快速对角阵权系数协方差交叉融合容积卡尔曼滤波器

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摘要: 针对互协方差信息未知的多传感器系统, 本文提出了一种快速对角阵权系数协方差交叉融合算法(FDCI). 本文首先提出了一种对角阵权系数协方差交叉融合(DCI)方案, 并证明了所提出DCI算法在融合估计精度上高于经典批处理CI融合(BCI)算法. 在此基础之上, 针对非线性等复杂的互协方差未知的多传感器系统, 提出FDCI算法, 并证明了所提出FDCI 算法的无偏性及鲁棒精度. FDCI融合算法虽然在融合估计精度上低于DCI, 但FDCI 无需进行多权系数的非线性代价函数的优化问题, 进而大大降低了计算负担, 提高了系统的实时性. 最后, 结合容积卡尔曼滤波算法(CKF)提出了快速对角阵权系数协方差交叉融合容积卡尔曼滤波算法. 仿真实例验证了所提出算法的正确性和有效性.

关键词: 非线性系统; 协方差交叉融合; 容积卡尔曼滤波器

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Fast covariance intersection fusion weighted by diagonal matrix cubature Kalman filter

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Abstract: For multi-sensor systems with unknown cross-covariance, a fast covariance intersection fusion algorithm weighted by diagonal matrix (FDCI) is proposed. First, the covariance intersection fusion algorithm weighted by diagonal matrix (DCI) is proposed in this paper, and it is proved that the fusion estimation accuracy of the DCI algorithm is higher than that of the classical batch CI fusion (BCI) algorithm. Furthermore, for complex multi-sensor systems with unknown cross-covariances such as nonlinear systems, the FDCI algorithm is proposed, and its unbiasedness and robust accuracy are proved. Although the FDCI fusion algorithm has lower fusion estimation accuracy than DCI, the FDCI algorithm does not involve the optimization of nonlinear cost function with multiple weight coefficients, which greatly reduces the computational burden and improves the real-time performance of the system. Finally, combining with the cubature Kalman filter algorithm (CKF), a fast covariance intersection fusion weighted by the diagonal matrix CKF algorithm is proposed. A simulation example verifies the correctness and effectiveness of the proposed algorithm.

Key words: nonlinear system; covariance intersection fusion; cubature Kalman filter

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1 引言

近年, 随着传感器和计算机技术不断发展, 系统感知能力不断提升, 其捕获信息量不断增加. 信息融合在此背景下成为多传感器系统不可或缺的重要环节之一. 多传感器融合技术不仅可以提高感知精度, 而且提高了系统的鲁棒性、灵活度和可信度. 因此信息融合被广泛地应用于目标识别、导航制导、工业控制、故障诊断等军事和民用领域.

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融合估计技术作为信息融合的重要组成部分, 其主要融合方式[1–5]分为集中式融合和分布式融合两种. 集中式融合方法由于没有信息损失因此是最优的, 但当传感器数量或者观测数据维度较大时, 会给系统带来严重的计算负担. 分布式融合方法以某种融合准则对局部估计进行融合. 相比于集中式融合, 尽管分布式融合不是最优的, 但它降低了计算量, 增加了系统的容错性和鲁棒性, 因此备受学者们的关注. 常用的分布式融合方法有矩阵加权[6]、对角阵加权[7]和标量加权[8]等. 在线性最小方差意义下, 上述3种融合算法是最优的并具有很好的融合精度, 且融合精度关系为矩阵加权高于对角阵加权高于标量加权. 分布式观测融合是将各子系统的观测进行最优加权得到一个融合的观测方程, 然后联立状态方程, 进行分布式融合[9]. 文献[9–10]给出了加权观测融合算法, 并通过信息滤波算法证明了加权观测融合与集中式融合数值等价. 但计算最优权值需要准确的知道各系统间的互协方差. 然而在许多情况下, 尤其是针对非线性系统而言, 互协方差很难获得或者是不可得到的. 因此, 文献[11–12]提出了协方差交叉(covariance intersection, CI)融合算法, CI融合可以有效地解决实际工程中互协方差未知或很难辨识的融合估计问题[11–16].

CI融合算法不需要获取子系统间的相关性, 通过局部系统的估计误差方差阵的凸组合得到一个融合误差方差上界, 并且通过附加的权系数加强了估计的一致性, 从而得到一个较为合理的融合估计. 由于CI融合结构简单, 易于实施, 国内外学者提出了多种CI融合方案. 文献[16]提出了协方差交叉融合稳态卡尔曼滤波器, 证明了它的一致性, 同时指出相比于矩阵, 对角阵和标量加权融合方法, CI融合能够避免互协方差的计算, 不仅可以显著降低计算量, 而且适用性更强. 文献[17]证明了在处理两个相关性未知的变量时, CI融合可以给出最优解. 并且在融合估计协方差上界的迹最小的条件下求解最优加权系数是在一维曲线上进行搜索, 而不是在整个参数空间内进行搜索. 文献[18]证明了CI融合算法是两个估计在完全未知相关性的条件下的最优边界算法, 但当融合3个或3个以上变量时, CI融合不一定能给出满意的估计.

CI融合按融合方式可分为批处理CI融合(batch covariance intersection, BCI) 和序贯 CI 融合 (sequential covariance intersection, SCI)[19]. 序贯CI融合是将多个估计融合过程分解为多次两个估计的融合过程, 因此只需要解决若干个一维非线性代价函数的优化问题, 大大降低了计算负担, 但融合顺序会影响融合精度. BCI融合是将多个估计一次性融合, 但需要解决关于权系数的高维非线性代价函数优化问题, 因此当传感器数目较大时, 计算量和复杂度倍增. 为了解决权系数的优化问题, 文献[20]通过求解线性方程组得到权系数的近似解, 从而提出了一种快速协方差交叉融合算法, 该方法避免了优化过程, 大大减少了计算量. 文献[21]在文献[20]的基础上提出了改进的快速协方差交叉融合算法, 虽然它的计算量稍有增加, 但在某些情况下可以得到更好的融合效果. 文献[22]提出了一种改进的快速协方差交叉融合算法, 用逆方差代替文献[20]中的方差计算加权系数. 基于该算法得到的序贯快速协方差交叉融合算法是一致的, 并且融合精度与融合顺序无关.

非线性环节广泛存在于各种系统, 因此非线性系统的融合估计适应度更广, 更具有实际工程应用价值. 常用的非线性滤波算法包括扩展卡尔曼滤波算法(extend Kalman filter, EKF)[23], 无迹卡尔曼滤波算法(unscented Kalman filter, UKF)[24], 容积卡尔曼滤波算法(cubature Kalman filter, CKF)[25–26], 粒子滤波算法(particle filter, PF)[27] 等. 由于非线性多传感器系统互协方差不易求得, 因此非线性多传感器系统的融合估计问题一直得不到很好的解决[28–31]. 正因如此, Julier等人将CI融合引入非线性多传感器系统[14].

本文针对互协方差未知的多传感器系统, 提出了一种快速对角阵权系数协方差交叉融合(fast covariance intersection fusion weighted by diagonal matrix, FDCI)算法. 主要创新点如下: 1) 基于批处理CI融合框架, 提出了对角阵权系数CI融合(covariance intersection fusion weighted by diagonal matrix, DCI)算法. 该算法用正定对角阵权系数代替标量权系数, 充分考虑了子系统各分量间的差异, 较经典CI融合可以获得更高的融合精度; 2) 基于快速CI 融合算法和所提出DCI融合算法, 提出了FDCI算法. 证明了FDCI算法的无偏性和鲁棒精度, 在理论上证明了FDCI融合算法的可行性. 该算法避免了权系数的优化问题, 大大降低了计算负担, 更适用于互协方差矩阵难以获得且估计误差方差不稳定的非线性多传感器系统; 3) 结合CKF滤波算法, 提出了快速对角阵权系数CI融合CKF滤波算法 (fast covariance intersection fusion cubature Kalman filter weighted by diagonal matrix, FDCI-CKF), 该算法可以处理互协方差未知的非线性多传感器系统的融合估计问题, 且具有很好的实时性和较高的融合精度.

2 问题阐述

考虑多传感器非线性系统

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k) + \boldsymbol{w}_k, \tag{1}$$

$$\boldsymbol{z}_k^{(j)} = \boldsymbol{h}^{(j)}(\boldsymbol{x}_k) + \boldsymbol{v}_k^{(j)},\ j = 1, 2, \cdots, L, \tag{2}$$

其中: $\boldsymbol{x}_k \in \mathbb{R}^{n\times 1}$表示$k$时刻的状态, $\boldsymbol{z}_k^{(j)}$为k时刻第j个传感器的观测, $\boldsymbol{f}(\boldsymbol{x}_k)$和$\boldsymbol{h}^{(j)}(\boldsymbol{x}_k)$分别是已知的非线性过程传递函数和观测方程. 过程噪声$\boldsymbol{w}_k \in \mathbb{R}^{m\times 1}$

和第j个传感器的观测噪声$\boldsymbol{v}_k^{(j)}\in\mathbb{R}^{m\times 1}$是零均值, 方差分别为$\boldsymbol{Q}_k$和$\boldsymbol{R}_k^{(j)}$的不相关高斯白噪声, 即

$$
\mathrm{E}\{\begin{bmatrix}\boldsymbol{w}_k\\ \boldsymbol{v}_k^{(i)}\end{bmatrix}[\boldsymbol{w}_t^{\mathrm{T}}\ (\boldsymbol{v}_t^{(j)})^{\mathrm{T}}]\}=
\begin{bmatrix}\boldsymbol{Q}_k & 0\\ 0 & \boldsymbol{R}_k^{(j)}\boldsymbol{\delta}_{ij}\end{bmatrix}\delta_{kt},\ \ i,j=1,2,\cdots,L, \tag{3}
$$

其中: E是数学期望运算, T代表矩阵转置, $\boldsymbol{\delta}_{ii}=1$ $(i=j)$, $\boldsymbol{\delta}_{ij}=0\ (i\neq j)$.

在实际应用中, 对于线性系统, 误差方差$\boldsymbol{P}_{k|k}^{(j)}$可通过Riccati方程迭代求解且达到稳态时存在稳态误差方差和互协方差[19]. 而对于非线性系统, 由于包含非线性环节, 系统辨识困难, 计算$\boldsymbol{P}_{k|k}^{(ij)}$涉及很难求解的复杂的高维积分运算, 并且不存在稳定值[26], 因此带未知互协方差信息的非线性多传感器系统具有较高的研究价值. Julier等人[14]率先将CI融合引入到非线性系统融合估计之中. 该分布式融合结构简单易于实施, 更适用于情况复杂的非线性融合问题.

3　经典CI融合算法和对角阵权系数CI融合算法

3.1　批处理CI融合(BCI)算法

当局部估计的估计误差方差阵$\boldsymbol{P}_{k|k}^{(j)}(j=1,\cdots,L)$已知, 互协方差$\boldsymbol{P}_{k|k}^{(ij)}\ (i,j=1,\cdots,L,i\neq j)$未知时, 则批处理CI融合(BCI)估计及其误差方差上界分别为[19]

$$
(\boldsymbol{P}_{k|k}^{(\mathrm{BCI})})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{BCI})}=\sum_{j=1}^{L}\omega_k^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(j)}, \tag{4}
$$

$$
(\boldsymbol{P}_{k|k}^{(\mathrm{BCI})})^{-1}=\sum_{j=1}^{L}\omega_k^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}, \tag{5}
$$

关于$\omega_k^{(j)}\in[0,1]$, $\sum_{j=1}^{L}\omega_k^{(j)}=1$的极小代价函数为

$$
\min J=\min_{\omega_k^{(j)}\in[0,1]}\mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{BCI})})=
\min_{\omega_k^{(j)}\in[0,1],\sum_{j=1}^{L}\omega_k^{(j)}=1}\mathrm{tr}\{[\sum_{j=1}^{L}\omega_k^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}]^{-1}\}, \tag{6}
$$

批处理CI融合实际的融合误差方差阵$\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{BCI})}$为

$$
\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{BCI})}=\boldsymbol{P}_{k|k}^{(\mathrm{BCI})}[\sum_{i=1}^{L}\sum_{j=1}^{L}\omega_k^{(i)}\omega_k^{(j)}(\boldsymbol{P}_{k|k}^{(i)})^{-1}(\bar{\boldsymbol{P}}_{k|k}^{(ij)})\times
(\boldsymbol{P}_{k|k}^{(j)})^{-1}]\boldsymbol{P}_{k|k}^{(\mathrm{BCI})}. \tag{7}
$$

3.2　快速CI融合(FCI)

当局部估计的估计误差方差阵$\boldsymbol{P}_{k|k}^{(j)}\ (j=1,\cdots,L)$已知, 互协方差$\boldsymbol{P}_{k|k}^{(ij)}\ (i,j=1,\cdots,L,i\neq j)$未知时, 则快速CI(fast covariance intersection fusion, FCI)融合估计及其误差方差上界分别为[20]

$$
(\boldsymbol{P}_{k|k}^{(\mathrm{FCI})})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FCI})}=\sum_{j=1}^{L}\omega_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(j)}, \tag{8}
$$

$$
(\boldsymbol{P}_{k|k}^{(\mathrm{FCI})})^{-1}=\sum_{j=1}^{L}\omega_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}, \tag{9}
$$

其中$\omega_{f,k}^{(j)}\ (j=1,\cdots,L)$有

$$
\omega_{f,k}^{(j)}=\frac{1/\mathrm{tr}(\boldsymbol{P}_{k|k}^{(j)})}{\sum_{i=1}^{L}1/\mathrm{tr}(\boldsymbol{P}_{k|k}^{(i)})}. \tag{10}
$$

在BCI融合算法中, 涉及关于权系数的优化问题, 当系统维数和复杂程度过高时, 往往不能得到较合理的权系数, 甚至还会陷入局部最优解, 而且优化过程极为耗时. 每时刻进行优化求解将给整个系统带来严重的计算负担且不能保证其不陷入局部最优. 而快速CI融合算法尽管理论上融合精度低于BCI, 但该算法通过局部误差方差的迹来计算权值, 不涉及复杂的优化问题, 避免了因陷入局部最优而带来的精度损失. 因此快速CI融合更适用于非线性等复杂的多传感器系统的融合估计问题.

在经典CI融合算法中, 由于加权系数为标量, 因此局部估计的不同分量之间的差异没有被充分考虑. 本文基于BCI融合框架, 用对角阵权系数代替标量权系数, 提出了对角阵权系数CI融合DCI算法, 并进行了精度对比分析. 考虑到更复杂的系统如估计误差方差的不稳定的非线性多传感器系统, 优化过程更加困难, 因此提出了快速对角阵权系数CI融合(FDCI)算法, 并证明了FDCI算法的无偏性和鲁棒精度.

3.3　对角阵权系数CI融合(DCI)算法

算法1　当局部估计的估计误差方差阵$\boldsymbol{P}_{k|k}^{(j)}\ (j=1,\cdots,L)$已知, 互协方差$\boldsymbol{P}_{k|k}^{(ij)}\ (i,j=1,\cdots,L,i\neq j)$未知时, 对角阵权系数CI (DCI)融合估计及其误差方差的上界分别为

$$
(\boldsymbol{P}_{k|k}^{(\mathrm{DCI})})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{DCI})}=\sum_{j=1}^{L}\boldsymbol{W}_k^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(j)}, \tag{11}
$$

$$
(\boldsymbol{P}_{k|k}^{(\mathrm{DCI})})^{-1}=\sum_{j=1}^{L}\boldsymbol{W}_k^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}, \tag{12}
$$

对角阵加权系数$\boldsymbol{W}_k^{(j)}=\mathrm{diag}\{\omega_k^{(j1)},\cdots,\omega_k^{(jn)}\}$, $\sum_{j=1}^{L}\boldsymbol{W}_k^{(j)}=\boldsymbol{I}_n$的极小代价函数为

$$
\min J=\min_{\sum_{j=1}^{L}\boldsymbol{W}_k^{(j)}=\boldsymbol{I}_n}\mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{DCI})})=
\min_{\sum_{j=1}^{L}\boldsymbol{W}_k^{(j)}=\boldsymbol{I}_n}\mathrm{tr}\{[\sum_{j=1}^{L}\boldsymbol{W}_k^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}]^{-1}\}. \tag{13}
$$

定理 1　对角阵权系数CI融合DCI算法的鲁棒融合精度高于批处理CI融合(BCI)算法的鲁棒融合精度, 即

$$
\mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{DCI})})\leqslant\mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{BCI})}). \tag{14}
$$

证 由式(13)有

$$\mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{DCI})}) = \min J = \mathrm{tr}\{[\sum_{j=1}^{L} \boldsymbol{W}_k^{(j)} (\boldsymbol{P}_{k|k}^{(j)})^{-1}]^{-1}\}, \tag{15}$$

令$\boldsymbol{\theta}_k^{(j)} = \omega_k^{(j)} \boldsymbol{I}_n$, $\boldsymbol{\theta}_k^{(j)}$代替式(6)中的$\omega_k^{(j)}$等式不变, 所以有

$$\mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{BCI})}) = \min J = \min_{\omega_k^{(j)}\in[0,1], \sum_{j=1}^{L} \boldsymbol{\theta}_k^{(j)} = \boldsymbol{I}_n} \mathrm{tr}\{[\sum_{j=1}^{L} \boldsymbol{\theta}_k^{(j)} (\boldsymbol{P}_{k|k}^{(j)})^{-1}]^{-1}\}, \tag{16}$$

取特殊值$\boldsymbol{W}_k^{(j)} = \boldsymbol{\theta}_k^{(j)}, j = 1, \cdots, L$时, 有

$$\min J = \mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{DCI})}) = \mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{BCI})}).$$

由于$\mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{DCI})})$是$J$相对于$\boldsymbol{W}_k^{(j)}, j=1,\cdots,L$的最小值, 所以有

$$\mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{DCI})}) \leqslant \mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{BCI})}). \tag{17}$$

证毕.

注 1 尽管定理1证明了对角阵权系数CI融合的精度优于批处理CI融合的精度, 但由式(13)可知, 对角阵权系数CI融合算法涉及更多的变量优化问题, 因此在进行优化时会因为传感器的数量, 估计的维度, 系统的复杂度等因素的影响找不到满意的权值而影响融合精度. 此外计算量和计算时间会随着传感器的数量和估计的维度成倍增长. 因此针对上述问题, 结合快速CI融合算法[20], 提出了快速对角阵权系数CI融合算法.

3.4 快速对角阵权系数CI融合(FDCI)算法

算法2 当局部估计的估计误差方差阵$\boldsymbol{P}_{k|k}^{(j)}$ $(j=1,\cdots,L)$已知, 互协方差$\boldsymbol{P}_{k|k}^{(ij)}$ $(i,j=1,\cdots,L, i\neq j)$未知时, 快速对角阵权系数CI融合(FDCI) 估计及其误差方差的上界分别为

$$(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{-1} \hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})} = \sum_{j=1}^{L} \boldsymbol{W}_{f,k}^{(j)} (\boldsymbol{P}_{k|k}^{(j)})^{-1} \hat{\boldsymbol{x}}_{k|k}^{(j)}, \tag{18}$$

$$(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{-1} = \sum_{j=1}^{L} \boldsymbol{W}_{f,k}^{(j)} (\boldsymbol{P}_{k|k}^{(j)})^{-1}, \tag{19}$$

其中对角阵加权系数$\boldsymbol{W}_{f,k}^{(j)} (j=1,\cdots,L)$满足$\sum_{j=1}^{L} \boldsymbol{W}_{f,k}^{(j)} = \boldsymbol{I}_n$, 每个对角阵加权系数的对角元素为

$$\begin{cases} \boldsymbol{W}_{f,k}^{(j)} = \mathrm{diag}\{\varepsilon_k^{(j1)}, \cdots, \varepsilon_k^{(jn)}\}, \\ \varepsilon_k^{(ju)} = \dfrac{1/\boldsymbol{P}_{k|k}^{(j)}(u,u)}{\sum_{i=1}^{L} 1/\boldsymbol{P}_{k|k}^{(i)}(u,u)}, u = 1, 2, \cdots, n. \end{cases} \tag{20}$$

其中n为系统维数.

证 类似文献[20], 设$L = 2, n = 2$, 两对角阵加权系数满足

$$\boldsymbol{W}_{f,k}^{(1)} + \boldsymbol{W}_{f,k}^{(2)} = \boldsymbol{I}_2, \tag{21}$$

其中: $\boldsymbol{W}_{f,k}^{(1)} = \mathrm{diag}\{\varepsilon_k^{(11)}, \varepsilon_k^{(12)}\}, \boldsymbol{W}_{f,k}^{(2)} = \mathrm{diag}\{\varepsilon_k^{(21)}, \varepsilon_k^{(22)}\}$. 状态估计$\hat{\boldsymbol{x}}_k^{(j)}$的误差方差矩阵的对角元素表征了估计分量的精度. 所以当$\boldsymbol{P}_{k|k}^{(1)}(u,u) = \boldsymbol{P}_{k|k}^{(2)}(u,u)$, $u=1,2,\cdots,n$时, 有理由认为

$$\boldsymbol{W}_{f,k}^{(1)} = \boldsymbol{W}_{f,k}^{(2)}, \tag{22}$$

而当$\boldsymbol{P}_{k|k}^{(1)}(u,u) \to 0$且$\boldsymbol{P}_{k|k}^{(1)}(u,u) \ll \boldsymbol{P}_{k|k}^{(2)}(u,u)$时有理由认为

$$\boldsymbol{W}_{f,k}^{(1)} \to \mathrm{diag}\{1,1\}, \boldsymbol{W}_{f,k}^{(2)} \to \mathrm{diag}\{0,0\}, \tag{23}$$

所以结合式(22)–(23)可得

$$\boldsymbol{W}_{f,k}^{(1)}(u,u)\boldsymbol{P}_{k|k}^{(1)}(u,u) - \boldsymbol{W}_{f,k}^{(2)}(u,u)\boldsymbol{P}_{k|k}^{(2)}(u,u) = 0, \quad u = 1, 2, \cdots, n, \tag{24}$$

即

$$\begin{cases} \varepsilon_k^{(11)} \boldsymbol{P}_{k|k}^{(1)}(1,1) - \varepsilon_k^{(21)} \boldsymbol{P}_{k|k}^{(2)}(1,1) = 0, \\ \varepsilon_k^{(12)} \boldsymbol{P}_{k|k}^{(1)}(2,2) - \varepsilon_k^{(22)} \boldsymbol{P}_{k|k}^{(2)}(2,2) = 0, \end{cases} \tag{25}$$

由式(21)和式(25)有

$$\begin{cases} \begin{bmatrix} \boldsymbol{P}_{k|k}^{(1)}(1,1) & -\boldsymbol{P}_{k|k}^{(2)}(1,1) \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_k^{(11)} \\ \varepsilon_k^{(21)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ \begin{bmatrix} \boldsymbol{P}_{k|k}^{(1)}(2,2) & -\boldsymbol{P}_{k|k}^{(2)}(2,2) \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_k^{(12)} \\ \varepsilon_k^{(22)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \end{cases} \tag{26}$$

解方程组(26)可得

$$\begin{cases} \varepsilon_k^{(11)} = \boldsymbol{P}_{k|k}^{(2)}(1,1)/(\boldsymbol{P}_{k|k}^{(1)}(1,1) + \boldsymbol{P}_{k|k}^{(2)}(1,1)), \\ \varepsilon_k^{(21)} = \boldsymbol{P}_{k|k}^{(1)}(1,1)/(\boldsymbol{P}_{k|k}^{(1)}(1,1) + \boldsymbol{P}_{k|k}^{(2)}(1,1)), \end{cases}$$
$$\begin{cases} \varepsilon_k^{(12)} = \boldsymbol{P}_{k|k}^{(2)}(2,2)/(\boldsymbol{P}_{k|k}^{(1)}(2,2) + \boldsymbol{P}_{k|k}^{(2)}(2,2)), \\ \varepsilon_k^{(22)} = \boldsymbol{P}_{k|k}^{(1)}(2,2)/(\boldsymbol{P}_{k|k}^{(1)}(2,2) + \boldsymbol{P}_{k|k}^{(2)}(2,2)), \end{cases} \tag{27}$$

同理, 当$L > 2, n > 2$时, 有

$$\boldsymbol{W}_{f,k}^{(1)} + \boldsymbol{W}_{f,k}^{(2)} + \cdots + \boldsymbol{W}_{f,k}^{(L)} = \boldsymbol{I}_n, \tag{28}$$

其中$\boldsymbol{W}_{f,k}^{(j)} = \mathrm{diag}\{\varepsilon_k^{(j1)}, \varepsilon_k^{(j2)}, \cdots, \varepsilon_k^{(jn)}\}$.

扩展式(24)可以得到

$$\varepsilon_k^{(iu)} \boldsymbol{P}_{k|k}^{(i)}(u,u) - \varepsilon_k^{(ju)} \boldsymbol{P}_{k|k}^{(j)}(u,u) = 0, \quad u = 1, 2, \cdots, n;\ i, j = 1, 2, \cdots, L, \tag{29}$$

观察式(29)可发现, 式(29)是一个由nL^2个方程构成的高度冗余的齐次线性方程组. 因此为了避免冗余[18], 可以用它的最大的线性无关子集表示为

$$\varepsilon_k^{(ju)} \boldsymbol{P}_{k|k}^{(j)}(u,u) - \varepsilon_k^{((j+1)u)} \boldsymbol{P}_{k|k}^{(j+1)}(u,u) = 0, \quad u = 1, 2, \cdots, n;\ j = 1, 2, \cdots, L-1, \tag{30}$$

所以结合式(28)和式(30)可以得到如下方程组:

$$\begin{bmatrix} \boldsymbol{P}_{k|k}^{(1)}(u,u) & -\boldsymbol{P}_{k|k}^{(2)}(u,u) & 0 & \cdots & 0 & 0 \\ 0 & \boldsymbol{P}_{k|k}^{(2)}(u,u) & -\boldsymbol{P}_{k|k}^{(3)}(u,u) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \boldsymbol{P}_{k|k}^{(L-1)}(u,u) & -\boldsymbol{P}_{k|k}^{(L)}(u,u) \\ 1 & 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_k^{(1u)} \\ \varepsilon_k^{(2u)} \\ \vdots \\ \varepsilon_k^{((L-1)u)} \\ \varepsilon_k^{(Lu)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$u=1,\ 2,\cdots,\ n, \tag{31}$$

由克莱姆法则求解方程组(31)可以得到

$$\varepsilon_k^{(ju)} = \frac{\prod\limits_{\substack{i=1\\i\neq j}}^{L} \boldsymbol{P}_{k|k}^{(i)}(u,u)}{\sum\limits_{i=1}^{L} \prod\limits_{\substack{m=1\\m\neq i}}^{L} \boldsymbol{P}_{k|k}^{(m)}(u,u)} = \frac{1/\boldsymbol{P}_{k|k}^{(j)}(u,u)}{\sum\limits_{i=1}^{L} 1/\boldsymbol{P}_{k|k}^{(i)}(u,u)},$$

$$u=1,2,\cdots,n,\ j=1,2,\cdots,L. \tag{32}$$

证毕.

定理 2　快速对角阵权系数CI融合估计$\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})}$的实际融合误差方差$\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{FDCI})}$为

$$\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{FDCI})} = \boldsymbol{P}_{k|k}^{(\mathrm{FDCI})}[\sum_{i=1}^{L}\sum_{j=1}^{L} \boldsymbol{W}_{f,k}^{(i)}(\boldsymbol{P}_{k|k}^{(i)})^{-1} \times \bar{\boldsymbol{P}}_{k|k}^{(ij)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\boldsymbol{W}_{f,k}^{(j)}](\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{\mathrm{T}}. \tag{33}$$

其中: $\boldsymbol{P}_{k|k}^{(i)}, \boldsymbol{P}_{k|k}^{(j)}, \boldsymbol{W}_{f,k}^{(i)}, \boldsymbol{W}_{f,k}^{(j)}$分别表示第$i$个和第$j$个传感器的误差方差阵和对应的融合加权系数, $\bar{\boldsymbol{P}}_{k|k}^{(ij)}$为第$i$个和第$j$个传感器之间的实际互协方差阵.

证　由式(18)–(19)可得

$$(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{-1}\boldsymbol{x}_k = \sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\boldsymbol{x}_k, \tag{34}$$

$$(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})} = \sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(j)}, \tag{35}$$

式(34)减式(35), 有

$$(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})} = \sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(j)}, \tag{36}$$

$$\tilde{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})} = \boldsymbol{P}_{k|k}^{(\mathrm{FDCI})}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(j)}, \tag{37}$$

则对角阵权系数CI融合实际融合误差方差$\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{FDCI})}$可计算为

$$\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{FDCI})} = \mathrm{E}[\tilde{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})}(\tilde{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})})^{\mathrm{T}}] = \boldsymbol{P}_{k|k}^{(\mathrm{FDCI})} \times [\sum_{i=1}^{L}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(i)}(\boldsymbol{P}_{k|k}^{(i)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(ij)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\boldsymbol{W}_{f,k}^{(j)}] \times (\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{\mathrm{T}}. \tag{38}$$

证毕.

定理 3　快速对角阵权系数CI融合估计$\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})}$具有如下性质:

1) 无偏性: $\mathrm{E}\{\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})}\} = \mathrm{E}\{\boldsymbol{x}_k\}$;

2) 鲁棒精度: $\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})}$的实际融合误差方差$\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{FDCI})}$与误差上界$\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})}$具有鲁棒精度关系

$$\mathrm{tr}(\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{FDCI})}) \leqslant \mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})}). \tag{39}$$

证　证明之前, 首先引入结论: 如果$\boldsymbol{A}=\mathrm{diag}\{[a_1,\cdots,a_n]\}$, $\boldsymbol{C}=\mathrm{diag}\{[c_1,\cdots,c_n]\}$, $\boldsymbol{B}\in\mathbb{R}^{n\times n}$, 则

$$\mathrm{tr}(\boldsymbol{ABC}) = \mathrm{tr}(\boldsymbol{CBA}), \tag{40}$$

由式(37)以及局部估计的无偏性有

$$\mathrm{E}\{\tilde{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})}\} = \mathrm{E}\{\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(j)}\} = \boldsymbol{P}_{k|k}^{(\mathrm{FDCI})}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\mathrm{E}\{\tilde{\boldsymbol{x}}_{k|k}^{(j)}\} = 0. \tag{41}$$

因而性质1成立.

由式(38), $\mathrm{tr}(\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{FDCI})}) \leqslant \mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})$可改写为

$$\mathrm{tr}\{\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})}[\sum_{i=1}^{L}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(i)}(\boldsymbol{P}_{k|k}^{(i)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(ij)}(\boldsymbol{P}_{k|k}^{(j)})^{-1} \times \boldsymbol{W}_{f,k}^{(j)}](\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{\mathrm{T}}\} \leqslant \mathrm{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})}). \tag{42}$$

不等式(42)两边同时左乘$(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{-1}$和右乘$((\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{\mathrm{T}})^{-1}$可得

$$\mathrm{tr}\{((\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{\mathrm{T}})^{-1} - \sum_{i=1}^{L}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(i)}(\boldsymbol{P}_{k|k}^{(i)})^{-1} \times \bar{\boldsymbol{P}}_{k|k}^{(ij)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\boldsymbol{W}_{f,k}^{(j)}\} \geqslant 0, \tag{43}$$

其中: $\boldsymbol{P}_{k|k}^{(j)}$为滤波估计误差方差矩阵, $\bar{\boldsymbol{P}}_{k|k}^{(j)}$表示$\hat{\boldsymbol{x}}_{k|k}^{(j)}$的实际估计协方差矩阵, 即

$$\bar{\boldsymbol{P}}_{k|k}^{(j)} = \mathrm{E}[(\hat{\boldsymbol{x}}_{k|k}^{(j)} - \boldsymbol{x})(\hat{\boldsymbol{x}}_{k|k}^{(j)} - \boldsymbol{x})^{\mathrm{T}}] = \mathrm{E}[\tilde{\boldsymbol{x}}_{k|k}^{(j)}\tilde{\boldsymbol{x}}_{k|k}^{(j)\mathrm{T}}], \tag{44}$$

由于估计是一致的, 所以有

$$\boldsymbol{P}_{k|k}^{(j)} - \bar{\boldsymbol{P}}_{k|k}^{(j)} \geqslant 0,\ j=1,\cdots,L, \tag{45}$$

式(45)两边同时左乘和右乘$(\boldsymbol{P}_{k|k}^{(j)})^{-1}$可得

$$(\boldsymbol{P}_{k|k}^{(j)})^{-1} \geqslant (\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}, \tag{46}$$

结合式(19)和式(46)有

$$\mathrm{tr}\{((\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{\mathrm{T}})^{-1}\} \geqslant$$

$$\operatorname{tr}\{\sum_{j=1}^{L}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\boldsymbol{W}_{f,k}^{(j)}\},\tag{47}$$

式(47)代入到式(43)得

$$\operatorname{tr}\{\sum_{j=1}^{L}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\boldsymbol{W}_{f,k}^{(j)}-\sum_{i=1}^{L}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(i)}(\boldsymbol{P}_{k|k}^{(i)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(ij)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\boldsymbol{W}_{f,k}^{(j)}\}\geqslant 0,\tag{48}$$

因此只要证明式(48)成立就等价于证明式(43)成立. 由$\sum_{i=1}^{L}\boldsymbol{W}_{f,k}^{(i)}=\boldsymbol{I}_n$有

$$\sum_{j=1}^{L}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\boldsymbol{W}_{f,k}^{(j)}=\sum_{i=1}^{L}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(i)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\boldsymbol{W}_{f,k}^{(j)},\tag{49}$$

将式(49)代入式(48)中, $\operatorname{tr}(\bar{\boldsymbol{P}}_{k|k}^{(\mathrm{FDCI})})\leqslant\operatorname{tr}(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})$可等价为

$$\operatorname{tr}(\boldsymbol{\Delta})=\operatorname{tr}\{\sum_{i=1}^{L}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(i)}[(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}-(\boldsymbol{P}_{k|k}^{(i)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(ij)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}]\boldsymbol{W}_{f,k}^{(j)}\}\geqslant 0,\tag{50}$$

交换下标i,j有

$$\operatorname{tr}(\boldsymbol{\Delta})=\operatorname{tr}\{\sum_{j=1}^{L}\sum_{i=1}^{L}\boldsymbol{W}_{f,k}^{(j)}[(\boldsymbol{P}_{k|k}^{(i)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(i)}(\boldsymbol{P}_{k|k}^{(i)})^{-1}-(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(ji)}(\boldsymbol{P}_{k|k}^{(i)})^{-1}]\boldsymbol{W}_{f,k}^{(i)}\}\geqslant 0,\tag{51}$$

式(50)和式(51)相加得到

$$\begin{aligned}2\operatorname{tr}(\boldsymbol{\Delta})=\operatorname{tr}\{&\sum_{i=1}^{L}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(i)}[(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}-\\&(\boldsymbol{P}_{k|k}^{(i)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(ij)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}]\boldsymbol{W}_{f,k}^{(j)}+\\&\boldsymbol{W}_{f,k}^{(j)}[(\boldsymbol{P}_{k|k}^{(i)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(i)}(\boldsymbol{P}_{k|k}^{(i)})^{-1}-\\&(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(ji)}(\boldsymbol{P}_{k|k}^{(i)})^{-1}]\boldsymbol{W}_{f,k}^{(i)}\}\geqslant 0,\end{aligned}\tag{52}$$

因为$\boldsymbol{W}_{f,k}^{(j)}\in\mathbb{R}^{n\times n}(j=1,2,\cdots,L)$为正定对角矩阵, 所以由式(40)和矩阵的迹的可加性, 式(52)可改写为

$$\begin{aligned}&\operatorname{tr}\{\sum_{i=1}^{L}\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(i)}[(\boldsymbol{P}_{k|k}^{(i)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(i)}(\mathbf{P}_{k|k}^{(i)})^{-1}-\\&(\boldsymbol{P}_{k|k}^{(i)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(ij)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}+(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}-\\&(\boldsymbol{P}_{k|k}^{(j)})^{-1}\bar{\boldsymbol{P}}_{k|k}^{(ji)}(\boldsymbol{P}_{k|k}^{(i)})^{-1}]\boldsymbol{W}_{f,k}^{(j)}\}\geqslant 0,\end{aligned}\tag{53}$$

式(53)可改写为

$$\begin{aligned}&\operatorname{tr}\{\boldsymbol{W}_{f,k}^{(i)}\mathrm{E}\{[(\boldsymbol{P}_{k|k}^{(i)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(i)}-(\boldsymbol{P}_{k|k}^{(j)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(j)}]\times\\&[(\boldsymbol{P}_{k|k}^{(i)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(i)}-(\boldsymbol{P}_{k|k}^{(j)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(j)}]^{\mathrm{T}}\}\boldsymbol{W}_{f,k}^{(j)}\}\geqslant 0.\end{aligned}\tag{54}$$

因式(54)中协方差阵$\mathrm{E}[(\boldsymbol{P}_{k|k}^{(i)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(i)}-(\boldsymbol{P}_{k|k}^{(j)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(j)}]\times[(\boldsymbol{P}_{k|k}^{(i)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(i)}-(\boldsymbol{P}_{k|k}^{(j)})^{-1}\tilde{\boldsymbol{x}}_{k|k}^{(j)}]^{\mathrm{T}}$是非负定矩阵, 且$\boldsymbol{W}_{f,k}^{(j)}\in\mathbb{R}^{n\times n}$ $(j=1,2,\cdots,L)$是实对称正定对角矩阵, 所以式(54)成立, 所以式(39)成立, 即性质2成立.

证毕.

注 2 由于BCI融合算法中权系数$\omega_k^{(j)}$是标量, $\omega_k^{(j)}$只是对式(4)中估计向量$(\boldsymbol{P}_{k|k}^{(j)})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(j)}$整体进行缩放, 即BCI融合算法是将各个局部估计整体进行凸组合. 这样对同一个局部估计的不同分量无差别的同等对待, 会导致局部估计分量间的精度差别被忽略. 而FDCI融合算法中权系数为对角阵, 式(18)可改写为

$$(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})}=\sum_{j=1}^{L}\boldsymbol{W}_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(j)}=\begin{bmatrix}\sum_{j=1}^{L}\varepsilon_k^{(j1)}(\boldsymbol{P}_{k|k,1}^{(j)})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(j)}\\\vdots\\\sum_{j=1}^{L}\varepsilon_k^{(jn)}(\boldsymbol{P}_{k|k,n}^{(j)})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(j)}\end{bmatrix},\tag{55}$$

其中$(\boldsymbol{P}_{k|k,i}^{(j)})^{-1}(j=1,2,\cdots,L;\ i=1,2,\cdots,n)$为误差方差阵$(\boldsymbol{P}_{k|k}^{(j)})^{-1}$的第$i$行元素. 显然$(\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})}$是$\boldsymbol{W}_{f,k}^{(j)}(\boldsymbol{P}_{k|k}^{(j)})^{-1}\hat{\boldsymbol{x}}_{k|k}^{(j)}$的分量的凸组合的形式, 既满足鲁棒性(定理3性质2), 又增加了系统的可调节性. FDCI融合算法不仅具有独特的对角结构, 区分了局部估计分量间的精度差别, 而且避免权系数优化过程带来的计算负担以及局部最优值对融合精度的影响.

注 3 文献[20]是在标准BCI算法的基础之上, 通过求解线性方程组得到权系数, 所得标量权系数与子系统误差方差的迹正相关. 而本文是在所提对角阵权系数CI融合算法的基础之上推导出来的. 整体思想与文献[20]相似, 区别在于: 首先本文算法的加权系数为对角阵, 融合的结构更优. 其次是通过找到加权系数的分量与子系统误差方差的对角线上分量之间的关系构造方程组求解, 加权系数的分量与对应的子系统误差方差的对角线上元素正相关, 充分考虑了分量之间的差异, 因此融合结果更优.

4 快速对角阵权系数CI融合容积卡尔曼滤波算法

相比于EKF, UKF和PF, CKF不仅计算量小, 且有较高的滤波精度, 稳定性较好. 因此, 本文结合CKF与所提出FDCI融合算法, 提出了快速对角阵权系数CI融合容积卡尔曼滤波算法, 用于解决互协方差信息未知的非线性多传感器系统的融合估计问题.

算法3 快速对角阵权系数CI融合容积卡尔曼滤波算法.

1. 局部CKF滤波估计$\hat{\boldsymbol{x}}_{k|k}^{(j)}$和误差方差矩阵$\boldsymbol{P}_{k|k}^{(j)}$.

1) 初始化:

$$\hat{\boldsymbol{x}}_{0|0}^{(j)}=\mathrm{E}(\boldsymbol{x}_0),\boldsymbol{P}_{0|0}^{(j)}=\mathrm{E}(\boldsymbol{x}_0-\hat{\boldsymbol{x}}_{0|0}^{(j)})(\boldsymbol{x}_0-\hat{\boldsymbol{x}}_{0|0}^{(j)})^{\mathrm{T}},\ j=1,\cdots,L.\tag{56}$$

2) 状态更新: 生成容积点, 并采用球面–相径容积

准则近似状态的后验均值和协方差[25], 从而得到状态估计和误差方差

$$\hat{\boldsymbol{x}}_{k|k}^{(j)}=\hat{\boldsymbol{x}}_{k|k-1}^{(j)}+\boldsymbol{K}_k^{(j)}(\boldsymbol{z}_k^{(j)}-\hat{\boldsymbol{z}}_{k|k-1}^{(j)}), \tag{57}$$

$$\boldsymbol{P}_{k|k}^{(j)}=\boldsymbol{P}_{k|k-1}^{(j)}-\boldsymbol{K}_k^{(j)}\boldsymbol{P}_{zz,k|k-1}^{(j)}(\boldsymbol{K}_k^{(j)})^{\mathrm{T}}, \tag{58}$$

$$\boldsymbol{K}_k^{(j)}=\boldsymbol{P}_{xz,k|k-1}^{(j)}(\boldsymbol{P}_{zz,k|k-1}^{(j)})^{-1}. \tag{59}$$

其中$\hat{\boldsymbol{z}}_{k|k-1}^{(j)}$, $\boldsymbol{P}_{k|k-1}^{(j)}$, $\boldsymbol{P}_{xz,k|k-1}^{(j)}$, $\boldsymbol{P}_{zz,k|k-1}^{(j)}$详细推导及计算过程见文献[25].

2. 融合: 计算对角阵加权系数: 根据式(20)和$\boldsymbol{P}_{k|k}^{(j)}$得到对角阵加权系数$\boldsymbol{W}_{f,k}^{(j)}$;

3. 根据式(18)–(19)以及$\boldsymbol{W}_{f,k}^{(j)}$, $\boldsymbol{P}_{k|k}^{(j)}$得到$\boldsymbol{P}_{k|k}^{(\mathrm{FDCI})}$和$\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{FDCI})}$.

5　仿真

考虑如下在水平面上做常转弯率运动的模型:

$$\boldsymbol{x}_k=\begin{bmatrix}1 & \dfrac{\sin\varOmega T}{\varOmega} & 0 & -(\dfrac{1-\cos\varOmega T}{\varOmega}) & 0\\ 0 & \cos\varOmega T & 0 & -\sin\varOmega T & 0\\ 0 & \dfrac{1-\cos\varOmega T}{\varOmega} & 1 & \dfrac{\sin\varOmega T}{\varOmega} & 0\\ 0 & \sin\varOmega T & 0 & \cos\varOmega T & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}\boldsymbol{x}_{k-1}+\boldsymbol{\varGamma}\boldsymbol{w}_{k-1}, \tag{60}$$

$$\boldsymbol{\varGamma}=\begin{bmatrix}T^2/2 & T & 0 & 0 & 0\\ 0 & 0 & T^2/2 & T & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}^{\mathrm{T}}, \tag{61}$$

其中$\boldsymbol{x}_k=[x_k\ \ y_k\ \ \dot{x}_k\ \ \dot{y}_k\ \ \varOmega]^{\mathrm{T}}$, x_k, $\dot{x}_k$, y_k, $\dot{y}_k$分别是x方向和y方向上的位置和速度. $\varOmega$为转弯率. $T=400$ ms为采样时间. 4个传感器的位置分别为(200 m, 200 m), (−200 m, 200 m), (200 m, −200 m), (−200 m, −200 m), 如图1所示. 因此4个传感器的观测方程为

$$\boldsymbol{z}_k^{(j)}=\begin{bmatrix}\sqrt{(x_k-x_{0j})^2+(y_k-y_{0j})^2}\\ \arctan((y_k-y_{0j})/(x_k-x_{0j}))\end{bmatrix}+\boldsymbol{v}_k^{(j)},\quad j=1,\cdots,4. \tag{62}$$

仿真时, 初始状态$\boldsymbol{x}_0=[0\ \ 20\ \ 0\ \ 20\ \ -0.01]^{\mathrm{T}}$, 过程噪声的方差为$\boldsymbol{Q}_k=\mathrm{diag}\{0.1^2\ \mathrm{m}^2/\mathrm{s}^4, 0.1^2\ \mathrm{m}^2/\mathrm{s}^4, 0.003^2\ \mathrm{mrad}^2/\mathrm{s}^3\}$, 4个传感器观测噪声$\boldsymbol{v}_k^{(j)}$的方差为

$$\boldsymbol{Q}_v^{(1)}=\mathrm{diag}\{0.5^2\ \mathrm{m}^2, 0.0033^2\ \mathrm{mrad}^2\};$$

$$\boldsymbol{Q}_v^{(2)}=\mathrm{diag}\{0.52^2\ \mathrm{m}^2, 0.003^2\ \mathrm{mrad}^2\};$$

$$\boldsymbol{Q}_v^{(3)}=\mathrm{diag}\{0.53^2\ \mathrm{m}^2, 0.0031^2\ \mathrm{mrad}^2\};$$

$$\boldsymbol{Q}_v^{(4)}=\mathrm{diag}\{0.55^2\ \mathrm{m}^2, 0.0034^2\ \mathrm{mrad}^2\}.$$

估计性能指标为k时刻位置的累积均方根误差(ARMSE)为

$$\mathrm{ARMSE}(k)=\sum_{t=0}^{k}\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left((x_t^{(i)}-\hat{x}_{t|t}^{(i)})^2+(y_t^{(i)}-\hat{y}_{t|t}^{(i)})^2\right)}. \tag{63}$$

其中$(x_t^{(i)}, y_t^{(i)})$和$(\hat{x}_{t|t}^{(i)}, \hat{y}_{t|t}^{(i)})$分别是第$i$次蒙特卡罗实验$t$时刻的真实位置和估计位置.

真实运动轨迹和FDCI-CKF估计跟踪轨迹如图1所示. 进行50次蒙特卡罗实验, 传感器1–4局部CKF (CKF-1-4)估计和FDCI-CKF估计的ARMSE曲线如图2所示. 快速CI融合[20]CKF(FCI-CKF)估计, 批处理CI融合CKF(BCI-CKF)估计(用fmincon工具箱[19]和遗传算法两种方式中效果最好的作为最终结果), FDCI-CKF估计和CF-CKF估计的ARMSE如图3所示. 从图2–3可以看出所提出的FDCI-CKF估计精度高于各个局部CKF估计器, 且高于经典FCI-CKF估计精度. 验证了所提出对角阵权系数CI融合算法的正确性, 以及对角阵权系数CI融合容积卡尔曼滤波算法的有效性. 从图3还可以看出FDCI-CKF估计精度高于经典BCI-CKF估计精度, 验证了注释2的结论. 此外从图3可以看出尽管理论上BCI精度高于FCI算法, 但由于非线性系统的维数和复杂程度高, 所以在寻优过程中不能保证每时刻都找到最优加权系数, 有时会陷入局部最优, 因此精度略低于FCI的精度.

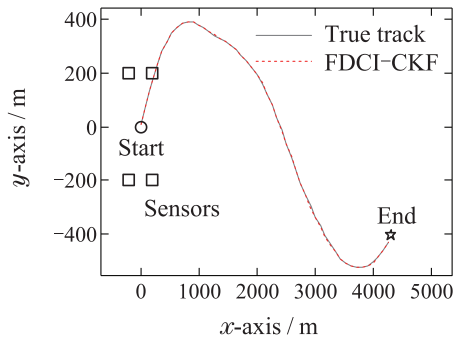

图 1　真实运动轨迹, FDCI-CKF和CF-CKF估计跟踪

Fig. 1　The true track and its estimations of FDCI-CKF and CF-CKF

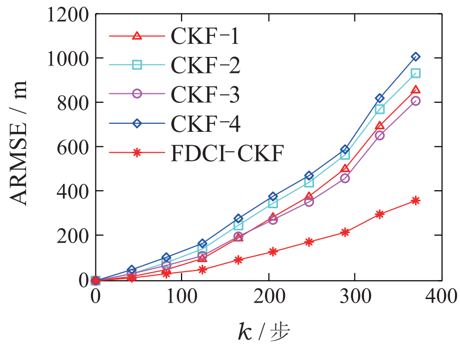

图 2　传感器1–4和FDCI-CKF的ARMSE曲线

Fig. 2　ARMSEs of sensors 1–4 and FDCI-CKF

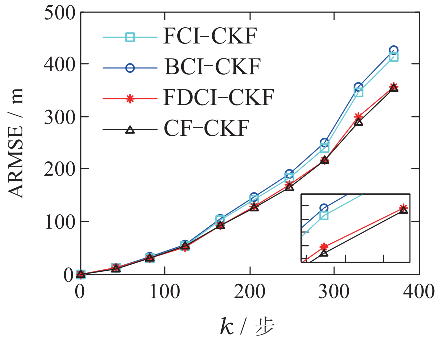

图 3 FCI-CKF, BCI-CKF, FDCI-CKF和CF-CKF的ARMSE曲线

Fig. 3 ARMSEs of FCI-CKF, BCI-CKF, FDCI-CKF and CF-CKF

为了进一步评估算法的性能, 取CKF1-4, BCI, FCI[20], FDCI和CF5种算法50次蒙特卡罗实验的平均CPU时间, 每次实验运行400步, 运行时间如表1所示. 从表1可以看出, 相比于BCI算法, FCI和FDCI算法由于不涉及权系数寻优过程, 因此具有更快的计算速度. FDCI算法与FCI算法的运行时间几乎一致, 因此FDCI算法不仅具有更高的融合精度, 而且计算速度快.

表 1 50次蒙特卡罗实验平均CPU时间对比

Table 1 Comparison of average CPU time of 50 Monte Carlo experiments

算法	CKF1-4	BCI	FCI	FDCI	CF
t/s	0. 1846	53. 5069	0. 2136	0. 2166	0. 2595

6 结论

对于带未知互协方差信息的多传感器系统, 提出了对角阵权系数CI融合(DCI)算法. 该算法充分考虑了子系统各分量间的差异, 较经典CI融合可以获得更高的融合精度. 接着, 提出了FDCI算法, 该算法更适用于复杂的非线性多传感器系统. 证明了FDCI算法的无偏性和鲁棒精度, 在理论上证明了FDCI融合算法的可行性. 该算法避免了权系数的优化问题, 大大降低了计算负担. 最后, 结合CKF滤波算法, 提出了快速对角阵权系数CI融合CKF滤波算法(FDCI-CKF), 该算法可以处理互协方差未知的非线性多传感器系统的融合估计问题, 且具有很好的实时性和较高的融合精度.

值得讨论的是, 在权值结构相同条件下, FCI算法理论上精度低于BCI算法, 但FCI避免了因优化过程陷入局部最优而带来的精度损失等情况, 增加了系统的鲁棒性和实用性. 同时系统复杂程度越高, 其在鲁棒性和实时性上的优势越明显. FDCI算法继承了FCI算法的优点, 更适合应用在高维, 传感器数量较多, 复杂程度更高的非线性系统.

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