具有随机网络攻击的切换信息物理系统的事件触发控制

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摘要: 本文研究了随机网络攻击下切换信息物理系统的事件触发控制问题. 将信息物理系统描述为一种切换线性系统形式. 引入事件触发机制来节省系统资源和减轻网络负载, 当误差超过给定阈值时传感器中的采样数据才通过通信网络传输到控制器中. 考虑在传感器与控制器的通信网络中受到两种不同特征的随机网络攻击. 在网络攻击和所设计的事件触发控制器下, 建立了切换随机信息物理系统模型. 利用模态依赖平均驻留时间方法构建了相应的切换信号. 在设计的事件触发控制器和模态依赖平均驻留时间切换信号下实现了系统的均方指数稳定性, 并给出了控制器增益. 最后, 通过实例验证了所得理论结果的有效性.

关键词: 切换信息物理系统; 事件触发控制; 模态依赖平均驻留时间; 网络攻击

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Event-triggered control of switched cyber-physical systems with stochastic cyber attacks

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Abstract: This paper focuses on the issue of event-triggered control against stochastic cyber attacks for switched cyber-physical systems. Cyber-physical systems are described as a form of switched linear systems. In order to conserve system resources and alleviate network load, an event-triggered scheme is introduced, under which the sampled data from the sensor is transmitted to the controller via the communication network when the defined error exceeds a given threshold. And two different characteristics of stochastic cyber attacks are considered in the sensor-controller communication network. Then, under cyber attacks and the designed event-triggered controller, a switched stochastic cyber-physical system model is established. In addition, by utilizing the mode-dependent average dwell time method, the corresponding switching signal is constructed. The mean-square exponential stability is guaranteed under the designed event-triggered controller and mode-dependent average dwell time switching signal, and the controller gain is presented. Finally, an example is exploited to verify the validity of the obtained theoretical results.

Key words: switched cyber-physical system; event-triggered control; mode-dependent average dwell time; cyber attacks

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1 Introduction

Cyber-physical systems (CPSs) are multiple dimensional complicated systems, which integrate advanced information technology and automatic control technology such as perception, calculation, communication, and control [1–3]. Due to the interaction and coordination of various elements in the physical space and the cyber space, CPSs realize on-demand response and dynamic optimization of resource allocation and operation [4–5]. CPSs have an extensive scope of applications, such as smart cities [6], electronic circuits [7], smart grid [8–9], electric vehicles [10], health care [11], water/gas distribution networks [12], to name a few. With the quick growth of computing, communication and intelligent control technology, CPSs have attracted widespread attention in the academic community in the past few years.

Although CPSs now have many advantages and

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bring great convenience to people's lives, they still face some challenges. One of the most challenging issues is the security of CPSs [13]. CPSs have become more sensitive to external environmental impacts due to the more open and shared networks and communications, in particular, they are vulnerable to malicious cyber attacks [14]. And CPSs are usually used in many large infrastructures that are critical, the occurrence of malicious cyber attacks may threaten national security and social stability and cause huge economic losses [15]. Thus, the security problem of CPSs need to be addressed urgently. Many scholars are devoted to studying the security issues of CPSs, and a large number of research results related to this issue have been proposed, see [16–17] and the references therein.

In recent years, the stability analysis and control synthesis of the switched system has become a hot spot and has achieved fruitful achievements because of its practical value. The switched systems are hybrid systems, which are composed of a limited number of subsystems and a switching signal coordinating these subsystems [18]. The switched system can approximate complex nonlinear processes, so it can be used to describe and model complex system behavior. As complex systems, CPSs can be considered as switched systems, see [19–20] and the references therein. Moreover, periodic control scheme sometimes has the insufficiency of occupying network bandwidth and wasting computing resources, therefore, event-triggered control (ETC) is proposed to reduce these unnecessary wastes [21]. A lot of research achievements have been made in ETC [22–25].

Inspired by the discussion above, the ETC problem with stochastic cyber attacks is explored for switched CPSs in this paper. Two different characteristics of stochastic attacks are considered, and the event-triggered mechanism is adopted to reduce network load and save system resources. The mean-square exponential stability (MSES) is guaranteed to the switched stochastic system by utilizing the multiple Lyapunov function technology and mode-dependent average dwell time (MDADT) method. Finally, an example is presented to illustrate the validity of the obtained results. The key contributions are summarized as below. 1) In view of the complexity of the physical layer system, the dynamics of CPSs is described as a switched system form. 2) An event-triggered mechanism is introduced to decrease unnecessary data transmission. 3) Considering two different characteristics of stochastic cyber attacks, a switched stochastic CPS model is established. Then under the designed event-triggered controller and MDADT switching strategy, the MSES is guaranteed.

Notations The notations used throughout the article are standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the n-dimensional Euclidean space and the set of $n\times m$ real matrices, respectively. The notation $P>0$ ($P\geqslant 0$) means that matrix P is positive definite (semi-positive definite). $\mathbb{N}$ denotes the set of natural numbers. I is an identity matrix with appropriate dimension. $\mathrm{diag}\{\cdot\}$ denotes a block-diagonal matrix. The superscript T stands for matrix transposition and the superscript -1 denotes the inverse of a matrix. $\mathrm{P}\{A\}$ denotes probability of event A to occur. $\mathrm{E}\{\cdot\}$ denotes the mathematical expectation operator. $*$ stands for the symmetric terms in symmetric block matrices. And $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ are the smallest and the largest eigenvalues of matrix P, respectively.

2 Problem formulation

Consider the following CPS described by switched linear form:

$$\dot{x}(t)=A_{\sigma(t)}x(t)+B_{\sigma(t)}u(t), \tag{1}$$

where $x(t)\in\mathbb{R}^n$ and $u(t)\in\mathbb{R}^m$ are the state and the control input, respectively. The switching signal $\sigma(t)$ takes its values in the set $\bar{M}=\{1,2,\cdots,\aleph\}$, where $\aleph$ stands for the number of subsystems. And the switching sequence is denoted as $\varXi=\{\,(h_0,s_0),(h_1,s_1),\cdots,(h_q,s_q),\cdots\mid h_q\in\bar{M},\ q\in\mathbb{N}\}$. The h_q-th subsystem is waked when $\sigma(s_q)=h_q$. Besides, for $\forall i\in\bar{M}$, $A_i\in\mathbb{R}^{n\times n}$ and $B_i\in\mathbb{R}^{n\times m}$ are known constant matrices with appropriate dimensions.

The controller is designed as follows:

$$u(t)=K_{\sigma(t)}x(t_kh), \tag{2}$$

where $K_{\sigma(t)}(\sigma(t)\in\bar{M})$ is the controller gain subsequently designed, t_kh is the sampling instant, and h is the sampling period.

Then, an event-triggered mechanism (ETM) is introduced to reduce communication burden of the network, and the system state $x(t_kh)$ is determined by the event-triggered scheme whether to be released. The event-triggered condition is described as

$$\begin{aligned}&t_{k_p+1}h=t_{k_p}h+\\&\min_{l_k\in\mathbb{N}}\{l_kh\mid e^{\mathrm{T}}(t_{k_p}h+l_kh)\varOmega_{1\sigma(t)}e(t_{k_p}h+l_kh)\geqslant\\&\wp_{\sigma(t)}x^{\mathrm{T}}(t_{k_p}h+l_kh)\varOmega_{2\sigma(t)}x(t_{k_p}h+l_kh)\},\end{aligned} \tag{3}$$

where $e(t_{k_p}h+l_kh)=x(t_{k_p}h)-x(t_{k_p}h+l_kh)$, $t_{k_p}h$ and $t_{k_p+1}h$ stand for the latest triggering instant and the next triggering instant, respectively. $t_{k_p}h+l_kh$ denotes the current sampling instant. Scalar $\wp_i\in(0,1)$, $\varOmega_{1i}>0$ and $\varOmega_{2i}>0(i\in\bar{M})$ are symmetric and positive definite weighting matrices. When the latest information $x(t_{k_p}h+l_kh)$ satisfies the event-triggered condition (3), the current sampled data $x(t_{k_p}h+l_kh)$ is transmitted.

When the stochastic cyber attacks occur, the controller is rewritten as

$$u(t)=K_{\sigma(t)}x(t_{k_p}h)+$$

$$\nu(t)K_{\sigma(t)}(\vartheta(t)h_1(x(t-\eta_1(t)))+ (1-\vartheta(t))h_2(x(t-\eta_2(t)))-x(t_{k_p}h)), \quad (4)$$

where $h_1(x(t))$ and $h_2(x(t))$ are two different forms of cyber attacks, $\eta_1(t) \in [0, \eta_{1M}]$ and $\eta_2(t) \in [0, \eta_{2M}]$ are the corresponding time delay of stochastic cyber attacks, where $\eta_{2M} > 0$ and $\eta_{1M} > 0$ represent a maximum cyber-attack time delay. Random variables $\nu(t)$ and $\vartheta(t)$ obey Bernoulli distribution with the probability $\mathrm{P}\{\nu(t) = 1)\} = \bar{\nu}$ and $\mathrm{P}\{\vartheta(t) = 1)\} = \bar{\vartheta}$.

Remark 1 The stochastic attacks $h_1(x(t))$ and $h_2(x(t))$ are supposed to satisfy the following inequalities [26]:

$$\begin{cases} h_1^{\mathrm{T}}(x(t))h_1(x(t)) \leqslant x^{\mathrm{T}}(t)G^{\mathrm{T}}Gx(t), \\ h_2^{\mathrm{T}}(x(t))h_2(x(t)) \leqslant x^{\mathrm{T}}(t)H^{\mathrm{T}}Hx(t), \end{cases} \quad (5)$$

where G and H are given constant matrices denoting the upper bound of cyber attacks.

Remark 2 The data submitted by the ETM is transmitted to the controller through the network. In this paper, assume that there are deception attacks on the network here. And whether a cyber attack occurs randomly is governed by $\nu(t)$. If $\nu(t) = 1$, the system is under cyber attacks launched by the adversary, and the control input is modified. $\nu(t) = 0$ represents that the cyber attack did not occur. The random variable $\vartheta(t)$ determines the form of attack on the system when an attack occurs. $\vartheta(t) = 1$ signifies that the cyber-attack $h_1(x(t))$ occurs and $\vartheta(t) = 0$ implies that $h_2(x(t))$ occurs.

Define $\hbar_k h = t_{k_p}h + \jmath h, \jmath = 0, 1, \cdots, l_k - 1$, then $[t_{k_p}h, t_{k_p+1}h)$ can be divided into $[t_{k_p}h + \jmath h, t_{k_p}h + (\jmath + 1)h), \jmath = 0, 1, \cdots, l_k - 2$ and $[t_{k_p}h + (l_k - 1)h, t_{k_p+1}h)$. Letting $\tau(t) = t - \hbar_k h$, we can obtain $x(\hbar_k h) = x(t - \tau(t))$ and

$$x(t_{k_p}h) = e(\hbar_k h) + x(t - \tau(t)). \quad (6)$$

Furthermore, one obtains

$$e^{\mathrm{T}}(\hbar_k h)\varOmega_{1\sigma(t)}e(\hbar_k h) < \wp_{\sigma(t)}x^{\mathrm{T}}(t-\tau(t))\varOmega_{2\sigma(t)}x(t-\tau(t)). \quad (7)$$

Substituting Eq. (6) into Eq. (4) yields

$$u(t) = (1-\nu(t))K_{\sigma(t)}(e(\hbar_k h) + x(t-\tau(t))) + \nu(t)K_{\sigma(t)}(\vartheta(t)h_1(x(t-\eta_1(t))) + (1-\vartheta(t))h_2(x(t-\eta_2(t)))). \quad (8)$$

By combining Eq. (1) with Eq. (8), the switched stochastic CPS is derived as

$$\begin{aligned}\dot{x}(t) = {} & A_{\sigma(t)}x(t) + \\ & (1-\bar{\nu})B_{\sigma(t)}K_{\sigma(t)}(x(t-\tau(t)) + e(\hbar_k h)) + \\ & \bar{\nu}(\bar{\vartheta}B_{\sigma(t)}K_{\sigma(t)}h_1(x(t-\eta_1(t))) + \\ & (1-\bar{\vartheta})B_{\sigma(t)}K_{\sigma(t)}h_2(x(t-\eta_2(t)))) + \\ & (\nu(t)-\bar{\nu})(\vartheta(t)-\bar{\vartheta})(B_{\sigma(t)}K_{\sigma(t)}h_1(x(t- \\ & \eta_1(t))) - B_{\sigma(t)}K_{\sigma(t)}h_2(x(t-\eta_2(t)))) + \\ & \bar{\nu}(\vartheta(t)-\bar{\vartheta})(B_{\sigma(t)}K_{\sigma(t)}h_1(x(t-\eta_1(t))) - \\ & B_{\sigma(t)}K_{\sigma(t)}h_2(x(t-\eta_2(t)))) + \\ & (\nu(t)-\bar{\nu})(\bar{\vartheta}B_{\sigma(t)}K_{\sigma(t)}h_1(x(t-\eta_1(t))) + \\ & (1-\bar{\vartheta})B_{\sigma(t)}K_{\sigma(t)}h_2(x(t-\eta_2(t)))) + (\bar{\nu} - \\ & \nu(t))B_{\sigma(t)}K_{\sigma(t)}(x(t-\tau(t)) + e(\hbar_k h)). \quad (9)\end{aligned}$$

In this paper, the primary objective is to construct a state feedback controller under the ETM and an MDADT switching signal such that system (9) can achieve MSES, that is, it can ensure that the switched cyber-physical system (1) can achieve MSES when it is attacked.

Some lemmas and definitions are given as below.

Lemma 1[27] For any symmetric and positive definite matrix U, P and positive scalars γ, the following inequality holds:

$$-PUP \leqslant -2\gamma P + \gamma^2 U^{-1}. \quad (10)$$

Definition 1[28] For $\forall t_2 \geqslant t_1 \geqslant 0$ and a switching signal $\sigma(t)$, $N_{\sigma s}(t_1, t_2)$ stands for the switching numbers that the s-th subsystem works over the interval $[t_1, t_2]$, $T_s(t_1, t_2)$ represents the total running time of the s-th subsystem over the interval $[t_1, t_2]$, $s \in \bar{M}$. If there exist $N_{0s} \geqslant 0$ and $\tau_{as} > 0$ such that

$$N_{\sigma s}(t_1, t_2) \leqslant N_{0s} + \frac{T_s(t_1, t_2)}{\tau_{as}}, \ \forall t_2 \geqslant t_1 \geqslant 0, \quad (11)$$

then τ_{as} is referred to as the mode-dependent average dwell time and N_{0s} is the mode-dependent chatter bounds.

Definition 2[29] Give scalars $\varsigma \geqslant 1$, $\varepsilon > 0$. System (9) can achieve MSES if

$$\mathrm{E}\{||x(t)||^2\} \leqslant \varsigma \mathrm{e}^{-\varepsilon(t-t_0)}||x(t_0)||^2 \quad (12)$$

for any initial condition $x(t_0)$ and the designed switching signal.

3 Main results

The sufficient conditions are given to ensure the MSES of switched system (9) in this section.

Theorem 1 Give positive scalars $\tau_M, \eta_{1M}, \eta_{2M}$, $\bar{\nu}$, $\bar{\vartheta}$, $\wp_i$, θ_i and $\mu_i > 1$. If there exist symmetric and positive definite matrices P_i, Q_{1i}, Q_{2i}, R_{1i}, R_{2i}, S_{1i}, S_{2i}, $\varOmega_{1i}$, $\varOmega_{2i}$ such that the following inequalities are true:

$$\begin{bmatrix} \varPsi_i & * \\ T_i & \varLambda_i \end{bmatrix} < 0, \quad (13)$$

$$\begin{cases} P_i \leqslant \mu_i P_j, Q_{1i} \leqslant \mu_i Q_{1j}, Q_{2i} \leqslant \mu_i Q_{2j}, \\ R_{1i} \leqslant \mu_i R_{1j}, R_{2i} \leqslant \mu_i R_{2j}, \\ S_{1i} \leqslant \mu_i S_{1j}, S_{2i} \leqslant \mu_i S_{2j}, \end{cases} \quad (14)$$

where

$$\varPsi_i = \begin{bmatrix} \varPsi_{1i} & * \\ \varPsi_{2i} & \varPsi_{3i} \end{bmatrix},$$

$$\Psi_{1i}=\begin{bmatrix}\Sigma_{1i} & * & * & * & * \\ \Sigma_{2i} & \wp_i\Omega_{2i}-2\mathrm{e}^{-\theta_i\tau_M}Q_{2i} & * & * & * \\ 0 & \mathrm{e}^{-\theta_i\tau_M}Q_{2i} & -\mathrm{e}^{-\theta_i\tau_M}(Q_{1i}+Q_{2i}) & * & * \\ (1-\bar{\nu})K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i & 0 & 0 & -\Omega_{1i} & * \\ \mathrm{e}^{-\theta_i\eta_{1M}}R_{2i} & 0 & 0 & 0 & -2\mathrm{e}^{-\theta_i\eta_{1M}}R_{2i}\end{bmatrix},$$

$$\Psi_{2i}=\begin{bmatrix}0 & 0 & 0 & 0 & \mathrm{e}^{-\theta_i\eta_{1M}}R_{2i} \\ \mathrm{e}^{-\theta_i\eta_{2M}}S_{2i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \bar{\nu}\bar{\vartheta}K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i & 0 & 0 & 0 & 0 \\ \bar{\nu}(1-\bar{\vartheta})K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Psi_{3i}=\begin{bmatrix}-\mathrm{e}^{-\theta_i\eta_{1M}}(R_{1i}+R_{2i}) & * & * & * & * \\ 0 & -2\mathrm{e}^{-\theta_i\eta_{2M}}S_{2i} & * & * & * \\ 0 & \mathrm{e}^{-\theta_i\eta_{2M}}S_{2i} & -\mathrm{e}^{-\theta_i\eta_{2M}}(S_{1i}+S_{2i}) & * & * \\ 0 & 0 & 0 & -I & * \\ 0 & 0 & 0 & 0 & -I\end{bmatrix},$$

$$\begin{aligned}\Sigma_{1i}=&A_i^{\mathrm{T}}P_i+P_iA_i+Q_{1i}+R_{1i}+S_{1i}+\theta_iP_i-\\&\mathrm{e}^{-\theta_i\tau_M}Q_{2i}-\mathrm{e}^{-\theta_i\eta_{1M}}R_{2i}-\mathrm{e}^{-\theta_i\eta_{2M}}S_{2i},\end{aligned}$$

$$\Sigma_{2i}=(1-\bar{\nu})K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i+\mathrm{e}^{-\theta_i\tau_M}Q_{2i},$$

$$T_i=\begin{bmatrix}\Phi_{1i}^{\mathrm{T}} & \Phi_{2i}^{\mathrm{T}} & \Phi_{3i}^{\mathrm{T}} & \Phi_{4i}^{\mathrm{T}} & \Phi_{3i}^{\mathrm{T}} & \Phi_{4i}^{\mathrm{T}} & \Phi_{5i}^{\mathrm{T}} & \Phi_{6i}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$

$$\Lambda_i=\mathrm{diag}\{\phi_i,\phi_i,\phi_i,\phi_i,\phi_i,\phi_i,-I,-I\},$$

$$\Phi_{1i}=\begin{bmatrix}\tau_M\varphi_{1i}\\ \eta_{1M}\varphi_{1i}\\ \eta_{2M}\varphi_{1i}\end{bmatrix},\ \Phi_{2i}=\begin{bmatrix}\tau_M\varphi_{2i}\\ \eta_{1M}\varphi_{2i}\\ \eta_{2M}\varphi_{2i}\end{bmatrix},$$

$$\Phi_{3i}=\begin{bmatrix}\tau_M\varphi_{3i}\\ \eta_{1M}\varphi_{3i}\\ \eta_{2M}\varphi_{3i}\end{bmatrix},\ \Phi_{4i}=\begin{bmatrix}\tau_M\varphi_{4i}\\ \eta_{1M}\varphi_{4i}\\ \eta_{2M}\varphi_{4i}\end{bmatrix},$$

$$\Phi_{5i}=\begin{bmatrix}0 & 0 & 0 & 0 & G & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Phi_{6i}=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & H & 0 & 0 & 0\end{bmatrix},$$

$$\begin{aligned}\varphi_{1i}=&\left[P_iA_i\ \ (1-\bar{\nu})P_iB_iK_i\ \ 0\ \ (1-\bar{\nu})P_iB_iK_i\right.\\&\left.0\ \ 0\ \ 0\ \ 0\ \ \bar{\nu}\bar{\vartheta}P_iB_iK_i\ \ \bar{\nu}(1-\bar{\vartheta})P_iB_iK_i\right],\end{aligned}$$

$$\varphi_{2i}=[0_{1\times 8}\ \ \sqrt{\tilde{\vartheta}\bar{\nu}}P_iB_iK_i\ \ -\sqrt{\tilde{\vartheta}\bar{\nu}}P_iB_iK_i],$$

$$\varphi_{3i}=\left[0_{1\times 8}\ \ \sqrt{\tilde{\nu}}\bar{\vartheta}P_iB_iK_i\ \ \sqrt{\tilde{\nu}}(1-\bar{\vartheta})P_iB_iK_i\right],$$

$$\varphi_{4i}=\left[0\ \sqrt{\tilde{\nu}}P_iB_iK_i\ 0\ \sqrt{\tilde{\nu}}P_iB_iK_i\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\right],$$

$$\tilde{\nu}=\bar{\nu}(1-\bar{\nu}),\ \tilde{\vartheta}=\bar{\vartheta}(1-\bar{\vartheta}),$$

$$\phi_i=\mathrm{diag}\{-P_iQ_{2i}^{-1}P_i,-P_iR_{2i}^{-1}P_i,-P_iS_{2i}^{-1}P_i\},$$

then system (9) can achieve MSES with the controller (8), the ETM (7) and the switching signal satisfying

$$\tau_{ai}\geqslant\tau_{ai}^*=\frac{\ln\mu_i}{\theta_i}. \tag{15}$$

Proof Construct the following Lyapunov function as:

$$V_{\sigma(t)}(t)=V_{1\sigma(t)}(t)+V_{2\sigma(t)}(t)+V_{3\sigma(t)}(t), \tag{16}$$

where

$$\begin{aligned}&V_{1\sigma(t)}(t)=x^{\mathrm{T}}(t)P_{\sigma(t)}x(t),\\&V_{2\sigma(t)}(t)=\int_{t-\tau_M}^{t}\mathrm{e}^{-\theta_{\sigma(t)}(t-s)}x^{\mathrm{T}}(s)Q_{1\sigma(t)}x(s)\mathrm{d}s+\\&\qquad\int_{t-\eta_{1M}}^{t}\mathrm{e}^{-\theta_{\sigma(t)}(t-s)}x^{\mathrm{T}}(s)R_{1\sigma(t)}x(s)\mathrm{d}s+\\&\qquad\int_{t-\eta_{2M}}^{t}\mathrm{e}^{-\theta_{\sigma(t)}(t-s)}x^{\mathrm{T}}(s)S_{1\sigma(t)}x(s)\mathrm{d}s,\\&V_{3\sigma(t)}(t)=\\&\tau_M\int_{-\tau_M}^{0}\int_{t+s}^{t}\mathrm{e}^{-\theta_{\sigma(t)}(t-v)}\dot{x}(v)Q_{2\sigma(t)}\dot{x}(v)\mathrm{d}v\mathrm{d}s+\\&\eta_{1M}\int_{-\eta_{1M}}^{0}\int_{t+s}^{t}\mathrm{e}^{-\theta_{\sigma(t)}(t-v)}\dot{x}(v)R_{2\sigma(t)}\dot{x}(v)\mathrm{d}v\mathrm{d}s+\\&\eta_{2M}\int_{-\eta_{2M}}^{0}\int_{t+s}^{t}\mathrm{e}^{-\theta_{\sigma(t)}(t-v)}\dot{x}(v)S_{2\sigma(t)}\dot{x}(v)\mathrm{d}v\mathrm{d}s.\end{aligned}$$

Besides, because the ETM is adopted in the switched system, there is a corresponding relationship between the switched instants and the event-triggered instants. Therefore, the stability analysis is discussed through the following different cases.

Case 1: The event-triggered instant does not occur in the switching interval $[s_q,s_{q+1}]$, i.e., $t_{k_p}h<(t_{k_p}+1)h<\cdots<(t_{k_p}+\ell)h\leqslant s_q<(t_{k_p}+\ell+1)h<\cdots<(t_{k_p}+\ell+c)h\leqslant s_{q+1}<t_{k_p+1}h$, where $\ell,c\in\mathbb{N}_+$, $\ell+c<l_k$. Let $\sigma(s_q)=i(i\in\bar{M})$ in the interval $t\in[s_q,s_{q+1}]$, and $e(t)=e(\hbar_kh)$.

By taking the derivative of (16), we have

$$\begin{aligned}&\mathrm{E}\{\dot{V}_i(t)\}=\mathrm{E}\{\dot{V}_{1i}(t)+\dot{V}_{2i}(t)+\dot{V}_{3i}(t)\}=\\&\mathrm{E}\{\dot{x}^{\mathrm{T}}(t)P_ix(t)+x^{\mathrm{T}}(t)P_i\dot{x}(t)-\theta_iV_{2i}(t)+\\&x^{\mathrm{T}}(t)Q_{1i}x(t)+x^{\mathrm{T}}(t)R_{1i}x(t)+x^{\mathrm{T}}(t)S_{1i}x(t)-\\&\mathrm{e}^{-\theta_i\tau_M}x^{\mathrm{T}}(t-\tau_M)Q_{1i}x(t-\tau_M)-\\&\mathrm{e}^{-\theta_i\eta_{1M}}x^{\mathrm{T}}(t-\eta_{1M})R_{1i}x(t-\eta_{1M})-\\&\mathrm{e}^{-\theta_i\eta_{2M}}x^{\mathrm{T}}(t-\eta_{2M})S_{1i}x(t-\eta_{2M})-\\&\theta_iV_{3i}(t)+\tau_M^2\dot{x}^{\mathrm{T}}(t)Q_{2i}\dot{x}(t)+\\&d_M^2\dot{x}^{\mathrm{T}}(t)R_{2i}\dot{x}(t)+\eta_M^2\dot{x}^{\mathrm{T}}(t)S_{2i}\dot{x}(t)-\\&\tau_M\mathrm{e}^{-\theta_i\tau_M}\int_{t-\tau_M}^{t}\dot{x}^{\mathrm{T}}(s)Q_{2i}\dot{x}(s)\mathrm{d}s-\\&\eta_{1M}\mathrm{e}^{-\theta_i\eta_{1M}}\int_{t-\eta_{1M}}^{t}\dot{x}^{\mathrm{T}}(s)R_{2i}\dot{x}(s)\mathrm{d}s-\\&\eta_{2M}\mathrm{e}^{-\theta_i\eta_{2M}}\int_{t-\eta_{2M}}^{t}\dot{x}^{\mathrm{T}}(s)S_{2i}\dot{x}(s)\mathrm{d}s\}.\end{aligned} \tag{17}$$

Using Jensen inequality, one obtains

$$\begin{aligned}&-\tau_M\mathrm{e}^{-\theta_i\tau_M}\int_{t-\tau_M}^{t}\dot{x}^{\mathrm{T}}(s)Q_{2i}\dot{x}(s)\mathrm{d}s\leqslant-\mathrm{e}^{-\theta_i\tau_M}\times\\&(x(t)-x(t-\tau_M))^{\mathrm{T}}Q_{2i}(x(t)-x(t-\tau_M)),\\&-\eta_{1M}\mathrm{e}^{-\theta_i\eta_{1M}}\int_{t-\eta_{1M}}^{t}\dot{x}^{\mathrm{T}}(s)R_{2i}\dot{x}(s)\mathrm{d}s\leqslant-\mathrm{e}^{-\theta_i\eta_{1M}}\times\\&(x(t)-x(t-\eta_{1M}))^{\mathrm{T}}R_{2i}(x(t)-x(t-\eta_{1M})),\end{aligned}$$

$$-\eta_{2M}\mathrm{e}^{-\theta_i\eta_{2M}}\int_{t-\eta_{2M}}^{t}\dot{x}^{\mathrm{T}}(s)S_{2i}\dot{x}(s)\mathrm{d}s\leqslant-\mathrm{e}^{-\theta_i\eta_{2M}}\times (x(t)-x(t-\eta_{2M}))^{\mathrm{T}}S_{2i}(x(t)-x(t-\eta_{2M})). \quad (18)$$

(5) implies

$$x^{\mathrm{T}}(t-\eta_1(t))G^{\mathrm{T}}Gx(t-\eta_1(t)-h_1^{\mathrm{T}}(x(t-\eta_1(t)))h_1(x(t-\eta_1(t))))\geqslant 0, \quad (19)$$

$$x^{\mathrm{T}}(t-\eta_2(t))H^{\mathrm{T}}Hx(t-\eta_2(t))-h_2^{\mathrm{T}}(x(t-\eta_2(t)))h_2(x(t-\eta_2(t)))\geqslant 0. \quad (20)$$

In addition, from (7), one has

$$\wp_i x^{\mathrm{T}}(t-\tau(t))\varOmega_{2i}x(t-\tau(t))-e^{\mathrm{T}}(t)\varOmega_{1i}e(t)>0. \quad (21)$$

On the basic of (9) and (17)–(21), the following inequality can be obtained

$$\mathrm{E}\{\dot{V}_i(t)\}=\mathrm{E}\{\dot{V}_{1i}(t)\}+\mathrm{E}\{\dot{V}_{2i}(t)\}+\mathrm{E}\{\dot{V}_{3i}(t)\}\leqslant \xi^{\mathrm{T}}(t)(\varPsi_i-T_i^{\mathrm{T}}\varLambda_i^{-1}T_i)\xi(t)-\theta_i\mathrm{E}\{V_i(t)\}, \quad (22)$$

where

$$\xi(t)=\begin{bmatrix}x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau(t)) & x^{\mathrm{T}}(t-\tau_M) & e^{\mathrm{T}}(t)\end{bmatrix.\\ \begin{matrix}x^{\mathrm{T}}(t-\eta_1(t)) & x^{\mathrm{T}}(t-\eta_{1M}) & \xi_1^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}},$$

$$\xi_1^{\mathrm{T}}(t)=\begin{bmatrix}x^{\mathrm{T}}(t-\eta_2(t)) & x^{\mathrm{T}}(t-\eta_{2M}) & h_1^{\mathrm{T}}(x(t-\eta_1(t))) & h_2^{\mathrm{T}}(x(t-\eta_2(t)))\end{bmatrix}^{\mathrm{T}}.$$

According to (13) and the Schur complement lemma, one gets

$$\mathrm{E}\{\dot{V}_i(t)\}\leqslant-\theta_i\mathrm{E}\{V_i(t)\}. \quad (23)$$

Then, integrating both sides of (23) from s_q to t ($t\in[s_q,s_{q+1})$) leads to

$$\mathrm{E}\{V_i(t)\}\leqslant \mathrm{e}^{-\theta_i(t-s_q)}\mathrm{E}\{V_i(s_q)\}. \quad (24)$$

Case 2: The event-triggered instant exists in the switching interval $[s_q,s_{q+1}]$, i.e., $(t_{k_p}+\ell')h\leqslant s_q< t_{k_p+1}h<\cdots<t_{k_p+L}h<(t_{k_p+L}+c')h\leqslant s_{q+1}< t_{k_p+L+1}h$, where $\ell',c'\in\mathbb{N}_+$, $\ell'<l_k$, $c'<l_{k+L}$. Furthermore, the switching interval $[s_q,s_{q+1}]$ is divided into $[s_q,t_{k_p+1}h)$, $[t_{k_p+\imath}h,t_{k_p+\imath+1}h)$, $[t_{k_p+L}h,s_{q+1})$, where $\imath=1,2,\cdots,L-1$. Let

$$\begin{cases}e(t)=\begin{cases}e(\hbar_k h), & t\in[s_q,t_{k_p+1}h),\\ e(\hbar_{k+\imath}h), & t\in[t_{k_p+\imath}h,t_{k_p+\imath+1}h),\\ e(\hbar_{k+L}h), & t\in[t_{k_p+L}h,s_{q+1}),\end{cases}\\ \xi(t)=\begin{cases}\xi_{\hbar_k}(t), & t\in[s_q,t_{k_p+1}h),\\ \xi_{\hbar_{k+\imath}}(t), & t\in[t_{k_p+\imath}h,t_{k_p+\imath+1}h),\\ \xi_{\hbar_{k+L}}(t), & t\in[t_{k_p+L}h,s_{q+1}),\end{cases}\end{cases} \quad (25)$$

where $\xi(t)=[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-\tau(t))\ x^{\mathrm{T}}(t-\tau_M)\ e^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-\eta_1(t))\ x^{\mathrm{T}}(t-\eta_{1M})\ x^{\mathrm{T}}(t-\eta_2(t))\ x^{\mathrm{T}}(t-\eta_{2M})\ h_1^{\mathrm{T}}(x(t-\eta_1(t)))\ h_2^{\mathrm{T}}(x(t-\eta_2(t)))]^{\mathrm{T}}$. Then, similar to Case 1, a same conclusion as (24) holds on any of the above subintervals.

From (14), it follows that

$$\mathrm{E}\{V_{\sigma(s_q)}(s_q)\}\leqslant\mu_{\sigma(s_q)}\mathrm{E}\{V_{\sigma(s_q^-)}(s_q^-)\}. \quad (26)$$

Suppose $0=s_0<s_1<\cdots<s_q=t_{N_\sigma(0,t)}<t$, where $N_\sigma(0,t)=\sum\limits_{i=1}^{\aleph}N_{\sigma i}(0,t)$. Combining (24) and (26), the following inequality can be obtained

$$\mathrm{E}\{V_{\sigma(s_q)}(t)\}\leqslant \mathrm{e}^{-\theta_{\sigma(s_q)}(t-s_q)}\mathrm{E}\{V_{\sigma(s_q)}(s_q)\}\leqslant \mu_{\sigma(s_q)}\mathrm{e}^{-\theta_{\sigma(s_q)}(t-s_q)}\mathrm{E}\{V_{\sigma(s_q^-)}(s_q^-)\}\leqslant \mu_{\sigma(s_q)}\mathrm{e}^{-\theta_{\sigma(s_q)}(t-s_q)}\mathrm{e}^{-\theta_{\sigma(s_{q-1})}(s_q-s_{q-1})}\mathrm{E}\{V_{\sigma(s_{q-1})}(s_{q-1})\}\leqslant\cdots\leqslant \mathrm{e}^{\sum\limits_{i=1}^{\aleph}(N_{\sigma i}(s_0,t)\ln\mu_i-\theta_iT_i(s_0,t))}V_{\sigma(s_0)}(s_0)\leqslant \mathrm{e}^{\sum\limits_{i=1}^{\aleph}N_{0i}\ln\mu_i}\mathrm{e}^{-\min(\theta_i-\frac{\ln\mu_i}{\tau_{ai}})T(s_0,t)}V_{\sigma(s_0)}(s_0). \quad (27)$$

Besides, for $i\in\bar{M}$, there exist scalars $a>0$ and $b>0$ such that

$$a||x(t)||^2\leqslant V_i(t)\leqslant b||x(t)||^2, \quad (28)$$

where

$$a=\min_{i\in\bar{M}}\{\lambda_{\min}(P_i)\},$$

$$b=\max_{i\in\bar{M}}\{\lambda_{\max}(P_i)\}+\tau_M\max_{i\in\bar{M}}\{\lambda_{\max}(Q_{1i})\}+\eta_{1M}\max_{i\in\bar{M}}\{\lambda_{\max}(R_{1i})\}+\eta_{2M}\max_{i\in\bar{M}}\{\lambda_{\max}(S_{1i})\}+\frac{\tau_M^3}{2}\max_{i\in\bar{M}}\{\lambda_{\max}(Q_{2i})\}+\frac{\eta_{1M}^3}{2}\max_{i\in\bar{M}}\{\lambda_{\max}(R_{2i})\}+\frac{\eta_{2M}^3}{2}\max_{i\in\bar{M}}\{\lambda_{\max}(S_{2i})\}.$$

From (27)–(28), and letting $t_0=s_0$, one yields

$$\mathrm{E}\{||x(t)||\}\leqslant\varsigma\mathrm{e}^{-\varepsilon(t-t_0)}||x(t_0)||, \quad (29)$$

where

$$\varsigma=\sqrt{\frac{b}{a}}\mathrm{e}^{\frac{1}{2}\sum\limits_{i=1}^{\aleph}N_{0i}\ln\mu_i},$$

$$\varepsilon=\frac{1}{2}\min_{i\in\bar{M}}(\theta_i-\frac{\ln\mu_i}{\tau_{ai}}).$$

Then from (29) and Definition 2, system (9) can achieve MSES. This completes the proof. □

Remark 3 System (9) achieves MSES under the designed state feedback controller, ETM and MDADT switching signal, which means that system (1) can guarantee its MSES when attacked.

Theorem 2 is presented to get the controller gains and event-triggering parameters.

Theorem 2 Give positive constants τ_M, η_{1M}, η_{2M}, $\bar{\nu}$, $\bar{\vartheta}$, $\wp_i$, θ_i, γ_q ($q=1,2,\cdots,10$) and $\mu_i>1$. If there exist symmetric and positive definite matrices X_i, $\hat{Q}_{1i}$, $\hat{Q}_{2i}$, $\hat{R}_{1i}$, $\hat{R}_{2i}$, $\hat{S}_{1i}$, $\hat{S}_{2i}$, $\hat{\varOmega}_{1i}$, $\hat{\varOmega}_{2i}$ and matrix Y_i such that the following inequalities hold

$$\begin{bmatrix}\bar{\varPsi}_i & *\\ \bar{T}_i & \bar{\varLambda}_i\end{bmatrix}<0, \quad (30)$$

$$\begin{bmatrix}-\mu_iX_j & X_j\\ * & -X_i\end{bmatrix}\leqslant 0,$$

$$\begin{bmatrix} -\mu_i\hat{Q}_{1j} & X_j \\ * & \gamma_5^2\hat{Q}_{1i}-2\gamma_5X_i \end{bmatrix} \leqslant 0,$$
$$\begin{bmatrix} -\mu_i\hat{Q}_{2j} & X_j \\ * & \gamma_6^2\hat{Q}_{2i}-2\gamma_6X_i \end{bmatrix} \leqslant 0,$$
$$\begin{bmatrix} -\mu_i\hat{R}_{1j} & X_j \\ * & \gamma_7^2\hat{R}_{1i}-2\gamma_7X_i \end{bmatrix} \leqslant 0,$$
$$\begin{bmatrix} -\mu_i\hat{R}_{2j} & X_j \\ * & \gamma_8^2\hat{R}_{2i}-2\gamma_8X_i \end{bmatrix} \leqslant 0,$$
$$\begin{bmatrix} -\mu_i\hat{S}_{1j} & X_j \\ * & \gamma_9^2\hat{S}_{1i}-2\gamma_9X_i \end{bmatrix} \leqslant 0,$$
$$\begin{bmatrix} -\mu_i\hat{S}_{2j} & X_j \\ * & \gamma_{10}^2\hat{S}_{2i}-2\gamma_{10}X_i \end{bmatrix} \leqslant 0, \tag{31}$$

where

$$\bar{\Psi}_i = \begin{bmatrix} \bar{\Psi}_{1i} & * \\ \bar{\Psi}_{2i} & \bar{\Psi}_{3i} \end{bmatrix},$$

$$\bar{\Psi}_{1i} = \begin{bmatrix} \bar{\Sigma}_{1i} & * & * & * & * \\ \bar{\Sigma}_{2i} & \wp_i\hat{\Omega}_{2i}-2\mathrm{e}^{-\theta_i\tau_M}\hat{Q}_{2i} & * & * & * \\ 0 & \mathrm{e}^{-\theta_i\tau_M}\hat{Q}_{2i} & -\mathrm{e}^{-\theta_i\tau_M}(\hat{Q}_{1i}+\hat{Q}_{2i}) & * & * \\ (1-\bar{\nu})Y_i^{\mathrm{T}}B_i^{\mathrm{T}} & 0 & 0 & -\hat{\Omega}_{1i} & * \\ \mathrm{e}^{-\theta_i\eta_{1M}}\hat{R}_{2i} & 0 & 0 & 0 & -2\mathrm{e}^{-\theta_i\eta_{1M}}\hat{R}_{2i} \end{bmatrix},$$

$$\bar{\Psi}_{2i} = \begin{bmatrix} 0 & 0 & 0 & 0 & \mathrm{e}^{-\theta_i\eta_{1M}}\hat{R}_{2i} \\ \mathrm{e}^{-\theta_i\eta_{2M}}\hat{S}_{2i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \bar{\nu}\bar{\vartheta}Y_i^{\mathrm{T}}B_i^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ \bar{\nu}(1-\bar{\vartheta})Y_i^{\mathrm{T}}B_i^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Psi}_{3i} = \begin{bmatrix} -\mathrm{e}^{-\theta_i\eta_{1M}}(\hat{R}_{1i}+\hat{R}_{2i}) & * & * & * & * \\ 0 & -2\mathrm{e}^{-\theta_i\eta_{2M}}\hat{S}_{2i} & * & * & * \\ 0 & \mathrm{e}^{-\theta_i\eta_{2M}}\hat{S}_{2i} & -\mathrm{e}^{-\theta_i\eta_{2M}}(\hat{S}_{1i}+\hat{S}_{2i}) & * & * \\ 0 & 0 & 0 & \gamma_4^2I-2\gamma_4X_i & * \\ 0 & 0 & 0 & 0 & \gamma_4^2I-2\gamma_4X_i \end{bmatrix},$$

$$\bar{\Sigma}_{1i} = X_iA_i^{\mathrm{T}}+A_iX_i+\hat{Q}_{1i}+\hat{R}_{1i}+\hat{S}_{1i}+\theta_iX_i - \mathrm{e}^{-\theta_i\tau_M}\hat{Q}_{2i}-\mathrm{e}^{-\theta_i\eta_{1M}}\hat{R}_{2i}-\mathrm{e}^{-\theta_i\eta_{2M}}\hat{S}_{2i},$$

$$\bar{\Sigma}_{2i} = (1-\bar{\nu})Y_i^{\mathrm{T}}B_i^{\mathrm{T}}+\mathrm{e}^{-\theta_i\tau_M}\hat{Q}_{2i},$$
$$\bar{T}_i = \begin{bmatrix} \bar{\Phi}_{1i}^{\mathrm{T}} & \bar{\Phi}_{2i}^{\mathrm{T}} & \bar{\Phi}_{3i}^{\mathrm{T}} & \bar{\Phi}_{4i}^{\mathrm{T}} & \bar{\Phi}_{3i}^{\mathrm{T}} & \bar{\Phi}_{4i}^{\mathrm{T}} & \bar{\Phi}_{5i}^{\mathrm{T}} & \bar{\Phi}_{6i}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$
$$\bar{\Lambda}_i = \mathrm{diag}\{\bar{\phi}_i,\bar{\phi}_i,\bar{\phi}_i,\bar{\phi}_i,\bar{\phi}_i,\bar{\phi}_i,-I,-I\},$$

$$\bar{\Phi}_{1i} = \begin{bmatrix} \tau_M\bar{\varphi}_{1i} \\ \eta_{1M}\bar{\varphi}_{1i} \\ \eta_{2M}\bar{\varphi}_{1i} \end{bmatrix},\ \bar{\Phi}_{2i} = \begin{bmatrix} \tau_M\bar{\varphi}_{2i} \\ \eta_{1M}\bar{\varphi}_{2i} \\ \eta_{2M}\bar{\varphi}_{2i} \end{bmatrix},$$
$$\bar{\Phi}_{3i} = \begin{bmatrix} \tau_M\bar{\varphi}_{3i} \\ \eta_{1M}\bar{\varphi}_{3i} \\ \eta_{2M}\bar{\varphi}_{3i} \end{bmatrix},\ \bar{\Phi}_{4i} = \begin{bmatrix} \tau_M\bar{\varphi}_{4i} \\ \eta_{1M}\bar{\varphi}_{4i} \\ \eta_{2M}\bar{\varphi}_{4i} \end{bmatrix},$$
$$\bar{\Phi}_{5i} = \begin{bmatrix} 0 & 0 & 0 & 0 & GX_i & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\bar{\Phi}_{6i} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & HX_i & 0 & 0 & 0 \end{bmatrix},$$
$$\bar{\varphi}_{1i} = [A_iX_i\ \ (1-\bar{\nu})B_iY_i\ \ 0\ \ (1-\bar{\nu})B_iY_i\ \ 0\ \ 0\ \ 0\ \ 0\ \ \bar{\nu}\bar{\vartheta}B_iY_i\ \ \bar{\nu}(1-\bar{\vartheta})B_iY_i],$$
$$\bar{\varphi}_{2i} = [0_{1\times 8}\ \ \sqrt{\tilde{\vartheta}\bar{\nu}}B_iY_i\ \ -\sqrt{\tilde{\vartheta}\bar{\nu}}B_iY_i],$$
$$\bar{\varphi}_{3i} = [0_{1\times 8}\ \ \sqrt{\tilde{\nu}}\bar{\vartheta}B_iY_i\ \ \sqrt{\tilde{\nu}}(1-\bar{\vartheta})B_iY_i],$$
$$\bar{\varphi}_{4i} = [0\ \ \sqrt{\tilde{\nu}}B_iY_i\ \ 0\ \ \sqrt{\tilde{\nu}}B_iY_i\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0],$$
$$\tilde{\nu} = \bar{\nu}(1-\bar{\nu}),\ \tilde{\vartheta} = \bar{\vartheta}(1-\bar{\vartheta}),$$
$$\bar{\phi}_i = \mathrm{diag}\{\gamma_1^2\hat{Q}_{2i}-2\gamma_1X_i,\gamma_2^2\hat{R}_{2i}-2\gamma_2X_i,\gamma_3^2\hat{S}_{2i}-2\gamma_3X_i\},$$

then system (9) can achieve MSES under the controller (8), the ETM (7) and the switching signal satisfying (15). Furthermore, the controller gains can be given by $K_i = Y_iX_i^{-1}$.

Proof From Lemma 1, it can be obtained that

$$-P_iQ_{2i}^{-1}P_i \leqslant \gamma_1^2Q_{2i}-2\gamma_1P_i, \tag{32}$$
$$-P_iR_{2i}^{-1}P_i \leqslant \gamma_2^2R_{2i}-2\gamma_2P_i, \tag{33}$$
$$-P_iS_{2i}^{-1}P_i \leqslant \gamma_3^2S_{2i}-2\gamma_3P_i, \tag{34}$$
$$-X_iIX_i \leqslant \gamma_4^2I-2\gamma_4X_i. \tag{35}$$

Then replacing $-P_iQ_{2i}^{-1}P_i$, $-P_iR_{2i}^{-1}P_i$, $-P_iS_{2i}^{-1}P_i$ in (13) by $\gamma_1^2Q_{2i}-2\gamma_1P_i$, $\gamma_2^2R_{2i}-2\gamma_2P_i$, and $\gamma_3^2S_{2i}-2\gamma_3P_i$, respectively, we have

$$\begin{bmatrix} \Psi_i & * \\ T_i & \tilde{\Lambda}_i \end{bmatrix} < 0, \tag{36}$$

where

$$\tilde{\Lambda}_i = \mathrm{diag}\{\tilde{\phi}_i,\tilde{\phi}_i,\tilde{\phi}_i,\tilde{\phi}_i,\tilde{\phi}_i,\tilde{\phi}_i,-I,-I\},$$
$$\tilde{\phi}_i = \mathrm{diag}\{\gamma_1^2Q_{2i}-2\gamma_1P_i,\gamma_2^2R_{2i}-2\gamma_2P_i,\gamma_3^2S_{2i}-2\gamma_3P_i\}.$$

Set $X_i = P_i^{-1}$ and $Y_i = K_iP_i^{-1}$, and pre- and post-multiply both sides of the inequality (36) with $[\underbrace{X_i,\cdots,X_i}_{28},I,I]$. Moreover, defining $X_iQ_{1i}X_i = \hat{Q}_{1i}$, $X_iQ_{2i}X_i = \hat{Q}_{2i}$, $X_iR_{1i}X_i = \hat{R}_{1i}$, $X_iR_{2i}X_i = \hat{R}_{2i}$, $X_iS_{1i}X_i = \hat{S}_{1i}$, $X_iS_{2i}X_i = \hat{S}_{2i}$, $X_i\Omega_{1i}X_i = \hat{\Omega}_{1i}$, $X_i\Omega_{2i}X_i = \hat{\Omega}_{2i}$. Then substituting $\gamma_4^2I-2\gamma_4X_i$ for $-X_iIX_i$, inequality (30) is derived. According to Lemma 1, we have

$$-Q_{1i}^{-1} = -X_i\hat{Q}_{1i}^{-1}X_i \leqslant \gamma_5^2\hat{Q}_{1i}-2\gamma_5X_i, \tag{37}$$
$$-Q_{2i}^{-1} = -X_i\hat{Q}_{2i}^{-1}X_i \leqslant \gamma_6^2\hat{Q}_{2i}-2\gamma_6X_i, \tag{38}$$
$$-R_{1i}^{-1} = -X_i\hat{R}_{1i}^{-1}X_i \leqslant \gamma_7^2\hat{R}_{1i}-2\gamma_7X_i, \tag{39}$$

$$-R_{2i}^{-1} = -X_i\hat{R}_{2i}^{-1}X_i \leqslant \gamma_8^2\hat{R}_{2i} - 2\gamma_8 X_i, \quad (40)$$

$$-S_{1i}^{-1} = -X_i\hat{S}_{1i}^{-1}X_i \leqslant \gamma_9^2\hat{S}_{1i} - 2\gamma_9 X_i, \quad (41)$$

$$-S_{2i}^{-1} = -X_i\hat{S}_{2i}^{-1}X_i \leqslant \gamma_{10}^2\hat{S}_{2i} - 2\gamma_{10} X_i. \quad (42)$$

Pre- and post-multiplying both sides of each inequality in (14) with X_j, and by using Schur complement lemma and (37)–(42), (31) is equivalent to (14). Controller gains can be obtained as $K_i = Y_iX_i^{-1}$ due to $Y_i = K_iP_i^{-1}$. This completes the proof. □

Assume $\bar{\nu} = 0$, that is, $\nu(t) \equiv 0$, system (1) works normally without cyber-attacks. Then the controller is as

$$u(t) = (1 - \nu(t))K_{\sigma(t)}(e(\hbar_k h) + x(t - \tau(t))), \quad (43)$$

and the corresponding switched system is described as follows:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}K_{\sigma(t)}(x(t-\tau(t)) + e(\hbar_k h)), \quad (44)$$

then the following result can be obtained.

Corollary 1 Given positive constants τ_M, η_{1M}, η_{2M}, $\wp_i$, θ_i, $\gamma_q(q = 1, 2, \cdots, 10)$ and $\mu_i > 1$, if there exist symmetric and positive definite matrices X_i, $\hat{Q}_{1i}$, $\hat{Q}_{2i}$, $\hat{R}_{1i}$, $\hat{R}_{2i}$, $\hat{S}_{1i}$, $\hat{S}_{2i}$, $\hat{\Omega}_{1i}$, $\hat{\Omega}_{2i}$ and matrix Y_i such that the following inequalities hold

$$\begin{bmatrix} \bar{\Psi}_i^1 & * \\ \bar{T}_i^1 & \Lambda_i \end{bmatrix} < 0, \quad (45)$$

$$\begin{bmatrix} -\mu_i X_j & X_j \\ * & -X_i \end{bmatrix} \leqslant 0,$$

$$\begin{bmatrix} -\mu_i \hat{Q}_{1j} & X_j \\ * & \gamma_5^2\hat{Q}_{1i} - 2\gamma_5 X_i \end{bmatrix} \leqslant 0,$$

$$\begin{bmatrix} -\mu_i \hat{Q}_{2j} & X_j \\ * & \gamma_6^2\hat{Q}_{2i} - 2\gamma_6 X_i \end{bmatrix} \leqslant 0,$$

$$\begin{bmatrix} -\mu_i \hat{R}_{1j} & X_j \\ * & \gamma_7^2\hat{R}_{1i} - 2\gamma_7 X_i \end{bmatrix} \leqslant 0,$$

$$\begin{bmatrix} -\mu_i \hat{R}_{2j} & X_j \\ * & \gamma_8^2\hat{R}_{2i} - 2\gamma_8 X_i \end{bmatrix} \leqslant 0,$$

$$\begin{bmatrix} -\mu_i \hat{S}_{1j} & X_j \\ * & \gamma_9^2\hat{S}_{1i} - 2\gamma_9 X_i \end{bmatrix} \leqslant 0,$$

$$\begin{bmatrix} -\mu_i \hat{S}_{2j} & X_j \\ * & \gamma_{10}^2\hat{S}_{2i} - 2\gamma_{10} X_i \end{bmatrix} \leqslant 0, \quad (46)$$

where

$$\bar{\Psi}_i^1 = \begin{bmatrix} \bar{\Psi}_{1i}^1 & * \\ \bar{\Psi}_{2i}^1 & \bar{\Psi}_{3i} \end{bmatrix},$$

$$\bar{\Psi}_{1i}^1 = \begin{bmatrix} \bar{\Sigma}_{1i} & * & * & * & * \\ Y_i^{\mathrm{T}}B_i^{\mathrm{T}} + \mathrm{e}^{-\theta_i\tau_M}\hat{Q}_{2i} & \wp_i\hat{\Omega}_{2i} - 2\mathrm{e}^{-\theta_i\tau_M}\hat{Q}_{2i} & * & * & * \\ 0 & \mathrm{e}^{-\theta_i\tau_M}\hat{Q}_{2i} & -\mathrm{e}^{-\theta_i\tau_M}(\hat{Q}_{1i} + \hat{Q}_{2i}) & * & * \\ Y_i^{\mathrm{T}}B_i^{\mathrm{T}} & 0 & 0 & -\hat{\Omega}_{1i} & * \\ \mathrm{e}^{-\theta_i\eta_{1M}}\hat{R}_{2i} & 0 & 0 & 0 & -2\mathrm{e}^{-\theta_i\eta_{1M}}\hat{R}_{2i} \end{bmatrix},$$

$$\bar{\Psi}_{2i}^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & \mathrm{e}^{-\theta_i\eta_{1M}}\hat{R}_{2i} \\ \mathrm{e}^{-\theta_i\eta_{2M}}\hat{S}_{2i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{T}_i^1 = \begin{bmatrix} \tilde{\Phi}_{1i}^{\mathrm{T}} & \tilde{\Phi}_{2i}^{\mathrm{T}} & \tilde{\Phi}_{3i}^{\mathrm{T}} & \tilde{\Phi}_{4i}^{\mathrm{T}} & \tilde{\Phi}_{3i}^{\mathrm{T}} & \tilde{\Phi}_{4i}^{\mathrm{T}} & \bar{\Phi}_{5i}^{\mathrm{T}} & \bar{\Phi}_{6i}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$

$$\tilde{\Phi}_{1i} = \begin{bmatrix} \tau_M\bar{\varphi}_{1i}^1 \\ \eta_{1M}\bar{\varphi}_{1i}^1 \\ \eta_{2M}\bar{\varphi}_{1i}^1 \end{bmatrix}, \ \tilde{\Phi}_{2i} = \begin{bmatrix} 0_{1\times10} \\ 0_{1\times10} \\ 0_{1\times10} \end{bmatrix},$$

$$\tilde{\Phi}_{3i} = \begin{bmatrix} 0_{1\times10} \\ 0_{1\times10} \\ 0_{1\times10} \end{bmatrix}, \ \tilde{\Phi}_{4i} = \begin{bmatrix} 0_{1\times10} \\ 0_{1\times10} \\ 0_{1\times10} \end{bmatrix},$$

$$\bar{\varphi}_{1i}^1 = \begin{bmatrix} A_iX_i & B_iY_i & 0 & B_iY_i & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

then system (1) is exponentially stable under the designed controller (43), the ETM (7) and the switching signal satisfying (15). Moreover, the controller gains can be given by $K_i = Y_iX_i^{-1}$.

4 Numerical example

An example is illustrated to demonstrate the effectiveness of the method mentioned above in this section.

Consider system (1) with three subsystems, where the corresponding parameters are as below:

$$A_1 = \begin{bmatrix} -1.3 & 0.5 \\ -0.7 & -1 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.9 & 2 \\ -1.2 & -1.5 \end{bmatrix}, \ B_2 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -1.4 & 0.8 \\ -1 & -0.5 \end{bmatrix}, \ B_3 = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}.$$

Choose the event-triggering parameters $\wp_1 = 0.1$, $\wp_2 = 0.2$, $\wp_3 = 0.3$ and other parameters $\tau_M = 0.3$, $\eta_{1M} = 0.2$, $\eta_{2M} = 0.1$, $\gamma_1 = 0.33$, $\gamma_2 = 0.21$, $\gamma_3 = 0.15$, $\gamma_4 = 1.5$, $\gamma_5 = 1.2$, $\gamma_6 = 0.33$, $\gamma_7 = 2.5$, $\gamma_8 = 0.21$, $\gamma_9 = 3.3$, $\gamma_{10} = 0.15$. The initial state of the system is set to $x(0) = [0.5 \ -0.2]^{\mathrm{T}}$. The nonlinear functions of the two kinds of random cyber attacks are $h_1(x(t)) = [-\tanh(0.03x_1(t)) \ -\tanh(0.5x_2(t))]^{\mathrm{T}}$ and $h_2(x(t)) = [-\tanh(0.5x_1(t)) \ -\tanh(0.2x_2(t))]^{\mathrm{T}}$, respectively, and $G = \mathrm{diag}\ \{0.03, 0.5\}$ and $H = \mathrm{diag}\{0.5, 0.2\}$ are given.

According to whether the attacks occur or not, two cases below are presented to demonstrate the validity of the method obtained above.

Case I The attacks occur and the MDADT method is adopted

Let $\mu_1 = 5$, $\mu_2 = 5$, $\mu_3 = 5$, $\theta_1 = 1.4$, $\theta_2 = 1$, $\theta_3 = 0.5$. According to (15), we have $\tau^*_{a1} = 1.1496$, $\tau^*_{a2} = 1.6094$, $\tau^*_{a3} = 3.2189$. Choosing $\tau_{a1} = 1.15$, $\tau_{a2} = 1.61$, $\tau_{a3} = 3.22$, and setting the switching sequence to $2 \to 3 \to 1 \to 2 \to 3 \to 1 \cdots$, the switching signal is shown in Fig. 1.

Assume $\bar{\nu} = 0.5$ and $\bar{\vartheta} = 0.5$, based on Theorem 2, we can obtain that

$$K_1 = \begin{bmatrix} -0.1262 & -1.3635 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.4408 & -0.8857 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -1.3004 & -0.9364 \end{bmatrix}.$$

Fig. 1 demonstrates the trajectories of the state $x(t)$, which shows that by using the designed controller and ETM, the impact of cyber-attacks can be effectively dealt with. The corresponding switching signal is presented in Fig. 1. Fig. 2 shows the response of $u(t)$. Event-triggered instants and release intervals are displayed in Fig. 3. Table 1 displays the relationship between the scalars $\bar{\nu}$ and τ_M. We can find out that the larger the value of $\bar{\nu}$, the smaller the maximum allowable time delay τ_M from Table 1.

Table 1 The relationship between $\bar{\nu}$ and τ_M

$\bar{\nu}$	0.1	0.3	0.5	0.7	0.9
τ_M	0.379	0.346	0.307	0.288	0.284

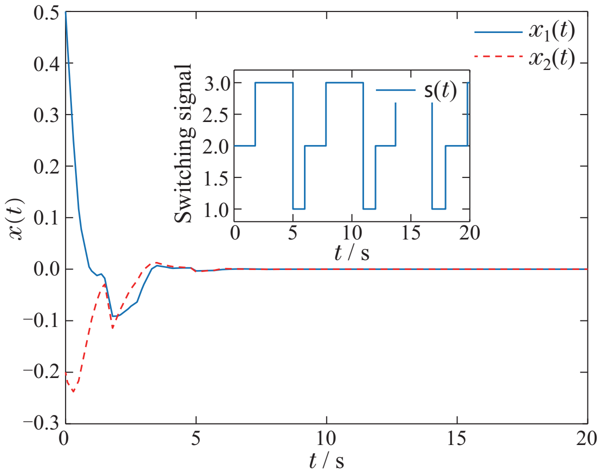

Fig. 1 The trajectory of the state $x(t)$

Case II The attacks don't occur and the MDADT method is adopted

Assume $\bar{\nu} = 0$, and other parameters are the same as in Case I, based on Corollary 1, we can obtain that

$$K_1 = \begin{bmatrix} -0.4159 & -1.4822 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.8386 & -1.7332 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -2.0664 & -0.9233 \end{bmatrix}.$$

Fig. 4 presents the response of state $x(t)$, which verifies that the obtained method is also effective for the switched CPSs without attacks.

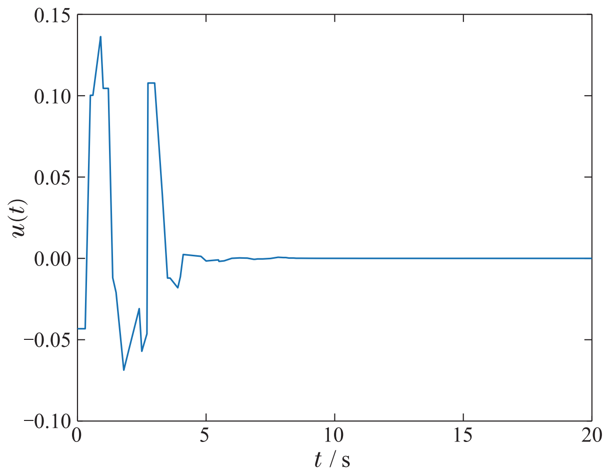

Fig. 2 The trajectory of the control input $u(t)$

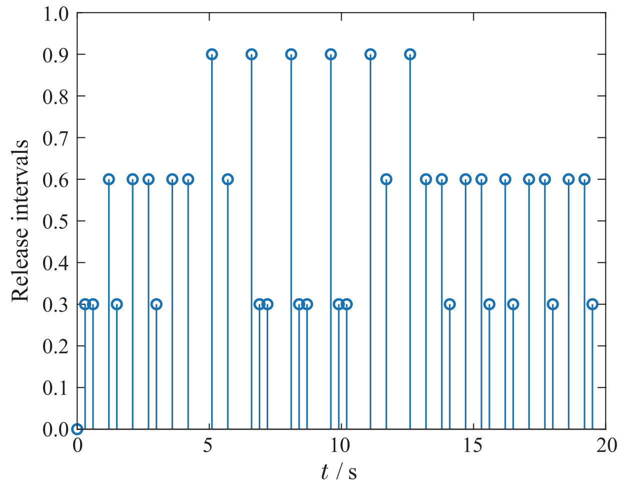

Fig. 3 Event-triggered instants and release intervals

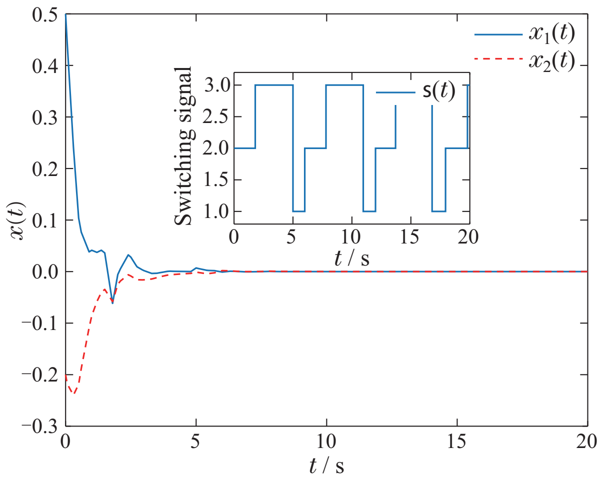

Fig. 4 The trajectory of the state $x(t)$

5 Conclusion

In this paper, the issue of ETC has been investigated for switched CPSs subject to stochastic cyber attacks.

The ETM and the state feedback controller have been used under the stochastic cyber attacks. In virtue of the multiple Lyapunov function technology and MDADT method, the mean-square exponential stability of CPSs is guaranteed. Finally, an example has been presented to illustrate the validity of the obtained results.

References:

[1] MAHMOUD M S, HAMDAN M M, BAROUDI U A. Modeling and control of cyber-physical systems subject to cyber attacks: A survey of recent advances and challenges. *Neurocomputing*, 2019, 338: 101 – 115.

[2] ZHANG D, WANG Q G, FENG G, et al. A survey on attack detection, estimation and control of industrial cyber-physical systems. *ISA Transactions*, 2021, 116: 1 – 16.

[3] ZHUANG Kangxi, SUN Ziwen. Establishing a detection model for denial of service attacks in industrialcyber physical systems. *Control Theory & Applications*, 2020, 37(3): 629 – 638.
(庄康熙, 孙子文. 针对工业信息物理系统中的拒绝服务攻击建立检测模型. 控制理论与应用, 2020, 37(3): 629 – 638.)

[4] GUO Nan, JIA Chao. Interpretation of "cyber physical systems whitepaper (2017)" (part one). *Information Technology & Standardization*, 2017, (4): 36 – 40.
(郭楠, 贾超. 《信息物理系统白皮书(2017)》解读(上). 信息技术与标准化, 2017, (4): 36 – 40.)

[5] DING D, HAN Q L, WANG Z D, et al. A survey on model-based distributed control and filtering for industrial cyber-physical systems. *IEEE Transactions on Industrial Informatics*, 2019, 15(5): 2483 – 2499.

[6] LÜ Z H, LLORET J, XIANG W. Introduction to the special section on smart city oriented cyber-physical systems (VSI-cps). *Computers & Electrical Engineering*, 2020, 87: 106869.

[7] LIU J L, YANG M, TIAN E G, et al. Event-based security control for state-dependent uncertain systems under hybrid-attacks and its application to electronic circuits. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 2019, 66(12): 4817 – 4828.

[8] HE H B, YAN J. Cyber-physical attacks and defences in the smart grid: a survey. *IET Cyber-Physical Systems: Theory & Applications*, 2016, 1(1): 13 – 27.

[9] GAVRILUTA C, BOUDINET C, KUPZOG F, et al. Cyber-physical framework for emulating distributed control systems in smart grids. *International Journal of Electrical Power & Energy Systems*, 2020, 114: 105375.

[10] AL-SHARMAN M, MURDOCH D, CAO D P, et al. A sensorless state estimation for a safety-oriented cyber-physical system in urban driving: Deep learning approach. *IEEE/CAA Journal of Automatica Sinica*, 2021, 8(1): 169 – 178.

[11] ZHANG Y, QIU M K, TSAI C W, et al. Health-CPS: Healthcare cyber-physical system assisted by cloud and big data. *IEEE Systems Journal*, 2017, 11(1): 88 – 95.

[12] LEITAO P, KARNOUSKOS S, RIBEIRO L, et al. Smart agents in industrial cyber-physical systems. *Proceedings of the IEEE*, 2016, 104(5): 1086 – 1101.

[13] LIU S, LI S B, XU B G. Finite horizon H_∞ controller design for time-varying cyber-physical system under hybrid attacks. *Control Theory & Applications*, 2020, 37(2): 331 – 339.

[14] FARIVAR F, HAGHIGHI M S, JOLFAEI A, et al. Artificial intelligence for detection, estimation, and compensation of malicious attacks in nonlinear cyber-physical systems and industrial IoT. *IEEE Transactions on Industrial Informatics*, 2020, 16(4): 2716 – 2725.

[15] WOLF M, SERPANOS D. Safety and security in cyber-physical systems and internet-of-things systems. *Proceedings of the IEEE*, 2018, 106(1): 9 – 20.

[16] HUMAYED A, LIN J Q, LI F J, et al. Cyber-physical systems security–a survey. *IEEE Internet of Things Journal*, 2017, 4(6): 1802 – 1831.

[17] JIN X, HADDAD W M, YUCELEN T. An adaptive control architecture for mitigating sensor and actuator attacks in cyber-physical systems. *IEEE Transactions on Automatic Control*, 2017, 62(11): 6058 – 6064.

[18] LIBERZON D, MORSE A S. Basic problems in stability and design of switched systems. *IEEE Control Systems*, 1999, 19(5): 59 – 70.

[19] LI Z J, ZHAO J. Resilient adaptive control of switched nonlinear cyber-physical systems under uncertain deception attacks. *Information Sciences*, 2021, 543: 398 – 409.

[20] ZHAO H J, NIU Y G, JIA T G. Security control of cyber-physical switched systems under round-robin protocol: Input-to-state stability in probability. *Information Sciences*, 2020, 508: 121 – 134.

[21] GE X H, HAN Q L. Distributed event-triggered H_∞ filtering over sensor networks with communication delays. *Information Sciences*, 2015, 291: 128 – 142.

[22] PENG C, LI F Q. A survey on recent advances in event-triggered communication and control. *Information Sciences*, 2018, 457 – 458: 113 – 125.

[23] ZHANG X M, HAN Q L, ZHANG B L. An overview and deep investigation on sampled-data-based event-triggered control and filtering for networked systems. *IEEE Transactions on Industrial Informatics*, 2017, 13(1): 4 – 16.

[24] LIU J L, TIAN E G, XIE X P, et al. Distributed event-triggered control for networked control systems with stochastic cyber-attacks. *Journal of the Franklin Institute*, 2019, 356(17): 10260 – 10276.

[25] HU S L, YUE D, HAN Q L, et al. Observer-based event-triggered control for networked linear systems subject to denial-of-service attacks. *IEEE Transactions on Cybernetics*, 2020, 50(5): 1952 – 1964.

[26] LIU J L, WEI L L, TIAN A G, et al. H_∞ filtering for networked systems with hybrid-triggered communication mechanism and stochastic cyber attacks. *Journal of the Franklin Institute*, 2017, 354(18): 8490 – 8512.

[27] XIONG J L, LAM J. Stabilization of networked control systems with a logic ZOH. *IEEE Transactions on Automatic Control*, 2009, 54(2): 358 – 363.

[28] ZHAO X D, ZHANG L X, SHI P, et al. Stability and stabilization of switched linear systems with mode-dependent average dwell time. *IEEE Transactions on Automatic Control*, 2012, 57(7): 1809 – 1815.

[29] WU L G, ZHENG W X, GAO H J. Dissipativity-based sliding mode control of switched stochastic systems. *IEEE Transactions on Automatic Control*, 2013, 58(3): 785 – 791.

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