线性切换系统的采样数据输出调节问题

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摘要: 本文基于采样控制, 解决了线性切换系统的输出调节问题.首先考虑了外部输入信号是常数的情况, 给出了采样周期与平均驻留时间的关系. 通过构造一类Lyapunov-Krasovskii泛函, 使得闭环系统内部稳定且调节输出趋于零. 其次考虑了外部输入信号是导数有界且时变的情况, 给出了采样数据状态反馈控制器, 并依据采样周期和平均驻留时间关系给出的切换条件得到了切换系统实用输出调节问题可解的充分条件. 最后通过两个数值例子验证了方法的有效性.

关键词: 切换系统; 采样控制; 输出调节

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Sampled-data output regulation of switched systems

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Abstract: The output regulation problem of the linear switched systems by which based on the sampled-data control is solved in this paper. Firstly, the constant exogenous signals are considered. The relationship between the sampling period and the average dwell time is given. By constructing a class of Lyapunov-Krasovskii functional, the internal stability of the closed-loop system is guaranteed and the regulated output tends to zero, where the asynchronous switching problem is solved. Secondly, the time-varying exogenous signals with bounded derivative are considered. The switching signals satisfy the average dwell time which is also obtained based on the sampling period. By designing the sampled-data state feedback controller, the sufficient conditions are got for the solvability of the practical output regulation problem of the switched system, the asynchronous switching problem is correspondingly solved in this situation. Finally, two numerical examples are provided to verify the effectiveness of the proposed main methods.

Key words: switched systems; sampled-data control; output regulation

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1 引言

切换系统是一类特殊的混杂系统, 它由有限个连续或离散的子系统和一个用于协调各子系统的切换规则组成[1–2]. 控制理论发展至今, 切换系统已广泛应用于实际生活, 如网络切换系统[3–4]、电路系统[5]和智能控制[6]. 目前, 切换系统的输出调节问题受到了广泛关注[7–8].

输出调节问题即通过设计一类反馈控制器来消除扰动的影响, 使得闭环系统稳定且调节输出趋于零[9]. 在20世纪70年代, 一般线性系统的输出调节问题因得益于内模原理得到了彻底的解决[10–11]. 相比非切换系统, 切换系统的输出调节问题更为复杂. Wang等[12]利用多重共轭李雅普诺夫函数方法研究了一类正切换系统的输出调节问题. 文献[13]中用同样的方法解决了切换系统在多重干扰下的输出调节问题. 李莉莉等[14]采用事件触发机制, 解决了线性切换系统的输出调节问题.

目前, 有多种实际系统由计算机实现控制, 因此采样控制问题备受关注[15–16]. 基于采样时刻数据设计的控制器通常由时间触发的连续阶跃信号采样器和零阶保持器构成, 因此可以直接在数字平台实现[17]. 此外, 基于时间触发的采样控制不会在有限区间内发

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生无限次触发的现象, 因此不会发生Zeno现象, 迎合了实际工程领域的需要. 基于采样数据的输出调节问题对于无线传感器网络、机械臂协调控制、移动机器人等实际系统的研究具有重要意义[18]. 采样控制下的被控系统, 其抗干扰性能和控制器的利用率大幅提高[19]. 所以将采样控制应用到解决切换系统的输出调节问题中具有广泛的实际意义. 鉴于上述优点, 不少学者采用采样数据控制的理论方法来解决输出调节问题. Liu和Huang[20]在非周期采样控制的框架下分析了线性系统输出调节问题的可解性, 分别解决了一类经典输出调节问题和一类实用输出调节问题. 对于切换中立系统, Fu等[21]利用固定采样周期和驻留时间的关系设计了满足条件的切换信号, 并通过采样控制器保证了系统稳定性. 文献[22]基于时变采样机制, 提出了一类新的泛函用以验证线性切换系统的稳定性. 但到目前为止, 基于采样数据控制的切换系统输出调节问题的研究成果还十分有限.

本文主要采用平均驻留时间方法研究了线性切换系统在采样数据控制下的输出调节问题. 首先, 利用采样周期与驻留时间的关系给出了使得闭环系统稳定的充分条件; 其次, 设计了采样控制器和满足平均驻留时间的切换信号, 当外部输入信号是常数时, 保证了输出调节问题可解; 最后, 通过引入由自由权矩阵, 解决了外部输入信号为时变信号(导数有界)的闭环系统的实用输出调节问题.

2 问题描述和预备知识

考虑如下线性切换系统:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + E_{\sigma(t)}\omega(t), \\ e = C_{\sigma(t)}x(t) + Q_{\sigma(t)}\omega(t), \end{cases} \tag{1}$$

其中: $\omega \in \mathbb{R}^r$代表扰动或参考输入信号, 满足方程

$$\dot{\omega} = S\omega, \tag{2}$$

称系统(2)为外系统; $x \in \mathbb{R}^n$系统的状态向量; 切换信号$\sigma : [0,\infty) \to I_N = \{1,\cdots,N\}$是一类分段右连续的常值函数; $u \in \mathbb{R}^m$表示控制输入; $e \in \mathbb{R}^P$代表调节输出; $A_i, B_i, E_i, C_i, Q_i, S$是合适维数的常矩阵, 其中$\sigma = i, i \in I_N$意味着第$i$个子系统被激活.

定义 1 如果存在$\tau_{\rm a} > \tau_{\rm d}$ 和$N_0 \geqslant 1$ 使得

$$N_\sigma(T,t) \leqslant N_0 + \frac{T-t}{\tau_{\rm a}},\ \forall T > t \geqslant t_0,$$

其中: $N_\sigma(T,t)$表示在区间(t,T)上的切换次数, 任意两个切换之间的间隔至少是驻留时间$\tau_{\rm d} > 0$, 那么称$\tau_{\rm a}$为平均驻留时间[22].

切换系统(1)满足输出调节问题的一般假设, 即

假设 1 矩阵S的所有特征值有非负实部, $\{(A_i, B_i) : i \in I_N\}$是可稳定的[20].

本文将设计如下采样数据状态反馈控制器:

$$u(t) = K_i x(t_k) + L_i\omega(t_k),\ \forall t \in [t_k, t_{k+1}), \tag{3}$$

其中: K_i, L_i也为常数矩阵.

对系统(1)作坐标变换: $\tilde{x}(t) = x(t) - x(t_k), \tilde{\omega}(t) = \omega(t) - \omega(t_k)$. 则由系统(1)和控制器(3)构成的闭环系统为

$$\begin{cases} \dot{x}(t) = A_{c_i}x(t) + B_{c_i}\omega(t) + E_{c_i}\tilde{x}(t) + F_{c_i}\tilde{\omega}(t), \\ \dot{\omega} = S\omega, \\ e = C_{c_i}x(t) + Q_{c_i}\omega(t), \end{cases} \tag{4}$$

其中: $C_{c_i} = C_i, Q_{c_i} = Q_i, A_{c_i} = A_i + B_iK_i, B_{c_i} = E_i + B_iL_i, E_{c_i} = -B_iK_i, F_{c_i} = -B_iL_i$.

下面给出两种形式的输出调节问题描述:

问题 1 给定$\{A_i, B_i, E_i, C_i, Q_i, S\}, i \in I_N$, 设计合适的采样控制器(3)和切换信号使得: 当$\dot{\omega}(t) = 0$时, 对于任意的$x(0)$和$\omega(0)$, 闭环系统(4)的输出调节问题可解且满足$\lim\limits_{t\to\infty} e(t) = 0$.

问题 2 给定$\{A_i, B_i, E_i, C_i, Q_i, S\}, i \in I_N$以及任意的$\varepsilon > 0$设计合适的采样控制器(3)和切换信号使得: 存在一个常数$a_1 > 0$满足$\|\dot{\omega}(t)\| \leqslant a_1$时, 对于任意的紧集$X$和$W$有$x(0) \in X$和$\omega(0) \in W$, 闭环系统(4)的实用输出调节问题可解且满足

$$\lim_{t\to\infty} \sup \|e(t)\| \leqslant \varepsilon.$$

注 1 本文结果采用了固定采样周期$\tau_{\rm s}$且满足$\tau_{\rm s} \leqslant \tau_{\rm d}$, 这样保证了在一个采样区间内最多发生一次切换. 在采样区间内发生切换后, 上一个子系统的控制器持续作用, 但该控制器与当前的系统模态不匹配, 从而产生异步, 该现象持续到下一个采样时刻为止. $\{t_k, k = 0, 1, \cdots\}$表示采样时间序列, 满足$t_0 < t_1 < \cdots$. $\{\hat{t}_s, s = 1, 2, \cdots\}$表示切换时间序列.

3 常数形式的外部输入信号

本节将针对外部输入信号满足$\dot{\omega}(t) = 0$的情况, 给出问题1可解的充分条件.

定理 1 对于$\dot{\omega}(t) = 0, t \geqslant 0$. 考虑闭环系统(4).

1) 对于给定的正常数$\lambda_1, \lambda_2, \tau_{\rm s}, \iota < 1, \mu \geqslant 1$, 若存在正定矩阵$\tilde{P}_\kappa, \tilde{M}_\kappa, \tilde{N}_\kappa, \tilde{W}_i$和任意矩阵$R_i, \kappa = \{i, j\} \in I_N, i \neq j$, 满足以下不等式:

$$\tilde{\varPhi}_i = \begin{bmatrix} \tilde{A}_{11} & 0 & 0 & \tilde{A}_{14} & \tilde{A}_{15} \\ * & \tilde{A}_{22} & 0 & 0 & 0 \\ * & * & \tilde{A}_{33} & 0 & 0 \\ * & * & * & \tilde{A}_{44} & \tilde{A}_{45} \\ * & * & * & * & \tilde{A}_{55} \end{bmatrix} < 0, \tag{5}$$

$$\tilde{A}_j=\begin{bmatrix}\tilde{B}_{11}&0&0&\tilde{B}_{14}&\tilde{B}_{15}&0&0\\ *&\tilde{B}_{22}&0&0&0&0&0\\ *&*&\tilde{B}_{33}&0&0&0&0\\ *&*&*&\tilde{B}_{44}&\tilde{B}_{45}&0&0\\ *&*&*&*&\tilde{B}_{55}&0&0\\ *&*&*&*&*&\tilde{B}_{66}&0\\ *&*&*&*&*&*&\tilde{B}_{77}\end{bmatrix}<0,\tag{6}$$

$$\tilde{P}_j\leqslant\mu\tilde{P}_i,\ \tilde{M}_j\leqslant\mu\tilde{M}_i,\ \tilde{N}_j\leqslant\mu\tilde{N}_i,\ \forall i,j\in I_N,\tag{7}$$

其中:

$\tilde{A}_{11}=\lambda_1\tilde{P}_i+\tilde{M}_i+\tilde{N}_i+A_i\tilde{W}_i+\tilde{W}_i^{\mathrm{T}}A_i^{\mathrm{T}}$,
$\tilde{A}_{14}=\tilde{P}_i+\tilde{W}_i^{\mathrm{T}}A_i^{\mathrm{T}}-\tilde{W}_i$,
$\tilde{A}_{15}=B_iR_i+\tilde{W}_i^{\mathrm{T}}A_i^{\mathrm{T}}$,
$\tilde{A}_{22}=-\mathrm{e}^{-\lambda_1\tau_{\mathrm{s}}}\tilde{M}_i$,
$\tilde{A}_{33}=-(1-\iota)\mathrm{e}^{-\iota\lambda_1\tau_{\mathrm{s}}}\tilde{N}_i$,
$\tilde{A}_{44}=-\tilde{W}_i-\tilde{W}_i^{\mathrm{T}}$,
$\tilde{A}_{45}=B_iR_i-\tilde{W}_i^{\mathrm{T}}$,
$\tilde{A}_{55}=B_iR_i+R_i^{\mathrm{T}}B_i^{\mathrm{T}}$,
$\tilde{B}_{11}=-\lambda_2\tilde{P}_j+\tilde{M}_j+\tilde{N}_j+A_j\tilde{W}_i+\tilde{W}_i^{\mathrm{T}}A_j^{\mathrm{T}}$,
$\tilde{B}_{14}=-\tilde{W}_i+\tilde{W}_i^{\mathrm{T}}A_j^{\mathrm{T}}+\tilde{P}_j$,
$\tilde{B}_{15}=B_jR_i+\tilde{W}_i^{\mathrm{T}}A_j^{\mathrm{T}}$,
$\tilde{B}_{22}=-\mathrm{e}^{-\lambda_1\tau_{\mathrm{s}}}\tilde{M}_j$,
$\tilde{B}_{33}=-(1-\iota)\mathrm{e}^{-\iota\lambda_1\tau_{\mathrm{s}}}\tilde{N}_j$,
$\tilde{B}_{44}=-\tilde{W}_i-\tilde{W}_i^{\mathrm{T}}$,
$\tilde{B}_{45}=B_jR_i-\tilde{W}_i^{\mathrm{T}}$,
$\tilde{B}_{55}=B_jR_i+R_i^{\mathrm{T}}B_j^{\mathrm{T}}$,
$\tilde{B}_{66}=-\dfrac{(\lambda_1+\lambda_2)\mathrm{e}^{-\lambda_1\tau_{\mathrm{s}}}}{\tau_{\mathrm{s}}}\tilde{M}_j$,
$\tilde{B}_{77}=-\dfrac{(\lambda_1+\lambda_2)\mathrm{e}^{-\iota\lambda_1\tau_{\mathrm{s}}}}{\tau_{\mathrm{s}}}\tilde{N}_j$.

2) 若存在矩阵Π, 满足调节方程

$$\begin{cases}\Pi S=A_{c_i}\Pi+B_{c_i},\\ 0=C_{c_i}\Pi+Q_{c_i},\end{cases}\tag{8}$$

则在满足平均驻留时间

$$\tau_{\mathrm{a}}>\frac{\ln\mu+(\lambda_1+\lambda_2)\tau_{\mathrm{s}}}{\lambda_1}\tag{9}$$

的任意切换下, 具有控制器增益$K_i=R_i\tilde{W}_i^{-1}$和$L_i=-K_i\Pi$的采样数据全信息反馈控制器可解决问题1.

证 根据式(8)对系统(4)再次作坐标变换$\bar{x}=x-\Pi\omega$, 则闭环系统(4)可写成

$$\begin{cases}\dot{\bar{x}}=A_{c_i}\bar{x}+E_{c_i}\tilde{\bar{x}}+(E_{c_i}\Pi+F_{c_i})\tilde{\omega},\\ e=C_{c_i}\bar{x}.\end{cases}\tag{10}$$

由于$\dot{\omega}(t)=0$, 容易得到$\omega(t)=\omega(0)$, $\tilde{\omega}(t)=0$, $t\geqslant0$, 并且根据之前的坐标变换$\tilde{x}(t)=x(t)-x(t_k)$, $\tilde{\omega}(t)=\omega(t)-\omega(t_k)$, 继而系统(10)变为以下形式:

$$\begin{cases}\dot{\bar{x}}(t)=A_i\bar{x}(t)+B_iK_i\bar{x}(t-\varphi(t)),\\ e=C_{c_i}\bar{x},\end{cases}\tag{11}$$

其中: $\varphi(t)=t-t_k<t_{k+1}-t_k=\tau_{\mathrm{s}}$并满足$\dot{\varphi}(t)=1$.

i) 采样区间内无切换.

假设子系统i在区间$[t_k,t_{k+1})$运转, 它与其相匹配的控制器在整个区间上同步. 对于系统(11), 定义如下Lyapunov-Krasovskii泛函:

$$\begin{aligned}V_i(t)=&\ \bar{x}^{\mathrm{T}}(t)P_i\bar{x}(t)+\\ &\int_{t-\tau_{\mathrm{s}}}^{t}\bar{x}^{\mathrm{T}}(s)M_i\mathrm{e}^{\lambda_1(s-t)}\bar{x}(s)\mathrm{d}s+\\ &\int_{t-\iota\varphi(t)}^{t}\bar{x}^{\mathrm{T}}(s)N_i\mathrm{e}^{\lambda_1(s-t)}\bar{x}(s)\mathrm{d}s,\end{aligned}\tag{12}$$

其中: P_i, M_i, N_i是正定对称矩阵. 且根据式(11), 对于任意矩阵W_i设松弛变量

$$-2(\dot{\bar{x}}^{\mathrm{T}}(t)\bar{x}^{\mathrm{T}}(t)\bar{x}^{\mathrm{T}}(t-\varphi(t)))W_i^{\mathrm{T}}(\dot{\bar{x}}(t)-A_i\bar{x}(t)-B_iK_i\bar{x}(t-\varphi(t)))=0,$$

结合上式对式(12)求导得到

$$\begin{aligned}&\dot{V}_i(t)+\lambda_1V_i(t)\leqslant\\ &2\bar{x}^{\mathrm{T}}(t)P_i\dot{\bar{x}}(t)+\lambda_1\bar{x}^{\mathrm{T}}(t)P_i\bar{x}(t)+\\ &\bar{x}^{\mathrm{T}}(t)M_i\bar{x}(t)+\bar{x}^{\mathrm{T}}(t)N_i\bar{x}(t)-2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\dot{\bar{x}}(t)+\\ &2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}A_i\bar{x}(t)-2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\dot{\bar{x}}(t)+\\ &2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}A_i\bar{x}(t)+2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}A_i\bar{x}(t)-\\ &(1-\iota)\mathrm{e}^{-\iota\tau_{\mathrm{s}}\lambda_1}\bar{x}^{\mathrm{T}}(t-\iota\varphi(t))N_i\bar{x}(t-\iota\varphi(t))-\\ &\mathrm{e}^{-\lambda_1\tau_{\mathrm{s}}}\bar{x}^{\mathrm{T}}(t-\tau_{\mathrm{s}})M_i\bar{x}(t-\tau_{\mathrm{s}})+\\ &2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}B_iK_i\bar{x}(t-\varphi(t))-\\ &2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}\dot{\bar{x}}(t)+\\ &2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}B_iK_i\bar{x}(t-\varphi(t))+\\ &2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}B_iK_i\bar{x}(t-\varphi(t)),\end{aligned}\tag{13}$$

可以推出

$$\dot{V}_i(t)+\lambda_1V_i(t)\leqslant\zeta^{\mathrm{T}}(t)\varPhi_i\zeta(t),\tag{14}$$

其中:

$$\zeta^{\mathrm{T}}(t)=(\bar{x}^{\mathrm{T}}(t)\bar{x}^{\mathrm{T}}(t-\tau_{\mathrm{s}})\bar{x}^{\mathrm{T}}(t-\iota\varphi(t))\times\dot{\bar{x}}^{\mathrm{T}}(t)\bar{x}^{\mathrm{T}}(t-\varphi(t))),$$

$$\varPhi_i=\begin{bmatrix}A_{11}&0&0&A_{14}&A_{15}\\ *&A_{22}&0&0&0\\ *&*&A_{33}&0&0\\ *&*&*&A_{44}&A_{45}\\ *&*&*&*&A_{55}\end{bmatrix},$$

$A_{11}=\lambda_1P_i+M_i+N_i+W_i^{\mathrm{T}}A_i+A_i^{\mathrm{T}}W_i$,
$A_{14}=P_i+A_i^{\mathrm{T}}W_i-W_i^{\mathrm{T}}$,
$A_{15}=W_i^{\mathrm{T}}B_iK_i+A_i^{\mathrm{T}}W_i$,

$A_{22}=-\mathrm{e}^{-\lambda_1\tau_\mathrm{s}}M_i$,
$A_{33}=-(1-\iota)\mathrm{e}^{-\iota\lambda_1\tau_\mathrm{s}}N_i$,
$A_{44}=-W_i^\mathrm{T}-W_i$,
$A_{45}=W_i^\mathrm{T}B_iK_i-W_i$,
$A_{55}=W_i^\mathrm{T}B_iK_i+K_i^\mathrm{T}B_i^\mathrm{T}W_i$.

根据式(14), 若$\varPhi_i<0$, 可得

$$\dot V_i(t)+\lambda_1V_i(t)\leqslant 0,\ t\in[t_k,t_{k+1}),$$

对上式两边由t_k到t积分得

$$V_i(t)\leqslant \mathrm{e}^{-\lambda_1(t-t_k)}V_i(t_k). \tag{15}$$

ii) 采样区间内包含切换.

对于$\forall t\in[t_k,t_{k+1})$, 假设在$\hat t_\mathrm{s}$时刻发生切换, 其中$\hat t_\mathrm{s}\in[t_k,t_{k+1})$. 于是, 由$\sigma((\hat t_\mathrm{s})^-)=i$切换到$\sigma(\hat t_\mathrm{s})=j\neq i$, 子系统$i$在区间$[t_k,\hat t_\mathrm{s})$上运转, 子系统$j$在区间$[\hat t_\mathrm{s},t_{k+1})$上运转, 且$t-\hat t_\mathrm{s}<\tau_\mathrm{s}$. 那么, 对于区间$[t_k,\hat t_\mathrm{s})$, 根据i)中的结果可知

$$V_i(t)\leqslant \mathrm{e}^{-\lambda_1(t-t_k)}V_i(t_k). \tag{16}$$

对$\forall t\in[\hat t_\mathrm{s},t_{k+1})$, 给出如下闭环系统:

$$\begin{cases}\dot{\bar x}(t)=A_j\bar x(t)+B_jK_i\bar x(t-\varphi(t)),\\ e=C_{c_j}\bar x.\end{cases} \tag{17}$$

定义如下Lyapunov-Krasovskii泛函:

$$\begin{aligned}V_j(t)=&\bar x^\mathrm{T}(t)P_j\bar x(t)+\\&\int_{t-\tau_\mathrm{s}}^t\bar x^\mathrm{T}(s)M_j\mathrm{e}^{\lambda_1(s-t)}\bar x(s)\mathrm{d}s+\\&\int_{t-\iota\varphi(t)}^t\bar x^\mathrm{T}(s)N_j\mathrm{e}^{\lambda_1(s-t)}\bar x(s)\mathrm{d}s,\end{aligned} \tag{18}$$

其中: P_j,M_j,N_j是正定对称矩阵. 根据式(17), 对于任意矩阵W_i设置松弛变量

$$\begin{aligned}&-2(\dot{\bar x}^\mathrm{T}(t)\bar x^\mathrm{T}(t)\bar x^\mathrm{T}(t-\varphi(t)))W_i^\mathrm{T}(\dot{\bar x}(t)-A_j\bar x(t)-\\&B_jK_i\bar x(t-\varphi(t)))=0,\end{aligned}$$

结合此变量, 对$\forall\lambda_2>0$可得

$$\begin{aligned}&\dot V_j(t)-\lambda_2V_j(t)\leqslant\\&2\bar x^\mathrm{T}(t)P_j\dot{\bar x}(t)-\lambda_2\bar x^\mathrm{T}(t)P_j\bar x(t)+\\&\bar x^\mathrm{T}(t)M_j\bar x(t)+\bar x^\mathrm{T}(t)N_j\bar x(t)-2\dot{\bar x}^\mathrm{T}(t)W_i^\mathrm{T}\dot{\bar x}(t)-\\&(1-\iota)\mathrm{e}^{-\iota\tau_\mathrm{s}\lambda_1}\bar x^\mathrm{T}(t-\iota\varphi(t))N_j\bar x(t-\iota\varphi(t))+\\&2\dot{\bar x}^\mathrm{T}(t)W_i^\mathrm{T}A_j\bar x(t)+2\bar x^\mathrm{T}(t)W_i^\mathrm{T}A_j\bar x(t)-\\&2\bar x^\mathrm{T}(t)W_i^\mathrm{T}\dot{\bar x}(t)+2\bar x^\mathrm{T}(t)W_i^\mathrm{T}B_jK_i\bar x(t-\varphi(t))-\\&\mathrm{e}^{-\lambda_1\tau_\mathrm{s}}\bar x^\mathrm{T}(t-\tau_\mathrm{s})M_j\bar x(t-\tau_\mathrm{s})+\\&2\dot{\bar x}^\mathrm{T}(t)W_i^\mathrm{T}B_jK_i\bar x(t-\varphi(t))-\\&2\bar x^\mathrm{T}(t-\varphi(t))W_i^\mathrm{T}\dot{\bar x}(t)+2\bar x^\mathrm{T}(t-\varphi(t))W_i^\mathrm{T}A_j\bar x(t)+\\&2\bar x^\mathrm{T}(t-\varphi(t))W_i^\mathrm{T}B_jK_i\bar x(t-\varphi(t))-\\&\frac{(\lambda_1+\lambda_2)\mathrm{e}^{-\lambda_1\tau_\mathrm{s}}}{\tau_\mathrm{s}}\int_{t-\tau_\mathrm{s}}^t\bar x^\mathrm{T}(s)\mathrm{d}sM_j\int_{t-\tau_\mathrm{s}}^t\bar x(s)\mathrm{d}s-\\&\frac{(\lambda_1+\lambda_2)\mathrm{e}^{-\iota\lambda_1\tau_\mathrm{s}}}{\tau_\mathrm{s}}\int_{t-\tau\varphi(t)}^t\bar x^\mathrm{T}(s)\mathrm{d}sN_j\times\\&\int_{t-\tau\varphi(t)}^t\bar x(s)\mathrm{d}s.\end{aligned} \tag{19}$$

可以推出

$$\dot V_j(t)-\lambda_2V_j(t)\leqslant\xi^\mathrm{T}(t)\varLambda_j\xi(t), \tag{20}$$

其中:

$$\begin{aligned}\xi^\mathrm{T}(t)=&(\bar x^\mathrm{T}(t)\bar x^\mathrm{T}(t-\tau_\mathrm{s})\bar x^\mathrm{T}(t-\iota\varphi(t))\dot{\bar x}^\mathrm{T}(t)\times\\&\bar x^\mathrm{T}(t-\varphi(t))\int_{t-\tau_\mathrm{s}}^t\bar x^\mathrm{T}(s)\mathrm{d}s\int_{t-\iota\varphi(t)}^t\bar x^\mathrm{T}(s)\mathrm{d}s),\end{aligned}$$

$$\varLambda_j=\begin{bmatrix}B_{11}&0&0&B_{14}&B_{15}&0&0\\ *&B_{22}&0&0&0&0&0\\ *&*&B_{33}&0&0&0&0\\ *&*&*&B_{44}&B_{45}&0&0\\ *&*&*&*&B_{55}&0&0\\ *&*&*&*&*&B_{66}&0\\ *&*&*&*&*&*&B_{77}\end{bmatrix}.$$

$B_{11}=-\lambda_2P_j+M_j+N_j+W_i^\mathrm{T}A_j+A_j^\mathrm{T}W_i$,
$B_{14}=-W_i^\mathrm{T}+A_j^\mathrm{T}W_i+P_j$,
$B_{15}=W_i^\mathrm{T}B_jK_i+A_j^\mathrm{T}W_i$,
$B_{22}=-\mathrm{e}^{-\lambda_1\tau_\mathrm{s}}M_j$,
$B_{33}=-(1-\iota)\mathrm{e}^{-\iota\lambda_1\tau_\mathrm{s}}N_j$,
$B_{44}=-W_i^\mathrm{T}-W_i$,
$B_{45}=W_i^\mathrm{T}B_jK_i-W_i$,
$B_{55}=W_i^\mathrm{T}B_jK_i+K_i^\mathrm{T}B_j^\mathrm{T}W_i$,
$B_{66}=-\dfrac{(\lambda_1+\lambda_2)\mathrm{e}^{-\lambda_1\tau_\mathrm{s}}}{\tau_\mathrm{s}}M_j$,
$B_{77}=-\dfrac{(\lambda_1+\lambda_2)\mathrm{e}^{-\iota\lambda_1\tau_\mathrm{s}}}{\tau_\mathrm{s}}N_j$.

根据式(20), 若$\varLambda_j<0$, 可得

$$\dot V_j(t)-\lambda_2V_j(t)\leqslant 0,\ t\in[\hat t_\mathrm{s},t_{k+1}),$$

对上式两边由$\hat t_\mathrm{s}$到t积分得

$$V_j(t)\leqslant\mathrm{e}^{\lambda_2(t-\hat t_\mathrm{s})}V_j(\hat t_\mathrm{s}). \tag{21}$$

对于采样区间$[t_k,t_{k+1})$, 结合式(16)(21)可以获得

$$\begin{aligned}V_j(t)\leqslant&\mathrm{e}^{\lambda_2(t-\hat t_\mathrm{s})}V_j(\hat t_\mathrm{s})\leqslant\\&\mu\mathrm{e}^{\lambda_2(t-\hat t_\mathrm{s})}V_i((\hat t_\mathrm{s})^-)\leqslant\\&\mu\mathrm{e}^{\lambda_2(t-\hat t_\mathrm{s})}\mathrm{e}^{-\lambda_1(\hat t_\mathrm{s}-t_k)}V_i(t_k)<\\&\mu\mathrm{e}^{-\lambda_1(t-t_k)+(\lambda_1+\lambda_2)\tau_\mathrm{s}}V_i(t_k),\end{aligned} \tag{22}$$

结合式(7)(22)以及定义1得到

$$\begin{aligned}&V(t)\leqslant\\&\mu^{N_0+\frac{t-t_0}{\tau_\mathrm{a}}}\mathrm{e}^{-\lambda_1(t-t_0)+(\lambda_1+\lambda_2)\tau_\mathrm{s}(N_0+\frac{t-t_0}{\tau_\mathrm{a}})}V(t_0)=\\&c\,\mathrm{e}^{(\frac{\ln\mu+(\lambda_1+\lambda_2)\tau_\mathrm{s}}{\tau_\mathrm{a}}-\lambda_1)(t-t_0)}V(t_0),\end{aligned} \tag{23}$$

其中$c = \mathrm{e}^{(\ln\mu+(\lambda_1+\lambda_2)\tau_\mathrm{s})N_0}$, 基于式(12)推出

$$\begin{cases}V(t)\geqslant\bar{x}^T(t)P_i\bar{x}(t)\geqslant\lambda_\mathrm{m}(P_i)\|\bar{x}(t)\|^2\geqslant a\|\bar{x}(t)\|^2,\\ V(t_0)\leqslant(b+\tau_\mathrm{s}c+\iota\tau_\mathrm{s}d)\|\bar{x}(t_0)\|^2=\hat{b}\|\bar{x}(t_0)\|^2,\end{cases}\tag{24}$$

其中:

$a=\inf\limits_{i\in I_N}\lambda_\mathrm{m}(P_i), b=\sup\limits_{i\in I_N}\lambda_M(P_i), c=\sup\limits_{i\in I_N}\lambda_M(M_i),$
$d=\sup\limits_{i\in I_N}\lambda_M(N_i), \hat{b}=b+\tau_\mathrm{s}c+\iota\tau_\mathrm{s}d.$

根据式(23)–(24)得出

$$\|\bar{x}(t)\|^2\leqslant\frac{\hat{b}}{a}c\,\mathrm{e}^{(\frac{\ln\mu+(\lambda_1+\lambda_2)\tau_\mathrm{s}}{\tau_\mathrm{a}}-\lambda_1)(t-t_0)}\|\bar{x}(t_0)\|^2.$$

受条件(9)约束, 使得$(\ln\mu+(\lambda_1+\lambda_2)\tau_\mathrm{s})\tau_\mathrm{a}^{-1}-\lambda_1<0$成立. 因此

$$\lim_{t\to\infty}e=\lim_{t\to\infty}C_{c_{\sigma(t)}}\bar{x}(t)=0,$$

则问题1可解.　证毕.

注 2　为了方便求得控制器增益K_i, 令$\tilde{W}_i=W_i^{-1}$且将以$\tilde{W}_i^\mathrm{T}$和$\tilde{W}_i$为对角线元素的对角矩阵分别乘到$\varPhi_i<0$和$\varLambda_j<0$的两侧, 因此有$R_i=K_i\tilde{W}_i, \tilde{P}_i=\tilde{W}_i^\mathrm{T}P_i\tilde{W}_i, \tilde{M}_i=\tilde{W}_i^\mathrm{T}M_i\tilde{W}_i, \tilde{N}_i=\tilde{W}_i^\mathrm{T}N_i\tilde{W}_i$, 模态$j$中的此类矩阵具有与上述相同的形式. 从而得到不等式(5)–(6).

4　具有有界导数的时变外部输入信号

本节将针对外部输入信号满足$\|\dot{\omega}(t)\|\leqslant a_1$的情况, 给出问题2可解的充分条件.

定理 2　存在常数$a_1>0$, 满足$\|\dot{\omega}(t)\|\leqslant a_1, t\geqslant 0$. 考虑闭环系统(4).

1) 对于给定的正常数$\bar{\lambda}_1, \bar{\lambda}_2, \breve{\lambda}_1, \breve{\lambda}_2, \tau_\mathrm{s}, \hat{\iota}<1, \mu\geqslant 1$和任意的$\varepsilon>0$, 若存在正定矩阵$\hat{\tilde{P}}_\kappa, \hat{\tilde{M}}_\kappa, \hat{\tilde{N}}_\kappa, \tilde{S}_\kappa, \tilde{W}_i$和任意矩阵$R_i, \kappa=\{i,j\}\in I_N, i\neq j$, 满足以下不等式:

$$\hat{\tilde{P}}_j\leqslant\mu\hat{\tilde{P}}_i, \hat{\tilde{M}}_j\leqslant\mu\hat{\tilde{M}}_i, \hat{\tilde{N}}_j\leqslant\mu\hat{\tilde{N}}_i, \forall i,j\in I_N,\tag{25}$$

$$\tilde{\varXi}_j=\begin{bmatrix}\tilde{D}_{11}&0&0\\ *&\tilde{D}_{22}&0\\ *&*&\tilde{D}_{33}\end{bmatrix}<0,\tag{26}$$

$$\tilde{\varTheta}_i=\begin{bmatrix}\tilde{C}_{11}&0&0&\tilde{C}_{14}&\tilde{C}_{15}&\tilde{C}_{16}\\ *&\tilde{C}_{22}&0&0&0&0\\ *&*&\tilde{C}_{33}&0&0&0\\ *&*&*&\tilde{C}_{44}&\tilde{C}_{45}&\tilde{C}_{46}\\ *&*&*&*&\tilde{C}_{55}&\tilde{C}_{56}\\ *&*&*&*&*&\tilde{C}_{66}\end{bmatrix}<0,\tag{27}$$

其中:

$$\tilde{D}_{11}=\begin{bmatrix}\varDelta_{11}&0&0&\varDelta_{15}&\varDelta_{16}&\varDelta_{17}\\ *&\varDelta_{22}&0&0&0&0\\ *&*&\varDelta_{33}&0&0&0\\ *&*&*&\varDelta_{44}&\varDelta_{45}&\varDelta_{46}\\ *&*&*&*&\varDelta_{55}&\varDelta_{56}\\ *&*&*&*&*&\varDelta_{66}\end{bmatrix},$$

$\tilde{C}_{11}=\bar{\lambda}_1\hat{\tilde{P}}_i+\hat{\tilde{M}}_i+\hat{\tilde{N}}_i+A_i\tilde{W}_i+\tilde{W}_i^\mathrm{T}A_i^\mathrm{T},$
$\tilde{C}_{14}=\hat{\tilde{P}}_i+\tilde{W}_i^\mathrm{T}A_i^\mathrm{T}-\tilde{W}_i,$
$\tilde{C}_{15}=B_iR_i+\tilde{W}_i^\mathrm{T}A_i^\mathrm{T},$
$\tilde{C}_{16}=-B_iR_i\varPi-B_iH_i+\tilde{W}_i^\mathrm{T}A_i^\mathrm{T},$
$\tilde{C}_{22}=-\mathrm{e}^{-\bar{\lambda}_1\tau_\mathrm{s}}\hat{\tilde{M}}_i,$
$\tilde{C}_{33}=-(1-\hat{\iota})\mathrm{e}^{-\hat{\iota}\bar{\lambda}_1\tau_\mathrm{s}}\hat{\tilde{N}}_i,$
$\tilde{C}_{44}=-\tilde{W}_i-\tilde{W}_i^\mathrm{T},$
$\tilde{C}_{45}=B_iR_i-\tilde{W}_i^\mathrm{T},$
$\tilde{C}_{46}=-B_iR_i\varPi-B_iH_i-\tilde{W}_i^\mathrm{T},$
$\tilde{C}_{55}=B_iR_i+R_i^\mathrm{T}B_i^\mathrm{T},$
$\tilde{C}_{56}=-B_iR_i\varPi-B_iH_i+R_i^\mathrm{T}B_i^\mathrm{T},$
$\tilde{C}_{66}=-B_iR_i\varPi-B_iH_i-\varPi^\mathrm{T}R_i^\mathrm{T}B_i^\mathrm{T}-H_i^\mathrm{T}B_i^\mathrm{T}-2\breve{\lambda}_1\tilde{S}_i,$
$\tilde{\varDelta}_{11}=-\bar{\lambda}_2\hat{\tilde{P}}_j+\hat{\tilde{M}}_j+\hat{\tilde{N}}_j+A_j\tilde{W}_i+\tilde{W}_i^\mathrm{T}A_j^\mathrm{T},$
$\tilde{\varDelta}_{15}=B_jR_i+\tilde{W}_i^\mathrm{T}A_j^\mathrm{T},$
$\tilde{\varDelta}_{16}=-B_jR_i\varPi-B_jH_i+\tilde{W}_i^\mathrm{T}A_j^\mathrm{T},$
$\tilde{\varDelta}_{22}=-\mathrm{e}^{-\bar{\lambda}_1\tau_\mathrm{s}}\hat{\tilde{M}}_j,$
$\tilde{\varDelta}_{33}=-(1-\hat{\iota})\mathrm{e}^{-\hat{\iota}\bar{\lambda}_1\tau_\mathrm{s}}\hat{\tilde{N}}_j,$
$\tilde{\varDelta}_{44}=-\tilde{W}_i-\tilde{W}_i^\mathrm{T},$
$\tilde{\varDelta}_{45}=B_jR_i-\tilde{W}_i^\mathrm{T},$
$\tilde{\varDelta}_{46}=-B_jR_i\varPi-B_jH_i-\tilde{W}_i^\mathrm{T},$
$\tilde{\varDelta}_{55}=B_jR_i+R_i^\mathrm{T}B_j^\mathrm{T},$
$\tilde{\varDelta}_{56}=-B_jR_i\varPi-B_jH_i+R_i^\mathrm{T}B_j^\mathrm{T},$
$\tilde{\varDelta}_{66}=-B_jR_i\varPi-B_jH_i-\varPi^\mathrm{T}R_i^\mathrm{T}B_j^\mathrm{T}-H_i^\mathrm{T}B_j^\mathrm{T}-2\breve{\lambda}_2\tilde{S}_j,$
$\tilde{D}_{22}=-\dfrac{(\bar{\lambda}_1+\bar{\lambda}_2)\mathrm{e}^{-\bar{\lambda}_1\tau_\mathrm{s}}}{\tau_\mathrm{s}}\hat{\tilde{M}}_j,$
$\tilde{D}_{33}=-\dfrac{(\bar{\lambda}_1+\bar{\lambda}_2)\mathrm{e}^{-\hat{\iota}\bar{\lambda}_1\tau_\mathrm{s}}}{\tau_\mathrm{s}}\hat{\tilde{N}}_j.$

2) 若存在矩阵$\varPi$满足矩阵方程(8), 那么在满足平均驻留时间

$$\tau_\mathrm{a}>\frac{(\bar{\lambda}_1+\bar{\lambda}_2)\tau_\mathrm{s}+\ln\mu}{\bar{\lambda}_1}\tag{28}$$

的任意切换下具有控制器增益$K_i=R_i\tilde{W}_i^{-1}$和$L_i=H_i\tilde{W}_i^{-1}$的采样数据状态反馈控制器可解决问题2.

证　对于满足式(8)的系统(4)进行与定理1中相同的坐标变换, 又根据$\|\dot{\omega}(t)\|\leqslant a_1, a_1>0$, 则给出闭环系统

$$\begin{cases}\dot{\bar{x}}(t)=A_i\bar{x}(t)+B_iK_i\bar{x}(t-\varphi(t))+\hat{E}_i\tilde{\omega}(t),\\ e=C_{c_i}\bar{x}(t).\end{cases}\tag{29}$$

为了方便表示, 令$\hat{E}_i=E_{c_i}\varPi+F_{c_i}$.

i) 采样区间内无切换.

假设子系统i在区间$[t_k, t_{k+1})$上运转, 它与其相匹配的控制器在整个区间上同步. 对系统(29)定义如下Lyapunov-Krasovskii泛函:

$$\begin{aligned}V_i(t) = &\bar{x}^{\mathrm{T}}(t)\hat{P}_i\bar{x}(t) + \\ &\int_{t-\tau_{\mathrm{s}}}^{t}\bar{x}^{\mathrm{T}}(s)\hat{M}_i\mathrm{e}^{\bar{\lambda}_1(s-t)}\bar{x}(s)\mathrm{d}s + \\ &\int_{t-\hat{\imath}\varphi(t)}^{t}\bar{x}^{\mathrm{T}}(s)\hat{N}_i\mathrm{e}^{\bar{\lambda}_1(s-t)}\bar{x}(s)\mathrm{d}s,\end{aligned} \quad (30)$$

其中: $\hat{P}_i, \hat{M}_i, \hat{N}_i$是正定对称矩阵.

对于任意矩阵W_i, 基于系统(29)有松弛变量

$$\begin{aligned}&-2(\dot{\bar{x}}^{\mathrm{T}}(t)\bar{x}^{\mathrm{T}}(t)\bar{x}^{\mathrm{T}}(t-\varphi(t))\tilde{\omega}^{\mathrm{T}}(t))W_i^{\mathrm{T}}(\dot{\bar{x}}(t) - \\ &A_i\bar{x}(t) - B_iK_i\bar{x}(t-\varphi(t)) - \hat{E}_i\tilde{\omega}(t)) = 0,\end{aligned}$$

对$\forall\lambda_1 > 0$和$\forall\breve{\lambda}_i > 0, i \in I_N$, 结合松弛变量对式(12)求导得到

$$\begin{aligned}&\dot{V}_i(t) + \bar{\lambda}_1V_i(t) \leqslant \\ &2\bar{x}^{\mathrm{T}}(t)\hat{P}_i\dot{\bar{x}}(t) + \bar{\lambda}_1\bar{x}^{\mathrm{T}}(t)\hat{P}_i\bar{x}(t) + \\ &\bar{x}^{\mathrm{T}}(t)\hat{M}_i\bar{x}(t) + \bar{x}^{\mathrm{T}}(t)\hat{N}_i\bar{x}(t) + \\ &2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}A_i\bar{x}(t) - 2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\dot{\bar{x}}(t) + \\ &2\tilde{\omega}^{\mathrm{T}}(t)(W_i^{\mathrm{T}}\hat{E}_i - \breve{\lambda}_iS_i)\tilde{\omega}(t) + \\ &2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\hat{E}_i\tilde{\omega}(t) - 2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\dot{\bar{x}}(t) + \\ &2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}A_i\bar{x}(t) + 2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\hat{E}_i\tilde{\omega}(t) + \\ &2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}\hat{E}_i\tilde{\omega}(t) - 2\tilde{\omega}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\dot{\bar{x}}(t) - \\ &(1-\hat{\imath})\mathrm{e}^{-\hat{\imath}\tau_{\mathrm{s}}\bar{\lambda}_1}\bar{x}^{\mathrm{T}}(t-\hat{\imath}\varphi(t))\hat{N}_i\bar{x}(t-\hat{\imath}\varphi(t)) + \\ &2\tilde{\omega}^{\mathrm{T}}(t)W_i^{\mathrm{T}}A_i\bar{x}(t) + 2\breve{\lambda}_i\tilde{\omega}^{\mathrm{T}}(t)S_i\tilde{\omega}(t) + \\ &2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}B_iK_i\bar{x}(t-\varphi(t)) + \\ &2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}B_iK_i\bar{x}(t-\varphi(t)) - \\ &2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}\dot{\bar{x}}(t) + \\ &2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}A_i\bar{x}(t) + \\ &2\tilde{\omega}^{\mathrm{T}}(t)W_i^{\mathrm{T}}B_iK_i\bar{x}(t-\varphi(t)) + \\ &2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}B_iK_i\bar{x}(t-\varphi(t)) - \\ &\mathrm{e}^{-\bar{\lambda}_1\tau_{\mathrm{s}}}\bar{x}^{\mathrm{T}}(t-\tau_{\mathrm{s}})\hat{M}_i\bar{x}(t-\tau_{\mathrm{s}}),\end{aligned} \quad (31)$$

可以推出

$$\dot{V}_i(t) + \bar{\lambda}_1V_i(t) \leqslant \varsigma^{\mathrm{T}}(t)\varTheta_i\varsigma(t) + 2\breve{\lambda}_1\tilde{\omega}^{\mathrm{T}}(t)S_i\tilde{\omega}(t), \quad (32)$$

其中:

$$\begin{aligned}\varsigma^{\mathrm{T}}(t) = &(\bar{x}^{\mathrm{T}}(t)\,\bar{x}^{\mathrm{T}}(t-\tau_{\mathrm{s}})\,\bar{x}^{\mathrm{T}}(t-\hat{\imath}\varphi(t))\,\dot{\bar{x}}^{\mathrm{T}}(t)\,\times \\ &\bar{x}^{\mathrm{T}}(t-\varphi(t))\,\tilde{\omega}^{\mathrm{T}}(t)),\end{aligned}$$

$C_{11} = \bar{\lambda}_1\hat{P}_i + \hat{M}_i + \hat{N}_i + W_i^{\mathrm{T}}A_i + A_i^{\mathrm{T}}W_i$,

$C_{14} = \hat{P}_i + A_i^{\mathrm{T}}W_i - W_i^{\mathrm{T}}$,

$C_{15} = W_i^{\mathrm{T}}B_iK_i + A_i^{\mathrm{T}}W_i$,

$C_{16} = W_i^{\mathrm{T}}\hat{E}_i + A_i^{\mathrm{T}}W_i$,

$C_{22} = -\mathrm{e}^{-\bar{\lambda}_1\tau_{\mathrm{s}}}\hat{M}_i$,

$C_{33} = -(1-\hat{\imath})\mathrm{e}^{-\hat{\imath}\bar{\lambda}_1\tau_{\mathrm{s}}}\hat{N}_i$,

$C_{44} = -W_i^{\mathrm{T}} - W_i$,

$C_{45} = W_i^{\mathrm{T}}B_iK_i - W$,

$C_{46} = W_i^{\mathrm{T}}\hat{E}_i - W_i$,

$C_{55} = W_i^{\mathrm{T}}B_iK_i + K_i^{\mathrm{T}}B_i^{\mathrm{T}}W_i$,

$C_{56} = W_i^{\mathrm{T}}\hat{E}_i + K_i^{\mathrm{T}}B_i^{\mathrm{T}}W_i$,

$\breve{\lambda}_1 = \max\limits_{i\in I_N}\breve{\lambda}_i$,

$C_{66} = W_i^{\mathrm{T}}\hat{E}_i + \hat{E}_i^{\mathrm{T}}W_i - 2\breve{\lambda}_1S_i$,

$$\varTheta_i = \begin{bmatrix} C_{11} & 0 & 0 & C_{14} & C_{15} & C_{16} \\ * & C_{22} & 0 & 0 & 0 & 0 \\ * & * & C_{33} & 0 & 0 & 0 \\ * & * & * & C_{44} & C_{45} & C_{46} \\ * & * & * & * & C_{55} & C_{56} \\ * & * & * & * & * & C_{66} \end{bmatrix}.$$

根据式(32), 令$\varPhi_i < 0$可得

$$\dot{V}_i(t) \leqslant -\bar{\lambda}_1V_i(t) + 2\breve{\lambda}_1\tilde{\omega}^{\mathrm{T}}(t)S_i\tilde{\omega}(t), \quad (33)$$

并且根据条件$\|\dot{\omega}(t)\| \leqslant a_1$推出

$$\|\tilde{\omega}(t)\| \leqslant \int_{t_k}^{t}\|\dot{\omega}(\tau)\|\mathrm{d}\tau \leqslant a_1(t-t_k) \leqslant a_1\tau_{\mathrm{s}},$$

因此式(33)变为

$$\dot{V}_i(t) \leqslant -\bar{\lambda}_1V_i(t) + \hat{\lambda}_1(a_1\tau_{\mathrm{s}})^2,$$

其中$\hat{\lambda}_1 = 2\max\limits_{i\in I_N}(\breve{\lambda}_1\|S_i\|)$. 对上式由$t$到$t_k$积分

$$\begin{aligned}V_i(t) \leqslant &\mathrm{e}^{-\bar{\lambda}_1(t-t_k)}V_i(t_k) + \\ &\int_{t_k}^{t}\mathrm{e}^{-\bar{\lambda}_1(t-\tau)}\hat{\lambda}_1(a_1\tau_{\mathrm{s}})^2\mathrm{d}\tau.\end{aligned} \quad (34)$$

ii) 采样区间内包含切换.

仍假设在区间$[t_k, t_{k+1})$内的$\hat{t}_{\mathrm{s}}$时刻发生切换. 那么对$\forall t \in [t_k, \hat{t}_{\mathrm{s}})$, 基于情况i)中结果有

$$\begin{aligned}V_i(t) \leqslant &\mathrm{e}^{-\bar{\lambda}_1(t-t_k)}V_i(t_k) + \\ &\int_{t_k}^{t}\mathrm{e}^{-\bar{\lambda}_1(t-\tau)}\hat{\lambda}_1(a_1\tau_{\mathrm{s}})^2\mathrm{d}\tau.\end{aligned} \quad (35)$$

对$\forall t \in [\hat{t}_{\mathrm{s}}, t_{k+1})$, 给出如下闭环系统:

$$\begin{cases}\dot{\bar{x}}(t) = A_j\bar{x}(t) + B_jK_i\bar{x}(t-\varphi(t)) + \hat{E}_j\tilde{\omega}(t), \\ e = C_{c_j}\bar{x},\end{cases} \quad (36)$$

同样的, 令$\hat{E}_j = E_{c_j}\varPi + F_{c_j}$.

针对系统(36)定义Lyapunov-Krasovskii泛函, 即

$$\begin{aligned}V_j(t) = &\bar{x}^{\mathrm{T}}(t)\hat{P}_j\bar{x}(t) + \\ &\int_{t-\tau_{\mathrm{s}}}^{t}\bar{x}^{\mathrm{T}}(s)\hat{M}_j\mathrm{e}^{\bar{\lambda}_1(s-t)}\bar{x}(s)\mathrm{d}s + \\ &\int_{t-\hat{\imath}\varphi(t)}^{t}\bar{x}^{\mathrm{T}}(s)\hat{N}_j\mathrm{e}^{\bar{\lambda}_1(s-t)}\bar{x}(s)\mathrm{d}s,\end{aligned} \quad (37)$$

其中: $\hat{P}_j, \hat{M}_j, \hat{N}_j$是正定对称矩阵.

根据式(36)有以下松弛变量:

$$-2(\dot{\bar{x}}^{\mathrm{T}}(t)\bar{x}^{\mathrm{T}}(t)\bar{x}^{\mathrm{T}}(t-\varphi(t))\tilde{\omega}^{\mathrm{T}}(t))W_i^{\mathrm{T}}(\dot{\bar{x}}(t)-A_j\bar{x}(t)-B_jK_i\bar{x}(t-\varphi(t))-\hat{E}_j\tilde{\omega}(t))=0,$$

对于$\forall\breve{\lambda}_i>0$, 结合上式对式(37)求导且放大, 得到

$$\begin{aligned}&\dot{V}_j(t)-\bar{\lambda}_2V_j(t)\leqslant\\&2\bar{x}^{\mathrm{T}}(t)\hat{P}_j\dot{\bar{x}}(t)-\bar{\lambda}_2\bar{x}^{\mathrm{T}}(t)\hat{P}_j\bar{x}(t)+\\&\bar{x}^{\mathrm{T}}(t)\hat{M}_j\bar{x}(t)+\bar{x}^{\mathrm{T}}(t)\hat{N}_j\bar{x}(t)-2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\dot{\bar{x}}(t)-\\&\mathrm{e}^{-\bar{\lambda}_1\tau_{\mathrm{s}}}\bar{x}^{\mathrm{T}}(t-\tau_{\mathrm{s}})\hat{M}_j\bar{x}(t-\tau_{\mathrm{s}})-\\&(1-\hat{\iota})\mathrm{e}^{-\hat{\iota}\tau_{\mathrm{s}}\bar{\lambda}_1}\bar{x}^{\mathrm{T}}(t-\hat{\iota}\varphi(t))\hat{N}_j\bar{x}(t-\hat{\iota}\varphi(t))+\\&2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}A_j\bar{x}(t)+2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}\hat{E}_j\tilde{\omega}(t)+\\&2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\hat{E}_j\tilde{\omega}(t)-2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\dot{\bar{x}}(t)+\\&2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}A_j\bar{x}(t)+2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\hat{E}_j\tilde{\omega}(t)+\\&2\tilde{\omega}^{\mathrm{T}}(t)W_i^{\mathrm{T}}A_j\bar{x}(t)+2\breve{\lambda}_j\tilde{\omega}^{\mathrm{T}}(t)S_j\tilde{\omega}(t)+\\&2\dot{\bar{x}}^{\mathrm{T}}(t)W_i^{\mathrm{T}}B_jK_i\bar{x}(t-\varphi(t))+\\&2\bar{x}^{\mathrm{T}}(t)W_i^{\mathrm{T}}B_jK_i\bar{x}(t-\varphi(t))-\\&2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}\dot{\bar{x}}(t)-2\tilde{\omega}^{\mathrm{T}}(t)W_i^{\mathrm{T}}\dot{\bar{x}}(t)+\\&2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}A_j\bar{x}(t)+\\&2\tilde{\omega}^{\mathrm{T}}(t)W_i^{\mathrm{T}}B_jK_i\bar{x}(t-\varphi(t))+\\&2\bar{x}^{\mathrm{T}}(t-\varphi(t))W_i^{\mathrm{T}}B_jK_i\bar{x}(t-\varphi(t))+\\&2\tilde{\omega}^{\mathrm{T}}(t)(W_i^{\mathrm{T}}\hat{E}_j-\breve{\lambda}_jS_j)\tilde{\omega}(t)-\\&\frac{(\bar{\lambda}_1+\bar{\lambda}_2)\mathrm{e}^{-\bar{\lambda}_1\tau_{\mathrm{s}}}}{\tau_{\mathrm{s}}}\int_{t-\tau_{\mathrm{s}}}^{t}\bar{x}^{\mathrm{T}}(s)\mathrm{d}s\hat{M}_j\int_{t-\tau_{\mathrm{s}}}^{t}\bar{x}(s)\mathrm{d}s-\\&\frac{(\bar{\lambda}_1+\bar{\lambda}_2)\mathrm{e}^{-\hat{\iota}\bar{\lambda}_1\tau_{\mathrm{s}}}}{\tau_{\mathrm{s}}}\int_{t-\hat{\iota}\varphi(t)}^{t}\bar{x}^{\mathrm{T}}(s)\mathrm{d}s\hat{N}_j\times\\&\int_{t-\hat{\iota}\varphi(t)}^{t}\bar{x}^{\mathrm{T}}(s)\mathrm{d}s,\end{aligned}\tag{38}$$

可以推出

$$\dot{V}_j(t)-\bar{\lambda}_2V_j(t)\leqslant\eta^{\mathrm{T}}(t)\varXi_j\eta(t)+2\breve{\lambda}_2\tilde{\omega}^{\mathrm{T}}(t)S_j\tilde{\omega}(t),\tag{39}$$

其中:

$$\eta^{\mathrm{T}}(t)=[\varsigma^{\mathrm{T}}(t)\int_{t-\tau_{\mathrm{s}}}^{t}\bar{x}^{\mathrm{T}}(s)\mathrm{d}s\int_{t-\hat{\iota}\varphi(t)}^{t}\bar{x}^{\mathrm{T}}(s)\mathrm{d}s],$$

$$\varXi_j=\begin{bmatrix}D_{11}&0&0\\ *&D_{22}&0\\ *&*&D_{33}\end{bmatrix},$$

$$D_{11}=\begin{bmatrix}\varDelta_{11}&0&0&\varDelta_{15}&\varDelta_{16}&\varDelta_{17}\\ *&\varDelta_{22}&0&0&0&0\\ *&*&\varDelta_{33}&0&0&0\\ *&*&*&\varDelta_{44}&\varDelta_{45}&\varDelta_{46}\\ *&*&*&*&\varDelta_{55}&\varDelta_{56}\\ *&*&*&*&*&\varDelta_{66}\end{bmatrix},$$

$$\varDelta_{11}=-\bar{\lambda}_2\hat{P}_j+\hat{M}_j+\hat{N}_j+W_i^{\mathrm{T}}A_j+A_j^{\mathrm{T}}W_i,$$

$$\varDelta_{14}=\hat{P}_j+A_j^{\mathrm{T}}W_i-W_i^{\mathrm{T}},$$

$$\varDelta_{15}=W_i^{\mathrm{T}}B_jK_i+A_j^{\mathrm{T}}W_i,$$

$$\varDelta_{16}=W_i^{\mathrm{T}}\hat{E}_j+A_j^{\mathrm{T}}W_i,$$

$$\varDelta_{22}=-\mathrm{e}^{-\bar{\lambda}_1\tau_{\mathrm{s}}}\hat{M}_j,$$

$$\varDelta_{33}=-(1-\hat{\iota})\mathrm{e}^{-\hat{\iota}\bar{\lambda}_1\tau_{\mathrm{s}}}\hat{N}_j,$$

$$\varDelta_{44}=-W_i^{\mathrm{T}}-W_i,$$

$$\varDelta_{45}=W_i^{\mathrm{T}}B_jK_i-W_i,$$

$$\varDelta_{46}=W_i^{\mathrm{T}}\hat{E}_j-W_i,$$

$$\varDelta_{55}=W_i^{\mathrm{T}}B_jK_i+K_i^{\mathrm{T}}B_j^{\mathrm{T}}W_i,$$

$$\varDelta_{56}=W_i^{\mathrm{T}}\hat{E}_j+K_i^{\mathrm{T}}B_j^{\mathrm{T}}W_i,$$

$$\varDelta_{66}=W_i^{\mathrm{T}}\hat{E}_j+\hat{E}_j^{\mathrm{T}}W_i-2\breve{\lambda}_2S_j,$$

$$\breve{\lambda}_2=\max_{j\in I_N}\breve{\lambda}_j,$$

$$D_{22}=-\frac{(\bar{\lambda}_1+\bar{\lambda}_2)}{\tau_{\mathrm{s}}}\hat{M}_j,$$

$$D_{33}=-\frac{(\bar{\lambda}_1+\bar{\lambda}_2)}{\tau_{\mathrm{s}}}\hat{N}_j.$$

根据式(39), 若$\varXi_j<0$可得

$$\dot{V}_j(t)\leqslant\bar{\lambda}_2V_j(t)+2\breve{\lambda}_2\tilde{\omega}^{\mathrm{T}}(t)S_j\tilde{\omega}(t),$$

结合$\|\dot{\omega}(t)\|\leqslant a_1$, 意味着下式成立:

$$\dot{V}_j(t)\leqslant\bar{\lambda}_2V_j(t)+\hat{\lambda}_2(a_1\tau_{\mathrm{s}})^2,$$

其中: $\hat{\lambda}_2=2\max\limits_{j\in I_N}(\breve{\lambda}_2\|S_j\|), t\in[\hat{t}_{\mathrm{s}},t_{k+1})$. 接着对上式由$\hat{t}_{\mathrm{s}}$到$t$积分求得

$$V_j(t)\leqslant\mathrm{e}^{\bar{\lambda}_2(t-\hat{t}_{\mathrm{s}})}V_j(\hat{t}_{\mathrm{s}})+\int_{\hat{t}_{\mathrm{s}}}^{t}\mathrm{e}^{\bar{\lambda}_2(t-\tau)}\hat{\lambda}_2(a_1\tau_{\mathrm{s}})^2\mathrm{d}\tau.\tag{40}$$

那么对于采样区间$[t_k,t_{k+1})$和$\hat{\lambda}=\max\{\hat{\lambda}_1,\hat{\lambda}_2\}$, 结合式(35)(40)以及条件(25)推出

$$\begin{aligned}V_j(t)\leqslant{}&\mu\mathrm{e}^{\bar{\lambda}_2(t-\hat{t}_{\mathrm{s}})}\mathrm{e}^{-\bar{\lambda}_1(\hat{t}_{\mathrm{s}}-t_k)}V_i(t_k)+\\&\mu\mathrm{e}^{\bar{\lambda}_2(t-\hat{t}_{\mathrm{s}})}\int_{t_k}^{\hat{t}_{\mathrm{s}}}\mathrm{e}^{-\bar{\lambda}_1(\hat{t}_{\mathrm{s}}-\tau)}\hat{\lambda}(a_1\tau_{\mathrm{s}})^2\mathrm{d}\tau+\\&\int_{\hat{t}_{\mathrm{s}}}^{t}\mathrm{e}^{\bar{\lambda}_2(t-\tau)}\hat{\lambda}(a_1\tau_{\mathrm{s}})^2\mathrm{d}\tau.\end{aligned}\tag{41}$$

由式(25)(41)联合得到

$$\begin{aligned}&V(t)\leqslant\\&\mu^2\int_{t_{k-1}}^{\hat{t}_{s-1}}\alpha_1\mathrm{d}\tau+\mu\int_{\hat{t}_{s-1}}^{t_k}\alpha_2\mathrm{d}\tau+\\&\mu^2\mathrm{e}^{-\bar{\lambda}_1(t-t_{k-1})+(\bar{\lambda}_1+\bar{\lambda}_2)[(t-\hat{t}_{\mathrm{s}})+(t_k-\hat{t}_{s-1})]}V(t_{k-1})+\\&\mu\int_{t_k}^{\hat{t}_{\mathrm{s}}}\mathrm{e}^{-\bar{\lambda}_1(t-\tau)+(\bar{\lambda}_1+\bar{\lambda}_2)(t-\hat{t}_{\mathrm{s}})}\hat{\lambda}(a_1\tau_{\mathrm{s}})^2\mathrm{d}\tau+\\&\int_{\hat{t}_{\mathrm{s}}}^{t}\mathrm{e}^{-\bar{\lambda}_1(t-\tau)+(\bar{\lambda}_1+\bar{\lambda}_2)(t-\tau)}\hat{\lambda}(a_1\tau_{\mathrm{s}})^2\mathrm{d}\tau\leqslant\cdots\leqslant\\&\mu^{N_\sigma(t,t_0)}\mathrm{e}^{-\bar{\lambda}_1(t-t_0)+(\bar{\lambda}_1+\bar{\lambda}_2)T_2(t,t_0)}V(t_0)+\\&\int_{t_0}^{t}\mathrm{e}^{-\bar{\lambda}_1(t-\tau)+(\bar{\lambda}_1+\bar{\lambda}_2)T_2(t,\tau)+N_\sigma(t,\tau)\ln\mu}\hat{\lambda}(a_1\tau_{\mathrm{s}})^2\mathrm{d}\tau,\end{aligned}\tag{42}$$

其中:
$\alpha_1 = \mathrm{e}^{-\bar\lambda_1(t-\tau)+(\bar\lambda_1+\bar\lambda_2)[(t-\hat t_s)+(t_k-\hat t_{s-1})]}\hat\lambda(a_1\tau_s)^2$,
$\alpha_2 = \mathrm{e}^{-\bar\lambda_1(t-\tau)+(\bar\lambda_1+\bar\lambda_2)(t_k-\tau)+(\bar\lambda_1+\bar\lambda_2)(t-\hat t_s)}\hat\lambda(a_1\tau_s)^2$,
$T_2(t,\tau)$代表区间$[\tau,t)$上子系统与控制器不匹配的总时间. 结合定义1和式(42), 可推出下式:

$$V(t)\leqslant \bar c\mathrm{e}^{-\bar\lambda(t-t_0)}V(t_0)+\int_{t_0}^{t}\bar c\mathrm{e}^{-\bar\lambda(t-\tau)}\hat\lambda(a_1\tau_s)^2\mathrm{d}\tau, \tag{43}$$

其中: $\bar\lambda=\bar\lambda_1-\tau_a^{-1}(\bar\lambda_1+\bar\lambda_2)\tau_s-\tau_a^{-1}\ln\mu$, $N_1=1+N_0$, $\bar c=\mathrm{e}^{N_0\ln\mu+(\bar\lambda_1+\bar\lambda_2)N_1}$.

根据所构造的Lyapunov-Krasovskii泛函得

$$\begin{cases} V(t)\geqslant\bar x^{\mathrm T}(t)\hat P_i\bar x(t)\geqslant\lambda_m(\hat P_i)\|\bar x(t)\|^2\geqslant\hat a\|\bar x(t)\|^2,\\ V(t_0)\leqslant(\hat b+\tau_s\hat c+\hat\iota\tau_s\hat d)\|\bar x(t_0)\|^2=\hat{\bar b}\|\bar x(t_0)\|^2, \end{cases} \tag{44}$$

其中:
$\hat a=\inf\limits_{i\in I_N}\lambda_m(\hat P_i)$, $\hat b=\sup\limits_{i\in I_N}\lambda_M(\hat P_i)$, $\hat c=\sup\limits_{i\in I_N}\lambda_M(\hat M_i)$,
$\hat d=\sup\limits_{i\in I_N}\lambda_M(\hat N_i)$, $\hat{\bar b}=\hat b+\tau_s\hat c+\hat\iota\tau_s\hat d$.
受式(28)约束, 使得$\bar\lambda>0$成立那么, 当时间t趋于无穷时, 得到

$$\lim_{t\to\infty}\sup V(t)\leqslant\lim_{t\to\infty}\sup[\bar c\mathrm{e}^{-\bar\lambda(t-t_0)}(V(t_0)-\frac{\hat\lambda(a_1\tau_s)^2}{\bar\lambda})+\frac{\bar c\hat\lambda(a_1\tau_s)^2}{\bar\lambda}]=\frac{\bar c\hat\lambda(a_1\tau_s)^2}{\bar\lambda}. \tag{45}$$

基于式(44), 对于给定的任意$\varepsilon_1>0$, 令

$$0<a_1\leqslant\frac{\varepsilon_1}{\tau_s}\sqrt{\frac{\bar\lambda\hat a}{\bar c\hat\lambda}}, \tag{46}$$

结合式(45)–(46)得

$$\lim_{t\to\infty}\sup\|\bar x(t)\|\leqslant\lim_{t\to\infty}\sup\sqrt{\frac{V(t)}{\hat a}}\leqslant\sqrt{\frac{\bar c\hat\lambda(a_1\tau_s)^2}{\hat a\bar\lambda}}\leqslant\varepsilon_1,$$

则调节输出满足

$$\lim_{t\to\infty}\sup\|e(t)\|\leqslant\max_{i\in I_N}\|C_i\|\varepsilon_1=\varepsilon.$$

综上所述, 问题2可解. 证毕.

注 3 与定理1和注2相似, 用同样的方法求解控制器增益K_i和L_i, 即将以$\tilde W_i^{\mathrm T}$和$\tilde W_i$为对角线元素的对角矩阵分别乘到$\Theta_i<0$和$\Xi_j<0$的两侧, 则有$R_i=K_i\tilde W_i$, $H_i=L_i\tilde W_i$, $\hat{\bar P}_i=\tilde W_i^{\mathrm T}\hat P_i\tilde W_i$, $\tilde S_i=\tilde W_i^{\mathrm T}S_i\tilde W_i$, $\hat{\bar M}_i=\tilde W_i^{\mathrm T}\hat M_i\tilde W_i$, $\hat{\bar N}_i=\tilde W_i^{\mathrm T}\hat N_i\tilde W_i$, 模态$j$中的此类矩阵具有与上述相同的形式. 从而得到不等式(26)–(27).

5 数值例子

本节通过两个数值例子, 说明了结果的可行性.

例 1 对应$\dot\omega(t)=0$的情况, 考虑切换系统(1), 其中:

$$A_1=\begin{bmatrix}-1.6&0.2\\0&-0.4\end{bmatrix},\ A_2=\begin{bmatrix}-2.2&0\\0.1&-0.2\end{bmatrix},$$
$$B_1=\begin{bmatrix}-0.5\\1.2\end{bmatrix},\ B_2=\begin{bmatrix}0.6\\-1.1\end{bmatrix},\ S=\begin{bmatrix}1&0\\0&1\end{bmatrix},$$
$$E_1=\begin{bmatrix}-1.4&0\\0.4&1.8\end{bmatrix},\ E_2=\begin{bmatrix}-1.2&0\\0.3&-0.2\end{bmatrix},$$

$C_1=C_2=[-0.2\ \ 0.2]$, $Q_1=Q_2=[-0.4\ \ 0.2]$, 且有满足输出调节方程的矩阵

$$\varPi=\begin{bmatrix}-1&2\\1&2\end{bmatrix},$$

令$\lambda_1=0.5$, $\lambda_2=0.8$, $\mu=1.6$, $\iota=0.5$和$\tau_s=0.4$, 求解定理1中的不等式(5)–(7)得到

$$K_1=[-0.0864\quad 0.0485],$$
$$K_2=[-0.0936\quad -0.0345],$$
$$L_1=[-0.1350\quad 0.0758],$$
$$L_2=[-0.0592\quad 0.2562].$$

根据式(9)计算出最小平均驻留时间$\tau_a^*=1.9800$, 选取切换规则$\tau_a=2.1>\tau_a^*$. 以$\bar x(0)=[-1.5\ \ 2.5]^{\mathrm T}$和$\omega(0)=[0\ \ 1]^{\mathrm T}$为初始状态进行仿真, 得到闭环系统指数稳定如图1所示. 图2显示了调节输出. 仿真结果表明了所设计的采样数据全信息反馈控制器在满足平均驻留时间的切换条件下可解决闭环系统(11)的输出调节问题.

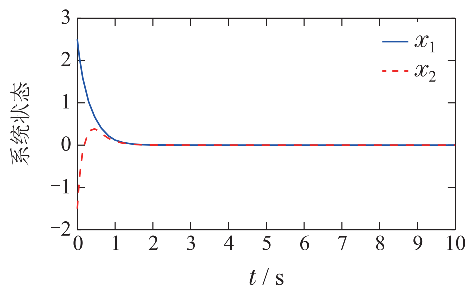

图 1 状态响应

Fig. 1 States responses

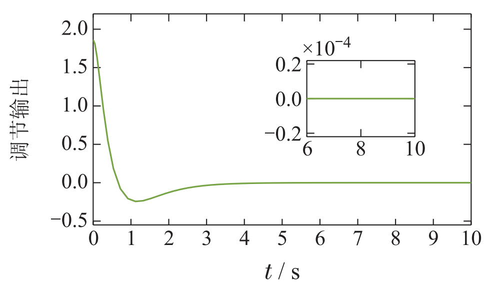

图 2 调节输出

Fig. 2 Regulated output

例 2　对应$\|\dot{\omega}(t)\|\leqslant a_1$的情况. 考虑切换系统(1), 其中:

$$A_1=\begin{bmatrix}-2.3 & 0\\ 0.4 & -0.9\end{bmatrix}, A_2=\begin{bmatrix}-2.8 & 0.3\\ 0 & -1.6\end{bmatrix},$$

$$B_1=\begin{bmatrix}-1.5\\ 0.2\end{bmatrix}, B_2=\begin{bmatrix}0.6\\ -1.1\end{bmatrix}, S=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix},$$

$C_1=C_2=[0.4\quad -1]$, $Q_1=Q_2=[-0.5\quad 0.2]$, 且有满足调节方程的矩阵

$$\Pi=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, E_1=\begin{bmatrix}1.8484 & 1.8122\\ -1.3398 & 0.7917\end{bmatrix},$$

$$E_2=\begin{bmatrix}1.6330 & 0.9064\\ 1.1395 & 1.2216\end{bmatrix},$$

令$\bar{\lambda}_1=0.5$, $\bar{\lambda}_2=1.2$, $\breve{\lambda}_1=\breve{\lambda}_2=0.4$, $\mu=1.3$, $\hat{\iota}=0.6$, $\tau_{\mathrm{s}}=0.5$, $a_1=0.7$和$\varepsilon=0.5$, 求解定理2中的矩阵不等式(25)–(27), 即

$K_1=[0.1950\quad 0.1741]$, $K_2=[0.0421\quad 0.0762]$,

$L_1=[-0.4961\quad 0.3674]$, $L_2=[1.9029\quad -0.4202]$.

根据式(28)计算出$\tau_{\mathrm{a}}^*=2.2247$, 选取平均驻留时间$\tau_{\mathrm{a}}=2.3>\tau_{\mathrm{a}}^*$. 在该切换规则下, 对于初始状态$\bar{x}(0)=[-0.4\ \ 0.5]^{\mathrm{T}}$和$\omega(0)=[-0.2\ \ 0.6]^{\mathrm{T}}$进行仿真, 得到闭环系统的状态稳定, 如图3所示. 调节输出在图4中显示. 仿真结果说明了闭环系统(29)的实用输出调节问题可解.

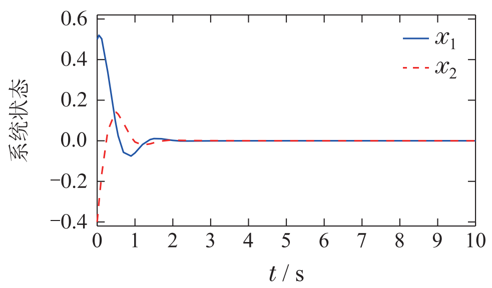

图 3　状态响应

Fig. 3　States responses

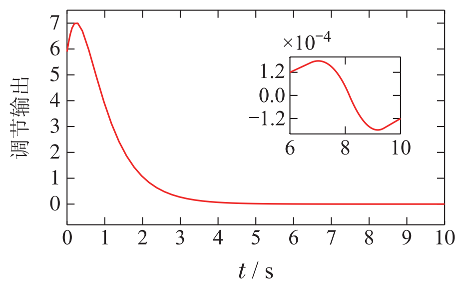

图 4　调节输出

Fig. 4　Regulated output

6　结论

本文针对线性切换系统研究了基于采样数据的输出调节问题, 分别考虑了常数形式的外部输入信号和具有有界导数且时变的外部输入信号. 针对这两种外部输入信号, 设计了合适的采样状态反馈控制器, 同时利用平均驻留时间方法给出了切换系统的输出调节问题可解的充分条件. 关于采样控制下的切换系统还有很多问题尚未研究, 接下来将基于本文结果进一步研究切换系统的事件驱动周期采样输出调节问题.

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