反馈非线性系统随机梯度辨识算法及其收敛性

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摘要: 反馈非线性受控自回归系统是由前向通道的受控自回归模型和反馈通道的静态非线性构成, 这类系统经过参数化后得到双线性参数辨识模型. 本文通过对辨识模型中双线性参数乘积项进行分解, 基于梯度搜索原理, 提出了反馈非线性系统的随机梯度辨识算法. 为了改善随机梯度算法的收敛速度, 引入遗忘因子, 文章给出了遗忘因子随机梯度算法, 利用随机过程理论, 建立了随机梯度算法的参数估计收敛定理, 证明了算法的收敛性. 最后, 通过数值仿真验证了算法的有效性.

关键词: 反馈非线性系统; 双线性参数模型; 随机梯度; 遗忘因子; 收敛性

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Stochastic gradient identification algorithm and its convergence for feedback nonlinear systems

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Abstract: Through the parameterization, we obtain the bilinear-parameter identification model of the feedback nonlinear controlled autoregressive system, which is composed of a controlled autoregressive model in the forward channel and a static nonlinearity in the feedback channel. By decomposing the product term of the bilinear parameters in the identification model, a stochastic gradient identification algorithm is developed to estimate the unknown parameters of the feedback nonlinear system based on the gradient search. In order to improve the convergence rate of the proposed stochastic gradient algorithm, the forgetting factor stochastic gradient algorithm is given by introducing the forgetting factor. Also, this paper establishes the convergence theorem of the parameter estimation and proves the convergence of the proposed stochastic gradient algorithm by means of the stochastic process theory. Finally, the simulation example is carried out to verify the effectiveness of the proposed algorithms from the aspects of parameter estimation accuracy and prediction performance.

Key words: feedback nonlinear system; bilinear-parameter model; stochastic gradient; forgetting factor; convergence

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1 引言

系统辨识是通过采集系统的输入输出观测数据, 利用优化准则, 研究建立系统数学模型的理论与方法[1–2]. 线性系统结构简单, 辨识方法已经十分成熟[3], 而非线性系统[4–5]结构复杂, 其系统辨识极为困难. 如何将线性系统辨识方法推广至非线性系统是目前辨识领域研究的重点[6]. 针对输入非线性系统, 文献[7]利用开关函数描述输入变增益非线性块, 结合迭代辨识理论, 提出了基于梯度的辅助模型迭代算法. 文献[8]研究了非平稳干扰和高斯白噪声干扰下的维纳非线性系统的鲁棒递归辨识问题. 文献[9]利用数据滤波技术和Levenberg-Marquardt算法辨识多输入单输出Hammerstein非线性系统. 文献[10]基于关键项分离技术, 研究有色噪声干扰下的分数阶非线性有限脉冲响应系统的参数估计问题. 反馈非线性系统是由前向通道的动态线性模块和反馈通道的静态非线性模块共同构成, 是一类典型的非线性系统. 此类系统通过在反馈通道引入非线性环节, 使得非线性控制系统在外部等效于线性系统, 但在控制系统内部, 控制对象和控制器仍然保持非线性, 从而在大范围内改善系统

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的稳定性. 这种理论已被应用于机器人控制和飞机控制等方面. 在继电控制系统中采用非线性继电元件构成非线性反馈, 来简化控制系统的结构, 实现可靠工作. 在最速控制系统, 最小能耗控制系统和最省燃料控制系统中, 通常采用非线性反馈代替线性反馈来实现性能指标的最优控制. 针对反馈非线性系统, 文献[11]利用递阶辨识原理, 数据滤波技术和多新息辨识思想, 研究了滤波递阶广义随机梯度辨识算法和滤波递阶多新息广义随机梯度辨识算法. 文献[12]结合最小二乘原理和谐波平衡法, 提出了一种具有时滞反馈控制的非线性系统辨识算法. 文献[13]利用辅助模型辨识思想和分解技术, 推导了基于分解的辅助模型递推最小二乘估计算法, 与基于辅助模型的递推最小二乘算法相比, 具有较小的计算量.

递推算法[14]和迭代算法[15–16]是系统辨识的重要分支. 迭代算法适用于离线辨识[17], 递推算法适用于在线辨识[18–20]. 递推辨识的主要思想是: 下一时刻的参数估计等于前一时刻参数估计加上一校正值. 随机梯度辨识算法和递推最小二乘辨识算法是递推辨识的两大重要方法. 在电动汽车电池荷电状态测量方面, 针对微机电系统中的惯性导航系统, 陈光武等人[21]提出了一种基于随机加权的递推最小二乘的陀螺去噪算法, 提高了姿态估计精度. 文献[22]以风洞系统为研究对象, 搭建面向块的模型, 提出一种递推在线估计未知模型参数算法. 文献[23]针对有源自回归 (auto-regressive with extra inputs, ARX)模型, 提出了加权递阶随机梯度算法, 该算法的修正项在递推过程中以加权形式出现. 文献[24]利用系统阶跃响应的观测数据, 引入多新息辨识理论解决传递函数的直接辨识问题, 推导出多新息随机梯度算法.

针对反馈非线性受控自回归系统, 本文首先推导出反馈非线性受控自回归系统辨识模型, 针对模型中存在乘积的特性, 对参数集进行分解, 提出了反馈非线性受控自回归系统的随机梯度算法和遗忘因子随机梯度算法, 然后, 证明了提出的算法参数估计收敛性, 最后, 通过仿真验证算法的有效性.

2 反馈非线性受控自回归系统辨识模型

首先定义下文需要使用的符号: “$X := A$”或“$A := X$”表示 A 被定义为X; 矩阵范数表示为$\|\boldsymbol{X}\|^2 := \mathrm{tr}[\boldsymbol{X}\,\boldsymbol{X}^{\mathrm{T}}]$; 上标T表示矩阵的转置; $\boldsymbol{I}_n$表示n维的单位阵, 其对角线元素全为1, 其余元素全为0; $\mathbf{1}_n$表示所有元全为1的向量.

反馈非线性受控自回归系统结构如图1所示, 其前向通道是受控自回归 (controlled auto-regression, CAR)模型, 表达式为

$$A(z)y(k) = B(z)h(k) + v(k), \tag{1}$$

其中: $y(k) \in \mathbb{R}$表示系统的可测输出; $h(k) \in \mathbb{R}$表示线性动态子系统的输入; $v(k) \in \mathbb{R}$表示均值为零、方差为σ^2的随机白噪声; $A(z)$和$B(z)$分别表示单位后移算子z^{-1}的多项式, 定义为

$$A(z) := 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a},$$
$$B(z) := b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b}.$$

设阶次n_a和n_b是已知的, 且当$k \leqslant 0$时, 有$y(k) = 0$和$v(k) = 0$.

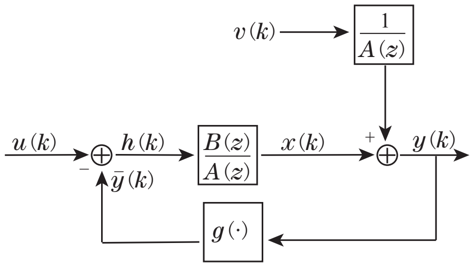

图 1 反馈非线性受控自回归系统框图

Fig. 1 The feedback nonlinear CAR system

反馈通道是无记忆非线性环节, 其输出$\bar{y}(k)$可以看作系数为$c_j(j = 1, 2, \cdots, m)$的已知非线性基$\boldsymbol{g} = (g_1, g_2, \cdots, g_m)$的非线性函数, 即

$$\bar{y}(k) = c_1 g_1(y(k)) + \cdots + c_m g_m(y(k)) = \boldsymbol{g}(y(k))\boldsymbol{c}, \tag{2}$$

其中: $\boldsymbol{g}(y(k)) := [g_1(y(k)) \ \cdots \ g_m(y(k))] \in \mathbb{R}^{1\times m}$是基函数构成的行向量; $\boldsymbol{c} := [c_1 \ c_2 \ \cdots \ c_m]^{\mathrm{T}} \in \mathbb{R}^m$是非线性环节的参数向量. 线性动态子系统的输入$h(k)$与非线性环节的输出$\bar{y}(k)$满足关系

$$h(k) = u(k) - \bar{y}(k), \tag{3}$$

其中: $u(k) \in \mathbb{R}$表示可测输入; $\bar{y}(k) \in \mathbb{R}$表示反馈非线性输出. 当$k \leqslant 0$时, 令$u(k) = 0$.

定义输出信息向量$\boldsymbol{\varphi}_y(k)$, 输入信息向量$\boldsymbol{\varphi}_u(k)$, 信息矩阵$\boldsymbol{G}(k)$如下:

$$\boldsymbol{\varphi}_y(k) := [-y(k-1) \ \cdots \ y(k-n_a)]^{\mathrm{T}} \in \mathbb{R}^{n_a}, \tag{4}$$

$$\boldsymbol{\varphi}_u(k) := [u(k-1) \ \cdots \ u(k-n_b)]^{\mathrm{T}} \in \mathbb{R}^{n_b}, \tag{5}$$

$$\boldsymbol{G}(k) := [-\boldsymbol{g}^{\mathrm{T}}(y(k-1)) \ \cdots \ -\boldsymbol{g}^{\mathrm{T}}(y(k-n_b))]^{\mathrm{T}} \in \mathbb{R}^{n_b\times m}. \tag{6}$$

定义线性动态环节参数向量$\boldsymbol{a}$和$\boldsymbol{b}$, 非线性环节参数向量$\boldsymbol{c}$如下:

$$\boldsymbol{a} := [a_1 \ a_2 \ \cdots \ a_{n_a}]^{\mathrm{T}} \in \mathbb{R}^{n_a},$$
$$\boldsymbol{b} := [b_1 \ b_2 \ \cdots \ b_{n_b}]^{\mathrm{T}} \in \mathbb{R}^{n_b},$$
$$\boldsymbol{c} := [c_1 \ c_2 \ \cdots \ c_m]^{\mathrm{T}} \in \mathbb{R}^{m}.$$

将式(2)–(6)代入式(1), 得到反馈非线性受控自回归系统的辨识模型为

$$y(k) =$$

$$(1-A(z))y(k)+B(z)(u(k)-\bar{y}(k))+v(k)=$$
$$-\sum_{i=1}^{n_a}a_iy(k-i)+\sum_{i=1}^{n_b}b_iu(k-i)-$$
$$\sum_{i=1}^{n_b}b_i\sum_{j=1}^{m}c_jg_j(y(k-i))+v(k)=$$
$$\boldsymbol{\varphi}_y^{\mathrm{T}}(k)\boldsymbol{a}+\boldsymbol{\varphi}_u^{\mathrm{T}}(k)\boldsymbol{b}+\boldsymbol{b}^{\mathrm{T}}\boldsymbol{G}(k)\boldsymbol{c}+v(k),\tag{7}$$

式(7)中出现线性动态环节参数向量$\boldsymbol{b}$与非线性环节参数向量$\boldsymbol{c}$的乘积, 是典型的双线性参数模型. 双线性参数模型是线性参数模型的直接推广, 乘积项的出现加大了辨识的难度, 如何将线性系统辨识方法推广到这类双线性参数非线性系统辨识是本文研究的重点.

3 随机梯度辨识算法

对于反馈非线性系统辨识模型(7), 其包含的输出信息向量$\boldsymbol{\varphi}_y(k)$、输入信息向量$\boldsymbol{\varphi}_u(k)$和信息矩阵$\boldsymbol{G}(k)$都是已知的, 主要的辨识难点在于模型中存在线性动态环节参数向量$\boldsymbol{b}$与非线性环节参数向量$\boldsymbol{c}$的乘积, 使得线性系统的辨识方法不再适用. 为了解决此问题, 可以将$\boldsymbol{b}^{\mathrm{T}}\boldsymbol{G}(k)$作为信息向量, 则系统输出可以表示为参数的线性组合形式, 即双线性参数辨识模型可以等价为伪线性回归模型形式, 辨识时未知变量利用前一时刻的估计值进行代替. 梯度类辨识方法计算量小, 计算过程中没有协方差矩阵, 且参数估计也是收敛的, 因此, 下面针对反馈非线性受控自回归系统, 推导该系统随机梯度辨识算法, 具体过程如下.

反馈非线性系统辨识模型(7)可等价为

$$y(k)=\boldsymbol{\varphi}_y^{\mathrm{T}}(k)\boldsymbol{a}+\boldsymbol{\varphi}_u^{\mathrm{T}}(k)\boldsymbol{b}+\boldsymbol{b}^{\mathrm{T}}\boldsymbol{G}(k)\boldsymbol{c}+v(k)=$$
$$[\boldsymbol{\varphi}_y^{\mathrm{T}}(k)\ \ \boldsymbol{\varphi}_u^{\mathrm{T}}(k)\ \ \boldsymbol{b}^{\mathrm{T}}\boldsymbol{G}(k)]\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\\ \boldsymbol{c}\end{bmatrix}+v(k)=$$
$$\boldsymbol{\varphi}^{\mathrm{T}}(\boldsymbol{b},k)\boldsymbol{\vartheta}+v(k),\tag{8}$$

其中: 系统的信息向量$\boldsymbol{\varphi}(\boldsymbol{b},k):=[\boldsymbol{\varphi}_y^{\mathrm{T}}(k)\ \ \boldsymbol{\varphi}_u^{\mathrm{T}}(k)\ \ \boldsymbol{b}^{\mathrm{T}}\boldsymbol{G}(k)]^{\mathrm{T}}\in\mathbb{R}^{n_a+n_b+m}$; 系统的参数向量$\boldsymbol{\vartheta}:=[\boldsymbol{a}^{\mathrm{T}}\ \ \boldsymbol{b}^{\mathrm{T}}\ \ \boldsymbol{c}^{\mathrm{T}}]^{\mathrm{T}}\in\mathbb{R}^{n_a+n_b+m}$. 模型(8)是一类伪线性回归模型形式, 其包含了$n:=n_a+n_b+m$个参数, 且信息向量$\boldsymbol{\varphi}(\boldsymbol{b},k)$中有部分项不可测. 解决的方法是: 辨识过程中构造出信息向量$\boldsymbol{\varphi}(\boldsymbol{b},k)$的估计值, 信息向量中包含的未知值$\boldsymbol{b}$用其估计值代替. 根据等价辨识模型(8), 构造梯度准则函数为

$$J(\boldsymbol{\vartheta}):=\frac{1}{2}(y(k)-\boldsymbol{\varphi}^{\mathrm{T}}(\boldsymbol{b},k)\boldsymbol{\vartheta})^2.$$

设$\hat{\boldsymbol{\vartheta}}(k)=[\hat{\boldsymbol{a}}^{\mathrm{T}}(k)\ \ \hat{\boldsymbol{b}}^{\mathrm{T}}(k)\ \ \hat{\boldsymbol{c}}^{\mathrm{T}}(k)]^{\mathrm{T}}$是$\boldsymbol{\vartheta}$在时刻$k$的估计值, $1/\tau(k)$为收敛因子(步长). 使用负梯度搜索, 极小化梯度准则函数, 可以得到下列递推关系:

$$\hat{\boldsymbol{\vartheta}}(k)=\hat{\boldsymbol{\vartheta}}(k-1)-\frac{1}{\tau(k)}\mathrm{grad}(J(\hat{\boldsymbol{\vartheta}}(k-1)))=$$
$$\hat{\boldsymbol{\vartheta}}(k-1)+\frac{\boldsymbol{\varphi}(\boldsymbol{b},k)}{\tau(k)}(y(k)-$$
$$\boldsymbol{\varphi}^{\mathrm{T}}(\boldsymbol{b},k)\hat{\boldsymbol{\vartheta}}(k-1)),\tag{9}$$
$$\tau(k)=\tau(k-1)+\|\boldsymbol{\varphi}(\boldsymbol{b},k)\|^2.\tag{10}$$

在递推关系式(9)和式(10)中, 由于信息向量$\boldsymbol{\varphi}(\boldsymbol{b},k)$中存在未知参数$\boldsymbol{b}$, 因此无法直接计算参数估计$\hat{\boldsymbol{\vartheta}}(k)$. 解决的方法是利用未知参数$\boldsymbol{b}$前一时刻$k-1$的估计值$\hat{\boldsymbol{b}}(k-1)$代替$\boldsymbol{b}$, 再构造$\boldsymbol{\varphi}(\boldsymbol{b},k)$的估计值为

$$\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1},k):=\boldsymbol{\varphi}(\hat{\boldsymbol{b}}(k-1),k)=$$
$$[\boldsymbol{\varphi}_y^{\mathrm{T}}(k)\ \ \boldsymbol{\varphi}_u^{\mathrm{T}}(k)\ \ \hat{\boldsymbol{b}}^{\mathrm{T}}(k-1)\boldsymbol{G}(k)]^{\mathrm{T}}.$$

式(9)–(10)中的未知向量$\boldsymbol{\varphi}(\boldsymbol{b},k)$用其估计值代替, 再联立式(4)–(6), 就得到估计反馈非线性受控自回归系统参数向量$\boldsymbol{\vartheta}$的随机梯度算法(简称FN–SG算法)为

$$\hat{\boldsymbol{\vartheta}}(k)=\hat{\boldsymbol{\vartheta}}(k-1)+\frac{\boldsymbol{\varphi}(\hat{\boldsymbol{b}}(k-1),k)}{\tau(k)}e(k),\tag{11}$$
$$e(k):=y(k)-\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{b}}(k-1),k)\hat{\boldsymbol{\vartheta}}(k-1),\tag{12}$$
$$\tau(k)=\tau(k-1)+\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}(k-1),k)\|^2,\tag{13}$$
$$\varphi(\hat{\boldsymbol{b}}_{k-1},k)=\boldsymbol{\varphi}(\hat{\boldsymbol{b}}(k-1),k)=$$
$$[\boldsymbol{\varphi}_y^{\mathrm{T}}(k)\ \ \boldsymbol{\varphi}_u^{\mathrm{T}}(k)\ \ \hat{\boldsymbol{b}}^{\mathrm{T}}(k-1)\boldsymbol{G}(k)]^{\mathrm{T}},\tag{14}$$
$$\boldsymbol{\varphi}_y(k)=[-y(k-1)\ \ \cdots\ \ -y(k-n_a)]^{\mathrm{T}},\tag{15}$$
$$\boldsymbol{\varphi}_u(k)=[u(k-1)\ \ \cdots\ \ u(k-n_b)]^{\mathrm{T}},\tag{16}$$
$$\boldsymbol{G}(k)=[-\boldsymbol{g}^{\mathrm{T}}(y(k-1))\ \ -\boldsymbol{g}^{\mathrm{T}}(y(k-2))\ \ \cdots$$
$$-\boldsymbol{g}^{\mathrm{T}}(y(k-n_b))]^{\mathrm{T}},\tag{17}$$
$$\hat{\boldsymbol{\vartheta}}(k)=[\hat{\boldsymbol{a}}^{\mathrm{T}}(k)\ \ \hat{\boldsymbol{b}}^{\mathrm{T}}(k)\ \ \hat{\boldsymbol{c}}^{\mathrm{T}}(k)]^{\mathrm{T}}.\tag{18}$$

FN–SG算法计算步骤如下:

步骤 1 令$k=1$. 设置初值$\hat{\boldsymbol{\vartheta}}(0)=[\hat{\boldsymbol{a}}^{\mathrm{T}}(0)\ \ \hat{\boldsymbol{b}}^{\mathrm{T}}(0)\ \ \hat{\boldsymbol{c}}^{\mathrm{T}}(0)]^{\mathrm{T}}=\mathbf{1}_n/p_0$, $p_0=10^6$, $\tau(0)=1$, $y(k-i)=0$, $u(k-i)=0$, $i=1,2,\cdots,\max[n_a\ \ n_b]$, 设置基函数$g(\cdot)$和数据长度$L$.

步骤 2 采集输入输出数据$u(k)$和$y(k)$. 用式(15)构造输出信息向量$\boldsymbol{\varphi}_y(k)$, 用式(16)构造输入信息向量$\boldsymbol{\varphi}_u(k)$, 用式(17)构造信息矩阵$\boldsymbol{G}(k)$.

步骤 3 用式(14)构造系统信息向量$\boldsymbol{\varphi}\times(\hat{\boldsymbol{b}}(k-1),k)$, 用式(13)计算$\tau(k)$.

步骤 4 根据式(11)–(12)刷新参数估计向量$\hat{\boldsymbol{\vartheta}}(k)$, 从式(18)的$\hat{\boldsymbol{\vartheta}}(k)$中读出线性动态环节参数估计$\hat{\boldsymbol{a}}(k)$, $\hat{\boldsymbol{b}}(k)$以及非线性环节参数估计$\hat{\boldsymbol{c}}(k)$.

步骤 5 若$k<L$, 则k增加1, 转到步骤2, 否则输出参数估计$\hat{\boldsymbol{a}}(k)$, $\hat{\boldsymbol{b}}(k)$和$\hat{\boldsymbol{c}}(k)$, 结束计算.

一般来说, 随机梯度算法参数估计收敛速度很慢. 为了加快FN–SG算法的收敛速度, 可以在式(13)中引入遗忘因子$\lambda\ (0<\lambda\leqslant 1)$得到

$$\tau(k)=\lambda\tau(k-1)+\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}(k-1),k)\|^2.\tag{19}$$

再联立式(11)–(12), 构成反馈非线性受控自回归系统的遗忘因子随机梯度算法(简称FN–FFSG算法). FN–FFSG算法通过引入遗忘因子改善暂态辨识效果, 提高算法开始运行阶段参数估计收敛速度. 当式(19)中$\lambda = 1$时, FN–FFSG算法退化为FN–SG算法.

4 算法性能分析

辨识算法的收敛性分析一直是系统辨识领域的重点. 下面针对反馈非线性受控自回归系统, 利用随机过程理论, 证明FN–SG算法的收敛性.

引理 1[25] 设$\{T(k)\}$, $\{\beta(k)\}$和$\{\eta(k)\}$均为非负随机变量序列, 且满足不等式

$$T(k+1) \leqslant T(k) - \beta(k) + \eta(k),$$

若$\sum\limits_{k=1}^{\infty} \eta(k) < \infty$, 则$\sum\limits_{k=1}^{\infty} \beta(k) < \infty$, 且$T(k)$收敛于一有限的随机变量$T_0$.

引理 2 对于FN–SG算法式(11)–(18), 下式成立:

$$\sum_{k=1}^{\infty} \frac{\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)\|^2}{\tau^2(k)} < \infty.$$

证 根据$\tau(k)$的定义式(13), 有

$$\sum_{k=1}^{\infty} \frac{\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)\|^2}{\tau^2(k)} \leqslant \sum_{k=1}^{\infty} \frac{\tau(k) - \tau(k-1)}{\tau(k-1)\tau(k)} = \sum_{k=1}^{\infty} (\frac{1}{\tau(k-1)} - \frac{1}{\tau(k)}) = 1 - \frac{1}{\tau(\infty)} < \infty.$$

证毕.

定理 1 对于反馈非线性受控自回归系统辨识模型(7)和FN–SG算法式(11)–(18), 假设$\{v(k)\}$是一个零均值, 方差为σ^2的白噪声序列, 即

1) $\mathrm{E}[v(k)] = 0$;

2) $\mathrm{E}[v^2(k)] = \sigma^2 < \infty$;

3) $\mathrm{E}[v(k)v(j)] = 0,\ j \neq k$.

若存在整数N和与$k(k > 0)$无关的正常数α, 使得下列持续激励条件4)成立:

4) $$\sum_{i=0}^{N-1} \frac{\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+i-1}, k+i)\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{b}}_{k+i-1}, k+i)}{\tau(k+i)} \geqslant \alpha \boldsymbol{I}_n,$$

那么FN–SG算法参数估计误差收敛于零.

证 定义参数估计误差向量$\tilde{\boldsymbol{\vartheta}}(k)$为

$$\tilde{\boldsymbol{\vartheta}}(k) := \hat{\boldsymbol{\vartheta}}(k) - \boldsymbol{\vartheta}, \tag{20}$$

将式(11)–(12)代入式(20)得到

$$\begin{aligned}\tilde{\boldsymbol{\vartheta}}(k) &= \tilde{\boldsymbol{\vartheta}}(k-1) + \frac{\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)}{\tau(k)} e(k) = \\ &\tilde{\boldsymbol{\vartheta}}(k-1) + \frac{\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)}{\tau(k)} (-\tilde{y}(k) + \\ &\xi(k) + v(k)),\end{aligned} \tag{21}$$

式中:

$$\begin{cases} \tilde{y}(k) := \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{b}}_{k-1}, k)\tilde{\boldsymbol{\vartheta}}(k-1) \in \mathbb{R}, \\ \xi(k) := [\boldsymbol{\varphi}(\boldsymbol{b}, k) - \boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)]^{\mathrm{T}} \boldsymbol{\vartheta} \in \mathbb{R}. \end{cases} \tag{22}$$

式(21)两边取范数得到

$$\begin{aligned}&\|\tilde{\boldsymbol{\vartheta}}(k)\|^2 = \\ &\|\tilde{\boldsymbol{\vartheta}}(k-1)\|^2 - \frac{\tau(k) + \tau(k-1)}{\tau^2(k)} \tilde{y}^2(k) + \\ &2\frac{\tau(k-1)}{\tau^2(k)} \tilde{y}(k)(\xi(k) + v(k)) + \\ &\frac{\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)\|^2}{\tau^2(k)} (\xi^2(k) + v^2(k) + 2\xi(k)v(k)) \leqslant \\ &\|\tilde{\boldsymbol{\vartheta}}(k-1)\|^2 - \frac{1}{\tau(k)} \tilde{y}^2(k) + 2\frac{\tau(k-1)}{\tau^2(k)} \tilde{y}(k) \times \\ &(\xi(k) + v(k)) + \frac{\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)\|^2}{\tau^2(k)} (\xi^2(k) + \\ &v^2(k) + 2\xi(k)v(k)),\end{aligned} \tag{23}$$

其中$\{v(k)\}$是零均值, 方差为σ^2的不相关白噪声序列, 如果$\xi^2(k) > \varDelta$, 则$\hat{\boldsymbol{\vartheta}}(k) := \hat{\boldsymbol{\vartheta}}(k-1)$. 如果$\xi^2(k) \leqslant \varDelta$, 因为变量$\tilde{\boldsymbol{\vartheta}}(k-1)$, $\tilde{y}(k)$, $\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)$, $\tau(k)$和$\xi(k)$与$v(k)$不相关, 式(23)两边取期望, 使用定理1的条件1)–3)可得

$$\begin{aligned}\mathrm{E}[\|\tilde{\boldsymbol{\vartheta}}(k)\|^2] \leqslant\ &\mathrm{E}[\|\tilde{\boldsymbol{\vartheta}}(k-1)\|^2] - \mathrm{E}[\frac{1}{\tau(k)} \tilde{y}^2(k)] + \\ &\mathrm{E}[\frac{\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)\|^2}{\tau^2(k)}](\sigma^2 + \varDelta).\end{aligned} \tag{24}$$

对式(24)右边最后一项从$k = 1$到$k = \infty$求和, 应用引理2可得

$$\sum_{k=1}^{\infty} \frac{\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k-1}, k)\|^2}{\tau^2(k)} (\sigma^2 + \varDelta) < \infty, \tag{25}$$

应用引理1于式(25), 得到$\mathrm{E}[\|\tilde{\boldsymbol{\vartheta}}(k)\|^2]$收敛于一有限随机变量$S_0$, 即$\mathrm{E}[\|\tilde{\boldsymbol{\vartheta}}(k)\|^2] \leqslant S_0$, 且

$$\mathrm{E}[\sum_{k=1}^{\infty} \frac{\tilde{y}^2(k)}{\tau(k)}] < \infty. \tag{26}$$

由式(11)可得

$$\begin{aligned}&\tilde{\boldsymbol{\vartheta}}(k+i-1) = \\ &\tilde{\boldsymbol{\vartheta}}(k) + \sum_{j=1}^{i-1} \frac{\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+j-1}, k+j)}{\tau(k+j)} e(k+j).\end{aligned} \tag{27}$$

由式(22)有

$$\tilde{y}(k+i) = \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{b}}_{k+i-1}, k+i)\tilde{\boldsymbol{\vartheta}}(k+i-1).$$

利用式(27)得到

$$\begin{aligned}&\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{b}}_{k+i-1}, k+i)\tilde{\boldsymbol{\vartheta}}(k) = \\ &\tilde{y}(k+i) - \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{b}}_{k+i-1}, k+i) \times \\ &\sum_{j=1}^{i-1} \frac{\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+j-1}, k+j)}{\tau(k+j)} e(k+j).\end{aligned} \tag{28}$$

利用不等式关系$(a+b)^2 \leqslant 2a^2+2b^2$, 式(28)两边平方, 得到

$$\begin{aligned}&\|\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}(k)\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+i-1},k+i)\|^2 \leqslant \\ &2\tilde{y}^2(k+i)+2\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+i-1},k+i)\|^2 \times \\ &\|\sum_{j=1}^{i-1}\frac{\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+j-1},k+j)}{\tau(k+j)}e(k+j)\|^2,\end{aligned}$$

上式两边除以$\tau(k+i)$, 对i从$i=0$到$i=N-1$求和, 利用定理1条件4)和式(27)可得

$$\begin{aligned}&\alpha\|\tilde{\boldsymbol{\vartheta}}(k)\|^2 \leqslant \\ &\sum_{i=0}^{N-1}(\frac{2\tilde{y}^2(k+i)}{\tau(k+i)}+\frac{2\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+i-1},k+i)\|^2}{\tau(k+i)} \times \\ &\|\sum_{j=1}^{i-1}\frac{\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+j-1},k+j)}{\tau(k+j)}e(k+j)\|^2) \leqslant \\ &\sum_{i=0}^{N-1}(\frac{2\tilde{y}^2(k+i)}{\tau(k+i)}+\frac{2\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+i-1},k+i)\|^2}{\tau(k+i)} \times \\ &\|\tilde{\boldsymbol{\vartheta}}(k+i-1)-\tilde{\boldsymbol{\vartheta}}(k)\|^2) \leqslant \\ &\sum_{i=0}^{N-1}(\frac{2\tilde{y}^2(k+i)}{\tau(k+i)}+\frac{4\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+i-1},k+i)\|^2}{\tau(k+i)} \times \\ &(\|\tilde{\boldsymbol{\vartheta}}(k+i-1)\|^2+\|\tilde{\boldsymbol{\vartheta}}(k)\|^2)).\end{aligned}$$

上式两边取期望和极限, 且$\mathrm{E}[\|\tilde{\boldsymbol{\vartheta}}(k)\|]^2 \leqslant S_0$, 得到

$$\begin{aligned}&\lim_{k\to\infty}\mathrm{E}[\|\tilde{\boldsymbol{\vartheta}}(k)\|^2] \leqslant \\ &\lim_{k\to\infty}\frac{1}{\alpha}\mathrm{E}(\sum_{i=0}^{N-1}\frac{2\tilde{y}^2(k+i)}{\tau(k+i)}+ \\ &\frac{8S_0\|\boldsymbol{\varphi}(\hat{\boldsymbol{b}}_{k+i-1},k+i)\|^2}{\tau(k+i)}).\end{aligned}$$

由式(25)–(26)可得, $\lim\limits_{k\to\infty}\mathrm{E}[\|\tilde{\boldsymbol{\vartheta}}(k)\|^2]=0$. 定理1得证. 定理1的证明过程表明, 在假设持续激励定理1条件4)成立, 噪声方差及变量$\xi(k)$有界时, FN–SG算法是收敛的.　证毕.

5　数值仿真

数值仿真是验证算法有效性和准确性的重要方法. 设有反馈非线性受控自回归系统

$$\begin{aligned}&A(z)y(k)=B(z)h(k)+v(k),\\ &h(k)=u(k)-\bar{y}(k),\\ &A(z)=1+a_1z^{-1}+a_2z^{-2}=\\ &1-0.18z^{-1}-0.06z^{-2},\\ &B(z)=b_1z^{-1}+b_2z^{-2}=0.68z^{-1}+0.36z^{-2},\\ &\bar{y}(k)=c_1\sin y^2(k)+c_2\sin y^3(k)=\\ &0.20\sin y^2(k)-0.12\sin y^3(k),\\ &\boldsymbol{\vartheta}=[a_1\ \ a_2\ \ b_1\ \ b_2\ \ c_1\ \ c_2]^{\mathrm{T}}=\\ &[-0.18\ \ -0.06\ \ 0.68\ \ 0.36\ \ 0.20\ \ -0.12]^{\mathrm{T}}.\end{aligned}$$

仿真时, 系统输入$\{u(k)\}$采用零均值单位方差不相关的随机信号序列, $\{v(k)\}$采用零均值方差为σ^2的白噪声序列, 在给定模型参数下计算系统的输出$\{y(k)\}$. 利用$L_e=4000$组输入输出数据, 分别应用FN–SG算法和FN–FFSG算法估计系统参数. 表1给出了方差分别为$\sigma^2=0.40^2$和$\sigma^2=0.20^2$时, FN–SG算法的参数估计及其误差, 表2和表3给出了方差$\sigma^2=0.20^2$, 遗忘因子分别为$\lambda=0.99$和$\lambda=0.98$时, FN–FFSG算法的参数估计和误差, 图2给出了不同噪声方差时FN–SG算法参数估计误差$\delta:=\|\hat{\boldsymbol{\vartheta}}(k)-\boldsymbol{\vartheta}\|/\|\boldsymbol{\vartheta}\|$随$k$变化曲线, 其中$\boldsymbol{\vartheta}$为参数真值, $\hat{\boldsymbol{\vartheta}}(k)$为$\boldsymbol{\vartheta}$的估计值, 图3给出了不同遗忘因子时, FN–FFSG算法参数估计误差δ随k变化曲线, 图4给出了FN–FFSG算法各参数估计值随k变化曲线图.

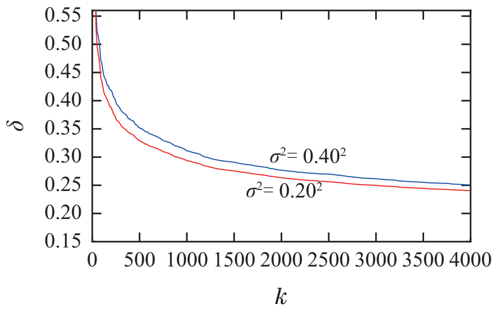

图 2　FN–SG算法参数估计误差δ随k变化曲线($\sigma^2=0.40^2$和$\sigma^2=0.20^2$)

Fig. 2　The FN–SG estimation errors δ versus k with $\sigma^2=0.40^2$ and $\sigma^2=0.20^2$

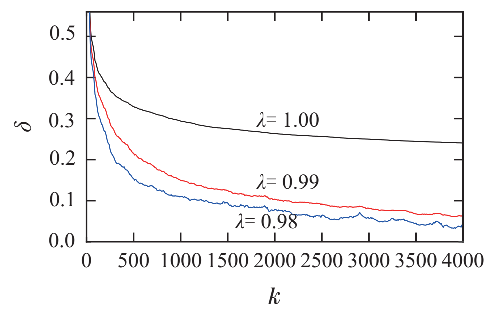

图 3　FN–FFSG算法参数估计误差δ随k变化曲线($\sigma^2=0.20^2$, $\lambda=1$, $\lambda=0.99$和$\lambda=0.98$)

Fig. 3　The FN–FFSG estimation errors δ versus k with $\sigma^2=0.20^2$, $\lambda=1$, $\lambda=0.99$ and $\lambda=0.98$

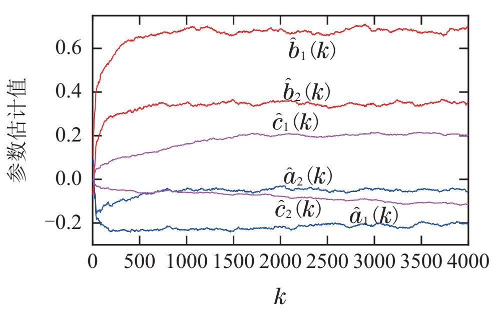

图 4　FN–FFSG算法参数估计值随k变化曲线($\sigma^2=0.20^2$和$\lambda=0.98$)

Fig. 4　The FN–FFSG parameter estimates versus k with $\sigma^2=0.20^2$ and $\lambda=0.98$

对于模型验证, 使用从$k=L_e+1=4001$到$k=$

$L_e + L_r = 4400$的$L_r = 400$组输入输出数据进行验证. 图5给出了系统输入输出数据曲线. 利用噪声方差$\sigma^2 = 0.20^2$, $k = 4000$时得到的参数估计作为最终的模型估计, 分别得到由FN–SG算法和FN–FFSG算法($\lambda = 0.98$)所获得的预测模型, 其对应的模型输出分别为$\hat{y}_1(k)$和$\hat{y}_2(k)$, 即

$$\begin{aligned}
&\hat{y}_1(k) = \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{b}}(L_e-1),k)\hat{\boldsymbol{\vartheta}}(L_e) = \\
&-0.22570y(k-1) - 0.09715y(k-2) + \\
&0.58164u(k-1) + 0.29207u(k-2) + \\
&0.58161\,(0.07359\sin(y^2(k-1)) - \\
&0.04446\sin(y^3(k-1))) + \\
&0.29210\,(0.07359\sin(y^2(k-2)) - \\
&0.04446\sin(y^3(k-2))), \\
&\hat{y}_2(k) = \boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{b}}(L_e-1),k)\hat{\boldsymbol{\vartheta}}(L_e) = \\
&-0.20218y(k-1) - 0.05060y(k-2) + \\
&0.69814u(k-1) + 0.35007u(k-2) + \\
&0.69651\,(0.19756\sin(y^2(k-1)) - \\
&0.11197\sin(y^3(k-1))) + \\
&0.35194\,(0.19756\sin(y^2(k-2)) - \\
&0.11197\sin(y^3(k-2))).
\end{aligned}$$

通过计算估计模型输出误差均方根(mean square error, MSE)

$$\mathrm{MSE}(\hat{y}_i(k)) = \Big(\frac{1}{L_r}\sum_{k=L_e+1}^{L_e+L_r}(\hat{y}_i(k) - y(k))^2\Big)^{\frac{1}{2}}$$

评价估计模型质量, 并得到$\mathrm{MSE}(\hat{y}_1(k)) = 0.24908$, $\mathrm{MSE}(\hat{y}_2(k)) = 0.20427$. 图6–7给出了FN–SG算法和FN–FFSG算法系统输出与模型输出及其误差随k变化情况曲线图.

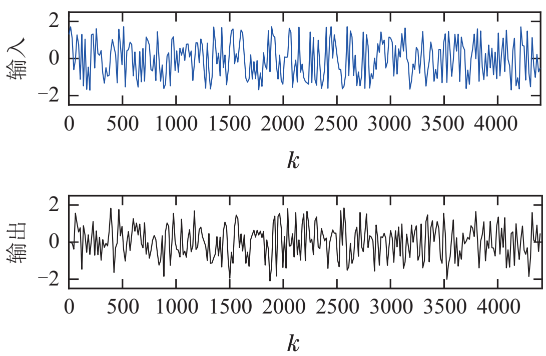

图 5 系统输入和输出曲线图

Fig. 5 The input and output signals

从表1–3可以看出, 随着数据长度的增加, FN–SG算法和FN–FFSG算法参数估计误差越小, 参数估计精度越高. 由图2–4 可知, FN–SG算法收敛速度很慢且对噪声变化不敏感. 在引入遗忘因子后, FN–SG算法的收敛速度明显加快. 遗忘因子越小, FN–FFSG算法参数估计误差也越小, 且参数估计收敛速度越快. 在算法开始阶段, FN–FFSG 算法波动较为平缓, 随着k的增加, FN–FFSG 参数估计逐渐逼近真值. 由图 6–7 可知, FN–SG 算法和 FN–FFSG 算法所获得的预测模型输出都十分接近于系统输出, 模型输出能跟踪系统输出, 说明估计模型能很好地捕捉系统动态. 比较算法估计模型输出误差均方根大小, 发现FN–FFSG算法估计模型输出误差均方根数值小于FN–SG算法估计模型输出误差均方根, 且与噪声的标准差σ十分接近, 说明其对应的模型质量较好.

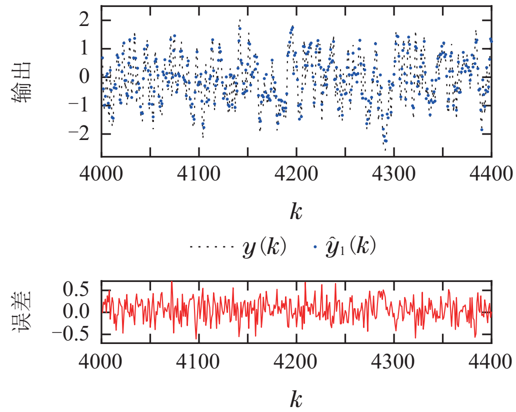

图 6 FN–SG算法获得的预测模型输出$\hat{y}_1(k)$与系统输出$y(k)$及其误差随k变化曲线图($\sigma^2 = 0.20^2$)

Fig. 6 The FN–SG model output $\hat{y}_1(k)$ and the system output $y(k)$ versus k with $\sigma^2 = 0.20^2$

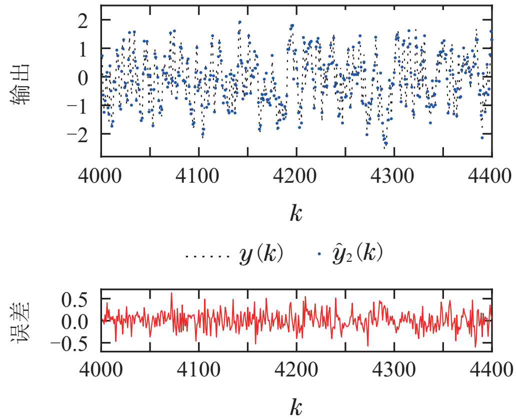

图 7 FN–FFSG算法获得的预测模型输出$\hat{y}_2(k)$与系统输出$y(k)$及其误差随k变化曲线图($\sigma^2 = 0.20^2$)

Fig. 7 The FN–FFSG model output $\hat{y}_2(k)$ and the system output $y(k)$ versus k with $\sigma^2 = 0.20^2$

6 结语

针对反馈非线性受控自回归系统, 本文通过极小化梯度准则函数, 提出了反馈非线性受控自回归系统随机梯度算法和遗忘因子随机梯度算法, 研究了持续激励条件下随机梯度算法的收敛性能, 证明了参数估计可以收敛到真值. 结合辅助模型辨识思想、递阶辨识理论等新辨识方法[26–28], 本文提出的算法可以推广到复杂干扰下随机系统[29–31]的辨识.

表 1 FN–SG算法参数估计及误差($\sigma^2 = 0.40^2$和$\sigma^2 = 0.20^2$)

Table 1 The FN–SG parameter estimates and their errors with $\sigma^2 = 0.40^2$ and $\sigma^2 = 0.20^2$

σ^2	k	a_1	a_2	b_1	b_2	c_1	c_2	δ/%
0.40^2	100	−0.15888	−0.15673	0.41742	0.16201	0.04737	−0.03037	46.77152
	200	−0.17802	−0.14241	0.44130	0.19783	0.05219	−0.03214	41.87076
	500	−0.19518	−0.13279	0.49252	0.23022	0.05740	−0.03712	35.23550
	1000	−0.20537	−0.12177	0.52248	0.25358	0.06271	−0.03891	31.10515
	2000	−0.21265	−0.11185	0.54776	0.27343	0.06861	−0.04111	27.66355
	3000	−0.21561	−0.10743	0.56004	0.28057	0.07168	−0.04241	26.16061
	4000	−0.21784	−0.10458	0.56924	0.28634	0.07398	−0.04397	25.03627
0.20^2	100	−0.17362	−0.13514	0.42985	0.18780	0.04574	−0.03177	43.56201
	200	−0.19317	−0.13053	0.45951	0.21865	0.05250	−0.03492	38.80114
	500	−0.20681	−0.11985	0.50737	0.24647	0.05655	−0.03884	32.96477
	1000	−0.21518	−0.11195	0.53661	0.26515	0.06201	−0.04031	29.37505
	2000	−0.22166	−0.10345	0.56139	0.28142	0.06812	−0.04224	26.32987
	3000	−0.22388	−0.09961	0.57322	0.28756	0.07125	−0.04335	24.98450
	4000	−0.22570	−0.09715	0.58164	0.29207	0.07359	−0.04446	24.04200
	真值	−0.18000	−0.06000	0.68000	0.36000	0.20000	−0.12000	–

表 2 FN–FFSG算法参数估计及误差($\sigma^2 = 0.20^2$和$\lambda = 0.99$)

Table 2 The FN–FFSG parameter estimates and their errors with $\sigma^2 = 0.20^2$ and $\lambda = 0.99$

k	a_1	a_2	b_1	b_2	c_1	c_2	δ/%
100	−0.19266	−0.13848	0.45896	0.21219	0.05481	−0.03674	39.23288
200	−0.21976	−0.12951	0.50531	0.25663	0.06790	−0.04139	32.31174
500	−0.23201	−0.09577	0.60264	0.30208	0.08338	−0.05081	21.55718
1000	−0.23442	−0.06060	0.66632	0.33559	0.11814	−0.05126	14.91116
2000	−0.22760	−0.04025	0.68399	0.34856	0.16554	−0.06207	10.37556
3000	−0.22071	−0.04519	0.68297	0.33791	0.18715	−0.07570	8.12135
4000	−0.21709	−0.04666	0.68913	0.34310	0.19161	−0.09205	6.37608
真值	−0.18000	−0.06000	0.68000	0.36000	0.20000	−0.12000	–

表 3 FN–FFSG算法参数估计及误差($\sigma^2 = 0.20^2$和$\lambda = 0.98$)

Table 3 The FN–FFSG parameter estimates and their errors with $\sigma^2 = 0.20^2$ and $\lambda = 0.98$

k	a_1	a_2	b_1	b_2	c_1	c_2	δ/%
100	−0.20921	−0.13955	0.48749	0.23530	0.06520	−0.04199	35.09446
200	−0.23770	−0.12208	0.54639	0.28750	0.08561	−0.04653	26.73416
500	−0.23612	−0.07334	0.64897	0.32734	0.11373	−0.06157	15.41409
1000	−0.23223	−0.04223	0.69172	0.35183	0.16265	−0.06034	10.96299
2000	−0.21849	−0.03179	0.68935	0.35802	0.20246	−0.07745	7.83157
3000	−0.20936	−0.04365	0.68066	0.34337	0.21230	−0.09423	5.70604
4000	−0.20218	−0.05060	0.69814	0.35077	0.19756	−0.11197	3.95122
真值	−0.18000	−0.06000	0.68000	0.36000	0.20000	−0.12000	–

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