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带有输出约束条件的随机多智能体系统容错控制

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摘要: 在有向通讯拓扑图下, 针对一类具有输出约束和执行器偏差增益故障的非严格反馈随机多智能体系统, 提出一种自适应神经网络容错控制设计方案. 采用神经网络逼近未知非线性函数, 构造障碍李雅普诺夫函数处理系统的输出约束问题, 以反步法和动态面技术为框架, 结合Nussbaum函数设计自适应神经网络容错控制方法. 基于李雅普诺夫稳定性理论, 证明所有跟随者输出与领导者输出达到一致, 闭环系统的所有信号依概率半全局一致最终有界且系统输出限制在给定紧集内. 论文最后通过仿真实验验证所给出控制方案的有效性.

关键词: 非严格反馈形式; 容错控制; 多智能体系统; 输出约束; 随机系统

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Fault-tolerant control for stochastic multi-agent systems with output constraints

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Abstract: In this paper, the adaptive neural network fault-tolerant control strategy is proposed for a class of nonstrict-feedback stochastic multi-agent systems with output constraints and actuator faults under a directed communication topology. Neural networks are utilized to approximate unknown nonlinear functions. Furthermore, the barrier Lyapunov function is employed to deal with the problem of output constraints. Combining backstepping method, dynamic surface control technique and Nussbaum function, an adaptive neural network fault-tolerant control method is proposed. Based on Lyapunov stability theory, it is proved that all followers' outputs can be consistent with the leader's output. All signals in the closed-loop systems are semiglobally uniform ultimate bounded in probability and the output of systems can be limited within the given compact set. Finally, the effectiveness of the proposed control scheme is verified through numerical simulation.

Key words: nonstrict-feedback form; fault-tolerant control; multi-agent systems; output constraints; stochastic systems

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1 引言

近年来, 多智能体系统协同控制在传感器网络、移动机器人和无人机编队等众多领域有着巨大的应用前景[1–6], 而一致性跟踪问题作为多智能体系统协同控制的基本问题之一, 更是受到了专家学者的广泛关注[7–13]. 文献[11]将代数图论与控制方法相结合研究了二阶多智能体的一致性问题. 针对高阶非线性多智能体系统, 文献[12–13]提出了自适应神经网络一致性控制方案. 众所周知, 随机干扰在工程系统中往往是不可避免的, 是影响系统稳定的主要因素之一, 因此对于随机系统的研究一直是控制领域的热点[14–17]. 文献[15]提出人工势场法解决了一类随机多智能体编队的避障问题. 文献[16–17]利用反步法技术讨论了一类随机多智能体系统的自适应控制. 但由于反步法的应用需要对系统中的虚拟控制器进行反复求导, 会造成“复杂性爆炸”问题. 为了克服这个困难, 文献[18]针对一类严格反馈非线性系统, 引入了动态面技术, 在此基础上设计了鲁棒自适应跟踪控制算法. 文献

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[19]则结合动态面技术解决了带有死区输入的严格反馈随机多智能体系统的控制问题.

值得注意的是, 以上提及的研究成果不能直接应用于非严格反馈形式的系统中, 相比于严格反馈系统, 非严格反馈系统中的非线性项包含了所有的状态变量, 采用上述控制方法如传统的反步法进行控制器设计会产生代数环问题. 文献[20–21]运用神经网络的性质突破了传统反步法控制设计应用于非严格反馈系统的限制. 尽管现有研究对非线性随机系统的自适应控制研究已经取得了很大进展, 但却很少有同时考虑输出约束和执行器故障对非严格反馈随机多智能体系统造成的影响.

事实上, 许多控制系统需要系统输出保持在一定范围内, 否则在系统运行过程中会导致性能下降. 对此, 利用障碍李雅普诺夫函数特性能为处理输出约束问题提供有效途径[22–25]. 另一方面, 控制系统会因复杂的工作环境、部件老化等因素的影响发生故障, 故障的存在严重影响了系统的稳定性, 甚至会造成安全隐患, 所以研究系统的故障问题是十分有必要的[25–29]. 基于以上讨论, 本文对含有输出约束和执行器故障的非严格反馈的随机多智体系统, 提出自适应神经网络容错控制的设计方案.

本文的主要贡献如下: 1) 采用的系统模型为非严格反馈形式的非线性随机多智能体系统, 更具有一般性. 此外, 系统中的随机干扰和非线性项是完全未知的, 且消除了一般研究中随机干扰的有界假设; 2) 在反步法设计中结合动态面技术解决“复杂性爆炸”问题的同时, 引入神经网络的性质[21], 克服了非严格反馈形式产生代数环的问题; 3) 基于自适应神经网络容错控制的设计方法, 同时考虑输出约束和执行器故障问题, 使本文的控制方案更具有适用性.

本文的组织结构如下: 第2节详细介绍了系统的数学模型和相关的准备工作; 第3节提出了一种自适应神经网络容错控制设计方案及其稳定性分析, 第4节通过仿真研究验证了该方法的有效性; 第5节对本文的工作进行总结.

2 预备知识与问题阐述

2.1 图论

为了方便系统分析, 引入图论知识来描述多智能体之间的通讯拓扑关系. 本文用$G=(\aleph, I)$表示有向图, 其中: $\aleph=\{\aleph_1,\cdots,\aleph_N\}$代表智能体的非空节点集, $I\subseteq\aleph\times\aleph$代表边集合. $(\aleph_j,\aleph_i)\in I$表示第j个智能体能接收到第i个智能体的信息, 邻接节点集合表示为$N_i=\{\aleph_j|(\aleph_j,\aleph_i)\in I\}$. 有向图$G$的通讯拓扑关系用邻接矩阵$A=[a_{ij}]\in\mathbb{R}^{N\times N}$表示, 其中$a_{ij}\geqslant 0$. 有向图$G$的拉普拉斯矩阵定义为$L=\varLambda-A$, 其中$\varLambda=\operatorname{diag}\{d_1,\cdots,d_N\}\in\mathbb{R}^{N\times N}$为入度矩阵, $d_i=\sum\limits_{j=1}^{N}a_{ij}$为第$i$个智能体的入度. 如果有向图$G$含有生成树, 说明至少存在一个根节点, 该根节点到其它任一节点都有有向路径. 如果节点0是生成树的根节点, 定义$H=\operatorname{diag}\{a_{10},a_{20},\cdots,a_{N0}\}$, 则矩阵$L+H$为非奇异.

2.2 问题阐述

考虑由N个跟随者和1个领导者组成的非严格反馈随机多智能体系统, 第i个多智能体的动态模型如下:

$$\begin{cases}\mathrm{d}x_{i,1}=(x_{i,2}+f_{i,1}(x_i))\mathrm{d}t+g_{i,1}(x_i)\mathrm{d}\varpi,\\ \mathrm{d}x_{i,k}=(x_{i,k+1}+f_{i,k}(x_i))\mathrm{d}t+g_{i,k}(x_i)\mathrm{d}\varpi,\\ \mathrm{d}x_{i,n_i}=(b_{m_i}u_i^{\mathrm{f}}+f_{i,n_i}(x_i))\mathrm{d}t+g_{i,n_i}(x_i)\mathrm{d}\varpi,\\ y_i=x_{i,1},\end{cases}\tag{1}$$

其中: $x_i=[x_{i,1}\ \ x_{i,2}\ \ \cdots\ \ x_{i,n_i}]^{\mathrm{T}}\in\mathbb{R}^{n_i}$为第$i$个智能体$t$时刻的状态向量, $k=2,\cdots,n_i-1$, $f_{i,k}(x_i)$和$g_{i,k}(x_i)$为未知光滑的非线性函数, b_{m_i}为未知控制参数, u_i^{f}和y_i为第i个智能体的控制输入和控制输出, 其中$i=1,\cdots,N$. ϖ为r维的定义在完全概率空间($\varOmega$, $F,\{F_t\}_{t\geqslant 0}$)上的标准维纳过程.

本文考虑的执行器故障类型为偏差和增益故障, 偏差故障的表达式如下: $u_i^{\mathrm{f}}(t)=u_i(t)+w_i(t)$, 其中$w_i(t)$为有界函数, 且增益故障的表达式如下:

$$u_i^{\mathrm{f}}(t)=(1-r_i)u_i(t),$$

其中r_i为未知失控率, 满足$0\leqslant r_i<1$. 因此, 根据文献[26], 结合两种故障类型得

$$u_i^{\mathrm{f}}(t)=(1-r_i)u_i(t)+w_i(t).\tag{2}$$

控制目标: 针对含有输出约束和执行器故障的非严格反馈随机多智体系统(1), 提出一种自适应神经网络容错控制的设计方案, 使闭环系统中所有信号依概率有界, 系统一致跟踪误差收敛到原点的邻域内.

假设 1[26] 领导者的输出信号y_{r}是光滑函数, 且y_{r}, $\dot y_{\mathrm{r}}$和$\ddot y_{\mathrm{r}}$都有界, 即

$$\prod_{i0}=\{(y_{\mathrm{r}},\dot y_{\mathrm{r}},\ddot y_{\mathrm{r}}):y_{\mathrm{r}}^2+\dot y_{\mathrm{r}}^2+\ddot y_{\mathrm{r}}^2\leqslant M_0\},$$

其中$M_0>0$.

为了处理系统(1)中的执行器故障问题, 本文引入了Nussbaum增益技术, Nussbaum函数一般具有以下性质:

$$\lim_{s\to\infty}\sup\frac{1}{s}\int_0^s N(\xi)\mathrm{d}\xi=\infty,$$
$$\lim_{s\to\infty}\inf\frac{1}{s}\int_0^s N(\xi)\mathrm{d}\xi=-\infty,$$

许多函数满足以上性质, 本文选择$\exp(\xi^2)\cos\xi^2$.

引理 1[20] 定义$N(\xi_i)=\exp(\xi_i^2)\cos\xi_i^2$为Nussbaum函数, 其中$\xi_i(t)$在区间$[0,t_{\mathrm{f}}]$为光滑函数, 如果存在正定径向无界的函数$V(t,x)$满足以下不等式:

$$\ell V(t,x)\leqslant-CV(t,x)+\sum_{i=1}^{N}k_i(\beta_iN'(\xi_i)+1)\dot\xi_i+\bar D,$$

则$\mathrm{E}(V(t,x))$, $\xi_i(t)$和$\sum_{i=1}^{N} k_i(\beta_i N'(\xi_i)+1)\dot{\xi}_i$在区间$[0, t_{\mathrm{f}}]$是有界的. 其中: ℓ为微分算子, $\mathrm{E}(\cdot)$ 为期望算子, k_i为设计参数, 同时$C>0$和$\bar{D}>0$.

2.3 随机系统

考虑随机系统如下:

$$\mathrm{d}x = f(x)\mathrm{d}t + g(x)\mathrm{d}\varpi, \tag{3}$$

其中: $x\in\mathbb{R}^n$为系统状态向量, ϖ是r维的标准维纳过程.

定义 1[21]　考虑系统(3), 假设存在一个正定、径向无界并且二次连续可微的函数$V(x)$, 其微分算子ℓ定义如下:

$$\ell V(x) = \frac{\partial V}{\partial x}f(x) + \frac{1}{2}\mathrm{tr}\{g^{\mathrm{T}}(x)\frac{\partial^2 V}{\partial x^2}g(x)\},$$

其中$\mathrm{tr}\{\cdot\}$代表矩阵的迹.

引理 2[21]　给定任意精度$\varepsilon>0$且$t\to\infty$时, 存在$\mathrm{E}|y_i(t)-y_{\mathrm{r}}(t)|^4\leqslant\varepsilon$, 则表明在有向图下, 系统(1)中的领导者和跟随者之间的跟踪误差依概率半全局一致最终有界.

2.4 障碍李雅普诺夫函数

定义 2[24]　定义在包含原点的开放域$\varXi$的系统$\mathrm{d}x=f(x)\mathrm{d}t+g(x)\mathrm{d}\varpi$中, 障碍李雅普诺夫函数$V(x)$为正定连续的标量函数, 且其在$\varXi$中的每一个点都存在一阶偏导数, 根据$V(x)\to\infty$时, x收敛于$\varXi$的边界的性质, 则存在b为正常数且$\forall t\geqslant 0$时, $V(x(t))\leqslant b$成立.

本文采用的障碍李雅普诺夫函数形式如下:

$$\bar{V}_{i,1} = \frac{1}{4}\log\frac{k_{i,b}^4}{k_{i,b}^4 - s_{i,1}^4},$$

其中: $\log(\cdot)$表示自然对数, $k_{i,b}$为$s_{i,1}$的约束.

引理 3[24]　对于任意的正常数$k_{i,b}$, 如果所有的$s_{i,1}$都能够满足$|s_{i,1}|<k_{i,b}$, 则以下不等式成立:

$$\log\frac{k_{i,b}^4}{k_{i,b}^4 - s_{i,1}^4} < \frac{s_{i,1}^4}{k_{i,b}^4 - s_{i,1}^4}.$$

2.5 径向基神经网络

系统(1)中的未知非线性函数可以用如下径向基神经网络来逼近处理, 具体形式如下:

$$f_n(\zeta) = \theta^{\mathrm{T}}\varphi(\zeta), \tag{4}$$

其中: $\theta = [\theta_1\ \ \theta_2\ \ \cdots\ \ \theta_\kappa]^{\mathrm{T}}$为权重向量, 基函数向量表示为$\varphi(\zeta)=[\varphi_1(\zeta)\ \ \varphi_2(\zeta)\ \ \cdots\ \ \varphi_\kappa(\zeta)]^{\mathrm{T}}$, 并且高斯函数$\varphi_i(\zeta)$ 的形式如下:

$$\varphi_i(\zeta) = \exp[-\frac{(\zeta-\iota_i)^{\mathrm{T}}(\zeta-\iota_i)}{\omega_i^2}],$$

其中: $i=[1\ \ 2\ \ \cdots\ \ \kappa]$, $\iota_i = [\iota_{i1}\ \ \iota_{i2}\ \ \cdots\ \ \iota_{iq}]^{\mathrm{T}}$为高斯函数的中心向量, ω_i表示高斯函数的宽度. 一般来说, 对于定义在紧集$\varOmega\subset\mathbb{R}^q$上的未知非线性函数$f(\zeta)$, 能用具有足够多节点的径向基神经网络(4)逼近到任意精度$\epsilon>0$, 即

$$f(\zeta) = \theta^{*\mathrm{T}}\varphi(\zeta) + \epsilon(\zeta),\ |\epsilon(\zeta)|\leqslant\epsilon^*, \tag{5}$$

其中: $\epsilon(\zeta)$表示逼近误差, ϵ^*为常数, θ^*为理想加权向量, 具体定义为$\theta^* := \arg\min\limits_{\theta\in\mathbb{R}^\kappa}\{\sup\limits_{\zeta\in\varOmega}|f(\zeta)-\theta^{\mathrm{T}}\varphi(\zeta)|\}$.

引理 4[21]　状态向量$\bar{x}_q = [x_1\ \ x_2\ \ \cdots\ \ x_q]^{\mathrm{T}}$, 且$\varphi(\bar{x}_q) = [\varphi_1(\bar{x}_q)\ \ \varphi_2(\bar{x}_q)\ \ \cdots\ \ \varphi_\kappa(\bar{x}_q)]^{\mathrm{T}}$为径向基神经网络的基函数向量, 对于任意正数$p\leqslant q$, 则

$$\|\varphi(\bar{x}_q)\|^2 \leqslant \|\varphi(\bar{x}_p)\|^2.$$

3 神经网络控制器设计和稳定性分析

本节将对系统(1)设计神经网络控制器, 首先, 基于通信拓扑, 对第i个跟随者系统进行坐标变换如下:

$$\begin{cases} s_{i,1} = \sum\limits_{j\in N_i} a_{ij}(y_i - y_j) + a_{i0}(y_i - y_{\mathrm{r}}), \\ s_{i,h} = x_{i,h} - \bar{\alpha}_{i,h},\ h = 2,\cdots,n_i, \\ z_{i,h} = \bar{\alpha}_{i,h} - \alpha_{i,h}, \end{cases} \tag{6}$$

其中: $s_{i,h}$为误差面, $z_{i,h}$为滤波误差, $\alpha_{i,h}$和$\bar{\alpha}_{i,h}$分别为虚拟控制器和滤波器输出信号. 令$q_{i,1}=\sum\limits_{j\in N_i} a_{ij}+a_{i0}$, $q_{i,k}=1,\ k=2,\cdots,n_i-1$. a_{i0}为正常数, 表示领导者和跟随者之间的通讯关系, 当且仅当第i个跟随者能够接收到领导者的信息时, $a_{i0}=1$, 否则$a_{i0}=0$.

为了设计方便, 定义以下符号和参数: $\lambda_{i,\varsigma}$, $l_{i,\varsigma}$, $\mu_{i,\varsigma}$, $c_{i,\varsigma}$, $\sigma_{i,\varsigma}$表示正设计参数; 定义$\varTheta_{i,\varsigma}=\|\theta_{i,\varsigma}^*\|^2$, 令$\hat{\varTheta}_{i,\varsigma}$为$\varTheta_{i,\varsigma}$的估计, 则估计误差$\tilde{\varTheta}_{i,\varsigma} = \varTheta_{i,\varsigma} - \hat{\varTheta}_{i,\varsigma}$; 定义$\bar{x}_{i,\varsigma} = [x_{i,1}\ \ x_{i,2}\ \ \cdots\ \ x_{i,\varsigma}]^{\mathrm{T}}$, 其中$\varsigma = 1,\cdots,k$.

步骤 1　结合式(1)和式(6)可得

$$\begin{aligned}\mathrm{d}s_{i,1} = {} & (q_{i,1}(s_{i,2}+z_{i,2}+\alpha_{i,2}+f_{i,1}(x_i))- \\ & \sum_{j\in N_i} a_{ij}(x_{j,2}+f_{j,1}(x_j)) - a_{i0}\dot{y}_{\mathrm{r}})\mathrm{d}t+ \\ & (q_{i,1}g_{i,1}(x_i) - \sum_{j\in N_i} a_{ij}g_{j,1}(x_j))\mathrm{d}\varpi.\end{aligned} \tag{7}$$

选取李雅普诺夫函数为

$$V_{i,1} = \frac{1}{4}\log\frac{k_{i,b}^4}{k_{i,b}^4 - s_{i,1}^4} + \frac{1}{2\lambda_{i,1}}\tilde{\varTheta}_{i,1}^2.$$

结合式(1)和式(7), 则$\ell V_{i,1}$为

$$\begin{aligned}\ell V_{i,1}= {} & \frac{s_{i,1}^3}{k_{i,b}^4 - s_{i,1}^4}(q_{i,1}(s_{i,2}+z_{i,2}+\alpha_{i,2}+f_{i,1}(x_i)) - \\ & \sum_{j\in N_i} a_{ij}(x_{j,2}+f_{j,1}(x_j)) - a_{i0}\dot{y}_{\mathrm{r}}) + \\ & \frac{(3k_{i,b}^4 + s_{i,1}^4)s_{i,1}^2}{2(k_{i,b}^4 - s_{i,1}^4)^2}\psi_{i,1}\psi_{i,1}^{\mathrm{T}} - \frac{1}{\lambda_{i,1}}\tilde{\varTheta}_{i,1}\dot{\hat{\varTheta}}_{i,1},\end{aligned} \tag{8}$$

其中$\psi_{i,1} = q_{i,1}g_{i,1}(x_i) - \sum\limits_{j\in N_i} a_{ij}g_{j,1}(x_j)$.

根据Young's不等式得

$$\frac{s_{i,1}^3}{k_{i,b}^4 - s_{i,1}^4}q_{i,1}s_{i,2} \leqslant \frac{3q_{i,1}}{4}(\frac{s_{i,1}^3}{k_{i,b}^4 - s_{i,1}^4})^{\frac{4}{3}} + \frac{q_{i,1}}{4}s_{i,2}^4, \tag{9}$$

$$\frac{s_{i,1}^3}{k_{i,b}^4-s_{i,1}^4}q_{i,1}z_{i,2}\leqslant\frac{3q_{i,1}}{4}(\frac{s_{i,1}^3}{k_{i,b}^4-s_{i,1}^4})^{\frac{4}{3}}+\frac{q_{i,1}}{4}z_{i,2}^4, \quad (10)$$

$$\frac{(3k_{i,b}^4+s_{i,1}^4)s_{i,1}^2}{2(k_{i,b}^4-s_{i,1}^4)^2}\psi_{i,1}\psi_{i,1}^{\mathrm{T}}\leqslant \frac{1}{2}l_{i,1}^2+\frac{(3k_{i,b}^4+s_{i,1}^4)^2s_{i,1}^4}{8l_{i,1}^2(k_{i,b}^4-s_{i,1}^4)^4}\|\psi_{i,1}\|^4. \quad (11)$$

将式(9)–(11)代入式(8)中得

$$\ell V_{i,1}\leqslant\frac{s_{i,1}^3}{k_{i,b}^4-s_{i,1}^4}(q_{i,1}\alpha_{i,2}+\bar{f}_{i,1})+\frac{q_{i,1}}{4}s_{i,2}^4+\frac{q_{i,1}}{4}z_{i,2}^4+\frac{1}{2}l_{i,1}^2-\frac{1}{\lambda_{i,1}}\tilde{\Theta}_{i,1}\dot{\hat{\Theta}}_{i,1}, \quad (12)$$

其中:

$$\bar{f}_{i,1}=q_{i,1}(f_{i,1}(x_i)+\frac{3}{2}(\frac{s_{i,1}^3}{k_{i,b}^4-s_{i,1}^4})^{\frac{1}{3}})+\frac{(3k_{i,b}^4+s_{i,1}^4)^2s_{i,1}}{8l_{i,1}^2(k_{i,b}^4-s_{i,1}^4)^3}\|\psi_{i,1}\|^4-a_{i0}\dot{y}_{\mathrm{r}}-\sum_{j\in N_i}a_{ij}(x_{j,2}+f_{j,1}(x_j)).$$

由式(5)可得: $\bar{f}_{i,1}=\theta_{i,1}^{*\mathrm{T}}\varphi_{i,1}(\vartheta_{i,1})+\epsilon_{i,1}(\vartheta_{i,1})$, 其中: $|\epsilon_{i,1}(\vartheta_{i,1})|\leqslant\epsilon_{i,1}^*$, $\vartheta_{i,1}=[x_i^{\mathrm{T}}\ \ x_j^{\mathrm{T}}\ \ y_{\mathrm{r}}\ \ \dot{y}_{\mathrm{r}}]^{\mathrm{T}}$.

由Young's不等式和引理4可得

$$\frac{s_{i,1}^3}{k_{i,b}^4-s_{i,1}^4}\bar{f}_{i,1}=\frac{s_{i,1}^3}{k_{i,b}^4-s_{i,1}^4}(\theta_{i,1}^{*\mathrm{T}}\varphi_{i,1}(\vartheta_{i,1})+\epsilon_{i,1}(\vartheta_{i,1}))\leqslant \frac{s_{i,1}^6\Theta_{i,1}\varphi_{i,1}^{\mathrm{T}}(X_{i,1})\varphi_{i,1}(X_{i,1})}{2\mu_{i,1}^2(k_{i,b}^4-s_{i,1}^4)^2}+\frac{\mu_{i,1}^2}{2}+\frac{s_{i,1}^6}{2(k_{i,b}^4-s_{i,1}^4)^2}+\frac{\epsilon_{i,1}^{*2}}{2}, \quad (13)$$

其中$X_{i,1}=[x_{i,1}\ \ y_{\mathrm{r}}\ \ \dot{y}_{\mathrm{r}}]^{\mathrm{T}}$.

虚拟控制器$\alpha_{i,2}$和参数自适应律$\dot{\hat{\Theta}}_{i,1}$的设计如下:

$$\alpha_{i,2}=\frac{1}{q_{i,1}}(-c_{i,1}s_{i,1}-\frac{s_{i,1}^3}{2(k_{i,b}^4-s_{i,1}^4)}-\frac{s_{i,1}^3\hat{\Theta}_{i,1}\varphi_{i,1}^{\mathrm{T}}(X_{i,1})\varphi_{i,1}(X_{i,1})}{2\mu_{i,1}^2(k_{i,b}^4-s_{i,1}^4)}), \quad (14)$$

$$\dot{\hat{\Theta}}_{i,1}=\frac{\lambda_{i,1}s_{i,1}^6\varphi_{i,1}^{\mathrm{T}}(X_{i,1})\varphi_{i,1}(X_{i,1})}{2\mu_{i,1}^2(k_{i,b}^4-s_{i,1}^4)^2}-\sigma_{i,1}\hat{\Theta}_{i,1}. \quad (15)$$

由Young's不等式得

$$\frac{\sigma_{i,1}}{\lambda_{i,1}}\tilde{\Theta}_{i,1}\hat{\Theta}_{i,1}\leqslant-\frac{\sigma_{i,1}}{2\lambda_{i,1}}\tilde{\Theta}_{i,1}^2+\frac{\sigma_{i,1}}{2\lambda_{i,1}}\Theta_{i,1}^2. \quad (16)$$

结合式(13)–(16), 则式(12)可重写如下:

$$\ell V_{i,1}\leqslant-\frac{c_{i,1}s_{i,1}^4}{k_{i,b}^4-s_{i,1}^4}-\frac{\sigma_{i,1}}{2\lambda_{i,1}}\tilde{\Theta}_{i,1}^2+\frac{q_{i,1}}{4}s_{i,2}^4+\frac{q_{i,1}}{4}z_{i,2}^4+D_{i,1},$$

其中$D_{i,1}=\frac{l_{i,1}^2}{2}+\frac{\mu_{i,1}^2}{2}+\frac{\epsilon_{i,1}^{*2}}{2}+\frac{\sigma_{i,1}}{2\lambda_{i,1}}\Theta_{i,1}^2$.

为了避免对$\alpha_{i,2}$反复偏微分, 定义在时间常数$\tau_{i,2}$下, 让$\alpha_{i,2}$通过一阶滤波器得到$\bar{\alpha}_{i,2}$, 则

$$\tau_{i,2}\dot{\bar{\alpha}}_{i,2}+\bar{\alpha}_{i,2}=\alpha_{i,2},\ \bar{\alpha}_{i,2}(0)=\alpha_{i,2}(0).$$

根据文献[18], 滤波误差$z_{i,2}$的动态方程为

$$\mathrm{d}z_{i,2}=(-\frac{z_{i,2}}{\tau_{i,2}}+B_{i,2})\mathrm{d}t+C_{i,2}\mathrm{d}\varpi,$$

其中:

$$B_{i,2}=-\frac{\partial\alpha_{i,2}}{\partial x_{i,1}}(x_{i,2}+f_{i,1}(x_i))-\frac{\partial\alpha_{i,2}}{\partial\hat{\Theta}_{i,1}}\dot{\hat{\Theta}}_{i,1}-\frac{1}{2}\frac{\partial^2\alpha_{i,2}}{\partial x_{i,1}^2}g_{i,1}(x_i)g_{i,1}^{\mathrm{T}}(x_i)-\frac{\partial\alpha_{i,2}}{\partial y_{\mathrm{r}}}\dot{y}_{\mathrm{r}}-\frac{\partial\alpha_{i,2}}{\partial\dot{y}_{\mathrm{r}}}\ddot{y}_{\mathrm{r}},$$

$$C_{i,2}=-\frac{\partial\alpha_{i,2}}{\partial x_{i,1}}g_{i,1}(x_i).$$

上式中$B_{i,2}$和$C_{i,2}$是基于式(14)中$\alpha_{i,2}$的偏导获得. 故存在常数$\bar{B}_{i,2}>0$和$\bar{C}_{i,2}>0$, 满足以下不等式:

$$|B_{i,2}|\leqslant\bar{B}_{i,2},\ |\mathrm{tr}\{C_{i,2}^{\mathrm{T}}C_{i,2}\}|\leqslant\bar{C}_{i,2}.$$

步骤 2 $(2\leqslant k\leqslant n_i-1)$ 由式(1)和式(6)可得

$$\mathrm{d}s_{i,k}=(s_{i,k+1}+z_{i,k+1}+\alpha_{i,k+1}+f_{i,k}(x_i)-\dot{\bar{\alpha}}_{i,k})\mathrm{d}t+g_{i,k}(x_i)\mathrm{d}\varpi. \quad (17)$$

选取李雅普诺夫函数为

$$V_{i,k}=V_{i,k-1}+\frac{1}{4}s_{i,k}^4+\frac{1}{4}z_{i,k}^4+\frac{1}{2\lambda_{i,k}}\tilde{\Theta}_{i,k}^2.$$

结合式(1)和式(17), 则$\ell V_{i,k}$为

$$\ell V_{i,k}=\ell V_{i,k-1}+s_{i,k}^3(s_{i,k+1}+z_{i,k+1}+\alpha_{i,k+1}+f_{i,k}(x_i)-\dot{\bar{\alpha}}_{i,k})+z_{i,k}^3(-\frac{z_{i,k}}{\tau_{i,k}}+B_{i,k})+\frac{3}{2}s_{i,k}^2\mathrm{tr}\{g_{i,k}^{\mathrm{T}}(x_i)g_{i,k}(x_i)\}-\frac{1}{\lambda_{i,k}}\tilde{\Theta}_{i,k}\dot{\hat{\Theta}}_{i,k}+\frac{3}{2}z_{i,k}^2\mathrm{tr}\{C_{i,k}^{\mathrm{T}}C_{i,k}\}. \quad (18)$$

根据Young's不等式得

$$s_{i,k}^3s_{i,k+1}\leqslant\frac{3}{4}s_{i,k}^4+\frac{q_{i,k}}{4}s_{i,k+1}^4, \quad (19)$$

$$s_{i,k}^3z_{i,k+1}\leqslant\frac{3}{4}s_{i,k}^4+\frac{q_{i,k}}{4}z_{i,k+1}^4, \quad (20)$$

$$z_{i,k}^3B_{i,k}\leqslant\frac{3}{4}\bar{B}_{i,k}^{\frac{4}{3}}z_{i,k}^4+\frac{1}{4}, \quad (21)$$

$$\frac{3}{2}s_{i,k}^2\mathrm{tr}\{g_{i,k}^{\mathrm{T}}(x_i)g_{i,k}(x_i)\}\leqslant\frac{3}{4l_{i,k}^2}+\frac{3l_{i,k}^2}{4}s_{i,k}^4\|g_{i,k}(x_i)\|^4, \quad (22)$$

$$\frac{3}{2}z_{i,k}^2\mathrm{tr}\{C_{i,k}^{\mathrm{T}}C_{i,k}\}\leqslant\frac{3\eta_{i,k}^2}{4}\bar{C}_{i,k}^2z_{i,k}^4+\frac{3}{4\eta_{i,k}^2}. \quad (23)$$

将式(19)–(23)代入式(18)得

$$
\begin{aligned}
\ell V_{i,k}\leqslant & \ell V_{i,k-1}-\frac{q_{i,k-1}}{4}s_{i,k}^4-\frac{q_{i,k-1}}{4}z_{i,k}^4+s_{i,k}^3(\bar f_{i,k}+\\
&\alpha_{i,k+1}-\dot{\bar\alpha}_{i,k})-(\frac{1}{\tau_{i,k}}-\frac{3\eta_{i,k}^2}{4}\bar C_{i,k}^2-\frac{3}{4}\bar B_{i,k}^{\frac{4}{3}}-\\
&\frac{q_{i,k-1}}{4})z_{i,k}^4-\frac{1}{\lambda_{i,k}}\tilde\Theta_{i,k}\dot{\hat\Theta}_{i,k}+\frac{3}{4l_{i,k}^2}+\frac{3}{4\eta_{i,k}^2}+\\
&\frac{q_{i,k}}{4}s_{i,k+1}^4+\frac{q_{i,k}}{4}z_{i,k+1}^4+\frac{1}{4},
\end{aligned}\tag{24}
$$

其中$\eta_{i,k}$为正设计参数, 且

$$
\begin{aligned}
\bar f_{i,k}=&f_{i,k}(x_i)+\frac{q_{i,k-1}}{4}s_{i,k}+\frac{3}{2}s_{i,k}+\\
&\frac{3l_{i,k}^2}{4}s_{i,k}\|g_{i,k}(x_i)\|^4.
\end{aligned}
$$

由式(5)得

$$
\bar f_{i,k}=\theta_{i,k}^{*\mathrm{T}}\varphi_{i,k}(\vartheta_{i,k})+\epsilon_{i,k}(\vartheta_{i,k}),\tag{25}
$$

其中: $|\epsilon_{i,k}(\vartheta_{i,k})|\leqslant\epsilon_{i,k}^*$, $\vartheta_{i,k}=[x_i^{\mathrm{T}}\ \ y_{\mathrm{r}}\ \ \dot y_{\mathrm{r}}]^{\mathrm{T}}$.

由Young's不等式和引理4可得

$$
\begin{aligned}
s_{i,k}^3\bar f_{i,k}=&s_{i,k}^3(\theta_{i,k}^{*\mathrm{T}}\varphi_{i,k}(\vartheta_{i,k})+\epsilon_{i,k}(\vartheta_{i,k}))\leqslant\\
&\frac{s_{i,k}^6\Theta_{i,k}\varphi_{i,k}^{\mathrm{T}}(X_{i,k})\varphi_{i,k}(X_{i,k})}{2\mu_{i,k}^2}+\\
&\frac{\mu_{i,k}^2}{2}+\frac{3}{4}s_{i,k}^4+\frac{\epsilon_{i,k}^{*4}}{4},
\end{aligned}\tag{26}
$$

其中$X_{i,k}=[\bar x_{i,k}^{\mathrm{T}}\ \ y_{\mathrm{r}}\ \ \dot y_{\mathrm{r}}]^{\mathrm{T}}$.

虚拟控制器$\alpha_{i,k+1}$设计如下:

$$
\begin{aligned}
\alpha_{i,k+1}=&-c_{i,k}s_{i,k}-\frac{3}{4}s_{i,k}+\dot{\bar\alpha}_{i,k}-\\
&\frac{s_{i,k}^3\hat\Theta_{i,k}\varphi_{i,k}^{\mathrm{T}}(X_{i,k})\varphi_{i,k}(X_{i,k})}{2\mu_{i,k}^2},
\end{aligned}\tag{27}
$$

令$\bar\tau_{i,k}=\dfrac{1}{\tau_{i,k}}-\dfrac{3\eta_{i,k}^2}{4}\bar C_{i,k}^2-\dfrac{3}{4}\bar B_{i,k}^{\frac{4}{3}}-\dfrac{q_{i,k-1}}{4}$. 设计参数自适应律如下:

$$
\dot{\hat\Theta}_{i,k}=\frac{\lambda_{i,k}s_{i,k}^6\varphi_{i,k}^{\mathrm{T}}(X_{i,k})\varphi_{i,k}(X_{i,k})}{2\mu_{i,k}^2}-\sigma_{i,k}\hat\Theta_{i,k}.\tag{28}
$$

再由Young's不等式得

$$
\frac{\sigma_{i,k}}{\lambda_{i,k}}\tilde\Theta_{i,k}\hat\Theta_{i,k}\leqslant-\frac{\sigma_{i,k}}{2\lambda_{i,k}}\tilde\Theta_{i,k}^2+\frac{\sigma_{i,k}}{2\lambda_{i,k}}\Theta_{i,k}^2.\tag{29}
$$

结合式(25)–(29), 式(24)可以重写为

$$
\begin{aligned}
\ell V_{i,k}\leqslant&-\frac{c_{i,1}s_{i,1}^4}{k_{i,b}^4-s_{i,1}^4}-\sum_{j=2}^{k}c_{i,j}s_{i,j}^4-\sum_{j=2}^{k}\bar\tau_{i,j}z_{i,j}^4-\\
&\sum_{j=1}^{k}\frac{\sigma_{i,j}}{2\lambda_{i,j}}\tilde\Theta_{i,j}^2+\frac{q_{i,k}}{4}s_{i,k+1}^4+\\
&\frac{q_{i,k}}{4}z_{i,k+1}^4+D_{i,k},
\end{aligned}\tag{30}
$$

其中 $D_{i,k}=D_{i,k-1}+\dfrac{3}{4l_{i,k}^2}+\dfrac{3}{4\eta_{i,k}^2}+\dfrac{1}{4}+\dfrac{\mu_{i,k}^2}{2}+\dfrac{\epsilon_{i,k}^{*4}}{4}+\dfrac{\sigma_{i,k}}{2\lambda_{i,k}}\Theta_{i,k}^2$.

在时间常数$\tau_{i,k+1}$下, 让$\alpha_{i,k+1}$通过一阶滤波器得到$\bar\alpha_{i,k+1}$, 则

$$
\tau_{i,k+1}\dot{\bar\alpha}_{i,k+1}+\bar\alpha_{i,k+1}=\alpha_{i,k+1},\ \bar\alpha_{i,k+1}(0)=\alpha_{i,k+1}(0).
$$

滤波误差$z_{i,k+1}$的动态方程表示如下:

$$
\mathrm{d}z_{i,k+1}=(-\frac{z_{i,k+1}}{\tau_{i,k+1}}+B_{i,k+1})\mathrm{d}t+C_{i,k+1}\mathrm{d}\varpi,
$$

其中:

$$
\begin{aligned}
B_{i,k+1}=&-\sum_{m=1}^{k}\frac{\partial\alpha_{i,k+1}}{\partial x_{i,m}}(x_{i,m+1}+f_{i,m}(x_i))-\\
&\frac{1}{2}\sum_{m=1}^{k}\sum_{j=1}^{k}\frac{\partial^2\alpha_{i,k+1}}{\partial x_{i,m}x_{i,j}}g_{i,j}(x_i)g_{i,m}^{\mathrm{T}}(x_i)-\\
&\sum_{m=1}^{k}\frac{\partial\alpha_{i,k+1}}{\partial\hat\Theta_{i,m}}\dot{\hat\Theta}_{i,m}-\sum_{m=2}^{k}\frac{\partial\alpha_{i,k+1}}{\partial\bar\alpha_{i,m}}\dot{\bar\alpha}_{i,m}-\\
&\frac{\partial\alpha_{i,k+1}}{\partial y_{\mathrm{r}}}\dot y_{\mathrm{r}}-\frac{\partial\alpha_{i,k+1}}{\partial\dot y_{\mathrm{r}}}\ddot y_{\mathrm{r}},\\
C_{i,k+1}=&-\sum_{m=1}^{k}\frac{\partial\alpha_{i,k+1}}{\partial x_{i,m}}g_{i,m}(x_i),
\end{aligned}
$$

故存在正常数$\bar B_{i,k+1}$和$\bar C_{i,k+1}$, 满足$|B_{i,k+1}|\leqslant\bar B_{i,k+1}$, $|\mathrm{tr}\{C_{i,k+1}^{\mathrm{T}}C_{i,k+1}\}|\leqslant\bar C_{i,k+1}$.

步骤 3　由式(1)–(2)和式(6)可得

$$
\begin{aligned}
\mathrm{d}s_{i,n_i}=&(b_{m_i}((1-r_i)u_i+w_i)+f_{i,n_i}(x_i)-\\
&\dot{\bar\alpha}_{i,n_i})\mathrm{d}t+g_{i,n_i}(x_i)\mathrm{d}\varpi.
\end{aligned}\tag{31}
$$

选取李雅普诺夫函数为

$$
V_{i,n_i}=V_{i,n_i-1}+\frac{1}{4}s_{i,n_i}^4+\frac{1}{4}z_{i,n_i}^4+\frac{1}{2\lambda_{i,n_i}}\tilde\Theta_{i,n_i}^2.
$$

结合式(1)和式(31), 则$\ell V_{i,n_i}$为

$$
\begin{aligned}
\ell V_{i,n_i}=&\ell V_{i,n_i-1}+s_{i,n_i}^3(\beta_iu_i+b_{m_i}w_i+f_{i,n_i}(x_i)-\\
&\dot{\bar\alpha}_{i,n_i})+\frac{3}{2}s_{i,n_i}^2\mathrm{tr}\{g_{i,n_i}^{\mathrm{T}}(x_i)g_{i,n_i}(x_i)\}+\\
&z_{i,n_i}^3(-\frac{z_{i,n_i}}{\tau_{i,n_i}}+B_{i,n_i})-\frac{1}{\lambda_{i,n_i}}\tilde\Theta_{i,n_i}\dot{\hat\Theta}_{i,n_i}+\\
&\frac{3}{2}z_{i,n_i}^2\mathrm{tr}\{C_{i,n_i}^{\mathrm{T}}C_{i,n_i}\},
\end{aligned}\tag{32}
$$

其中$\beta_i=b_{m_i}(1-r_i)$, 令$\bar w_i\geqslant|b_{m_i}w_i|$.

当$k=n_i$时, 应用不等式(21)–(23)的结果, 以及$s_{i,n_i}^3b_{m_i}w_i\leqslant\dfrac{3}{4}s_{i,n_i}^4+\dfrac{1}{4}\bar w_i^4$, 可得

$$
\begin{aligned}
\ell V_{i,n_i}\leqslant&-\frac{c_{i,1}s_{i,1}^4}{k_{i,b}^4-s_{i,1}^4}-\sum_{j=2}^{n_i-1}c_{i,j}s_{i,j}^4-\sum_{j=2}^{n_i-1}\bar\tau_{i,j}z_{i,j}^4-\\
&\sum_{j=1}^{n_i-1}\frac{\sigma_{i,j}}{2\lambda_{i,j}}\tilde\Theta_{i,j}^2+D_{i,n_i-1}+s_{i,n_i}^3(\beta_iu_i+\\
&\bar f_{i,n_i}-\dot{\bar\alpha}_{i,n_i})-(\frac{1}{\tau_{i,n_i}}-\frac{3\eta_{i,n_i}^2}{4}\bar C_{i,n_i}^2-\\
&\frac{3}{4}\bar B_{i,n_i}^{\frac{3}{4}}-\frac{q_{i,n_i-1}}{4})z_{i,n_i}^4-\frac{1}{\lambda_{i,n_i}}\tilde\Theta_{i,n_i}\dot{\hat\Theta}_{i,n_i}+\\
&\frac{3}{4l_{i,n_i}^2}+\frac{3}{4\eta_{i,n_i}^2}+\frac{1}{4}+\frac{1}{4}\bar w_i^4,
\end{aligned}\tag{33}
$$

其中

$$\bar{f}_{i,n_i}=\frac{3l_{i,n_i}^2}{4}s_{i,n_i}\|g_{i,n_i}(x_i)\|^4+\frac{q_{i,n_i-1}}{4}s_{i,n_i}+\frac{3}{4}s_{i,n_i}+f_{i,n_i}(x_i).$$

当$k=n_i$时, 将式(25)–(26)代入式(33)中, 则控制器u_i和ξ_i的动态方程如下:

$$u_i=N'(\xi_i)(c_{i,n_i}s_{i,n_i}+\frac{3}{4}s_{i,n_i}-\dot{\bar{\alpha}}_{i,n_i}+\frac{s_{i,n_i}^3\hat{\Theta}_{i,n_i}\varphi_{i,n_i}^{\mathrm{T}}(X_{i,n_i})\varphi_{i,n_i}(X_{i,n_i})}{2\mu_{i,n_i}^2}),\tag{34}$$

$$\dot{\xi}_i=\frac{s_{i,n_i}^3}{k_i}(c_{i,n_i}s_{i,n_i}+\frac{3}{4}s_{i,n_i}-\dot{\bar{\alpha}}_{i,n_i}+\frac{s_{i,n_i}^3\hat{\Theta}_{i,n_i}\varphi_{i,n_i}^{\mathrm{T}}(X_{i,n_i})\varphi_{i,n_i}(X_{i,n_i})}{2\mu_{i,n_i}^2}),\tag{35}$$

其中: $X_{i,n_i}=[x_i^{\mathrm{T}}\ \ y_{\mathrm{r}}\ \ \dot{y}_{\mathrm{r}}]^{\mathrm{T}}$, $k_i>0$为设计参数, 则自适应律如下:

$$\dot{\hat{\Theta}}_{i,n_i}=\frac{\lambda_{i,n_i}s_{i,n_i}^6\varphi_{i,n_i}^{\mathrm{T}}(X_{i,n_i})\varphi_{i,n_i}(X_{i,n_i})}{2\mu_{i,n_i}^2}-\sigma_{i,n_i}\hat{\Theta}_{i,n_i}.\tag{36}$$

当$k=n_i$时, 应用式(29), 同时结合式(34)–(36), 式(33)可化简为

$$\ell V_{i,n_i}\leqslant-\frac{c_{i,1}s_{i,1}^4}{k_{i,b}^4-s_{i,1}^4}-\sum_{j=2}^{n_i}c_{i,j}s_{i,j}^4-\sum_{j=2}^{n_i}\bar{\tau}_{i,j}z_{i,j}^4-\sum_{j=1}^{n_i}\frac{\sigma_{i,j}}{2\lambda_{i,j}}\tilde{\Theta}_{i,j}^2+D_{i,n_i}+k_i(\beta_iN'(\xi_i)+1)\dot{\xi}_i,\tag{37}$$

其中:

$$D_{i,n_i}=D_{i,n_i-1}+\frac{3}{4l_{i,n_i}^2}+\frac{3}{4\eta_{i,n_i}^2}+\frac{1}{4}+\frac{1}{4}\bar{w}_i^4+\frac{\mu_{i,n_i}^2}{2}+\frac{\epsilon_{i,n_i}^{*4}}{4}+\frac{\sigma_{i,n_i}}{2\lambda_{i,n_i}}\Theta_{i,n_i}^2,$$

$$\bar{\tau}_{i,n_i}=\frac{1}{\tau_{i,n_i}}-\frac{3\eta_{i,n_i}^2}{4}\bar{C}_{i,n_i}^2-\frac{3}{4}\bar{B}_{i,n_i}^{\frac{4}{3}}-\frac{q_{i,n_i-1}}{4}.$$

根据引理3可得

$$-\frac{c_{i,1}s_{i,1}^4}{k_{i,b}^4-s_{i,1}^4}<-c_{i,1}\log\frac{k_{i,b}^4}{k_{i,b}^4-s_{i,1}^4}.\tag{38}$$

令$C_i=\min\limits_{k=2,\cdots,n_i}\{4c_{i,1},4c_{i,k},\sigma_{i,1},\sigma_{i,k},4\bar{\tau}_{i,k}\}$, 则$C=\min\limits_{1\leqslant i\leqslant N}\{C_i\}$和$D=\sum\limits_{i=1}^{N}D_{i,n_i}$. 选取如下李雅普诺夫函数:

$$V=\sum_{i=1}^{N}\frac{1}{4}\log\frac{k_{i,b}^4}{k_{i,b}^4-s_{i,1}^4}+\sum_{i=1}^{N}\sum_{j=2}^{n_i}\frac{1}{4}s_{i,j}^4+\sum_{i=1}^{N}\sum_{j=1}^{n_i}\frac{1}{2\lambda_{i,j}}\tilde{\Theta}_{i,j}^2+\sum_{i=1}^{N}\sum_{j=2}^{n_i}\frac{1}{4}z_{i,j}^4.\tag{39}$$

根据式(37)–(39), 则ℓV为

$$\ell V\leqslant-CV+\sum_{i=1}^{N}k_i(\beta_iN'(\xi_i)+1)\dot{\xi}_i+D.\tag{40}$$

因此, 根据引理1可得, $\sum\limits_{i=1}^{N}k_i(\beta_iN'(\xi_i)+1)\dot{\xi}_i$在区间$[0,t_{\mathrm{f}}]$上是有界的, 同时定义

$$D_{\max}=\max_{t\in[0,t_{\mathrm{f}}]}\sum_{i=1}^{N}k_i(\beta_iN'(\xi_i)+1)\dot{\xi}_i,$$

则式(40)可重写为

$$\ell V\leqslant-CV+\bar{D},\tag{41}$$

其中$\bar{D}=D+D_{\max}$. 由式(41)可以得到闭环系统中的所有信号是依概率半全局一致最终有界. 根据文献[30], 式(41)可以重写如下:

$$\mathrm{E}(V(t))\leqslant V(0)\mathrm{e}^{-Ct}+\frac{\bar{D}}{C},\tag{42}$$

故式(42)中$\mathrm{E}(\sum\limits_{i=1}^{N}\sum\limits_{k=1}^{n_i}|s_{i,k}|^4)\leqslant 4\mathrm{E}(V(t))$成立. 同时如果选择适当的设计参数, 即让$\bar{D}$足够小或者让$C$足够大, 则能保证系统的跟踪误差收敛原点附近的紧集中. 上述控制方案的设计和分析总结为如下定理.

定理 1 对于含有输出约束和执行器故障的非严格反馈随机多智体系统(1), 通过设计实际控制器(34), 中间虚拟控制器(14)和(27)以及参数自适应律(15)(28)(36), 保证了闭环系统中的所有信号是依概率半全局一致最终有界, 且通过合理选择设计参数, 跟踪误差能够收敛到原点的邻域内.

4 仿真研究

考虑由4个跟随者和1个领导者组成的非严格反馈形式的随机多智能体系统, 具体形式如下:

$$\begin{aligned}&\mathrm{d}x_{i,1}=(x_{i,2}+0.5x_{i,1}^2x_{i,2})\mathrm{d}t+0.2\cos(x_{i,1}^2x_{i,2})\mathrm{d}\varpi,\\&\mathrm{d}x_{i,2}=(b_{m_i}u_i^{\mathrm{f}}-x_{i,2})\mathrm{d}t-1.96x_{i,2}\sin x_{i,1}\mathrm{d}\varpi,\\&y_i=x_{i,1},\end{aligned}$$

领导者的输出信号$y_{\mathrm{r}}=\sin(0.5t)+0.5\sin(1.5t)$. 图1表示智能体之间的通讯拓扑图, 由图1可知, 跟随者的邻接矩阵

$$A=\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix},$$

且$H=\mathrm{diag}\{0,1,0,0\}$.

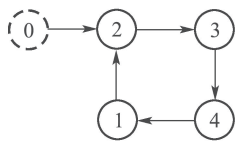

图 1 通讯拓扑图

Fig. 1 Communication topology

选择适当的设计参数为

$$c_{i,1}=c_{i,2}=60,\ k_{i,b}=0.4,\ \mu_{i,1}=\mu_{i,2}=10,$$
$$\tau_{i,1}=0.01,\ \lambda_{i,1}=\lambda_{i,2}=20,\ \sigma_{i,1}=\sigma_{i,2}=1,$$
$$k_i=1,\ b_{m_i}=2,\ w_i=\mathrm{e}^{-t^2},\ r_i=0.2,$$

其中$i=1,2,3,4$. 设置跟随者的初始状态如下:

$$x_1(0)=[-0.1\ \ -0.2]^{\mathrm{T}},\ x_2(0)=[-0.01\ \ -0.1]^{\mathrm{T}},$$
$$x_3(0)=x_4(0)=[0.1\ \ 0.1]^{\mathrm{T}},$$

且设置自适应参数的初始状态如下:

$$\hat{\Theta}_{1,1}(0)=0.1,\ \hat{\Theta}_{1,2}(0)=0.2,\ \hat{\Theta}_{2,1}(0)=0.2,$$
$$\hat{\Theta}_{2,2}(0)=0.3,\ \hat{\Theta}_{3,1}(0)=0.2,\ \hat{\Theta}_{3,2}(0)=0.3,$$
$$\hat{\Theta}_{4,1}(0)=0.3,\ \hat{\Theta}_{4,2}(0)=0.1.$$

系统的仿真结果如图2–6所示. 图2为跟随者输出信号y_i与领导者输出信号y_{r}的跟踪轨迹曲线, 其跟踪误差轨迹则由图3表示, 图4描述了系统控制器u_i的轨迹曲线, 图5–6则为自适应参数的轨迹曲线. 由以上仿真结果可知, 本文所提出的方案保证了系统的稳定性, 同时验证了设计的控制器能保证跟踪性能, 系统的输出信号能够被限定在指定范围内.

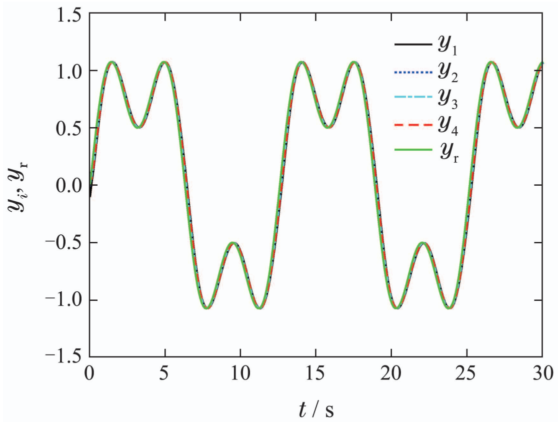

图 2　领导者和跟随者的输出信号

Fig. 2　Output signals of the followers and leader

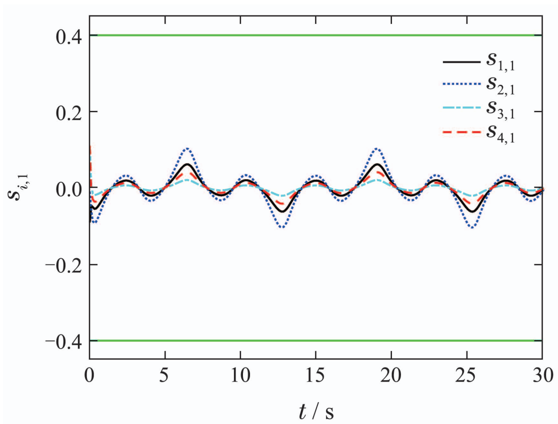

图 3　跟踪误差$s_{i,1}$

Fig. 3　Tacking errors $s_{i,1}$

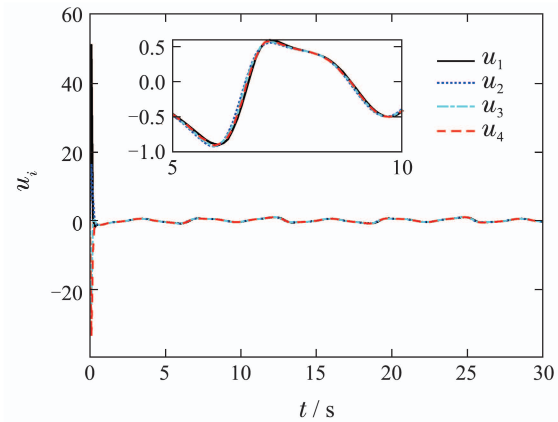

图 4　控制器u_i

Fig. 4　Controllers u_i

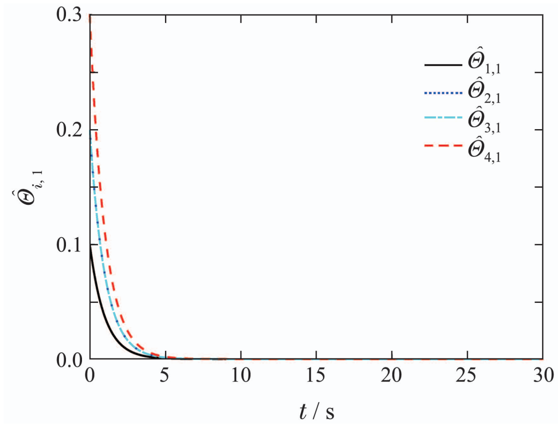

图 5　自适应参数$\hat{\Theta}_{i,1}$

Fig. 5　Adaptive parameters $\hat{\Theta}_{i,1}$

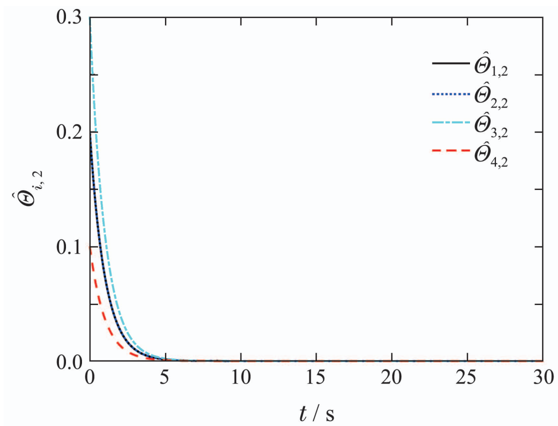

图 6　自适应参数$\hat{\Theta}_{i,2}$

Fig. 6　Adaptive parameters $\hat{\Theta}_{i,2}$

5　总结

本文针对一类非严格反馈的非线性随机多智能体系统, 考虑了在输出约束和执行器故障情况下的协同控制一致性问题, 结合有向拓扑图知识设计了一种基于神经网络的自适应容错控制方案. 在反步法的基础上, 构造障碍李雅普诺夫函数解决了输出约束问题, 综合考虑动态面技术和Nussbaum函数, 设计了一种

有效的自适应容错控制器. 最后通过李雅普诺夫稳定性定理, 证明了闭环系统的稳定性. 本文提出的自适应神经网络控制方案扩展了严格反馈非线性系统的研究成果, 未来的研究将把本文的结果推广到有限时间或固定时间的非线性随机多智能体中.

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