面向动态能源资源协调的分布式原始－对偶优化方法

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摘要: 分布式优化在电力系统中发挥着越来越重要的作用. 本文研究一类包含分布式发电机(DGs)和储能设备(ESs)的动态能源资源(DERs)协调问题, 其目标是在满足局部耦合物理约束的前提下, 使得总成本(包括发电成本, 储能成本和环境成本)最小化. 首先, 本文将动态DERs协调问题等价转换为更具一般性的分布式复合约束优化模型, 并利用拉格朗日对偶理论分析得到原问题的对偶形式. 其次, 提出一种新的分布式原对偶优化算法. 特别地, 所提算法使用局部常数步长, 同时采用基于边的通信方式, 这本质上区别于基于节点的一致性优化方法. 最后, 利用基于IEEE 39-bus系统的仿真实验进一步验证了所提算法在求解DERs协调问题上的有效性与可行性.

关键词: 分布式优化; 能源资源协调; 智能电网; 原始－对偶算法

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Distributed primal-dual algorithm for dynamic energy resources coordination

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Abstract: Distributed optimization plays an increasingly significant role in power systems. This paper investigates a dynamic energy resources (DERs) coordination problem with distributed generations (DGs) and energy storages (ESs), the goal of which is to minimize the total cost including generation (or storage) and environmental cost meanwhile satisfying local and coupled physical constraints. Firstly, the dynamic DERs coordination problem is equivalently converted into a more generalized distributed composite constrained optimization form, and its dual setting is also constructed, through exploiting the Lagrangian duality. Then, a novel distributed primal dual algorithm is developed to deal with the considered problem. It is worth mentioning that the proposed algorithm has edge-based communicating phrase and local constant step-sizes, essentially different from some node-based consensus methods. Finally, the simulation setting based on the IEEE 39-bus system are conducted to verify the effectiveness and feasibility of the proposed algorithm for solving the considered problem.

Key words: distributed optimization; energy resources coordination; smart grid; primal-dual algorithm

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1 Introduction

With the development of digital smart grid sensing, the efficiency, the reliability and the security of power systems have been substantially improved [1]. As important components of dynamic energy resources (DERs), distributed generation (DG) and energy storage (ES) play a crucial role in fast responding. Thus, DERs can work as a valuable system by coordinating with system requirements and control processes [1]. The traditional centralized control mechanisms for DERs require a center receiving information from the entire network and sending control signals back to the system [2–3]. However, there exit some limitations, such as the heavy communication burden and single-point-of-failure, un-

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der the centralized methods. In contrast, distributed methods possess stronger robustness and extensibility, which are widely applied in smart grid, resource allocation, Nash equilibrium, machine learning, and other scenarios [4–8].

In distributed algorithms, each agent (distributed generation or storage device) receives information from neighbors and makes its own decision. Some existing studies only consider a single type of DERs, such as DGs in [9–10] and ESs in [11–12]. To coordinate multiple types of DERs, the authors in [13] proposed a distributed energy management algorithm based on consensus + innovations method. In [14], the Laplacian gradient dynamics and dynamic average consensus was employed to design a distributed algorithm for coordinate DGs and ESs. The authors in [15] improved the distributed DERs coordination algorithms by considering the charging and discharging efficiency. In recent years, environmental issues have attracted attention, and it would be more practical to consider environmental factors in the DERs model. Inspired by this fact, we introduce environmental cost functions in the DERs coordination problem, which further refines the DERs model.

This paper focuses on a DERs coordination problem with DGs and ESs, where the involved constraints include local output capacity limitations, coupled supply-demand equality, ramping up/down constraints, storage capacity limitations for energy storages devices, and the limits of state-of-charge of the storages. Unlike recent works [1, 9–17] that only consider the cost of generations, we additionally introduce the environmental cost. Considering that the complexity of the DERs model may make itself difficult to be solved, we first simplify it into a generalized minimization problem and further construct its dual setup by using the Lagrangian function. Inspired by the distributed primal-dual method [18], we put forward a novel distributed splitting algorithm to deal with DERs problem, essentially distinguishing from the consensus methods [16–17, 19]. The proposed algorithm only relies on local step-sizes rather than coordinated ones [1], guaranteeing the privacy of agents. Meanwhile, the involved relaxed constant enhances the flexibility of algorithm implementability through setting different values. To our knowledge, there only are a few works for solving the DERs coordination. Finally, we verify the correctness and effectiveness of the proposed algorithm by using the IEEE 39-bus system.

The remainder of this paper is arranged as follows: Section 2 formulates the DERs coordination problem, and Section 3 describes its dual setup. Then, the development of distributed algorithm is presented in Section 4. In Section 5, simulation results are reported. A conclusion and future work are given in Section 6.

2 Problem formulation

In this paper, an optimal coordination of DERs containing s DGs and v ESs is considered, the objective of which aims to collaboratively minimize the total cost, while all agents (including generations and storages) has to satisfy a given supply-demand through communication exchange during certain time slot $\mathcal{T} = \{1, \cdots, n\}$. Next, more details on the considered problem are described below.

The total cost over the time slot $\mathcal{T}$ takes form of

$$\sum_{t=1}^{n}\sum_{i=1}^{s+v}(C_i^{\mathrm{c}}(p_{i,t})+C_i^{\mathrm{e}}(p_{i,t})), \tag{1}$$

where $C_i^{\mathrm{c}}(\cdot)$ and $C_i^{\mathrm{e}}(\cdot)$ are respectively generation cost and environmental cost functions of agent i, while $p_{i,t}$ is its output power at the instant $t \in \mathcal{T}$. In this paper, we assume that both of the cost functions are strongly convex, which is common in recent literature [21, 24]. At time period $t \in \mathcal{T}$, all agents need to meet the supply-demand constraint

$$\sum_{i=1}^{s+v} p_{i,t} = D_t, \tag{2}$$

with the time-varying demand D_t. For each generation, the output $p_{i,t}$ is restricted by the capacity limitation

$$p_i^{\min} \leqslant p_{i,t} \leqslant p_i^{\max},\ \forall t \in \mathcal{T}, \tag{3}$$

and the ramping up/down constraint

$$p_i^{\text{down}} \leqslant p_{i,t} - p_{i,t-1} \leqslant p_i^{\text{up}},\ \forall t \in \mathcal{T}. \tag{4}$$

For each storage, the output $p_{i,t}$ is given by

$$p_{i,t} = p_{i,t}^{+} - p_{i,t}^{-}, \tag{5}$$

where $p_{i,t}^{+}$ is charging efficiency and $p_{i,t}^{-}$ discharging one, both of which, for $t \in \mathcal{T}$, also are bounded by

$$\begin{cases} 0 \leqslant p_{i,t}^{+} \leqslant p_i^{\max}, \\ 0 \leqslant p_{i,t}^{-} \leqslant p_i^{\max}. \end{cases} \tag{6}$$

Furthermore, each storage dynamically stores the energy $E_{i,t}$ at $t \in \mathcal{T}$

$$E_{i,t} = E_{i,t-1} - p_{i,t}^{\text{batt}} \nabla t, \tag{7}$$

where $p_{i,t}^{\text{batt}} = p_{i,t}^{+}/\eta_i^{+} - \eta_i^{-} p_{i,t}^{-}$ with $\eta_i^{-}, \eta_i^{+} \in (0,1)$ is the rate of change of energy, ∇t is the size of slot. Meanwhile, $E_{i,t}$ has to be restricted within

$$0 \leqslant E_{i,t} \leqslant E_i^{\max}. \tag{8}$$

As stated above, we can formulate the DERs coordination problem as the following constrained optimization one:

$$\min_{p_{i,t}} \sum_{t=1}^{n}\sum_{i=1}^{s+v} C_i(p_{i,t}), \tag{9}$$

which subjects to (2)–(8). To address this problem (9), we define the local objective function

$$C_i(p_i) = \sum_{t=1}^{n} (C_i^{\mathrm{c}}(p_{i,t}) + C_i^{\mathrm{e}}(p_{i,t}))$$

with $p_i = [p_{i,1}\ p_{i,2} \cdots p_{i,n}]^{\mathrm{T}} \in \mathbb{R}^n$. Let S and M be the sets of DGs and ESs, respecti- vely. Then, the local constraint corresponding to p_i is given by

$$\mathcal{J}_i=\mathcal{J}_i^{\mathrm{Q}}\cup\mathcal{J}_i^{\mathrm{R}}\cup\mathcal{J}_i^{\mathrm{W}}, \tag{10}$$

where $\mathcal{J}_i^{\mathrm{Q}}=\{p_i\in\mathbb{R}^n: p_i^{\min}\leqslant p_{i,t}\leqslant p_i^{\max},\forall t\in\mathcal{T}\}$, $\mathcal{J}_i^{\mathrm{R}}=\{p_i\in\mathbb{R}^n: p_i^{\mathrm{down}}\leqslant p_{i,t}-p_{i,t-1}\leqslant p_i^{\mathrm{up}},\forall t\in\mathcal{T}\}$, and $\mathcal{J}_i^{\mathrm{W}}=\{p_i=p_i^+-p_i^-: E_{i,t}=E_{i,t-1}-p_{i,t}^{\mathrm{batt}}\nabla t,\ 0\leqslant E_{i,t}\leqslant E^{\max},\forall t\in\mathcal{T}\}$. Therefore, problem (9) can be reformulated into the following general form:

$$\begin{aligned}&\underset{p_1,\cdots,p_m}{\text{minimize}}\ \sum_{i=1}^m C_i(p_i),\\&\text{subject to}\ \sum_{i=1}^m A_ip_i=\sum_{i=1}^m b_i,\ p_i\in\mathcal{J}_i,\end{aligned} \tag{11}$$

where $m=s+v$ is the total number of agents, $A_i\in\mathbb{R}^{n\times n}$ is a linear transformer and $b_i=[D_1/m\ \cdots\ D_n/m]^{\mathrm{T}}\in\mathbb{R}^n$ is the virtue local demand, some discussion of which can refer to[19].

As a result, the dynamic optimal coordination of DERs (9) can be addressed through solving the general model (11).

3 Dual problem

This section formally constructs the dual setup of (11). First of all, let's consider the Lagrangian function

$$\varPhi(p,\tilde\lambda)=\sum_{i=1}^m f_i(p_i)+\delta_{\mathcal{J}_i}(p_i)+\sum_{i=1}^m\tilde\lambda^{\mathrm{T}}(A_ip_i-b_i), \tag{12}$$

where the stacked vector $p=\mathrm{col}\{p_i\}_{i=1}^m$ ($\mathrm{col}\{\cdot\}_{i=1}^m$ denotes the stacked vector from 1 to m) is the Lagrange multiplier, and $\delta_{\mathcal{J}_i}(p_i)$ is the indicator function of $\mathcal{J}_i$, expressed by

$$\delta_{\mathcal{J}_i}(p_i)=\begin{cases}0, & p_i\in\mathcal{J}_i,\\+\infty, & p_i\notin\mathcal{J}_i.\end{cases}$$

Next, construct the dual function

$$\begin{aligned}q(\tilde\lambda)=&\min_{p\in\mathbb{R}^{mn}}\varPhi(p,\tilde\lambda)=\\&\min_{p\in\mathbb{R}^{mn}}\sum_{i=1}^m(C_i(p_i)+\delta_{\mathcal{J}_i}(p_i)+\tilde\lambda^{\mathrm{T}}(A_ip_i-b_i))=\\&-\sum_{i=1}^m(\max_{p_i\in\mathcal{J}_i}(-\tilde\lambda^{\mathrm{T}}A_ip_i-C_i(p_i))+\tilde\lambda^{\mathrm{T}}b_i).\end{aligned} \tag{13}$$

For ease of analysis processing, we define the conjugate function $F_i^*(z)=\max_{p_i\in\mathcal{J}_i}\{z^{\mathrm{T}}A_ip_i-C_i(p_i)\}$. Therefore, one obtains the dual problem of (11), given by

$$\max_{\tilde\lambda\in\mathbb{R}^r}q(\tilde\lambda)=-\sum_{i=1}^m(F_i^*(-\tilde\lambda)+\tilde\lambda^{\mathrm{T}}b_i). \tag{14}$$

The strong convexity assumption on C_i implies that $F_i^*(-\tilde\lambda)$ has Lipschitz continuous gradient [22] and its gradient is attainable, i.e.,

$$\nabla F_i^*(-\tilde\lambda)=-A_i\cdot\underset{p_i\in\mathcal{J}_i}{\operatorname{argmin}}\{\tilde\lambda^{\mathrm{T}}A_ip_i+C_i(p_i)\}, \tag{15}$$

from of which the unique optimal solution of (11) can be given by

$$p_i^*(\tilde\lambda^*)=\underset{p_i\in\mathcal{J}_i}{\operatorname{argmin}}\{(\tilde\lambda^*)^{\mathrm{T}}A_ip_i+C_i(p_i)\}. \tag{16}$$

Consequently, we can solve the DERs coordination problem (9), by addressing the following minimization dual problem:

$$\underset{\tilde\lambda\in\mathbb{R}^r}{\text{minimize}}\sum_{i=1}^m(F_i^*(-\tilde\lambda)+\tilde\lambda^{\mathrm{T}}b_i). \tag{17}$$

Note that the Lagrange multiplier $\tilde\lambda$ is known by all agents, which prevent us from solving the DERs coordination problem in the distributed manner. To this end, each agent has to maintain its own local variables λ_i to estimate $\tilde\lambda$, and further communicate with its neighboring agents to achieve the consensus state.

In what follows, we consider m agents communicating over an undirected graph $\mathcal{G}$, consisting of the vertex set $\mathcal{V}=\{1,\cdots,m\}$ and edge set $\mathcal{E}=\mathcal{V}\times\mathcal{V}$. Then, the obtained dual problem (17) can be equivalently rewritten as the following constrained problem:

$$\begin{aligned}&\underset{\lambda\in\mathbb{R}^{mr}}{\text{minimize}}\ \sum_{i=1}^m(F_i^*(-\lambda_i)+\lambda_i^{\mathrm{T}}b_i),\\&\text{subject to}\ U_{ij}\lambda_i+U_{ji}\lambda_j=0,\ (i,j)\in\mathcal{E},\end{aligned} \tag{18}$$

where $\lambda=\mathrm{col}\{\lambda_i\}_{i=1}^m$, the linear operator $U_{ij}=I$ if $i<j$ and $-I$ if $i>j$. The coupled edge constraint reveals that neighboring agents, linked by $(i,j)\in\mathcal{E}$, will locally share $U_{ij}\lambda_i$ and $U_{ji}\lambda_j$ with each other. Next, we define the following operator:

$$N:\lambda\to\left(N_{(i,j)}\lambda\right)_{(i,j)\in\mathcal{E}}, \tag{19}$$

where the edge-based linear operator $N_{(i,j)}$ is expressed by

$$N_{(i,j)}:\lambda\to(U_{ij}\lambda_i,U_{ji}\lambda_j). \tag{20}$$

Define the set

$$\mathcal{Z}_{(i,j)}=\{(z_1,z_2)\in\mathbb{R}^r\times\mathbb{R}^r\,|z_1+z_2=0\}. \tag{21}$$

The coupled edge constraint in (18) can be constrained by the following indictor function

$$\delta_{M_{(i,j)}}\left(N_{(i,j)}\lambda\right)=\begin{cases}0, & N_{(i,j)}\lambda\in\mathcal{Z}_{(i,j)},\\+\infty, & \text{others}.\end{cases} \tag{22}$$

Thus, we obtain the unconstrained formulation of (18)

$$\underset{\lambda\in\mathbb{R}^{mr}}{\text{minimize}}\ \sum_{i=1}^m\tilde f_i(\lambda_i)+\sum_{i=1}^m\sum_{(i,j)\in\mathcal{E}}\delta_{\mathcal{Z}_{(i,j)}}\left(N_{(i,j)}\lambda\right), \tag{23}$$

where $\tilde f_i(\lambda_i)=F_i^*(-\lambda_i)+\lambda_i^{\mathrm{T}}b_i$.

4 Distributed algorithm for DERC

Inspired by the forward-backward splitting[18], this section develops a new distributed algorithm to deal with the globally optimal solution of optimization problem (23). According to[23, Proposition 19.18], one can derive the Lagrangian function of (23) as follows:

$$\begin{aligned}&\mathcal{L}(\lambda, w)=\\&\sum_{i=1}^{m}\tilde{f}_i(\lambda_i)+\sum_{i=1}^{m}\sum_{(i,j)\in\mathcal{E}}({w_{(i,j)}}^{\mathrm{T}}N_{(i,j)}\lambda-\\&\delta^*_{\mathcal{Z}_{(i,j)}}(w_{(i,j)}))=\\&\sum_{i=1}^{m}(\tilde{f}_i(\lambda_i)+\sum_{j\in\mathcal{N}_i}(w^{\mathrm{T}}_{(i,j)}N_{(i,j)}\lambda-\delta^*_{\mathcal{Z}_{(i,j)}}(w_{(i,j)}))),\end{aligned}$$

where $w_{(i,j)}=\mathrm{col}\{w_{(i,j),i}, w_{(i,j),j}\}_{(i,j)\in\mathcal{E}}$ is the edge-based variable, $w=\mathrm{col}\{w_{(i,j)}\}_{(i,j)\in\mathcal{E}}$ and $\delta^*_{\mathcal{Z}_{(i,j)}}$ is the conjugate function of $\delta_{\mathcal{Z}_{(i,j)}}$. Then, applying the forward-backward method, we can develop a distributed iterative algorithm to address (23) as follows:

$$\begin{cases}\bar{w}^k_{(i,j)}=\mathrm{prox}_{\kappa_{(i,j)}\delta^*_{\mathcal{Z}_{(i,j)}}}(w^k_{(i,j)}+\kappa_{(i,j)}N_{(i,j)}\lambda^k),\\ \bar{\lambda}^k_i=\lambda^k_i-\tau_i(\partial_{\lambda_i}\tilde{f}_i(\lambda^k_i)+\\ \qquad\sum_{j\in\mathcal{N}_i}U^{\mathrm{T}}_{ij}(2\bar{w}^k_{(i,j),i}-w^k_{(i,j),i}),\\ w^{k+1}_{(i,j)}=\alpha\bar{w}^k_{(i,j)}+(1-\alpha)\,w^k_{(i,j)},\\ \lambda^{k+1}_i=\alpha\bar{\lambda}^k_i+(1-\alpha)\,\lambda^k_i,\end{cases}\tag{24}$$

where $\kappa_{(i,j)}$ is the edge step-size, τ_i is the local step-size, and α is a relaxed factor. In (24), it should be pointed out that: i) the update of $\bar{w}^k_{(i,j)}$ is not explicitly distributed manner owing to it containing $w_{(i,j),i}$ and $w_{(i,j),j}$; ii) the local function $\tilde{f}_i$ involves the conjugate function F^*_i (see (23)). Both of the mentioned block the implementation of the distributed form of (24). Therefore, it is necessary for us to tackle such issues.

Using Moreau decomposition [41], one can decompose the first line of (24) into

$$\begin{aligned}\bar{w}^k_{(i,j)}=&\,w^k_{(i,j)}+\kappa_{(i,j)}N_{(i,j)}\lambda^k-\\&\kappa_{(i,j)}\mathcal{P}_{\mathcal{Z}_{(i,j)}}(\kappa^{-1}_{(i,j)}w^k_{(i,j)}+N_{(i,j)}\lambda^k).\end{aligned}\tag{25}$$

Combing the projection of $\mathcal{Z}_{(i,j)}$

$$P_{\mathcal{Z}_{(i,j)}}(z_1, z_2)=\frac{1}{2}\mathrm{col}\{z_1-z_2, z_2-z_1\},$$

we obtain the update of $\bar{w}_{(i,j),i}$ at the iteration k

$$\bar{w}^k_{(i,j),i}=\frac{1}{2}(w^k_{(i,j),i}+w^k_{(i,j),j})+\frac{\kappa_{(i,j)}}{2}(U_{ij}\lambda^k_i+U_{ji}\lambda^k_j).\tag{26}$$

On the other hand, $\tilde{f}_i$ is endowed with Lipschitz differentiability due to C_i with strong convexity [24, Lemma V.8]. Calculating the derivatives of $\tilde{f}_i$ obtains

$$\partial_{\lambda_i}\tilde{f}_i(\lambda^k_i)=-A_i\operatorname*{argmin}_{p_i\in\mathcal{J}_i}((\lambda^k_i)^{\mathrm{T}}A_ip_i+C_i(p_i))+b_i.\tag{27}$$

For ease of computation, let

$$p^k_i=\operatorname*{argmin}_{p_i\in\mathcal{J}_i}((\lambda^k_i)^{\mathrm{T}}A_ip_i+C_i(p_i))+b_i.$$

Consequently, replacing $\mathrm{prox}_{\kappa_{(i,j)}\delta^*_{\mathcal{Z}_{(i,j)}}}(\cdot)$ and $\partial_{\lambda_i}\tilde{f}_i(\lambda^k_i)$ of (24) with (26) and (27), we develop a new distributed algorithm and the detailed updates are described in (28)–(29).

Each agent i initializes p^0_i, λ^0_i, $w^0_{(i,j),i}$ for $j\in\mathcal{N}_i$ and

$$p^0_i=\operatorname*{argmin}_{p_i\in\mathcal{J}_i}\{(\lambda^0_i)^{\mathrm{T}}A_ip_i+C_i(p_i)\},$$

then chooses the parameters τ_i, $\kappa_{(i,j)}$ and α. For $k>0$, each agent $i\in\mathcal{V}$ carries out the local updates

$$\begin{cases}\bar{w}^k_{(i,j),i}=\frac{1}{2}(w^k_{(i,j),i}+\\ \qquad w^k_{(i,j),j})+\frac{\kappa_{(i,j)}}{2}(U_{ij}\lambda^k_i+U_{ji}\lambda^k_j),\\ \bar{\lambda}^k_i=\lambda^k_i+\tau_i(A_ip^k_i-b_i-\\ \qquad\sum_{j\in\mathcal{N}_i}U^{\mathrm{T}}_{ij}(2\bar{w}^k_{(i,j),i}-w^k_{(i,j),i})),\\ w^{k+1}_{(i,j),i}=\alpha\bar{w}^k_{(i,j),i}+(1-\alpha)\,w^k_{(i,j),i},\\ \lambda^{k+1}_i=\alpha\bar{\lambda}^k_i+(1-\alpha)\,\lambda^k_i,\end{cases}\tag{28}$$

and broadcasts $w^{k+1}_{(i,j),i}$ and $U_{ij}\lambda^{k+1}_i$ to neighbors $j\in\mathcal{N}_i$, and computes

$$p^{k+1}_i=\operatorname*{argmin}_{p_i\in\mathcal{J}_i}\{{(\lambda^{k+1}_i)}^{\mathrm{T}}A_ip_i+C_i(p_i)\}.\tag{29}$$

Remark 1 The proposed algorithm flowchart is shown in Fig. 1. Notice that, each agent requires $U_{ji}\lambda^k_j$ receiving from its neighbors $j\in\mathcal{N}_i$. For each edge $(i,j)\in\mathcal{E}$, there exist two edge variables $w^k_{(i,j),i}$ and $w^k_{(i,j),j}$, which are respectively maintained by i and j. The local step-size τ_i is only knew by agent i, while the edge step-size $\kappa_{(i,j)}$ is shared commonly between i and j because of the communication phrase.

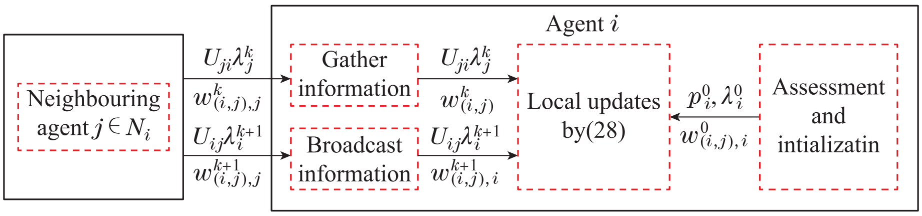

Fig. 1 The flowchart of operations of an agent

Remark 2 Although the proposed algorithm (see (28)–(29)) is based on the forward-backward splitting [18], its update formulations are totally different from that of the distributed forward-backward algorithm. In particular, the distributed algorithm (24) cannot be accessible to deal with problem (23), since the complex composite function $\tilde{f}_i$ contains a conjugate function. Fortunately, using the strong convexity assumption on cost function C_i and the equality (27), we can smoothly and explicitly evaluate the derivatives of $\tilde{f}_i$ and obtain the new algorithm (28)–(29), which is the essential difference compared with (24). Finally, as a convergence rate is not established in [18], we explore the possible superiority of our proposed algorithm that adopts the constant relaxed factor, through the simulations. The work in terms of this issue are still studied in the future.

Theorem 1 [18, Theorem 1] Suppose that the local step-sizes $\sigma_i > 0$, $\kappa_{(i,j)} > 0$, and $0 < \alpha < 1$. Meanwhile, τ_i needs to meet

$$0 < \tau_i < \frac{2\mu_C^i}{\sqrt{2} + 2\mu_C^i \sum\limits_{j \in \mathcal{N}_i} \kappa_{(i,j)}},$$

where μ_C^i is the strong convexity factor associated with C_i. Then, the sequence $\{\lambda_i^k\}_{k\in\mathbb{N}}$, generated by the proposed algorithm (i.e., (28) and (29)) converge to an optimal solution $\lambda_1^* = \cdots = \lambda_m^* = \tilde{\lambda}^*$ of (18), i.e., $\lim\limits_{k\to+\infty} \lambda_i^k = \tilde{\lambda}^*$. Meanwhile, the optimal responses $p_1^*, \cdots, p_m^*$ of problem (11) is obtained.

Remark 3 It is worth mentioning that the algorithm (28)–(29) inherits the convergence of the distributed one [18], the key analysis line of which is to establish the Krasnosel'skii-Mann iteration [18, Lemma 1]. Here we report the convergence results by Theorem 1 and omit the detailed proof. Other comments on the convergence results can refer to [18].

5 Illustrative example

In this section, we use an example of DERs in power system to illustrate the effectiveness of the proposed algorithm. As shown in Fig. 2, this example is carried out on the IEEE 39-bus system given in [21]. The buses 30–39 and buses 17–18 are connected with DGs and ESs, respectively. The communication network between DGs and ESs is a connected undirected graph. The cost function of DG is $C_i^{\mathrm{c}}(p_{i,t}) = \alpha_i p_{i,t}^2 + \beta_i p_{i,t} + \mu_i$, the cost function of ES is $C_i^{\mathrm{c}}(p_{i,t}) = \gamma_i p_{i,t}^2$, and the environmental cost function is $C_i^{\mathrm{e}}(p_{i,t}) = 0.01(a_i + b_i p_{i,t} + c_i p_{i,t}^2) + d_i \exp(\theta_i p_{i,t})$, where α_i, β_i and μ_i denote the coefficients of DG, γ_i indicates the coefficients of ES, while a_i, b_i, c_i, d_i and θ_i are the coefficients of the environmental cost function. The specific parameters of DGs and ESs are derived from [21]. Let Alg. 1 denote the proposed algorithm (28)–(29) and Alg. 2 indicate the algorithm in [18].

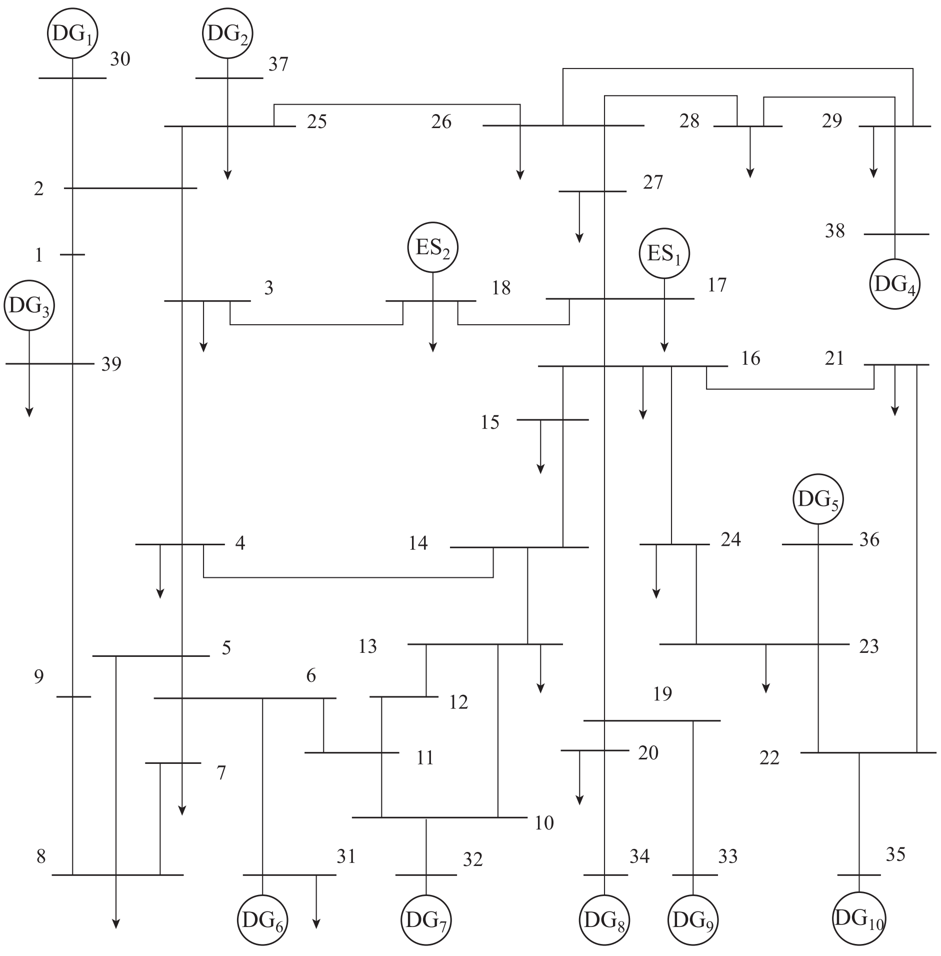

Fig. 2 IEEE 39-bus system.

The results are shown in Figs. 3–9. Fig. 3 presents the role of ESs in cutting the peak and filling the valley. Specifically, the ESs discharge at peak periods (hours 12–16) and charge at the valley periods (hours 2–4). Fig. 4 plots the optimal power generated by the DGs and ESs for 24 hours, and Fig. 5 indicates that the total generation meets the total demand at hour 18 after 1500 iterations for Alg. 1. The state of charge (SOC) and power output for the ESs are plotted in Figs. 6 and 7, where SOC is represented by a percentage of the rated capacity of ES, while discharging power by a positive value and charging power of ES by a negative one. In addition, the total charging power is not less than the total discharging one due to the charging and discharging efficiency.

The evolution of dual variables λ_i^k at hour 18 is given in Fig. 8, from which we can obtain that all dual variables converge to $-0.0883\$$ /kWh. Fig. 9 plots the residual iteration evolution of Alg. 1 and Alg. 2 in solving the DERs, respectively. The residual is expressed as $(1/m)\sum_{i=1}^{m}\|p_{i,k}-p_i^*\|^2$, where $p_i^*\in\mathbb{R}^{24}$ is the optimal power generated by DG or ES. It is shown that Alg. 1 converges faster than Alg. 2.

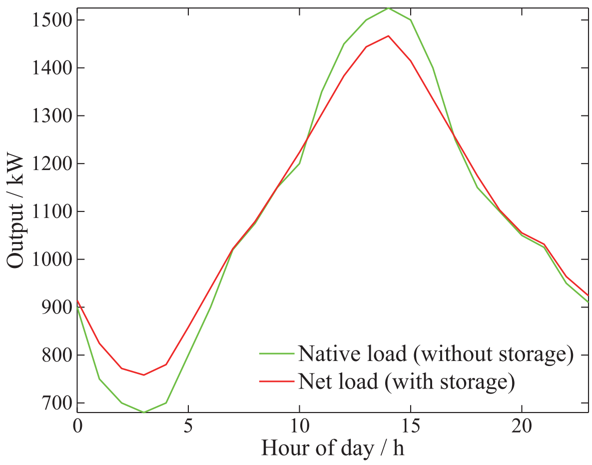

Fig. 3 Cut the peak and fill the valley

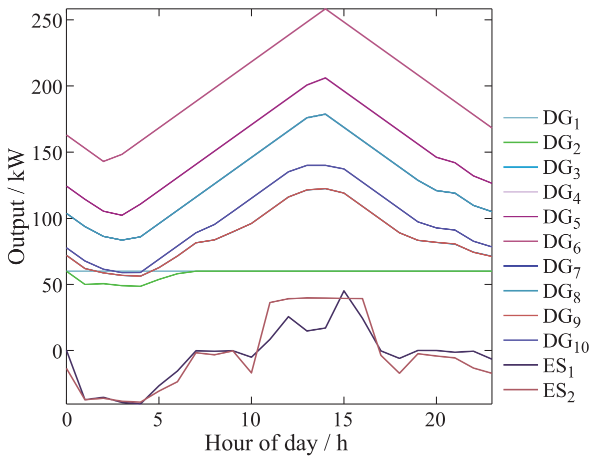

Fig. 4 Optimal power output for DGs and ESs over a day

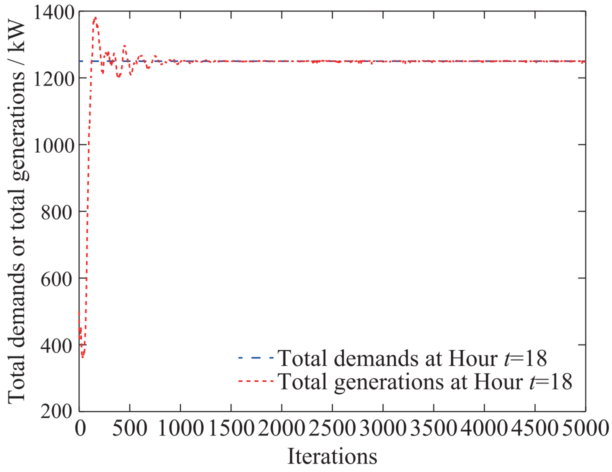

Fig. 5 Total generations and total demands at hour 18

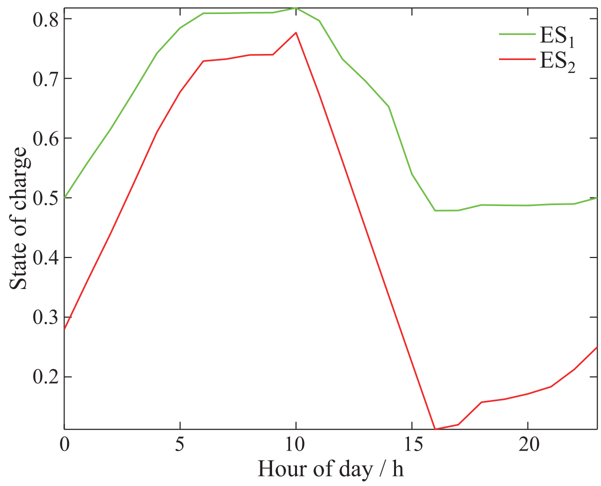

Fig. 6 State of charge

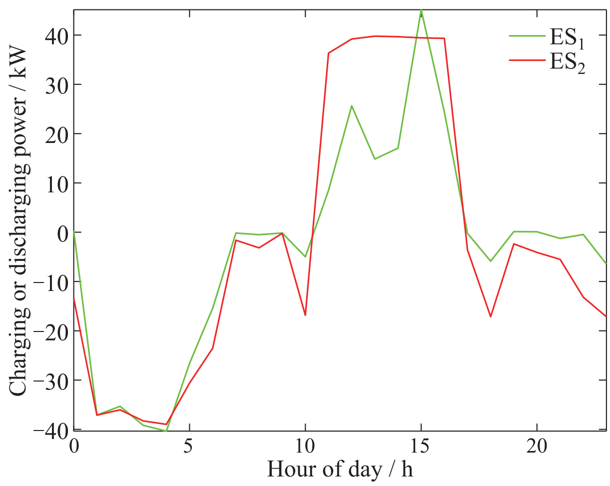

Fig. 7 Charging and discharging power

6 Conclusion

This paper has presented a distributed splitting algorithm for solving the optimal DERs coordination problem with both generation (storage) and environmental cost. The proposed algorithm enjoys certain flexibility due to the introduction of the relaxed factor and local step-sizes. Future work is to consider more practical physical constraints (such as the power flow capacities) and solve the considered problem over more complex scenarios, including time-varying or asynchronous networks, stochastic noises, etc.

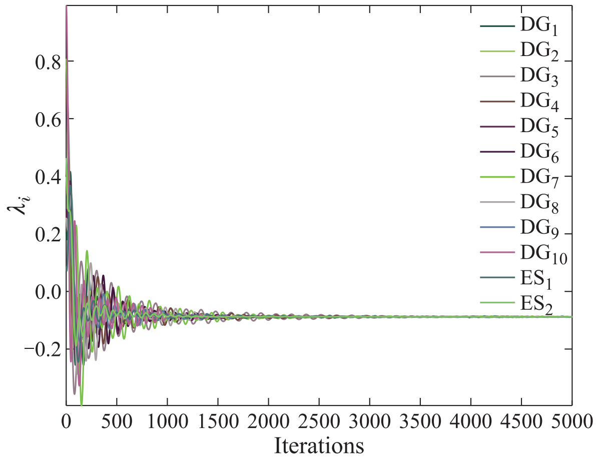

Fig. 8 Evolution of dual variables λ_i^k at hour 18

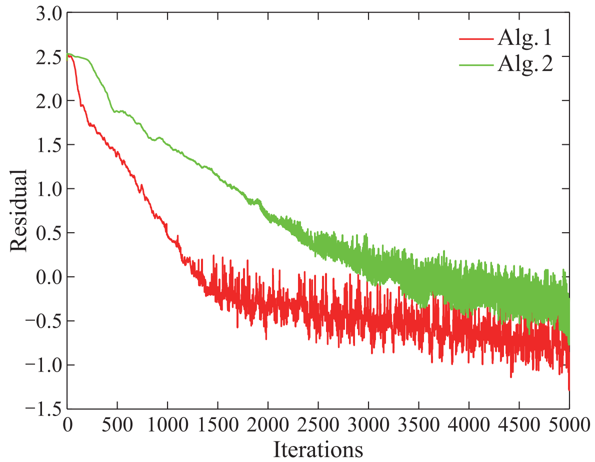

Fig. 9 Performance comparison between Alg. 1 and Alg. 2

References:

[1] YANG T, WU D, FANG H, et al. Distributed energy resources coordination over time-varying directed communication networks. *IEEE Transactions on Control of Network Systems*, 2019, 6(3): 1124 – 1134.

[2] LOPOES J P, MOREIRA C L, MADUREIRA A G. Defining control strategies for microgrids islanded operation. *IEEE Transaction on Power System*, 2006, 21(2): 916 – 924.

[3] LI H, LIAO X, HUANG T, et al. Event-triggering sampling based leader-following consensus in second-order multi-agent systems. *IEEE Transactions on Automatic Control*, 2015, 60(7): 1998 – 2003.

[4] TANG Z, HILL D J, LIU T. A novel consensus-based economic dispatch for microgrids. *IEEE Transactions on Smart Grid*, 2018, 9(4): 3920 – 3922.

[5] WANG Y, TU Z, QIN H. Distributed stochastic mirror descent algorithm for resource allocation problem. *Control Theory & Technology*, 2020, 18(4): 339 – 347.

[6] XIONG H, HAN J, NIAN X, et al. Distributed projection subgradient algorithm for two-network zero-sum game with random sleep scheme. *Control Theory & Technology*, 2021, 19(3): 405 – 417.

[7] PENG Z, HU J, LUO R, et al. Distributed multi-agent temporal-difference learning with full neighbor information. *Control Theory & Technology*, 2020, 18(4): 379 – 389.

[8] SAYED A H. Adaptation, learning and optimization over networks. *Foundations and Trends in Machine Learning*, 2014, 7(45): 311 – 801.

[9] YANG S, TAN S, XU J X. Consensus based approach for economic dispatch problem in a smart grid. *IEEE Transactions on Power Systems*, 2013, 28(4): 4416 – 4426.

[10] ZHAO C, TOPCU U, LI N, et al. Design and stability of load-side primary frequency control in power systems. *IEEE Transactions on Automatic Control*, 2014, 59(5): 1177 – 1189.

[11] XU Y, ZHANG W, HUG G, et al. Cooperative control of distributed energy storage systems in a microgrid. *IEEE Transactions on Smart Grid*, 2015, 1(6): 238 – 248.

[12] FANG H, WU D, YANG T. Cooperative management of a Lithium-Ion battery energy storage network: A distributed MPC approach. *Processing IEEE Conference on Decision and Control*. Las Vegas, NV: IEEE, 2016: 4226 – 4232.

[13] HUG G, KAR S, WU C. Consensus + innovations approach for distributed multiagent coordination in a microgrid. *IEEE Transanctions on Smart Grid*, 2015, 6(4): 1893 – 1903.

[14] CHERUKURI A, CORTES J. Distributed coordination of DERs with storage for dynamic economic dispatch. *IEEE Transactions on Automatic Control*, 2018, 63(3): 835 – 842.

[15] YANG T, WU D, STOORVOGEL A A, et al. Distributed coordination of energy storage with distributed generators. *Processing 2016 IEEE Power and Energy Society General Meeting*. Boston, MA, USA: IEEE, 2016: 1 – 5.

[16] LI C, YU X, HUANG T, et al. Distributed optimal consensus over resource allocation network and its application to dynamical economic dispatch. *IEEE Transactions on Neural Networks and Learning Systems*, 2018, 29(6): 2407 – 2418.

[17] YU W, LI C, YU X, et al. Economic power dispatch in smart grids: A framework for distributed optimization and consensus dynamics. *Science China Information Sciences*, 2018, 61(1): 1 – 16.

[18] LI H, SU E, WANG C, et al. A primal-dual forward-backward splitting algorithm for distributed convex optimization. *IEEE Transactions on Emerging Topics in Computational Intelligence*, 2021, DOI: 10.1109/TETCI.2021.3098831.

[19] LI H, WANG Z, CHEN G, et al. Distributed robust algorithm for economic dispatch in smart grids over general unbalanced directed networks. *IEEE Transactions on Industrial Informatics*, 2020, 16(7): 4322 – 4332.

[20] IVANO N, GIUSEPPE N. Asynchronous distributed optimization via randomized dual proximal gradient. *IEEE Transactions on Automatic Control*, 2017, 62(5): 2095 – 2106.

[21] LI H, HU J, RAN L, et al. Decentralized dual proximal gradient algorithms for non-smooth constrained composite optimization problems. *IEEE Transactions on Parallel and Distributed Systems*, 2021, 32(10): 2594 – 2605.

[22] STEPHEN B, LIEVEN V. *Convex Optimization*. England: Cambridge University Press, 2004.

[23] BAUSCHK H H, COMBETTES P L. *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*. Cham, Switzerland: Springer, 2011.

[24] NOTARNICOLA I, NOTARSTEFANO G. Asynchronous distributed optimization via randomized dual proximal gradient. *IEEE Transactions on Automatic Control*, 2017, 62(5): 2095 – 2106.

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