

具有预设性能的切换系统输出反馈自适应动态面控制

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摘要: 本文针对一类具有未建模动态和预设性能的输出反馈非线性切换系统, 提出基于公共Lyapunov函数法的自适应输出反馈动态面控制方案. 通过设计 K 滤波器和观测器估计不可测量的状态. 引入动态信号处理动态不确定性. 利用Nussbaum函数解决增益符号未知的问题. 神经网络用于逼近由设计过程和理论分析所产生的未知连续函数. 引入性能函数和误差转换器将预设性能控制问题转换为稳定性问题. 通过适当选取切换子系统的初值, 并利用动态面控制系统证明的特点, 证明了闭环切换系统所有信号半全局一致终结有界. 仿真例子验证了所提方案的有效性.

关键词: 未建模动态; 预设性能; Nussbaum函数; 公共李雅普诺夫函数; 动态面控制

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Adaptive output feedback dynamic surface control for switched systems with prescribed performance

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Abstract: In this paper, an adaptive output feedback dynamic surface control scheme is proposed based on common Lyapunov function method for a class of nonlinear switched systems with unmodeled dynamics and prescribed performance. By designing the K -filters and the observer, the unmeasured states are estimated and the dynamic uncertainties are dealt with by the designed dynamic signal. The Nussbaum function is introduced to solve the unknown control gain sign problem. Neural networks are used to approximate the unknown continuous functions produced by design process and theoretical analysis. The control issue of prescribed performance can be transformed into the stability problem by using performance function and error transformer. By appropriately choosing the initial values of all switched subsystems and using the proof characteristics of dynamic surface control, all the signals in the closed-loop switched system are proved to be semi-globally uniformly ultimately bounded. Simulation example is provided to illustrate the effectiveness of the proposed control approach.

Key words: unmodeled dynamics; prescribed performance; Nussbaum function; common Lyapunov function; dynamic surface control

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1 引言

切换系统是一类重要的混杂系统, 是一个由切换逻辑有机结合起来的子系统的集合. 由于在实际工程系统中应用广泛所以被单独提出并引起了控制领域专家学者的极大关注. 他们针对切换系统的分析与设计提出了一系列的方法和策略. 文献[1–3]给出了公共

李雅普诺夫函数(common Lyapunov function, CLF)存在的充分条件及构造方法. 文献[4]利用后推技术和CLF法, 针对具有输出约束和时滞的下三角形切换系统设计了模糊跟踪控制方案. 文献[5]在后推技术和CLF法的基础上, 利用变量分离定理和神经网络逼近, 解决了非严格反馈切换系统在任意切换下的控制

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问题. 文献[6]在此基础上, 研究了一类带有输入时滞的严格反馈非线性切换系统在任意切换下的控制问题. 文献[7]针对一类耦合的非线性切换系统, 利用多Lyapunov函数法(multiple Lyapunov function, MLF)和小增益定理, 根据平均驻留时间法, 通过构造合适的切换信号确保控制系统稳定. 文献[8]结合了输出反馈动态面控制方法和平均驻留时间法, 提出了针对一类具有未知控制增益和不可量测状态的非线性切换大系统的控制策略.

过去, 人们常常选取稳态误差作为描述系统稳态性能的一种性能指标. 随着科技的发展, 系统的瞬态性能越来越受到研究者的关注. 文献[9]提出了预设性能的概念, 预设性能要求跟踪误差必须收敛到预定义的任意小的残差集内. 同时, 收敛速度、超调量必须满足之前设置的条件. 预设性能同时考虑了系统的稳态和瞬态性能, 因此得到了广泛关注. 文献[10–12]在文献[9]的基础上进一步推广, 研究了带有预设性能的线性化反馈多输入多输出(multi-input multi-output, MIMO)非线性系统的鲁棒自适应控制. 文献[13–14]结合了输出反馈控制和预设性能控制, 针对不确定非线性系统提出了相应的自适应控制方案. 文献[15]研究了具有输入约束和不可量测状态的随机非线性单输入单输出系统的预设性能自适应模糊输出反馈控制. 文献[16]研究了一类非严格反馈形式的非线性切换系统的预设性能控制. 但是, 文献[9–16]所考虑的系统均不含未建模动态. 而在实际非线性系统中, 由测量噪声、模型误差、外部扰动和模型简化等因素造成的未建模动态经常出现, 它的存在严重降低了闭环系统的性能. 由于未建模动态问题的普遍性, 一经提出便引起了控制领域的广泛关注. 国内外学者针对带有未建模动态的非线性系统进行了深入研究, 提出了多种解决方案. 文献[17–18]假设未建模动态是指数输入状态实用稳定的, 利用已知的Lyapunov函数衰减, 引入一个动态信号处理未建模动态. 文献[19–20]利用模糊逼近和小增益定理方法, 对具有动态扰动的非线性系统分别提出了两种鲁棒自适应控制方案. 文献[21]研究了一类具有输入未建模动态的输出反馈自适应预设性能控制. 文献[22–23]分别考虑了含有未建模动态的不确定非线性切换系统和非下三角形切换系统的跟踪控制问题. 然而, 具有未建模动态的输出反馈形式的非线性切换系统的预设性能控制问题尚未被研究.

在上述文献的基础上, 受文献[24–28]的启发, 本文致力于解决一类具有未建模动态和预设性能的输出反馈非线性切换系统的跟踪控制问题, 主要贡献如下: 1) 与文献[16]相比, 本文考虑了具有未建模动态、动态不确定性以及增益符号未知的输出反馈非线性切换系统的预设性能控制问题, 观测器的设计采用了

广泛应用的 K 滤波器设计方法, 并且子系统的未建模动态维数可以不相等, 而文献[16]中依据研究对象的特点, 直接设计了具有输出反馈的线性观测器; 2) 与文献[28]所采用的后推方法相比, 通过在切换系统中采用动态面控制方法, 不仅避免了对虚拟控制律的反复求导, 降低了参数计算和控制器设计的复杂性; 同时在由单个子系统到多个子系统的稳定性证明上更加清晰直观; 3) 与文献[24]相比, 通过引入Nussbaum函数解决了切换系统中增益符号未知的问题, 同时弥补了其稳定性证明的缺陷.

2 问题的描述与预备知识

2.1 问题的描述与基本假设

考虑如下具有未建模动态的输出反馈非线性切换系统:

$$\begin{cases} \dot{z}_{\sigma(t)} = q_{\sigma(t)}(z_{\sigma(t)}, y), \\ \dot{x} = Ax + f_{\sigma(t)}(y) + g_{\sigma(t)}(y) + \\ \quad \Delta_{\sigma(t)}(z_{\sigma(t)}, y, t) + bu, \\ y = x_1, \end{cases} \quad (1)$$

其中: $x = [x_1 \ x_2 \ \cdots \ x_n]^T \in \mathbb{R}^n$ 是系统的状态向量; u 是实际的控制输入; $y \in \mathbb{R}$ 是系统的输出; $z_{\sigma(t)} \in \mathbb{R}^{n_{\sigma(t)}}$ 称为未建模动态, $q_{\sigma(t)}(z_{\sigma(t)}, y)$ 是满足Lipschitz条件的未知函数; $\sigma(t): [0, +\infty) \rightarrow M = \{1, \cdots, m\}$ 表示逐段常数的切换信号, 当 $\sigma(t) = k$ 时, 表示系统(1)的第 k 个子系统在 t 时刻运行.

$$A = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{n \times n},$$

$$f_k(y) = [f_{k,1}(y) \ f_{k,2}(y) \ \cdots \ f_{k,n}(y)]^T$$

是已知的光滑非线性向量函数;

$$g_k(y) = [g_{k,1}(y) \ g_{k,2}(y) \ \cdots \ g_{k,n}(y)]^T$$

是未知的光滑非线性向量函数;

$$\Delta_k(z_k, y, t) = [\Delta_{k,1}(z_k, y, t) \ \Delta_{k,2}(z_k, y, t) \ \cdots \ \Delta_{k,n}(z_k, y, t)]^T$$

是未知的光滑非线性动态扰动;

$$b = [0 \ \cdots \ 0 \ b_m \ \cdots \ b_0]^T \in \mathbb{R}^n$$

是未知参数向量; $B(s) = b_m s^m + \cdots + b_1 s + b_0$ 是Hurwitz多项式; 假设系统只有输出 y 是可量测的, 其余状态均不可测.

控制目标: 设计一个自适应输出反馈控制器 u , 使得系统输出 y 能够跟踪期望轨迹 y_d , 并且保证闭环系统所有信号有界且输出误差 $e(t) = y(t) - y_d(t)$ 满足预设性能.

假设 1 存在已知的正常数 $b_{\max}$, 使得 $0 < |b_m| \leq b_{\max}$.

假设 2 未知非线性动态扰动 $\Delta_{k,i}(z_k, y, t)$, $i =$

1, 2, \dots, n 满足如下不等式:

$$|\Delta_{k,i}(z_k, y, t)| \leq \varphi_{k,i1}(|y|) + \varphi_{k,i2}(\|z_k\|),$$

其中: $\varphi_{k,i1}(\cdot)$ 是未知光滑函数, $\varphi_{k,i2}(\cdot)$ 是未知非负连续单调增函数, $\|\cdot\|$ 表示向量的欧氏范数.

假设 3^[21] 假设系统 $\dot{z}_k = q_k(z_k, y)$ 是指输入状态实用稳定的, 即存在一个 Lyapunov 函数 $V_k(z_k)$ 满足如下不等式:

$$\beta_{k,1}(\|z_k\|) \leq V_k(z_k) \leq \beta_{k,2}(\|z_k\|), \quad (2)$$

$$\frac{\partial V_k(z_k)}{\partial z_k} q_k(z_k, y) \leq -c_k V_k(z_k) + \gamma_k(|y|) + d_k, \quad (3)$$

其中: $\beta_{k,1}(\cdot), \beta_{k,2}(\cdot)$ 是 K_∞ 类函数, $\gamma_k(\cdot)$ 是已知的 K_∞ 类函数, $c_k > 0, d_k \geq 0$ 是已知常数.

引理 1^[21] 如果 V_k 是系统 $\dot{z}_k = q_k(z_k, y)$ 的一个指数输入状态实用稳定 Lyapunov 函数, 即式(2)–(3)成立, 则对于任意常数 $\bar{c}_k \in (0, c_k)$, 任意初始时刻 $t_0 > 0$, 任意初始状态 $z_{k,0} = z_k(t_0), v_{k,0} > 0$ 以及任意连续函数 $\bar{\gamma}_k(\cdot)$ 满足 $\bar{\gamma}_k(|y|) \geq \gamma_k(|y|)$, 存在有限时间

$$T_{k,0} = \max\left\{0, \frac{\ln\left[\frac{V_k(z_{k,0})}{v_{k,0}}\right]}{(c_k - \bar{c}_k)}\right\} \geq 0.$$

对于非负函数 $D_k(t, t_0)$, 定义动态信号 $v_k > 0, \dot{v}_k = -\bar{c}_k v_k + \bar{\gamma}_k(|y|) + d_k, v_k(t_0) = v_{k,0} > 0$, 当 $t \geq t_0 + T_{k,0}$ 时, 存在 $D_k(t, t_0) = 0$, 使得 $V_k(z_k) \leq D_k(t_0, t) + v_k(t)$. 不失一般性, 本文取 $\bar{\gamma}_k(|y|) = \gamma_k(|y|)$.

假设 4^[15] 期望轨迹向量 $x_d = [y_d \ \dot{y}_d \ \ddot{y}_d]^T \in \Omega_d$ 已知, 其中 $\Omega_d = \{x_d : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq B_0\}$, 且 B_0 为已知正常数.

注 1 假设 4 的含义是要求参考信号是 2 阶连续可导的; 需要指出的是, 这个对参考信号进行约束的假设是解决输出反馈系统跟踪控制问题时普遍用到的, 类似的假设在文献[16]和文献[27]中都能见到. 另外, 在使用后推设计进行控制器的设计时, 会要求期望轨迹是 n 阶连续可导的, 而本文使用的动态面的方法, 不仅降低了控制器设计的复杂性, 还只要求 y_d 的 1 阶和 2 阶导数有界, 放宽了 y_d 需要 1 至 n 阶导数有界的假设.

引理 2^[12] 对于任意连续实函数 $f(x, y)$, 存在光滑标量函数 $\phi(x) \geq 0$ 和 $\psi(y) \geq 0$, 满足如下不等式:

$$|f(x, y)| \leq \phi(x) + \psi(y),$$

其中: $x \in \mathbb{R}^m, y \in \mathbb{R}^n$.

2.2 Nussbaum 函数

如果连续函数 $N(\varsigma)$ 满足如下性质:

$$\limsup_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\varsigma) d\varsigma = +\infty,$$

$$\liminf_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\varsigma) d\varsigma = -\infty,$$

则称 $N(\varsigma)$ 为具有 Nussbaum 形式的函数. 本文选用如下形式的 Nussbaum 型函数:

$$N(\varsigma) = e^{\varsigma^2} \cos \frac{\pi \varsigma}{2}.$$

引理 3^[22] 设 $V(\cdot)$ 和 $\varsigma(\cdot)$ 是定义在 $[0, t_f]$ 上的光滑函数, 满足 $\forall t \in [0, t_f], V(t) \geq 0$. $N(\varsigma)$ 为一个光滑的 Nussbaum 型函数, 若 $\forall t \in [0, t_f]$ 下述不等式成立:

$$0 \leq V(t) \leq \kappa_0 + e^{-\kappa_1 t} \int_0^t (g(x(\tau))N(\varsigma) + 1) \varsigma e^{\kappa_1 \tau} d\tau,$$

则 $V(t), \varsigma(t), \int_0^t g(x(\tau))N(\varsigma) \varsigma d\tau$ 有界, 其中: κ_0 为适当的常数; κ_1 为正常数, $g(x(\tau))$ 为时变参数, 在闭区间 $[I^-, I^+]$ 上取值, 且 $0 \notin [I^-, I^+]$.

2.3 预设性能

如果跟踪误差 $e(t) = y(t) - y_d(t)$ 满足如下不等式, 则输出信号的预设性能可以实现

$$-\delta \rho(t) < e(t) < \bar{\delta} \rho(t),$$

其中 $\rho(t) = (\rho_0 - \rho_\infty)e^{-nt} + \rho_\infty$ 是表示瞬态以及稳态的输出性能的预设函数, 是一个正的光滑递减函数, 且 $\rho_0 > \rho_\infty > 0, \underline{\delta}, n$ 和 $\bar{\delta}$ 是选取的正常数. 为了将预设性能跟踪控制问题转换成一般无约束问题, 本文定义具有如下性质的转换函数:

- 1) $S^{-1}(\frac{e(t)}{\rho(t)})$ 是光滑并严格递增的;
- 2) 当 $-\delta < \frac{e(t)}{\rho(t)} < \bar{\delta}$ 时, $-\infty < S^{-1}(\frac{e(t)}{\rho(t)}) < +\infty$;
- 3) $\lim_{\frac{e}{\rho} \rightarrow -\delta} S^{-1}(\frac{e(t)}{\rho(t)}) = -\infty, \lim_{\frac{e}{\rho} \rightarrow \bar{\delta}} S^{-1}(\frac{e(t)}{\rho(t)}) = +\infty$.

所以本文选取满足上述所有性质的如下函数作为转换函数:

$$\xi = S^{-1}(\frac{e(t)}{\rho(t)}) = \frac{1}{2} \ln \frac{S + \bar{\delta}}{\bar{\delta} - S} - \frac{1}{2} \ln \frac{\bar{\delta}}{\underline{\delta}}. \quad (4)$$

因为它具有严格单调递增性质, 所以其反函数存在且具有如下形式:

$$S(\xi) = \frac{\bar{\delta} e^{\xi \underline{\delta}} - \underline{\delta} e^{-\xi \bar{\delta}}}{e^{\xi \underline{\delta}} + e^{-\xi \bar{\delta}}},$$

其中 $S(\xi) = \frac{e(t)}{\rho(t)}$, ξ 关于时间 t 的导数为

$$\dot{\xi} = r(\dot{e} - \frac{e\dot{\rho}}{\rho}),$$

其中 $r = \frac{1}{2\rho}(\frac{1}{S + \bar{\delta}} - \frac{1}{S - \bar{\delta}})$, $r > 0$.

为了进行控制器的设计, 引入如下状态转换器:

$$s_1 = \xi,$$

其关于时间的导数为

$$\dot{s}_1 = r(\dot{e} - \frac{e\dot{\rho}}{\rho}). \quad (5)$$

2.4 径向基函数神经网络

定义紧集 $\Pi_y = \{y | |y| \leq M_y\}$, 其中 $M_y > 0$ 是设计常数, 利用神经网络在紧集 Π_y 上对未知非线性函数 $g_{k,i}(y)$ 进行逼近, 即

$$g_{k,i}(y) = \theta_{k,i}^T \phi_{k,i}(y) + \delta_{k,i}(y),$$

其中: $\delta_{k,i}(y)$ 是逼近误差, 径向基函数

$$\phi_{k,i}(y) = [\phi_{k,i1}(y) \ \phi_{k,i2}(y) \ \cdots \ \phi_{k,iM_{k,i}}(y)]^T,$$

$M_{k,i}$ 表示第 i 个神经网络的节点数, 径向基函数 $\phi_{k,ij}(y)$ 通常取高斯函数

$$\phi_{k,ij}(y) = \exp\left[-\frac{(y - \mu_{k,ij})^2}{b_{k,ij}}\right],$$

其中: $\mu_{k,ij}, b_{k,ij}$ 分别为函数的中心和宽度, $1 \leq i \leq n$, $1 \leq j \leq M_{k,i}$, 理想权值 $\theta_{k,i}^*$ 表示为

$$\theta_{k,i}^* = \arg \min_{\theta \in \mathbb{R}^{M_{k,i}}} [\theta_{k,i}^T \phi_{k,i}(y) - g_{k,i}(y)].$$

为了方便后面滤波器和控制器的设计, 将式(1)改写成如下格式:

$$\begin{cases} \dot{z}_k = q_k(z_k, y), \\ \dot{x} = Ax + f_k(y) + F_k^T(y, u)\theta_k + \delta_k(y) + \Delta_k(z_k, y, t), \\ y = x_1, \end{cases} \quad (6)$$

其中:

$$F_k^T(y, u) = \left[\begin{matrix} 0_{(v-1) \times (m+1)} \\ I_{m+1} \end{matrix} \right] u, \Phi_k^T(y) \in \mathbb{R}^{n \times (m+1+N_k)},$$

$$\theta_k = [b^T \ \theta_{k,f}^T]^T \in \mathbb{R}^{(m+1+N_k) \times 1},$$

$$\Phi_k^T(y) = \begin{bmatrix} \phi_{k,1}^T(y) & 0 & 0 \\ 0 & \vdots & 0 \\ 0 & 0 & \phi_{k,n}^T(y) \end{bmatrix} \in \mathbb{R}^{n \times N_k},$$

$$\theta_{k,f} = [\theta_{k,1}^* \ \theta_{k,2}^* \ \cdots \ \theta_{k,n}^*]^T \in \mathbb{R}^{N_k \times 1},$$

$$\delta_k(y) = [\delta_{k,1}(y) \ \delta_{k,2}(y) \ \cdots \ \delta_{k,n}(y)]^T \in \mathbb{R}^{n \times 1},$$

$$N_k = \sum_{i=1}^n M_{k,i}.$$

3 基于神经网络逼近的 K 滤波器设计和观测器设计

由于系统只有输出 y 是可量测的, 其余状态均不可测, 因此设计如下状态观测器和滤波器去重构系统状态. 滤波器的设计如下:

$$\begin{cases} \dot{\zeta} = A_0 \zeta + Ly, & \zeta \in \mathbb{R}^n, \\ \dot{\Omega}_k^T = A_0 \Omega_k^T + F_k^T(y, u), & \Omega_k^T \in \mathbb{R}^{n \times (m+1+N_k)}, \end{cases} \quad (7)$$

其中: $L = [l_1 \ l_2 \ \cdots \ l_n]^T$, $A_0 = A - Le_1^T$ 为 Hurwitz 矩

阵, 即满足如下方程:

$$PA_0 + A_0^T P = -hI, \ P = P^T > 0, \quad (8)$$

其中 $h > 0$ 为设计常数.

定义矩阵 $\Omega_k = [v_{k,m} \ v_{k,m-1} \ \cdots \ v_{k,0} \ \Xi_k]^T$, 其中向量 $v_{k,j} (j = 0, 1, \cdots, m)$ 满足如下等式:

$$v_{k,j} = A_0^j \lambda_k, \quad (9)$$

$$A_0^j e_n = e_{n-j}, \quad (10)$$

$$\dot{v}_{k,j} = A_0 v_{k,j} + e_{n-j} u, \quad (11)$$

其中: $j = 0, 1, \cdots, m$, A_0^j 表示矩阵 A_0 的 j 次幂.

由式(7)得到矩阵 Ξ_k 满足

$$\dot{\Xi}_k = A_0 \Xi_k + \Phi_k^T(y). \quad (12)$$

由式(9)–(11)可得如下等式:

$$v_{k,i,j} = [* \ \cdots \ * \ 1] \begin{bmatrix} \lambda_{k,1} \\ \vdots \\ \lambda_{k,i+j} \end{bmatrix}, \quad (13)$$

其中: $i = 0, 1, \cdots, m, j = 1, 2, \cdots, n, \lambda_{k,j} = 0, j > n$.

定义虚拟状态估计如下:

$$\hat{x} = \zeta + \Omega_k^T \theta_k.$$

定义虚拟观测误差为

$$\varepsilon = x - \hat{x} \varepsilon,$$

则有 $x = \varepsilon + \hat{x}$, 对 ε 求导可得

$$\dot{\varepsilon} = \dot{x} - \dot{\hat{x}} = A_0 \varepsilon + \delta_k(y) + \Delta_k(z_k, y, t). \quad (14)$$

综上, 滤波器可以改写成如下形式:

$$\begin{cases} \dot{\zeta} = A_0 \zeta + Ly, & \zeta \in \mathbb{R}^n, \\ \dot{\Xi}_k = A_0 \Xi_k + \Phi_k^T(y), & \Xi_k \in \mathbb{R}^{n \times N_k}, \\ \dot{\lambda}_k = A_0 \lambda_k + e_n u, & \lambda_k \in \mathbb{R}^n. \end{cases} \quad (15)$$

根据上述观测器可以得到系统状态 x_2 的表达式

$$x_2 =$$

$$\zeta_2 + \Omega_{k,2}^T \theta_k + \varepsilon_2 =$$

$$\zeta_2 + [v_{k,m,2} \ v_{k,m-1,2} \ \cdots \ v_{k,0,2} \ \Xi_{k,(2)}] \theta_k + \varepsilon_2 =$$

$$[0 \ v_{k,m-1,2} \ \cdots \ v_{k,0,2} \ \Xi_{k,(2)}] \theta_k + b_m v_{k,m,2} + \zeta_2 + \varepsilon_2, \quad (16)$$

其中: $\Omega_{k,(2)}^T$ 表示 Ω_k^T 第2行的向量, $\Xi_{k,(2)}$ 表示 Ξ_k 第2行的向量, ζ_2 表示列向量 ζ 的第2个分量, ε_2 表示列向量 ε 的第2个分量. 将式(16)代入式(2)中, 可得

$$\begin{aligned} \dot{y} &= b_m v_{k,m,2} + \zeta_2 + \varpi_k^T \theta_k + \varepsilon_2 + \\ &\quad \delta_{k,1}(y) + \Delta_{k,1}(z_k, y, t), \end{aligned} \quad (17)$$

其中: $\varpi_k^T = [0 \ v_{k,m-1,2} \ \cdots \ v_{k,0,2} \ \Xi_{k,(2)} + \Phi_{k,(1)}^T]$, $\Phi_{k,(1)}^T$ 表示 Φ_k^T 的第1行. 综合式(11)和式(13), 可得到如下 $v = n - m$ 阶系统:

$$\begin{cases} \dot{y} = b_m v_{k,m,2} + \zeta_2 + \varpi_k^T \theta_k + \varepsilon_2 + \delta_{k,1}(y) + \\ f_{k,1}(y) + \Delta_{k,1}(z_k, y, t), \\ \dot{v}_{k,m,i} = v_{k,m,i+1} - l_i v_{k,m,1}, \quad i = 2, \dots, v-1, \\ \dot{v}_{k,m,v} = u + v_{k,m,v+1} - l_v v_{k,m,1}. \end{cases} \quad (18)$$

在以下设计过程中, 作者将用系统(18)替代系统(1)设计控制方案.

3.1 自适应动态面控制器设计

为了便于讨论, 定义一些符号如下:

$$\begin{aligned} \tilde{\theta}_k &= \hat{\theta}_k - \theta_k, \quad \tilde{\theta}_{k,0} = \hat{\theta}_{k,0} - \theta_{k,0}, \quad \tilde{b}_m = \hat{b}_m - b_m, \\ \bar{s}_i &= [s_1 \ s_2 \ \dots \ s_i]^T, \quad 1 \leq i \leq v+1, \\ \bar{y}_i &= [y_2 \ \dots \ y_i]^T, \quad 2 \leq i \leq v+1, \\ \bar{\lambda}_{k,i} &= [\lambda_{k,1} \ \lambda_{k,2} \ \dots \ \lambda_{k,i}]^T, \quad i = 1, \dots, n. \end{aligned}$$

令

$$V_\varepsilon = \varepsilon^T P \varepsilon, \quad (19)$$

对式(19)进行求导, 并将式(14)代入, 得

$$\begin{aligned} \dot{V}_\varepsilon &= \dot{\varepsilon}^T P \varepsilon + \varepsilon^T P \dot{\varepsilon} = \\ &- h \varepsilon^T \varepsilon + 2 \varepsilon^T P \delta_k + 2 \varepsilon^T P \Delta_k + 2 \varepsilon^T P f_k(y). \end{aligned} \quad (20)$$

同时利用假设2和Young's不等式对上式化简得

$$\begin{aligned} \dot{V}_\varepsilon &\leq - (h-3) \varepsilon^T \varepsilon + \sum_{j=1}^n \|P\|^2 \delta_{k,j}^2 + \sum_{j=1}^n \|P\|^2 f_{k,j}^2 + \\ &\sum_{j=1}^n 2 \|P\|^2 (\varphi_{k,j1}^2(|y|) + \varphi_{k,j2}^2(\|z_k\|)). \end{aligned} \quad (21)$$

由假设3和引理1可得

$$\|z_k\| \leq \beta_{k,1}^{-1}(v_k(t) + D_k(t_0, t)), \quad (22)$$

$$\varphi_{k,j2}^2(\|z_k\|) \leq \varphi_{k,j2}^2 \circ \beta_{k,1}^{-1}(v_k(t) + D_k(t_0, t)), \quad (23)$$

其中“ $\circ$ ”运算定义为 $\varphi_{k,j2}^2 \circ \beta_{k,1}^{-1}(\cdot) = \varphi_{k,j2}^2(\beta_{k,1}^{-1}(\cdot))$.

由于 $\varphi_{k,j2}^2 \circ \beta_{k,1}^{-1}(\cdot)$ 是非负连续函数, 则根据引理1有

$$\begin{aligned} \sum_{j=1}^n 2 \|P\|^2 \varphi_{k,j2}^2(\|z_k\|) &\leq \\ \|P\|^2 \vartheta_{k,0}(D_k(t_0, t)) + \|P\|^2 \vartheta_{k,0}(v_k), \end{aligned} \quad (24)$$

其中 $\varphi_{k,0}(\cdot)$ 和 $\vartheta_{k,0}(\cdot)$ 是未知连续函数. 由引理1可得, 当 $t \geq t_0 + T_{k,0}$ 时, $D_k(t_0, t) = 0$. 所以存在一个常数 ϑ_0^* , 使得不等式 $\vartheta_{k,0}(D_k(t_0, t)) \leq \vartheta_0^*$ 成立.

将式(22)–(24)代入式(21)中可得

$$\begin{aligned} \dot{V}_\varepsilon &\leq - (h-3) \varepsilon^T \varepsilon + \sum_{j=1}^n \|P\|^2 \delta_{k,j}^2 + \\ &\sum_{j=1}^n \|P\|^2 f_{k,j}^2 + \sum_{j=1}^n 2 \|P\|^2 \varphi_{k,j1}^2(|y|) + \\ &\|P\|^2 \varphi_{k,0}(v_k) + \|P\|^2 \vartheta_0^*, \end{aligned} \quad (25)$$

引入如下坐标变换:

其中: $i = 1, 2, \dots, v$, s_1 由式(5)确定, ω_{i+1} 是一阶滤波器的输出, $\alpha_{k,i}$ 是虚拟控制律. 下面逐步进行控制器设计.

构造若干Lyapunov函数如下:

$$V_{s_i} = \frac{s_i^2}{2}, \quad i = 1, \dots, v, \quad (26)$$

$$V_1 = V_\varepsilon + V_{s_1} + \frac{v_k}{\lambda_0}, \quad \lambda_0 > 0, \quad (27)$$

$$V_i = V_1 + \sum_{j=2}^i V_{s_j} + \frac{1}{2} \sum_{j=2}^i y_j^2. \quad (28)$$

Step 1 由式(18)得

$$\begin{aligned} \dot{s}_1 &= r(b_m v_{k,m,2} + \zeta_2 + \varpi_k^T \theta_k + \varepsilon_2 + \delta_{k,1}(y) + \\ &\Delta_{k,1}(z_k, y, t) + f_{k,1}(y) - \dot{y}_d - \frac{e\dot{\rho}}{\rho}). \end{aligned} \quad (29)$$

根据式(27), 对 V_{s_1} 求导并将上式代入得

$$\begin{aligned} \dot{V}_{s_1} &= r(b_m(s_2 + y_2 + \alpha_1) + \zeta_2 + \varpi^T \theta_k + \\ &\varepsilon_2 + \delta_{k,1}(y) + \Delta_{k,1}(z_k, y, t) + \\ &f_{k,1}(y) - \dot{y}_d - \frac{e\dot{\rho}}{\rho}) s_1. \end{aligned} \quad (30)$$

根据Young's不等式得

$$r s_1 \varepsilon_2 \leq r^2 s_1^2 + \frac{\varepsilon_2^2}{4}, \quad (31)$$

$$r b_m y_2 s_1 \leq r^2 s_1^2 + \frac{b_{\max}^2}{4} y_2^2, \quad (32)$$

$$r b_m s_2 s_1 \leq r^2 s_1^2 + \frac{b_{\max}^2}{4} s_2^2, \quad (33)$$

$$r s_1 \delta_{k,1} \leq r^2 s_1^2 + \frac{1}{4} \delta_{k,1}^2. \quad (34)$$

由假设3、引理2有

$$r s_1 \Delta_{k,1} \leq r |s_1| \varphi_{k,11}(|y|) + r |s_1| \varphi_{k,12}(\|z_k\|), \quad (35)$$

$$\begin{aligned} r |s_1| \varphi_{k,12}(\|z_k\|) &\leq r |s_1| \vartheta_{k,1}(D_k(t_0, t)) + \\ &r |s_1| \varphi_{k,1}(v_k), \end{aligned} \quad (36)$$

其中 $\varphi_{k,1}(\cdot)$ 和 $\vartheta_{k,1}(\cdot)$ 为未知连续函数.

利用Young's不等式进行化简得

$$r |s_1| \varphi_{k,11}(|x_1|) \leq \frac{r^2}{a_{11}^2} s_1^2 \varphi_{k,11}^2(|x_1|) + \frac{a_{11}^2}{4}, \quad (37)$$

$$r |s_1| \varphi_{k,1}(v_k) \leq \frac{r^2}{a_1^2} s_1^2 \varphi_{k,1}^2(v_k) + \frac{a_1^2}{4}, \quad (38)$$

$$r |s_1| \vartheta_{k,1}(D_k(t_0, t)) \leq \frac{\vartheta_{k,1}^2(D_k(t_0, t))}{4} + r^2 s_1^2, \quad (39)$$

其中: a_{11}, a_1 已知为正常数, 由于 $D_k(t_0, t)$ 与 $\vartheta_{k,1}(\cdot)$ 为连续函数, 于是存在一个正常数 $\vartheta_{k,1}^*$, 使得不等式 $\vartheta_{k,1}(D_k(t_0, t)) \leq \vartheta_{k,1}^*$ 成立.

将式(32)–(40)代入式(31)化简得

$$\begin{aligned} \dot{V}_{s_1} \leq & s_1 r (b_m \alpha_1 + \zeta_2 + \varpi_k^T \theta_k + f_{k,1}(y) - \dot{y}_d - \frac{e\dot{\rho}}{\rho}) + \\ & \frac{r^2}{a_{11}^2} s_1^2 \varphi_{k,11}^2(|x_1|) + \frac{r^2}{a_1^2} s_1^2 \varphi_{k,1}^2(v_k) + 5r^2 s_1^2 + \\ & \frac{b_{\max}^2}{4} y_2^2 + \frac{b_{\max}^2}{4} s_2^2 + \frac{\vartheta_{k,1}^{*2}}{4} + \frac{a_{11}^2}{4} + \frac{a_1^2}{4} + \\ & \frac{1}{4} \varepsilon_2^2 + \frac{1}{4} \delta_{k,1}^2. \end{aligned} \quad (41)$$

设计虚拟控制 $\alpha_{k,1}$ 和Nussbaun参数 ζ 如下:

$$\begin{aligned} \alpha_{k,1} = & N(\varsigma)(c_{k,1}s_1 + \zeta_2 + \varpi_k^T \hat{\theta}_k + f_{k,1}(y) - \\ & \dot{y}_d + \frac{s_1 \hat{\theta}_{k,0} \|\Phi_k(Z_1)\|^2}{2a_0^2} - \frac{e\dot{\rho}}{\rho}), \end{aligned} \quad (42)$$

$$\begin{aligned} \dot{\varsigma} = & r s_1 (c_{k,1}s_1 + \zeta_2 + \varpi_k^T \hat{\theta}_k + f_{k,1}(y) - \dot{y}_d + \\ & \frac{s_1 \hat{\theta}_{k,0} \|\Phi_k(Z_1)\|^2}{2a_0^2} - \frac{e\dot{\rho}}{\rho}), \end{aligned} \quad (43)$$

其中 $N(\varsigma) = e^{\varsigma} \cos \frac{\pi \varsigma}{2}$, $c_{k,1} > 0$ 为设计常数, 可得

$$\begin{aligned} \dot{V}_{s_1} \leq & -c_{k,1} r s_1^2 - \frac{s_1^2 r \hat{\theta}_{k,0} \|\Phi_k(Z_1)\|^2}{2a_0^2} + \\ & (b_m N(\varsigma) + 1) \dot{\varsigma} + \frac{r^2}{a_{11}^2} s_1^2 \varphi_{k,11}^2(|x_1|) + \\ & \frac{r^2}{a_1^2} s_1^2 \varphi_{k,1}^2(v_k) + 5r^2 s_1^2 + \frac{b_{\max}^2}{4} y_2^2 + \\ & \frac{b_{\max}^2}{4} s_2^2 - s_1 r \varpi_k^T \tilde{\theta}_k + \frac{\vartheta_{k,1}^{*2}}{4} + \frac{a_{11}^2}{4} + \\ & \frac{a_1^2}{4} + \frac{1}{4} \varepsilon_2^2 + \frac{1}{4} \delta_{k,1}^2. \end{aligned} \quad (44)$$

令

$$H_{k,1}(Z_1) = \frac{r^{\frac{3}{2}}}{a_{11}^2} s_1 \varphi_{k,11}^2(|x_1|) + \frac{r^{\frac{3}{2}}}{a_1^2} s_1 \varphi_{k,1}^2(v_k) + 5r^{\frac{3}{2}} s_1,$$

其中: $Z_1 = [x_1 \ s_1 \ v_k \ r]^T$ 利用神经网络对未知函数 $H_{k,1}(Z_1)$ 进行逼近, 有 $H_{k,1}(Z_1) = W_{k,1}^T \Phi_k(Z_1) + e_{k,1}$, 其中: $W_{k,1}$ 为理想的权向量, $\Phi_k(Z_1)$ 为基向量, $e_{k,1}$ 为逼近误差, 满足 $|e_{k,1}| \leq e_{k,1}^*$, 定义 $\theta_{k,0} = \|W_{k,1}\|^2$.

参数自适应律设计如下:

$$\dot{\hat{\theta}}_k = \mu_1 (r s_1 \varpi_k - \gamma_1 \hat{\theta}_k), \quad (45)$$

$$\dot{\hat{\theta}}_{k,0} = \mu_2 \left(\frac{s_1^2 r \|\Phi_k(Z_1)\|^2}{2a_0^2} - \gamma_2 \hat{\theta}_{k,0} \right), \quad (46)$$

其中 $\mu_1, \mu_2, \gamma_1, \gamma_2, a_0 > 0$ 为设计常数.

对式(44)进行化简, 得

$$\begin{aligned} \dot{V}_{s_1} \leq & -c_{k,1} r s_1^2 - \frac{s_1^2 r \hat{\theta}_{k,0} \|\Phi_k(Z_1)\|^2}{2a_0^2} + \\ & (b_m N(\varsigma) + 1) \dot{\varsigma} + H_{k,1}(Z_1) r^{\frac{1}{2}} s_1 - \\ & s_1 r \varpi_k^T \tilde{\theta}_k + \frac{b_{\max}^2}{4} y_2^2 + \frac{b_{\max}^2}{4} s_2^2 + \frac{\vartheta_{k,1}^{*2}}{4} + \end{aligned}$$

$$\frac{a_{11}^2}{4} + \frac{a_1^2}{4} + \frac{1}{4} \varepsilon_2^2 + \frac{1}{4} \delta_{k,1}^2. \quad (47)$$

根据Young's不等式, 有

$$W_{k,1}^T \Phi_k(Z_1) r^{\frac{1}{2}} s_1 \leq \frac{s_1^2 r \theta_{k,0}}{2a_0^2} \|\Phi_k(Z_1)\|^2 + \frac{a_0^2}{2}, \quad (48)$$

$$e_{k,1} r^{\frac{1}{2}} s_1 \leq \frac{1}{4} e_{k,1}^{*2} + r s_1^2. \quad (49)$$

利用式(48)–(49)对式(47)进行化简, 得

$$\begin{aligned} \dot{V}_{s_1} \leq & -(c_{k,1} - 1) r s_1^2 - \frac{s_1^2 r \tilde{\theta}_{k,0}}{2a_0^2} \|\Phi_k(Z_1)\|^2 - \\ & s_1 r \varpi_k^T \tilde{\theta}_k + (b_m N(\varsigma) + 1) \dot{\varsigma} + \frac{\delta_{k,1}^2}{4} + \\ & \frac{b_{\max}^2}{4} s_2^2 + \frac{b_{\max}^2}{4} y_2^2 + \frac{a_0^2}{2} + \frac{1}{4} e_{k,1}^{*2} + \\ & \frac{a_{11}^2}{4} + \frac{a_1^2}{4} + \frac{\vartheta_{k,1}^{*2}}{4} + \frac{\varepsilon_2^2}{4}. \end{aligned} \quad (50)$$

对式(28)求导, 并将式(25)(50)以及 $\dot{v}_k = -\bar{c}_k v_k + \bar{\gamma}_k(y) + d_k$ 代入得

$$\begin{aligned} \dot{V}_1 \leq & -(h - \frac{13}{4}) \varepsilon^T \varepsilon - (c_{k,1} - 1) r s_1^2 - s_1 r \varpi_k^T \tilde{\theta}_k - \\ & \frac{s_1^2 r \tilde{\theta}_{k,0}}{2a_0^2} \|\Phi_k(Z_1)\|^2 + (b_m N(\varsigma) + 1) \dot{\varsigma} - \\ & \frac{\bar{c}_k}{\lambda_0} v_k + \frac{b_{\max}^2}{4} s_2^2 + \frac{b_{\max}^2}{4} y_2^2 + \\ & Q_k(y, v_k) + D_{k,1}, \end{aligned} \quad (51)$$

其中:

$$\begin{aligned} Q_k(y, v_k) = & \sum_{j=1}^n \|P\|^2 \delta_{k,j}^2 + \frac{\delta_{k,1}^2}{4} \sum_{j=1}^n 2 \|P\|^2 \varphi_{k,j1}^2(|y|) + \\ & \sum_{j=1}^n \|P\|^2 f_{k,j}^2 + \|P\|^2 \varphi_{k,0}(v_k) + \frac{\bar{\gamma}_k(|y|)}{\lambda_0}, \\ D_{k,1} = & \frac{a_0^2}{2} + \frac{e_{k,1}^{*2}}{4} + \frac{a_{k,11}^2}{4} + \frac{a_{k,1}^2}{4} + \frac{\vartheta_{k,1}^{*2}}{4} + \\ & \frac{\varepsilon_2^2}{4} + \|p\|^2 \vartheta_{k,0}^* + \frac{d_k}{\lambda_0}. \end{aligned}$$

构造如下二阶滤波器:

$$\tau_{k,2} \dot{\omega}_2 + \omega_2 = \alpha_{k,1}, \quad \omega_2(t_{k,0}) = \alpha_{k,1}(t_{k,0}), \quad (52)$$

其中: $\tau_{k,2}$ 为时间常数, $t_{k,0}$ 是 $\sigma(t) = k$ 时切换子系统运行的初始时刻. 由 $y_2 = \omega_2 - \alpha_{k,1}$ 和式(52)可知: 存在非负连续函数 $\eta_{k,2}(\bar{s}_2, \bar{y}_2, \hat{\theta}_{k,0}, \hat{\theta}_k, v_k, \zeta, \Xi_k, \bar{\lambda}_{k,m+2}, \varepsilon_2, \varsigma, y_d, \dot{y}_d, \ddot{y}_d, \rho, \dot{\rho})$ 满足

$$|\dot{y}_2 + \frac{y_2}{\tau_{k,2}}| \leq \eta_{k,2}(\cdot). \quad (53)$$

利用Young's不等式对上式进行化简得

$$\dot{y}_2 y_2 \leq -\frac{y_2^2}{\tau_{k,2}} + y_2^2 + \frac{\eta_{k,2}^2(\cdot)}{4}. \quad (54)$$

Step i ($2 \leq i \leq v-1$) 定义第*i*个动态面误差为

$$s_i = v_{k,m,i} - \omega_i. \quad (55)$$

对上式求导可得

$$\begin{aligned} \dot{s}_i &= \dot{v}_{k,m,i} - \dot{\omega}_i = \\ &= -l_i v_{k,m,1} + v_{k,m,i+1} - \dot{\omega}_i = \\ &= s_{i+1} + y_{i+1} + \alpha_{k,i} - l_i v_{k,m,1} - \dot{\omega}_i. \end{aligned} \quad (56)$$

设计虚拟控制律如下:

$$\alpha_{k,i} = -c_{k,i} s_i + l_i v_{k,m,1} + \dot{\omega}_i, \quad (57)$$

其中 $c_{k,i} > 0$ 是设计常数. 令

$$V_{s_i} = \frac{1}{2} s_i^2, \quad (58)$$

对式(58)求导, 并将式(56)–(57)代入, 同时利用Young's不等式进行化简可得

$$\dot{V}_{s_i} \leq -(c_{k,i} - 2) s_i^2 + \frac{1}{4} s_{i+1}^2 + \frac{1}{4} y_{i+1}^2. \quad (59)$$

构造如下—阶滤波器:

$$\tau_{k,i+1} \dot{\omega}_{i+1} + \omega_{i+1} = \alpha_{k,i}, \quad \omega_{i+1}(t_{k,0}) = \alpha_{k,i}(t_{k,0}),$$

其中: $\tau_{k,i+1}$ 为时间常数. 取 $y_{i+1} = \omega_{i+1} - \alpha_{k,i}$, 可得 $\dot{\omega}_{i+1} = -\frac{y_{i+1}}{\tau_{k,i+1}}$. 根据 $y_{i+1} = \omega_{i+1} - \alpha_{k,i}$, 对 y_{i+1} 求得

$$\dot{y}_{i+1} = -\frac{y_{i+1}}{\tau_{k,i+1}} - (-c_{k,i} \dot{s}_i + l_i \dot{v}_{k,m,1} + \ddot{\omega}_i),$$

存在非负连续函数 $\eta_{k,i+1}(\bar{s}_{i+1}, \bar{y}_{i+1}, \hat{\theta}_{k,0}, \hat{\theta}_k, v_k, \zeta, \Xi_k, \bar{\lambda}_{k,m+2}, \varepsilon_2, \varsigma, y_d, \dot{y}_d, \ddot{y}_d, \rho, \dot{\rho}, \ddot{\rho})$ 满足

$$|\dot{y}_{i+1} + \frac{y_{i+1}}{\tau_{k,i+1}}| \leq \eta_{k,i+1}(\cdot),$$

再利用Young's不等式进行化简得

$$\dot{y}_{i+1} y_{i+1} \leq -\frac{y_{i+1}^2}{\tau_{k,i+1}} + y_{i+1}^2 + \frac{\eta_{k,i+1}^2(\cdot)}{4}. \quad (60)$$

Step v 定义第 v 个动态面误差

$$s_v = v_{k,m,v} - \omega_v, \quad (61)$$

对式(61)求导得

$$\begin{aligned} \dot{s}_v &= \dot{v}_{k,m,v} - \dot{\omega}_v = \\ &= v_{k,m,v+1} - l_v v_{k,m,1} + u - \dot{\omega}_v. \end{aligned} \quad (62)$$

由于 $v+1 > n$, 所以 $v_{k,m,v+1} = 0$. 取

$$V_{s_v} = \frac{1}{2} s_v^2. \quad (63)$$

设计控制律为

$$u = -c_{k,v} s_v - v_{k,m,v+1} + l_v v_{k,m,1} + \dot{\omega}_v, \quad (64)$$

其中 $c_{k,v} > 0$ 是设计常数. 则结合式(62)–(64)有

$$\dot{V}_{s_v} = -c_{k,v} s_v^2. \quad (65)$$

4 稳定性分析

定理 1 考虑由式(1), 虚拟控制律(42)(57), 参数

自适应律(45)–(46)和控制律(64)组成的闭环系统, 如果假设1–4成立, 则对于任意给定的正常数 p 和初始条件 $V(0) \leq p$ 以及满足式(66)的正常数 $c_{k,i}, \tau_{k,i}, c_0, h$, 那么该闭环系统所有信号是半全局一致终结有界的.

$$\left\{ \begin{aligned} c_{k,i} &\geq 2 + \frac{b_{\max}^2}{2}, \quad i = 3, 4, \dots, v, \\ \frac{1}{\tau_{k,2}} &\geq 1 + \frac{b_{\max}^2}{4} + \frac{c_0}{2}, \\ \frac{1}{\tau_{k,i}} &\geq \frac{5}{4} + \frac{c_0}{2}, \quad i = 3, 4, \dots, v, \\ h &\geq \frac{13}{4} + c_0 \lambda_{\max}(P), \\ c_0 &= \min(\mu_1 \gamma_1, \mu_2 \gamma_2, \mu_3 \gamma_3, \bar{c}_k). \end{aligned} \right. \quad (66)$$

证 定义总的李雅普诺夫函数如下:

$$\begin{aligned} V &= V_1 + \sum_{j=2}^v \frac{1}{2} V_{s_j} + \sum_{j=2}^v \frac{1}{2} y_j^2 + \frac{1}{2\mu_1} \tilde{\theta}_k^T \tilde{\theta}_k + \\ &\quad \frac{1}{2\mu_2} \tilde{\theta}_{k,0}^T \tilde{\theta}_{k,0}. \end{aligned} \quad (67)$$

对函数 V 进行求导, 并将式(51)(54)(59)–(60)和式(65)代入, 整理得

$$\begin{aligned} \dot{V} &\leq -(h - \frac{13}{4}) \varepsilon^T \varepsilon - (c_{k,1} - 1) r s_1^2 - c_{k,v} s_v^2 - \\ &\quad \frac{s_1^2 r \tilde{\theta}_{k,0}^T \|\Phi(Z_1)\|^2}{2a_0^2} - s_1 r \varpi_k \tilde{\theta}_k - \frac{\bar{c}}{\lambda_0} v + \\ &\quad (b_m N(\varsigma) + 1) \dot{\varsigma} + \frac{b_{\max}^2}{4} s_2^2 + \frac{b_{\max}^2}{4} y_2^2 - \\ &\quad \sum_{i=2}^{v-1} ((c_{k,i} - 2) s_i^2 + \frac{1}{4} s_{i+1}^2 + \frac{1}{4} y_{i+1}^2) + \\ &\quad \sum_{i=1}^{v-1} (-\frac{y_{i+1}^2}{\tau_{k,i+1}} + y_{i+1}^2 + \frac{\eta_{k,i+1}^2(\cdot)}{4}) + \\ &\quad \tilde{\theta}_k (r s_1 \varpi_k - \gamma_1 \hat{\theta}_k) + Q_k(y, v_k) + D_{k,1} + \\ &\quad \tilde{\theta}_{k,0} (\frac{s_1^2 r \|\Phi(Z_1)\|^2}{2a_0^2} - \gamma_2 \hat{\theta}_{k,0}), \end{aligned} \quad (68)$$

进行合并化简可得

$$\begin{aligned} \dot{V} &\leq -(h - \frac{13}{4}) \varepsilon^T \varepsilon - (c_{k,1} - 1) r s_1^2 + \\ &\quad (c_{k,2} - 2 - \frac{b_{\max}^2}{4}) s_2^2 - (\frac{1}{\tau_{k,2}} - 1 - \frac{b_{\max}^2}{4}) y_2^2 - \\ &\quad \frac{\bar{c}}{\lambda_0} v - \sum_{j=3}^{v-1} (c_{k,i} - \frac{9}{4}) s_j^2 - (c_{k,v} - \frac{1}{4}) s_v^2 - \\ &\quad \sum_{j=3}^v (\frac{1}{\tau_{k,j}} - \frac{5}{4}) y_j^2 + (b_m N(\varsigma) + 1) \dot{\varsigma} - \\ &\quad \gamma_1 \tilde{\theta}_k \hat{\theta}_k - \gamma_2 \tilde{\theta}_{k,0} \hat{\theta}_{k,0} + Q_k(y, v_k) + \\ &\quad \sum_{j=2}^v \frac{\eta_{k,j}^2}{4} + D_{k,1}. \end{aligned} \quad (69)$$

根据Young's不等式可知

$$-\gamma_1 \tilde{\theta}_k \hat{\theta}_k \leq -\frac{\gamma_1}{2} \tilde{\theta}_k^2 + \frac{\gamma_1}{2} \theta_k^2, \quad (70)$$

$$-\gamma_2 \tilde{\theta}_{k,0} \hat{\theta}_{k,0} \leq -\frac{\gamma_2}{2} \tilde{\theta}_{k,0}^2 + \frac{\gamma_2}{2} \theta_{k,0}^2. \quad (71)$$

将式(70)–(71)代入式(69)得

$$\begin{aligned} \dot{V} \leq & -(h - \frac{13}{4}) \varepsilon^T \varepsilon - (c_{k,1} - 1) r s_1^2 + \\ & (c_{k,2} - 2 - \frac{b_{\max}^2}{4}) s_2^2 - (\frac{1}{\tau_{k,2}} - 1 - \frac{b_{\max}^2}{4}) y_2^2 - \\ & \frac{\bar{c}_k}{\lambda_0} v_k - \sum_{j=3}^{v-1} (c_{k,i} - \frac{9}{4}) s_j^2 - (c_{k,v} - \frac{1}{4}) s_v^2 - \\ & \sum_{j=3}^v (\frac{1}{\tau_{k,j}} - \frac{5}{4}) y_j^2 + (b_m N(\varsigma) + 1) \dot{\varsigma} - \\ & \frac{\gamma_1}{2} \tilde{\theta}_k^2 - \frac{\gamma_2}{2} \tilde{\theta}_{k,0}^2 + \frac{\gamma_1}{2} \theta_k^2 + \frac{\gamma_2}{2} \theta_{k,0}^2 + \\ & Q_k(y, v_k) + \sum_{j=2}^v \frac{\eta_{k,j}^2}{4} + D_{k,1}. \end{aligned} \quad (72)$$

再根据式(66)对式(72)化简可得

$$\begin{aligned} \dot{V} \leq & -\bar{c}_0 V + \chi_1 + (b_m N(\varsigma) + 1) \dot{\varsigma} + \\ & Q_k(y, v_k) + \sum_{j=2}^v \frac{\eta_{k,j}^2}{4}, \end{aligned} \quad (73)$$

$$\text{其中 } \chi_1 = \sum_{j=2}^v \frac{\mu_1}{2} \theta_k^2 + \frac{\mu_2}{2} \theta_{k,0}^2 + D_{k,1}.$$

如果 $V(t) \leq p$, 则可以得到 $s_1, s_2, \dots, s_v, y_2, y_3, \dots, y_v, v_k, \hat{\theta}_k, \hat{\theta}_{k,0}, \varepsilon$ 都是有界的. 由于 v_k 有界, 根据式(22)可得 z_k 有界. 由 $s_1 \in L_\infty$ 可知 $\xi \in L_\infty$, 则 $-\delta < S = \frac{e}{\rho} < \bar{\delta}$, 所以 $-\delta\rho < e < \bar{\delta}\rho$, $e \in L_\infty$. 由 $e = y - y_d, y_d \in L_\infty$, 可以得到 $y \in L_\infty$. 再由式(15)可得 ζ, Ξ_k 有界. 由于 $v_k(t)$ 有界, 根据式(1)和式(15)中第3个子式可得

$$\begin{aligned} \lambda_{k,i} = & \frac{s^{i-1} + l_i s^{i-2} + \dots + l_{i-1}}{L(s)B(s)} \left\{ \frac{d^n y}{dt^n} - \right. \\ & \left. \sum_{i=1}^n \frac{d^{n-i}}{dt^{n-i}} (f_{k,i}(y) + g_{k,i}(y) + \Delta_{k,i}(z_k, y, t)) \right\}, \end{aligned}$$

其中 $L(s) = s^n + l_1 s^{n-1} + \dots + l_n$ 是 Hurwitz 多项式.

由 $f_{k,i}(y), g_{k,i}(y), \Delta_{k,i}(z_k, y, t)$ 均为光滑函数及假设2中的 $|\Delta_{k,i}(z_k, y, t)| \leq \varphi_{k,i1}(|y|) + \varphi_{k,i2}(\|z_k\|)$, 根据 $z_k \in L_\infty, y \in L_\infty$, 可推出 $\lambda_{k,1}, \lambda_{k,2}, \dots, \lambda_{k,m+1}$ 都有界. 再由式(13)可得 $v_{k,m-1,2}, \dots, v_{k,0,2}, v_{k,0,1} \in L_\infty$, 同时 $v_{k,m,1} \in L_\infty$,

$$\varpi_k^T = [0 \ v_{k,m-1,2} \ \dots \ v_{k,0,2} \ \Xi_{k,(2)} + \Phi_{k,(1)}^T],$$

显然 $\Xi_{k,(2)}, \Phi_{k,(1)}^T$ 是有界的, 所以可以得到 $\varpi_k \in L_\infty$, 由此式(51)可以化为如下形式:

$$\dot{V}_1 \leq -\bar{c}_0 V_1 + D^* + (b_m N(\varsigma) + 1) \dot{\varsigma}, \quad (74)$$

两边进行积分可得

$$V_1(t) \leq e^{-\bar{c}_0 t} \int_0^t (b_m N(\varsigma) + 1) \dot{\varsigma} e^{\bar{c}_0 \tau} d\tau +$$

$$V_1(0) e^{-\bar{c}_0 t} + \frac{D^*}{\bar{c}_0}.$$

由引理3可知, $V_1(t), \varsigma, \int_0^t (b_m N(\varsigma) + 1) \dot{\varsigma} d\tau \in L_\infty$, 进一步, $\dot{\varsigma}, N(\varsigma) \in L_\infty$, 然后由式(55)可得 $\alpha_{k,1} \in L_\infty$, 因为 $v_{k,m,2} = z_2 + y_2 + \alpha_{k,1}$ 可知 $v_{k,m,2}$ 有界. 又有

$$y_2 = \omega_2 - \alpha_{k,1}, \dot{\omega}_2 = -\frac{y_2}{\tau_{k,2}}, i = 1, 2, \dots, v-1,$$

所以 $\omega_2, \dot{\omega}_2$ 有界. 由式 $\alpha_{k,i} = -c_{k,i} s_i + l_i v_{k,m,1} + \dot{\omega}_i$ 可以得到 $\alpha_{k,i} \in L_\infty$, 与上面过程类似, 可以得到 $v_{k,m,3}, \dots, v_{k,m,v+1}, \omega_{i+1}, \dot{\omega}_{i+1}$ 有界. 最后可得 u 是有界的. 因为 $Q(y, v_k), \eta_{k,j}^2$ 都是非负连续函数, 且 $\eta_{k,i+1}(\bar{s}_{i+1}, \bar{y}_{i+1}, \hat{\theta}_{k,0}, \hat{\theta}_k, v_k, \zeta, \Xi_k, \bar{\lambda}_{k,m+2}, \varepsilon_2, \varsigma, y_d, \dot{y}_d, \ddot{y}_d, \rho, \dot{\rho}, \ddot{\rho})$ 是有界的. 所以存在一个未知正常数 M_1 , 使得

$$Q(y, v_k) + \sum_{j=2}^v \frac{\eta_{k,j}^2}{4} \leq M_1.$$

又因为 $N(\varsigma), \varsigma, \dot{\varsigma} \in L_\infty$, 则存在一个未知正常数 M_2 , 使得 $(b_m N(\varsigma) + 1) \dot{\varsigma} \leq M_2$, 则有

$$\dot{V} \leq -\bar{c}_0 V + M, \quad (75)$$

其中 $M = M_1 + M_2 + \chi_1$, 当 $\bar{c}_0 \geq \frac{M}{p}$, 则有 $\dot{V} \leq 0$. 因此, 当初始条件满足 $V(0) \leq p$ 时, $\forall t > 0$ 有 $V(t) \leq p$, 所以以上分析成立, 所有的闭环系统信号都是全局一致终结有界的.

注2 为了保证闭环系统的稳定性, 设计时应选取所有切换子系统对应的总的公共李亚普诺夫函数中所有变量的初值与该子系统首次运行时公共李亚普诺夫函数中所有变量的初值相同. 根据稳定性分析可知, 相同切换子系统的 Lyapunov 函数值不超过该子系统初次运行时 Lyapunov 函数的初值. 由此得, 相同切换子系统在整个切换过程中所涉及的信号有界. 进一步可知, 在有限个子系统的任意切换下, Lyapunov 函数值都是有界的, 从而所提控制策略能够保证整个闭环切换系统的稳定性.

5 数值仿真

在这一部分作者用数值仿真实例来验证本论文所提方法的有效性. 考虑如下具有未建模动态的输出反馈非线性切换系统:

第1个子系统:

$$\dot{z}_1 = -z_1 + x_1^2 \sin x_1 + 0.5,$$

$$\dot{x}_1 = x_2 + \frac{x_1 + x_1^2}{1 + x_1^4} \sin x_1 + x_1^2 \sin z_1,$$

$$\dot{x}_2 = u + x_1 \sin x_1 + 2x_1^2 + 0.5 \sin t + z_1 x_1,$$

$$y = x_1.$$

第2个子系统:

$$\dot{z}_2 = -z_2 + x_1^2 + 1,$$

$$\dot{x}_1 = x_2 + \frac{x_1 - x_1^3}{1 + x_1^2} \sin x_1 + x_1^2 \sin z_2,$$

$$\dot{x}_2 = u + x_1^3 \sin x_1^2 + x_1^2 \sin x_1 + z_2 e^{-x_1},$$

$$y = x_1,$$

取期望轨迹: $y_d = \sin(0.3t) \times \sin(0.6t)$, 预设性能:

$$\underline{\delta} = 0.2, \bar{\delta} = 0.2, \rho(t) = 0.9e^{-3t} + 1.$$

对两个子系统选取参数:

$$c_{k,1} = c_{k,2} = 60, \tau_{k,2} = 0.001, k = 1, 2, c_0 = 1,$$

$$\mu_1 = \mu_2 = 0.02, \gamma_1 = \gamma_2 = 0.01,$$

神经网络节点数都为10,

$$\mu_{k,ij} = 2 \times (j - \frac{M_{k,i}}{2}), b_{k,ij} = 1,$$

$$j = 1, \dots, M_{k,i}, i = 1, 2, k = 1, 2.$$

状态初值分别选取 $x(0) = [0.03 \ 0.02]^T$ 与 $x(0) = [0.2 \ 0.2]^T$ 两组, $z(0) = 0.1, \varsigma(0) = 0.1$, 其余参数初值均为零. 仿真结果如图1-6所示.

图1是状态初值选取 $x(0) = [0.03 \ 0.02]^T$ 时的跟踪结果. 图2是在相同控制条件下只将状态初值增大为 $x(0) = [0.2 \ 0.2]^T$ 时的仿真结果, 可以看出当初值偏离原点时, 虽然超调量增大但仍能快速有效地跟踪期望的轨迹. 图3表明所设计的自适应控制策略能够确保系统具有很好的稳态性能与瞬态性能. 图4表示控制律 u 的轨迹.

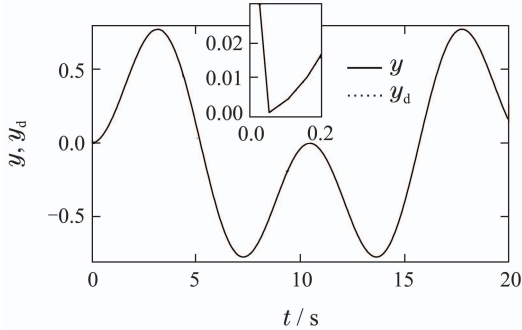


图1 初值 $x(0) = [0.03 \ 0.02]^T$ 时的输出 y 和期望轨迹 y_d

Fig. 1 Output y and desired trajectory y_d when the initial value is $x(0) = [0.03 \ 0.02]^T$

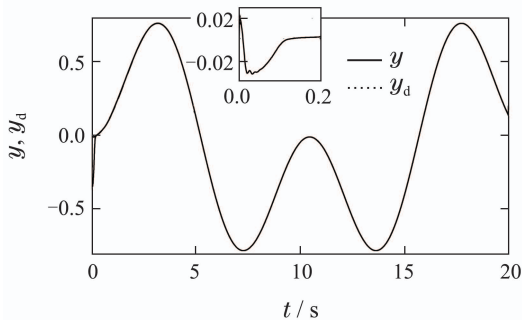


图2 初值 $x(0) = [0.2 \ 0.2]^T$ 时的输出 y 和期望轨迹 y_d

Fig. 2 Output y and desired trajectory y_d when the initial value is $x(0) = [0.2 \ 0.2]^T$

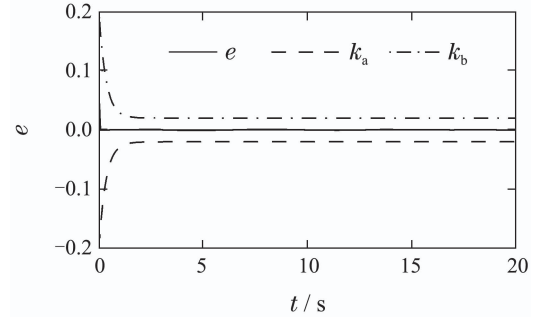


图3 跟踪误差 e , 预设上界和下界

Fig. 3 Tracking error e and prescribed upper bound and lower bound

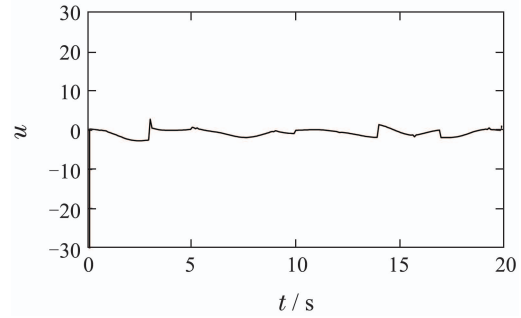


图4 控制律 u

Fig. 4 Control law u

图5为任意选取的切换信号. 另外, 与文献[30]控制方案相比, 本文考虑了未建模动态的影响, 所以当使用文献[30]的控制方案对本文的仿真系统进行控制时, 跟踪结果如图6所示, 通过对比可以看出使用该方案时有较大的跟踪误差且调节时间更长.

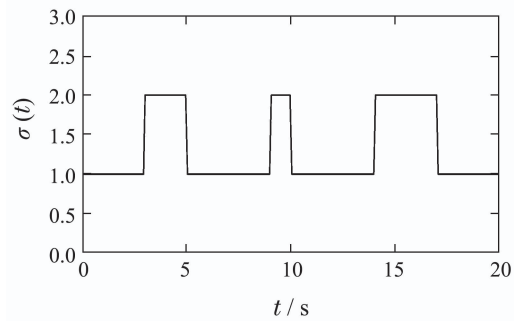


图5 切换信号 $\sigma(t)$

Fig. 5 Switching signal $\sigma(t)$

注3 令 $V_1(z_1) = 0.5z_1^2, V_2(z_2) = 0.5z_2^2$, 求导可得

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1(-z_1 + x_1^3 \sin x_1 + 0.5),$$

$$\dot{V}_2 = z_2 \dot{z}_2 = z_2(-z_2 + x_1^2 + 1).$$

根据Young's不等式可得

$$\dot{V}_1 \leq -z_1^2 + \frac{1}{4}z_1^2 + x_1^4 \sin^2(x_1) + \frac{1}{4}z_1^2 + 0.25 \leq$$

$$-V_1 + 2.5x_1^4 + 0.625,$$

$$\dot{V}_2 \leq -z_2^2 + \frac{1}{4}z_2^2 + x_1^4 + 1 \leq -V_2 + 1.1x_1^4 + 1.5.$$

于是, 取

$$\begin{aligned} \beta_{1,1}(|z_1|) &= 0.5z_1^2, \beta_{1,2}(|z_1|) = 1.5z_1^2, \beta_{2,1}(|z_2|) = 0.5z_2^2, \\ \beta_{2,2}(|z_2|) &= 1.5z_2^2, \gamma_1(|y|) = 2.5x_1^4, \gamma_2(|y|) = 1.1x_1^4, \\ c_1 &= c_2 = 1, d_1 = 0.625, d_2 = 1.5, \bar{c}_1 = \bar{c}_2 = 0.6 \in (0, c_1), \\ \bar{\gamma}_1(|y|) &= 2.5x_1^4, \bar{\gamma}_2(|y|) = 1.1x_1^4, \end{aligned}$$

则 $\beta_{k,1}(|z_k|) \leq V_k(Z_k) \leq \beta_{k,2}(|z_k|)$, 且 $\dot{V}_k \leq -c_k V_k + \gamma_k(|y|) + d_k, k = 1, 2$. 根据上述分析可知, 未建模动态满足假设3和引理1的条件, 因此, 系统仿真中的动态信号设计为

$$\begin{aligned} \dot{v}_1 &= -0.6v_1 + 2.5x_1^4 + 0.625, \\ \dot{v}_2 &= -0.6v_2 + 1.1x_1^4 + 1.5. \end{aligned}$$

注4 上述系统与其他文献不同的是, 两个子系统的未建模动态是不同的, 维数也可以不相同. 因此, 构造不同的动态信号处理未建模动态, 同时分别采用不同的神经网络对未知函数进行逼近. 由于在设计时选取切换子系统对应的总的公共李亚普诺夫函数中所有变量的初值与该子系统首次运行时李亚普诺夫函数中所有变量的初值相同, 所以可以得到如图1和2所示的切换时刻无跳变的跟踪轨迹.

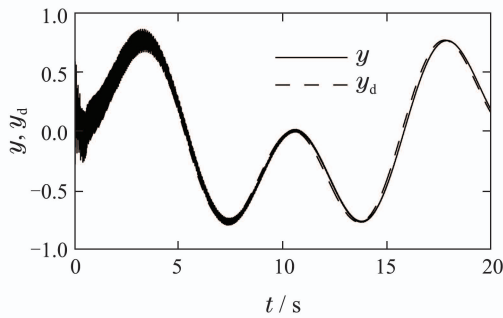


图6 对比控制方案^[30]的输出 y 和期望的轨迹 y_d

Fig. 6 Output y and desired trajectory y_d of the compared control scheme^[30]

6 结论

本文研究了一类具有未建模动态和预设性能的输出反馈非线性切换系统的跟踪控制问题. 通过将预设性能控制问题转化为稳定性问题, 设计 K 滤波器估计不可量测的状态, 引入动态信号处理系统的动态不确定性. 利用Nussbaum函数性质解决系统增益符号未知的问题. 同时采用动态面控制方法, 避免了对虚拟控制律的反复求导, 降低了参数计算和控制器设计的复杂性. 同时在公共Lyapunov函数的基础上, 动态面方法使得由单个子系统到切换系统的稳定性的证明更加清晰直观, 并且不需要对切换信号进行约束, 放宽了保证系统稳定的条件. 数值仿真结果验证了方案的有效性.

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