}(u-\bar{u}) \end{bmatrix} < 0,$$

$$\begin{bmatrix} b_1^{\mathrm{T}}(\underline{u}-u) & 0 & \cdots & 0 \\ 0 & b_2^{\mathrm{T}}(\underline{u}-u) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_m^{\mathrm{T}}(\underline{u}-u) \end{bmatrix} < 0$$

with the variables $u \in \mathbb{R}^m$, $\delta > 0$ and $\lambda > 0$, the control signal is $u^{\mathrm{T}} = [u_1 \ \cdots \ u_m]$, the upper and lower bounds of the control signal are $\bar{u}^{\mathrm{T}} = [\bar{u}_1 \ \cdots \ \bar{u}_m]$ and $\underline{u}^{\mathrm{T}} = [\underline{u}_1 \ \cdots \ \underline{u}_m]$, and $b_i (i = 1, 2, \cdots, m)$ are unit column vectors and satisfy $[b_1 \ b_2 \ \cdots \ b_m] = I_m$.

Proof Since the squared worst-case residual is represented as

$$r^2(u) = \max_{\delta\in\Delta_{\mathrm{E}}} (B_0u - T_{\mathrm{c}} + EFHu)^{\mathrm{T}}(B_0u - T_{\mathrm{c}} + EFHu).$$

To ensure $r^2(u) < \lambda$, it holds if

$$\begin{bmatrix} -I & B_0u - T_{\mathrm{c}} + EFHu \\ (B_0u - T_{\mathrm{c}} + EFHu)^{\mathrm{T}} & -\lambda I \end{bmatrix} < 0$$

and furthermore

$$\begin{bmatrix} -I & B_0u - T_{\mathrm{c}} \\ (B_0u - T_{\mathrm{c}})^{\mathrm{T}} & -\lambda I \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix} [0 \ \ Hu] + \begin{bmatrix} 0 \\ (Hu)^{\mathrm{T}} \end{bmatrix} [E^{\mathrm{T}} \ \ 0] < 0.$$

Then according to Lemma 1, (20) is obtained. The proof of the constraints on u is similar in Theorem 1; thus, the conclusion is obtained. QED.

5 Simulation

In order to demonstrate the proposed method, the simulation is conducted. Though the effectiveness of the inertia parameter identification method has been demonstrated in [22], we only show that the effectiveness of the proposed direct parameter control algorithm and the proposed RLSCA is effectively robust to an uncertain control effectiveness matrix in this paper.

To demonstrate the effectiveness of the direct parameter control algorithm for the combined spacecraft, without loss of generality, the inertia matrix J of the combined spacecraft after identification, the initial attitude MRP and the initial angular velocity of the combined spacecraft are set as follows:

$$J = \mathrm{diag}\{25, 20, 15\},$$
$$\sigma = [0.08381 \ \ 0.101 \ \ 0.1205]^{\mathrm{T}},$$
$$\omega = [0.087266 \ \ 0.043633 \ \ 0.05236]^{\mathrm{T}}.$$

Furthermore, we set the desired attitude MRP $\sigma_{\mathrm{d}} = [\sigma_{\mathrm{d}1} \ \ \sigma_{\mathrm{d}2} \ \ \sigma_{\mathrm{d}3}]^{\mathrm{T}}$ of the combined spacecraft as

$$\begin{aligned} \sigma_{\mathrm{d}1} &= -1.475\times10^{-8}t^4 + 4.559\times10^{-6}t^3 - \\ &\quad 5.105\times10^{-4}t^2 + 0.02439t + 0.08381, \\ \sigma_{\mathrm{d}2} &= 1.136\times10^{-6}t^3 - 2.5\times10^{-4}t^2 + \\ &\quad 0.01768t + 0.101, \\ \sigma_{\mathrm{d}3} &= 8.899\times10^{-7}t^3 - 2.088\times10^{-4} + \\ &\quad 0.01578t + 0.1205. \end{aligned}$$

During the simulation, we found that different choice of F_0 will lead to different magnitudes of tracking error and control tuques, define

$$E = \mathrm{Blockdiag}\{\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}, -3, -4, -5, -6\}$$

now consider three cases: Case 1: $F_0 = E$, $Z = [I_3 \ \ I_3]$; Case 2: $F_0 = 0.5E$, $Z = [I_3 \ \ I_3]$; Case 3: $F_0 = 2E$, $Z = [I_3 \ \ I_3]$. The time of the attitude control is 100 s. Fig. 2 and Fig. 3 provide restively the desired attitude trajectory σ_{d} of combined spacecraft and the desired velocity trajectory $\dot{\sigma}_{\mathrm{d}}$ of combined spacecraft.

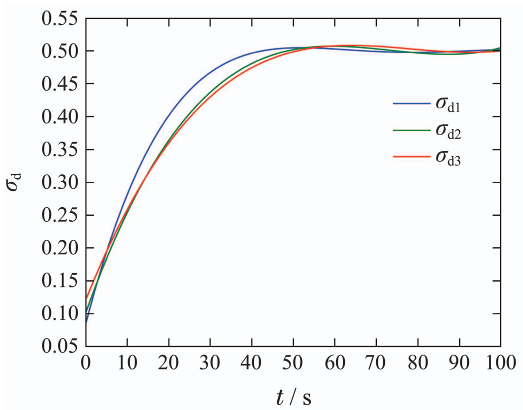


Fig. 2 Desired attitude trajectory σ_d of combined spacecraft

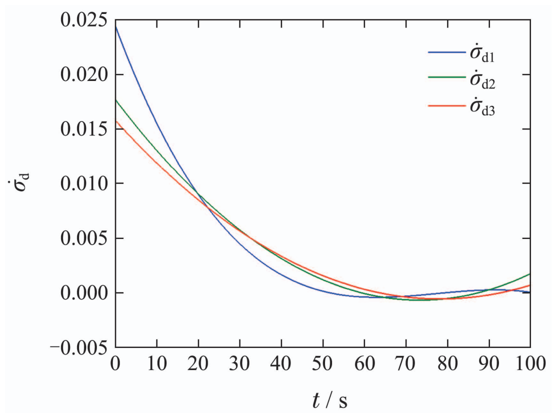
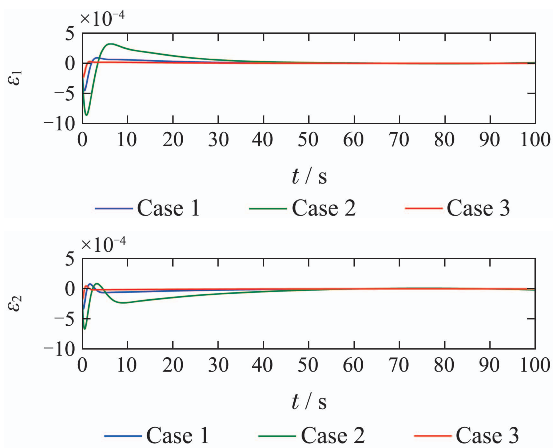


Fig. 3 Desired attitude trajectory $\dot{\sigma_d}$ of combined spacecraft

The tracking laws of direct parameter control of Case 1 to Case 3 are respectively employed to control the combined spacecraft in attempt to reach the desired position trajectory σ_d and the desired velocity trajectory $\dot{\sigma}_d$. Fig. 4 and Fig. 5 show respectively the comparison of position trajectory error ε and the velocity trajectory tracking errors $\dot{\varepsilon}$ of combined spacecraft, and Fig. 6 shows the control torques T_c of combined spacecraft generated by direct parameter control. It can be seen that, both the position trajectory tracking errors ε and the velocity trajectory tracking errors $\dot{\varepsilon}$ finally become zero. Furthermore, the larger magnitude of matrix F_0, the smaller magnitude of ε and $\dot{\varepsilon}$ and the larger magnitude of control torques T_c.



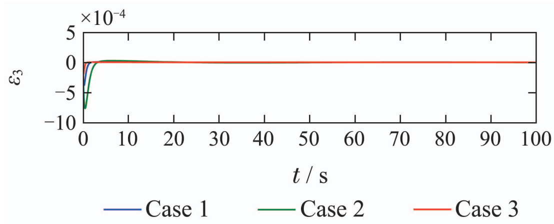


Fig. 4 Comparison of position trajectory tracking errors ε

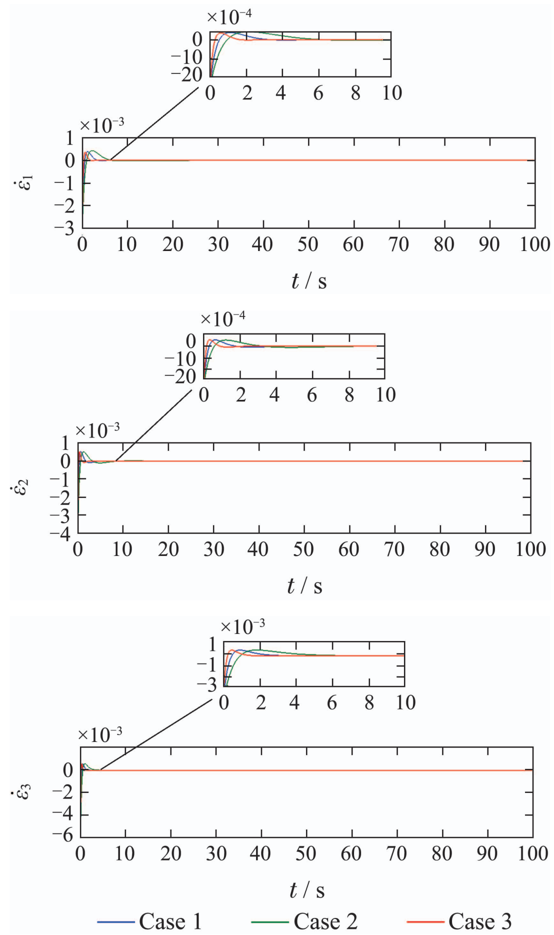
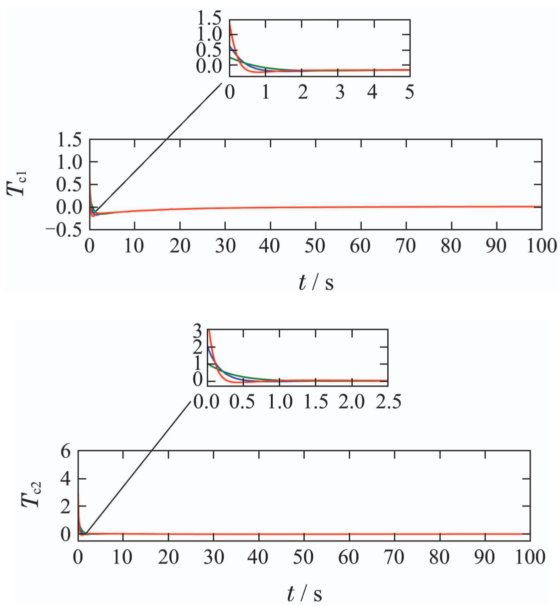


Fig. 5 Comparison of velocity trajectory tracking errors $\dot{\varepsilon}$



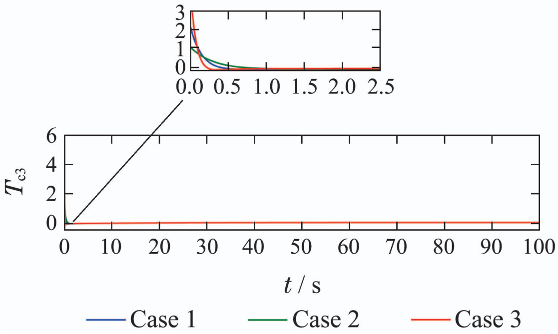


Fig. 6 Control torques T_c of combined spacecraft

Then, in order to demonstrate the effectiveness of the proposed RLSCA, the following points need to be known as follows:

1) The number of actuators $m = 8$ and the the number of the thruster is $n = 3$.

2) The upper bound of uncertainty in the control effectiveness matrix is adopted as $\rho = 0.1$.

3) Without loss of generality, the control torques in this simulation are selected from Case 1.

4) The bounds of force vector of the thrusters u are $-2.3e \leqslant \underline{u} \leqslant u \leqslant \bar{u} = 2.35e$ with $e = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1]^{\rm T}$.

5) All of the three proposed RLSCA approaches use the virtual control signals produced by the same controller. Besides, The position vectors and orientation vectors of the thrusters in the combined spacecraft are shown in Table 1.

Table 1 The positions and orientations of the thrusters

ith	Position/m	Orientation/rad
1	$[0\ \ -0.75\ \ 0.75]^{\rm T}$	$[\pi/2\ \ \pi/2\ \ \pi]^{\rm T}$
2	$[-0.75\ \ -0.75\ \ 0]^{\rm T}$	$[0\ \ \pi/2\ \ \pi/2]^{\rm T}$
3	$[0\ \ -0.75\ \ -0.75]^{\rm T}$	$[\pi/2\ \ \pi/2\ \ 0]^{\rm T}$
4	$[0.75\ \ -0.75\ \ 0]^{\rm T}$	$[\pi\ \ \pi/2\ \ \pi/2]^{\rm T}$
5	$[0.375\sqrt{2}\ \ -0.75\ \ 0.375\sqrt{2}]^{\rm T}$	$[\pi\ \ \pi/2\ \ \pi/2]^{\rm T}$
6	$[-0.375\sqrt{2}\ \ -0.75\ \ 0.375\sqrt{2}]^{\rm T}$	$[0\ \ \pi/2\ \ \pi/2]^{\rm T}$
7	$[-0.375\sqrt{2}\ \ -0.75\ \ -0.375\sqrt{2}]^{\rm T}$	$[0\ \ \pi/2\ \ \pi/2]^{\rm T}$
8	$[0.375\sqrt{2}\ \ -0.75\ \ -0.375\sqrt{2}]^{\rm T}$	$[\pi\ \ \pi/2\ \ \pi/2]^{\rm T}$

To illustrate the effectiveness of Theorem 1 and Theorem 2, we set

$$\Delta B(\delta) = \sum_{i=1}^{4} \delta_i(t) B_i$$

with

$$\begin{aligned}
&B_1 = 0.1 \times B,\ B_2 = 0.1 \times B,\\
&B_3 = 0.2 \times B,\ B_4 = 0.2 \times B,\\
&\delta_1^- = -1,\ \delta_1^+ = 1,\ \delta_2^- = -1,\ \delta_2^+ = 1,\\
&\delta_3^- = -1,\ \delta_3^+ = 1,\ \delta_4^- = -1,\ \delta_4^+ = 1
\end{aligned}$$

and the results are showed in Figs. 7–10.

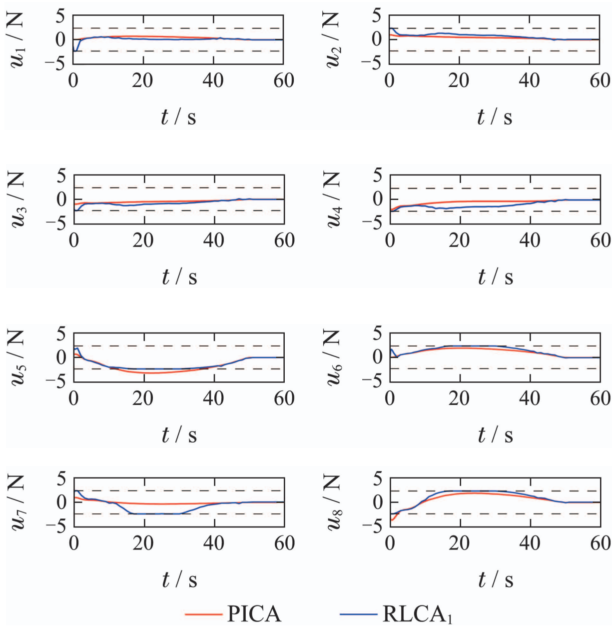


Fig. 7 Results of pseudo-inverse control allocation and SRLSCA in regular polyhedron form

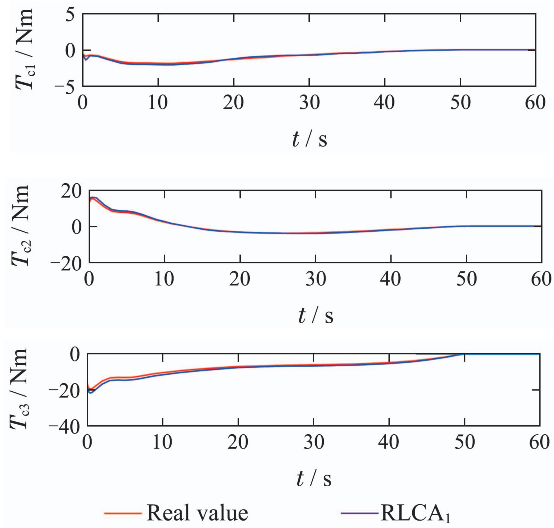
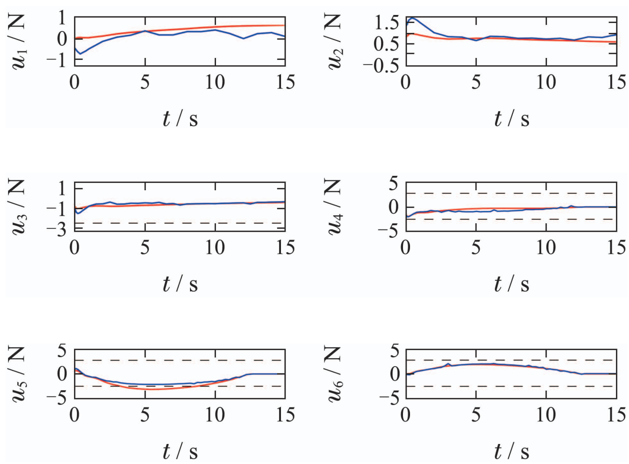


Fig. 8 The real torque value and value computed by SRLSCA in regular polyhedron form



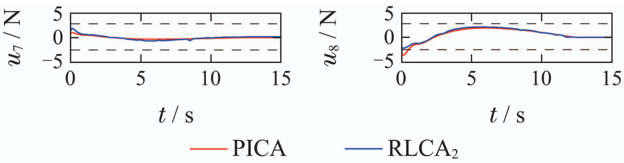


Fig. 9 Results of pseudo-inverse control allocation and SRLSCA in polytopic form

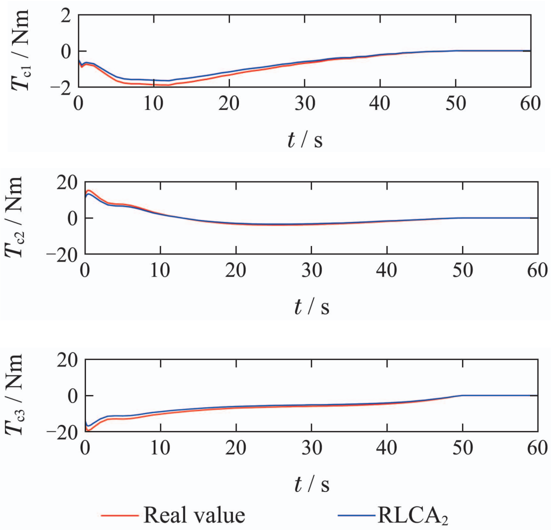


Fig. 10 The real torque value and value computed by SRLSCA in polytopic form

Figures 7 and 8 show that comparing with pseudo-inverse control allocation, the method proposed in Theorem 1 is limited in the bounds, and also conducted torque of thruster is mainly fitted with true value. Thus, the proposed SRLSCA is effectively robust to the uncertainty of regular polyhedron form in the control effectiveness matrix.

Figures 9 and 10 show that comparing with pseudo-inverse control allocation, the method proposed in Theorem 2 is limited in the bounds, and also conducted torque of thruster is mainly fitted with true value. Thus, the proposed SRLSCA is effectively robust to the uncertainty of polytopic form in the control effectiveness matrix.

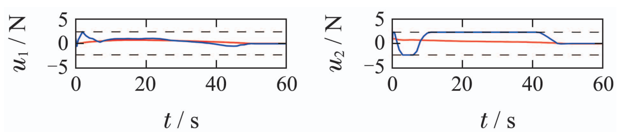
To illustrate the effectiveness of Theorem 3, we set

$$\Delta B = EFH$$

with $E = \mathrm{diag}\{0.1, 0.15, 0.1\}$ and

$$H = \begin{bmatrix} 0.1 & 0 & 0 & 0.1 & -0.1 & 0.1 & 0.1 & 0.1 \\ 0 & 0.1 & 0 & -0.1 & 0.1 & 0.1 & 0.1 & -0.1 \\ 0 & 0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.2 & -0.1 \end{bmatrix},$$

and the results are showed in Fig. 11 and Fig. 12.



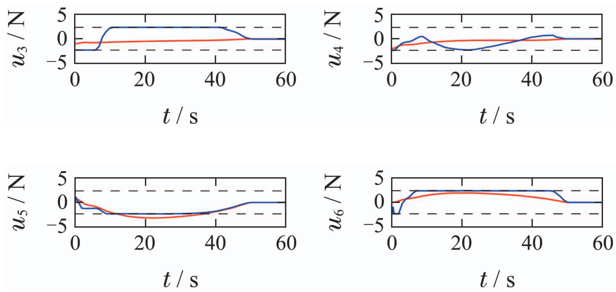


Fig. 11 Results of pseudo-inverse control allocation and SRLSCA with special linear fractional structured uncertainty

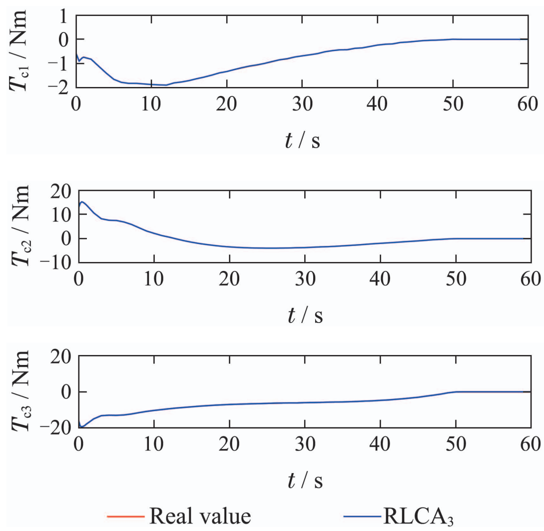


Fig. 12 The real torque value and value computed by SRLSCA with special linear fractional structured uncertainty

Figures 11 and 12 show that comparing with pseudo-inverse control allocation, the method proposed in Theorem 3 is limited in the bounds, and also conduced torque of thruster is almost perfectly fitted with true value. Thus, the proposed LFSRLSCA is effectively robust to the special linear fractional structured uncertainty in control effectiveness matrix, and the results are better than those obtained by the two previously proposed methods.

6 Conclusions

In this paper, a new algorithm for solving issues in the postcapture of an unknown tumbling target with a space manipulator is presented. The inertia parameters of the combined spacecraft are identified online depending on momentum-based estimation method and recursive least squares algorithm. Then a direct parametric approach for combined spacecraft attitude control is established. Different from many previously reported results, a simple controller parametrization is proposed in the form of state proportional plus derivative feedback for the second-order nonlinear format. An important consequence of this set of controllers is that the resulted in closed-loop system is a linear constant one with designed eigenstructure. Furthermore, a robust least-squares method is introduced to solve the problem of control allocation with the uncertain control effectiveness matrix subject to structured and linear fractional structured uncertainties. According to the simulation results, it is concluded that the control effectors can deflect smoothly to produce the required virtual control moments by use of the proposed RLSCA.

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