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时变输出约束非完整系统的非缩放预设时间镇定控制设计

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摘要: 本文针对一类具有时变输出约束链式非完整系统的预设时间镇定问题, 首先通过tan型障碍Lyapunov函数处理系统输出约束, 然后基于所给的新型切换时变函数, 直接应用于虚拟(实际)控制器设计, 提出了系统状态反馈镇定的非缩放变换设计方案. 本文所设计的控制器使得闭环系统状态不违反约束的同时, 可在任意给定的有限时间内收敛到零点. 与传统的基于缩放变换设计相比, 本文所提出的控制策略既有效解决了控制器的计算奇异性问题, 又减少了关于时变缩放函数的计算, 使控制器设计更为简单. 最后, 通过仿真结果验证了所提设方法的有效性.

关键词: 非完整系统; 时变输出约束; 非缩放变换; 预设时间镇定

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Prescribed-time stabilizing control of time-varying output constrained nonholonomic systems via non-scaling design

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Abstract: This paper reports the problem of prescribed-time stabilization (PTS) for a kind of uncertain nonholonomic systems (NSs) in chained form with time-varying output constraints. To handle the obstacle caused by the output constraints, a tan-type barrier lyapunov function (BLF) is exploited. By suitably introducing the time-varying function into the virtual (actual) controllers, a non-scaling transformation design scheme for state feedback is developed, which forces the states of the closed-loop system (CLS) to zero in any prescribed finite time without disobeying the constraints. In comparison with the traditional scaling transformation design, the advantages of the proposed control strategy are that it both solves the computationally singular problem effectively and leads to a simpler controller by reducing the computation burden of the time-varying scaling function. Finally, the effectiveness of the proposed scheme is confirmed by the simulation results.

Key words: nonholonomic systems (NSs); time-varying output constraints; non-scaling transformation; prescribed-time stabilization (PTS)

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1 Introduction

As a type of special nonlinear systems, nonholonomic systems (NSs) have received much attention in the last decades due to their widespread applications in practice, such as wheeled mobile robot, space robot, and underactuated satellites [1]. However, the existence of nonintegrable velocity constraint (i.e., nonholonomic constraint), makes such systems not to meet the famous Brockett necessary condition and their stabilization challenging [2]. Thanks to several constructive method mainly including discontinuous time-invariant feedback [3], smooth time-varying feedback [4] and hybrid feedback [5], lots of significant results have been gained, for instance, refer to [6–13] and the references therein.

From the point of view of convergence rate, the existing stabilization results can be divided into infinite-time stabilization (e.g., asymptotic or exponential stabilization) and finite-time stabilization. By comparison, the latter is more desired because it exhibits the appealing features of fast convergence and good disturbance

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rejection [14–16]. However, the existing finite-time stabilization results suffer from two shortcomings: one is that convergence rate is relatively slow when the system states are far away from the equilibrium points, and the other is that the settling time heavily relies on initial system conditions. To address these two shortcomings, Andrieu et al. in [17] put forward the idea of fixed-time stability that the involved settling time function is irrespective of initial system conditions. Soon afterwards, the research on fixed-time control has become a popular topic [18–23]. Roughly speaking, the existing methods on the topic of fixed-time control come down to two kinds: the bi-limit homogeneous-based one [17–18] and the Lyapunov-based one [19–23]. It is emphasized that the both methods suffer from some inherent defects, that is, the upper bound of the settling time (UBST) in the former exists but is unknown, and the UBST in the latter is bounded and adjustable, but it is so hard to be prespecified discretionarily in the light of requirements because the derived settling time functions currently depend on a few design parameters, whose selections are laborious to meet the prespecified settling time requirements [24].

However, prespecifiable settling time is indeed expected by some practical applications [25]. This fact urges that the prescribed-time stability [26](also called predefined-time stability [27]), where the UBST can be selected by the user, has been drew into to study the stabilization problem of the considered systems [28–34]. Especially, drawing support from scaling the system states by a function that grows unboundedly tending to the terminal time, a state-scaling design method to solve the prescribed-time stabilization (PTS) of Brunovsky systems in [26]. To reduce the computation burden of [26] which uses the time varying function to scale the states in all the transformations, a new non-scaling design framework was put forward in [33] by only scaling the virtual (scaling) controllers. However, the controller proposed in [33] is subject to the computationally singular problem at the terminal time. To address this trouble, a switching mechanism is recently introduced to study the PTS of parametric nonlinear systems in [34]. But the requirement that the nonlinear function must be smooth, renders the proposed technique [34] difficult to apply to nonsmooth nonlinear systems. Moreover, another common drawback of the above-mentioned works [26–34] is that the effect of state/output constraints is ignored. As we know, suffering from state/output constraints is ineluctable in many actual systems as a result of physical limitations and safe requirements. Violation of these constraints might impel system performance degradation even danger [35–38]. However, the presence of state/output constraints makes it difficult to deal with the PTS of nonlinear/nonholonomic systems using state-scaling-based control design, to date there is no related results about the PTS of constrained NSs.

Motivated by the above considerations, in this paper we concentrate on studying the PTS problem for a kind of uncertain NSs in chained form with time-varying output constraints. The significant contributions are as follows.

1) A novel switched scaling function whose switching rule dependent on both state and time is introduced to effectively overcome the computationally singular problem of the conventional scaling function-based design in [26].

2) Inspired the recent studies of [33] and [34], a nonsmooth framework of non-scaling transformation-based design is presented for constrained NSs. Different from the scaling design of [26], in which the time-varying function is adopted to scale the states in all the transformations, the proposed method employs the given switched time-varying function scaling the virtual (scaling) controllers to achieve the PTS. In this way, the BLF can be directly applied and the computation burden of the time-varying scaling function is reduced to a large extent, leading to a simpler controller.

3) Different from the PTS results in [28–34] where the effect of the state/output constraints is ignored, this paper includes output constraints in the considered system, making the developed control scheme more practical in engineering application.

The rest of this paper is organised as follows. Section 2 elaborates the problem to be investigated. Section 3 gives the design and analysis. Section 4 where the simulation study of the presented scheme is provided. Finally, some concluding remarks are given in Section 5.

Notations. The notations adopted in this paper are fairly standard. Specifically, for a vector $z=(z_1\ \cdots\ z_n)^{\mathrm{T}}\in\mathbb{R}^n$, denote $\bar{z}_j=(z_1\ \cdots\ z_j)^{\mathrm{T}}\in\mathbb{R}^j, j=1,\cdots,n$, and define $\lceil z\rfloor^{\delta}$ as $\lceil z\rfloor^{\delta}=\mathrm{sgn}(z)|z|^{\delta}$.

2 Problem formulation and preliminaries

2.1 Problem formulation

Consider the following kind of uncertain NSs in chained form:

$$\begin{cases}\dot{\zeta}_1=u_0\zeta_2+\varPhi_1(\zeta_0,\zeta_1,u_0),\\ \dot{\zeta}_i=\zeta_{i+1}u_0+\varPhi_i(\zeta_0,\bar{\zeta}_i,u_0),\\ \dot{\zeta}_n=u_1+\varPhi_n(\zeta_0,\bar{\zeta}_n,u_0),\\ \dot{\zeta}_0=u_0,\ i=2,\cdots,n-1,\end{cases}\tag{1}$$

where $(\zeta_0\ \ \zeta)^{\mathrm{T}}=(\zeta_0\ \ \zeta_1\ \ \cdots\ \ \zeta_n)^{\mathrm{T}}\in\mathbb{R}^{n+1}$, $u=(u_0\ \ u_1)^{\mathrm{T}}\in\mathbb{R}^2$, $y=(\zeta_0\ \ \zeta_1)^{\mathrm{T}}\in\mathbb{R}^2$ are the system state, control input and output, respectively. $\varPhi_i:\mathbb{R}\times\mathbb{R}^i\times\mathbb{R}\rightarrow\mathbb{R}, i=1,\cdots,n$ are continuous nonlinear functions satisfying $\varPhi_i(\zeta_0,0,u_0)=0$. Due to physical or performance limitations, this paper supposes the output y suffering from the following time-varying constraints

$$\Omega_i = \{\zeta_i(t) : -k_{i1}(t) < \zeta_i(t) < k_{i2}(t)\}\,,\ i = 0, 1, \tag{2}$$

with some pre-specified positive functions $k_{i1}(t)$ and $k_{i2}(t)$.

Remark 1 As reported by [37], such constraints are common in practice. For instance, a mobile robot working in a restricted area can be modeled as the system (1) with $n=2$, $\Phi_1 = \Phi_2 = 0$ and the output constraints (2) are equal to the space constraints of such robot. The appearance of the time-varying constraints makes the unconstrained control techniques not applicable to constrained system (1), for which new control techniques should be developed.

The control goal of this paper is to present a switched, non-scaling, state feedback control mechanism which stabilizes system (1) within prescribed finite time T_p, while fulfilling the time-varying constraints (2).

The following assumptions are needed in this paper.

Assumption 1 The time-varying output constraints $k_{ij}(t)$ $(i = 0, 1, j = 1, 2)$ are continuous differentiable and there are positive constants $\underline{k}_{i1}$, $\underline{k}_{i2}$, $\bar{k}_{i3}$ and $\bar{k}_{i4}$ such that $\underline{k}_{i1} \leqslant k_{i1}(t)$, $\underline{k}_{i2} \leqslant k_{i2}(t)$, $|\dot{k}_{i1}(t)| \leqslant \bar{k}_{i3}$ and $|\dot{k}_{i2}(t)| \leqslant \bar{k}_{i4}$.

Assumption 2 There are smooth functions $\varphi_i \geqslant 0$ and a constant $\tau \in (0, 1/n)$ such that

$$|\Phi_i(\zeta_0, \bar{\zeta}_i, u_0)| \leqslant \varphi_i(\zeta_0, \bar{\zeta}_i, u_0) \sum_{j=1}^{i} |\zeta_j|^{\frac{\lambda_i - \tau}{\lambda_j}}, \tag{3}$$

where $\lambda_i = 1 - (i-1)\tau > 0, i = 1, \cdots, n$.

Remark 2 Assumption 1 is similar to these used in [37], which slightly relaxes the corresponding assumptions in [35–36] by removing the upper bound restrictions. Assumption 2 is a generalized homogeneous-growth-like condition and can include the frequently-used ones on the practical systems (e.g., the Hölder-like growth condition and the Lipschitz-like growth condition) as special cases [15, 20].

2.2 Preliminary results

Consider the nonlinear system

$$\dot{z} = \mu(t, z),\ z(0) = z_0,\ \mu(t, 0) = 0, \tag{4}$$

where $\mu : \mathbb{R}^+ \times U \to \mathbb{R}^n$ is a (discontinuous) nonlinear vector field on an open neighborhood U of the origin.

Definition 1 ([14]) The origin of system (4) is named finite-time stable if it is asymptotically stable and for any $z_0 \in U$, a settling time function $T : U\backslash\{0\} \to (0, \infty)$ exists such that every solution $z(t, z_0)$ of (4) satisfies $z(t, z_0) = 0, \forall\, t \geqslant T(z_0)$.

Definition 2 ([34, 38]) The origin of system (4) is named prescribed-time stable if it is finite-time stable and a tunable designing parameter $\vartheta \in \mathbb{R}$ exists to ensure $T(z_0) \leqslant T_p$ for any prescribed finite time $T_p > 0$ and any $z_0 \in U$.

Lemma 1 ([14]) For system (4), if there exist a C^1 and positive definite function $V(z)$ defined $\hat{U}$ with $0 \subseteq \hat{U} \subseteq U$, some real numbers $c > 0$ and $0 < \alpha < 1$ such that

$$\dot{V}(z) \leqslant -cV^{\alpha}(z),\ \forall\, z \in \hat{U}.$$

Then, the origin of system (4) is finite-time stable with

$$T(z_0) \leqslant \frac{V^{1-\alpha}(0)}{c(1-\alpha)},\ \forall\, z \in \hat{U}.$$

Lemma 2 ([39]) For $\zeta_1 \in \mathbb{R}$, $\zeta_2 \in \mathbb{R}$, and a constant $m \geqslant 1$, one has 1) $|\zeta_1 + \zeta_2|^m \leqslant 2^{m-1}|\zeta_1^m + \zeta_2^m|$; 2) $(|\zeta_1| + |\zeta_2|)^{1/m} \leqslant |\zeta_1|^{1/m} + |\zeta_2|^{1/m} \leqslant 2^{(m-1)/m}(|\zeta_1| + |\zeta_2|)^{1/m}$.

Lemma 3 ([39]) If c, d are positive constant and $\gamma(\zeta_1, \zeta_2) > 0$ are real-valued function, then one has $|\zeta_1|^c|\zeta_2|^d \leqslant \dfrac{c}{c+d}\gamma(\zeta_1, \zeta_2)|\zeta_1|^{c+d} + \dfrac{d}{c+d}\gamma^{\frac{-c}{d}}(\zeta_1, \zeta_2)|\zeta_2|^{c+d}$.

Lemma 4 ([37]) For $\zeta_1 \in \mathbb{R}$, $\zeta_2 \in \mathbb{R}$ and constant $0 < m \leqslant 1$ and $a > 0$, one has $|\lceil\zeta_1\rceil^{am} - \lceil\zeta_2\rceil^{am}| \leqslant 2^{1-m}|\lceil\zeta_1\rceil^a - \lceil\zeta_2\rceil^a|^m$.

3 Prescribed-time stabilization

In this section, a non-scaling control strategy is designed to achieve the stabilization task of system (1) within any given prescribed finite time $T_p > 0$, while preventing the violation of the time-varying constraints (2).

3.1 Scaling function and tan-type BLF

For the object of this paper, we introduce the switched scaling function as in [38]

$$\digamma_1 = \begin{cases} \Gamma_1, & \zeta \in \{\mathbb{R}^n - \Xi_1\}\ \&\ t < T_{s1}, \\ 1, & \text{otherwise}, \end{cases} \tag{5}$$

where Ξ_1 is a small closed neighborhood of origin and

$$\Gamma_1 = \frac{T_{s1}}{T_{s1} - t}, \tag{6}$$

with the positive design parameter T_{s1} satisfying $0 < T_{s1} < T_p$.

Remark 3 It is clearly that Γ_1 monotonically increases on $[0, T_{s1})$ with $\Gamma_1(0) = 1$ and $\Gamma_i(T_{s1}) = +\infty$. To address the incapability of ensuring the closed-loop viability and stability behind T_{s1}, a new switched scaling function (5) is introduced in this paper. In comparision with the one used in [26], its novelty is that the switching rule dependent on both state and time, i.e, it uses a small closed neighborhood of origin Ξ_1 to replaces the origin, which renders the system trajectory $\zeta(t)$ to the switching set Ξ_1 at some moment before T_{s1} can effectively overcome the computationally singular problem ($\infty \times 0$ type) of the resulting controller as $t \to T_{s1}$.

To avoid the state ζ_i violating the constraints (2), an asymmetric BLF function $V_{bi} : \Omega_i \to \mathbb{R}$ is given as follows:

$$V_{bi}(\zeta_i) = \frac{k_{bi}^2}{\pi}\tan(\frac{\pi\zeta_i^2}{2k_{bi}^2}), \tag{7}$$

where $k_{bi} = k_{i2}$, if $\zeta_i > 0$, otherwise $k_{bi} = k_{i1}$.

From (16), it is clear that the function $V_{bi}(\zeta_i)$ is positive definite on Ω_i and satisfies $V_{bi}(\zeta_i) \to \infty$ as $\zeta_i \to -k_{i1}$ or $\zeta_i \to k_{i2}$. Besides, differentiating the function $V_{bi}(x_i)$ obtains that

$$\begin{cases} \dfrac{\partial V_{bi}(\zeta_i)}{\partial \zeta_i} = \Lambda_{bi}(\zeta_i)\zeta_i, \\ \dfrac{\partial V_{bi}(\zeta_i)}{\partial k_{bi}} = \dfrac{2k_{bi}}{\pi}\tan(\dfrac{\pi\zeta_i^2}{2k_{bi}^2}) - \dfrac{1}{k_{bi}}\Lambda_{bi}(\zeta_i)\zeta_i^2, \end{cases} \tag{8}$$

with $\Lambda_{bi}(\zeta_i)$ defined as

$$\Lambda_{bi}(\zeta_i) = \begin{cases} \sec^2(\dfrac{\pi\zeta_i^2}{2k_{i2}^2}),\ \zeta_i > 0, \\ \sec^2(\dfrac{\pi\zeta_i^2}{2k_{i1}^2}),\ \zeta_i \leqslant 0. \end{cases} \tag{9}$$

Remark 4 What needs to be emphasized is that $V_{bi}(\zeta_i)$ has an attractive property that

$$\lim_{k_{ij}\to\infty} V_{bi}(\zeta_i) = \lim_{k_{ij}\to\infty}\frac{k_{i1}^2}{\pi}\tan(\frac{\pi\zeta_i^2}{2k_{ij}^2}) = \frac{1}{2}\zeta_i^2,\ j = 1, 2. \tag{10}$$

which implies that when no constraints is required on the lower and/or upper bounds of ζ_i, by setting $k_{i1} \to \infty$ and/or $k_{i2} \to \infty$, $V_{bi}(\zeta_i)$ in (7) becomes the equivalent Lyapunov function which is widely used in the unconstrained control design. As a consequence, the presented asymmetric BLF $V_{bi}(\zeta_i)$ can serve as a unified tool to address the control problem simultaneously with asymmetric constraint or without constraint requirements.

3.2 PTS of the ζ-subsystem

For the ζ_0-subsystem, we pick up the control u_0 as

$$u_0 = (|\operatorname{sgn}(\zeta_0(0))| - \operatorname{sgn}(\zeta_0(0)) - 1)\, c_0^*, \tag{11}$$

where $c_0^* > 0$ is a design constant satisfying $c_0^* < k_{01}/(\varepsilon T_p)$ with $\varepsilon \in (0, 1)$. For simplicity, without loss of generality, in later use we assume $\zeta_0(0) < 0$, that is, the sign of u_0 is positive. Then, the ζ-subsystem is rewritten as

$$\begin{cases} \dot\zeta_1 = h_1 p_2 + \Phi_1(\zeta_0, \bar\zeta_n), \\ \dot\zeta_i = h_i p_{i+1} + \Phi_i(\zeta_0, \bar\zeta_n),\ i = 2, \cdots, n-1, \\ \dot\zeta_n = h_n u_1 + \Phi_n(\zeta_0, \bar\zeta_n), \end{cases} \tag{12}$$

with $h_i(t) = c_0^*, i = 1, \cdots, n-1$ and $h_n(t) = 1$. As a consequence, the following result is reaped by simple mathematical derivations.

Proposition 1 Under (11), the solution of the ζ_0-subsystem $\zeta_0(t)$ is well-defined on $[0, \varepsilon T_p)$ provided that $|\zeta_0(0)| < \underline{k}_0$.

Next a state feedback controller u_1 will be developed to stabilize system (12) within a settling time T_1 $(T_{s1} < T_1 \leqslant \varepsilon T_p)$ by the recursive idea.

Step 1 Select $V_1 = V_{b1}$ as the Lyapunov function for this step. Based on (3) and (8), the derivative of V_1 arrives

$$\begin{aligned} \dot V_1 = {} & \frac{\partial V_{b1}}{\partial \zeta_1}\dot\zeta_1 + \frac{\partial V_{b1}}{\partial k_{b1}}\dot k_{b1} = \\ & \Lambda_{b1}(\zeta_1)\zeta_1(h_1\zeta_2 + \Phi_1) + \frac{2k_{b1}}{\pi}\tan(\frac{\pi\zeta_1^2}{2k_{b1}^2})\dot k_{b1} - \\ & \frac{1}{k_{b1}}\Lambda_{b1}(\zeta_1)\zeta_1^2\dot k_{b1} \leqslant \\ & \Lambda_{b1}(\zeta_1)\zeta_1(h_1\zeta_2 + \Phi_1) + \frac{2}{k_{b1}}\Lambda_{b1}(\zeta_1)\zeta_1^2|\dot k_{b1}| \leqslant \\ & \Lambda_{b1}(\zeta_1)\left(h_1\zeta_1(\zeta_2 - \zeta_2^*) + h_1\zeta_1\zeta_2^* + |\zeta_1|^{2-\tau}\tilde\varphi_1\right), \end{aligned} \tag{13}$$

where $\tilde\varphi_1 \geqslant \rho_1 + (2\bar K_1|\zeta_1|^\tau)/\underline{K}_1$ with $\underline{K}_1 = \min\{\underline{k}_{11}, \underline{k}_{12}\}$ and $\bar K_1 = \max\{\bar k_{13}, \bar k_{14}\}$ is a smooth function and ζ_2^* is the virtual controller of ζ_2.

Take

$$\zeta_2^* = -F_1\beta_1\lceil\zeta_1\rfloor^{\lambda_2}, \tag{14}$$

where

$$\beta_1 = \frac{1 + c + \tilde\varphi_1}{h_1}, \tag{15}$$

with c being a positive constant. Then, by substituting (14) into (13), one has

$$\begin{aligned} \dot V_1 \leqslant {} & -(1+c)F_1\Lambda_{b1}(x_1)|\zeta_1|^{2-\tau} + \\ & \Lambda_{b1}(\zeta_1)h_1\zeta_1(\zeta_2 - \zeta_2^*). \end{aligned} \tag{16}$$

Step 2 Define $z_2 = \lceil\zeta_2\rfloor^{\frac{1}{\lambda_2}} - \lceil\zeta_2^*\rfloor^{\frac{1}{\lambda_2}}$ and take the Lyapunov function $V_2 = V_1 + W_2$ with

$$W_2 = \int_{\zeta_2^*}^{\zeta_2}\lceil\lceil s\rfloor^{\frac{1}{\lambda_2}} - \lceil\zeta_2^*\rfloor^{\frac{1}{\lambda_2}}\rfloor^{2-\lambda_2}\mathrm{d}s. \tag{17}$$

From

$$\begin{cases} \dfrac{\partial W_2}{\partial \zeta_2} = \lceil z_2\rfloor^{2-\lambda_2}, \\ \dfrac{\partial W_2}{\partial \theta} = -(2-\lambda_2)\dfrac{\partial(\lceil\zeta_2^*\rfloor^{\frac{1}{\lambda_2}})}{\partial\theta}\times \\ \qquad \displaystyle\int_{\zeta_2^*}^{\zeta_2}|\lceil s\rfloor^{\frac{1}{\lambda_2}} - \lceil\zeta_2^*\rfloor^{\frac{1}{\lambda_2}}|^{1-\lambda_2}\mathrm{d}s, \end{cases} \tag{18}$$

where $\theta = t$ or $\theta = \zeta_1$, a direct calculation gives

$$\begin{aligned} \dot V_2 \leqslant {} & -(1+c)F_1\Lambda_{b1}(\zeta_1)|\zeta_1|^{2-\tau} + \\ & \Lambda_{b1}(\zeta_1)h_1\zeta_1(\zeta_2 - \zeta_2^*) + \lceil z_2\rfloor^{2-\lambda_2}h_2\zeta_3 + \\ & \lceil z_2\rfloor^{2-\lambda_2}\Phi_2 + \frac{\partial W_2}{\partial \zeta_1}(h_1\zeta_2 + \Phi_1) + \frac{\partial W_2}{\partial t}. \end{aligned} \tag{19}$$

Based on the fact $F_1 \geqslant 1$ for all $t \geqslant 0$, we give the following estimates for some terms of (19). First, from the definitions of z_2 and ζ_2^* and Lemma 4, one has

$$\begin{aligned} |\zeta_2 - \zeta_2^*| \leqslant {} & 2^{1-\lambda_2}|\lceil\zeta_2\rfloor^{\frac{1}{\lambda_2}} - \lceil\zeta_2^*\rfloor^{\frac{1}{\lambda_2}}|^{\lambda_2} = \\ & 2^{1-\lambda_2}|z_2|^{\lambda_2}. \end{aligned} \tag{20}$$

Thus, from (20) and Lemma 3, it is obtained that

$$\varLambda_{b1}(\zeta_1)\lceil z_1\rfloor^{2-\lambda_1}h_2\left(\zeta_2-\zeta_2^*\right)\leqslant \frac{1}{4}|z_1|^{2-\tau}+|z_2|^{2-\tau}\varrho_{21},\tag{21}$$

where $\varrho_{21}\geqslant 0$ is a smooth function.

Secondly, from Assumption 2 and Lemma 2, one gets

$$|\varPhi_2|\leqslant\varphi_2(|\zeta_1|^{\frac{\lambda_2-\tau}{\lambda_1}}+|\zeta_2|^{\frac{\lambda_2-\tau}{\lambda_2}})\leqslant \varphi_2(|\zeta_1|^{\lambda_2-\tau}+|z_2|^{\lambda_2-\tau}+\digamma_1\beta_1|\zeta_1|^{\lambda_2-\tau}).\tag{22}$$

Using (22) and Lemma 3 yields

$$\lceil z_2\rfloor^{2-\lambda_2}\varPhi_2\leqslant\frac{1}{4}|\zeta_1|^{2-\tau}+\digamma_1^{\frac{2-\tau}{1+\tau}}|z_2|^{2-\tau}\varrho_{22},\tag{23}$$

where $\varrho_{22}\geqslant 0$ is a smooth function.

Finally, notice that

$$\int_{\zeta_2^*}^{\zeta_2}|\lceil s\rfloor^{\frac{1}{\lambda_2}}-\lceil\zeta_2^*\rfloor^{\frac{1}{\lambda_2}}|^{1-\lambda_2}\mathrm{d}s\leqslant 2^{1-\lambda_2}|z_2|,\tag{24}$$

$$|\frac{\partial(\lceil\zeta_2^*\rfloor^{\frac{1}{\lambda_2}})}{\partial\zeta_1}|\leqslant\digamma_1^{\frac{1}{\lambda_2}}\varpi_{21},\tag{25}$$

$$|\frac{\partial(\lceil\zeta_2^*\rfloor^{\frac{1}{\lambda_2}})}{\partial t}|\leqslant\digamma^{\frac{1+\lambda_2}{\lambda_2}}|\zeta_1|\varpi_{22},\tag{26}$$

where ϖ_{21} and ϖ_{22} are some nonnegative smooth functions.

Then, from (24)–(26) and Lemma 3, one arrives

$$\frac{\partial W_2}{\partial\zeta_1}(h_1\zeta_2+\varPhi_1)\leqslant\frac{1}{4}|\zeta_1|^{2-\tau}+\digamma_1^{\frac{(2-\tau)^2}{(1-\tau)^2}}|z_2|^{2-\tau}\varrho_{23},\tag{27}$$

$$\frac{\partial W_2}{\partial t}\leqslant\frac{1}{4}|\zeta_1|^{2-\tau}+\digamma_1^{\frac{(2-\tau)^2}{(1-\tau)^2}}|z_2|^{2-\tau}\varrho_{24},\tag{28}$$

where $\varrho_{23}\geqslant 0$ and $\varrho_{24}\geqslant 0$ are smooth functions.

As a result, by letting $\varrho_2=\varrho_{21}+\varrho_{22}+\varrho_{23}+\varrho_{24}$ and

$$\gamma_2=\max\{1,\frac{2-\tau}{1+\tau},\frac{(2-\tau)^2}{(1-\tau)^2}\},\tag{29}$$

one has

$$\varLambda_{b1}(\zeta_1)h_1\zeta_1\left(\zeta_2-\zeta_2^*\right)+\lceil z_2\rfloor^{2-\lambda_2}\varPhi_2+\frac{\partial W_2}{\partial t}+\frac{\partial W_2}{\partial\zeta_1}(h_1\zeta_2+\varPhi_1)\leqslant|\zeta_1|^{2-\tau}+\digamma_1^{\gamma_2}|z_2|^{2-\tau}\varrho_2.\tag{30}$$

Substituting (30) into (19) yields

$$\dot V_2\leqslant-c\digamma_1\varLambda_{b1}(\zeta_1)|\zeta_1|^{2-\tau}+\lceil z_2\rfloor^{2-\lambda_2}h_2\zeta_3+\digamma_1^{\gamma_2}|z_2|^{2-\tau}\varrho_2.\tag{31}$$

Hence, one can design the virtual controller

$$\zeta_3^*=-\digamma_1^{\gamma_2}\lceil z_2\rfloor^{\lambda_2-\tau}\beta_2,\tag{32}$$

with $\beta_2=(c+\varrho_2)$, which together with the fact that $\digamma_1\geqslant 1$ and $\varLambda_{b1}(\zeta_1)\geqslant 1$ for all $t\geqslant 0$ is such that

$$\dot V_2\leqslant-c\digamma_1\varLambda_{b1}(\zeta_1)|\zeta_1|^{2-\tau}-c\digamma_1|z_2|^{2-\tau}+\lceil z_2\rfloor^{2-\lambda_2}h_2(\zeta_3-\zeta_3^*).\tag{33}$$

Following the same arguments of Step 2, for Step i ($i=2,\cdots,n$), we can find a C^1 and positive definite Lyapunov function $V_i=V_{b_1}+\sum\limits_{j=2}^{i}W_j$ with

$$W_j=\int_{\zeta_j^*}^{\zeta_j}\lceil\lceil s\rfloor^{\frac{1}{\lambda_j}}-\lceil\zeta_j^*\rfloor^{\frac{1}{\lambda_j}}\rfloor^{2-\lambda_j}\mathrm{d}s,\tag{34}$$

and a group of continuous virtual controllers $\zeta_{j+1}^*=-\digamma_1^{\gamma_j}\lceil z_j\rfloor^{\lambda_j-\tau}\beta_j$, $j=1,\cdots,n$, such that

$$\dot V_j\leqslant-c\digamma_1\varLambda_{b1}(\zeta_1)|\zeta_1|^{2-\tau}-c\digamma_1\sum_{j=2}^{i}|z_j|^{2-\tau}+\lceil z_j\rfloor^{2-\lambda_j}h_j(\zeta_{j+1}-\zeta_{j+1}^*).\tag{35}$$

where $u_1=\zeta_{n+1}$. Consequently, the following result is obtained.

Theorem 1 Considering system (12) under Assumptions 1–2, the state feedback controller $u_1=\zeta_{+1}^*$ with a properly selection of the design parameters renders the following conclusions hold.

1) The state ζ_1 keeps in the set $\varOmega_1$ for all $t\geqslant 0$ without violating the constraints.

2) The equilibrium at the origin is prescribed-time stable within any given settling time T_1.

Proof The main proof is divided into three parts.

Part 1 Prescribed-time attractive without violating constraints: Since for all $\theta\in(0,1)$,

$$\tan(\frac{\pi\theta}{2})\leqslant\frac{\pi\theta}{2}\sec(\frac{\pi\theta}{2})\leqslant\frac{\pi\theta}{2}\sec^2(\frac{\pi\theta}{2})\tag{36}$$

holds, and then we have

$$V_{b1}=\frac{k_{b1}^2}{\pi}\tan(\frac{\pi\zeta_1^2}{2k_{b1}^2})\leqslant\frac{1}{2}\varLambda_{b1}(\zeta_1)|\zeta_1|^2.\tag{37}$$

Moreover, by Lemma 4, W_j can be calculated as

$$W_j=\int_{\zeta_2^*}^{\zeta_j}\lceil\lceil s\rfloor^{\frac{1}{\lambda_j}}-\lceil\zeta_j^*\rfloor^{\frac{1}{\lambda_j}}\rfloor^{2-\lambda_j}\mathrm{d}s\leqslant |z_j|^{2-\lambda_j}|\zeta_j-\zeta_j^*|\leqslant 2^{1-\lambda_j}|z_j|^2.\tag{38}$$

Therefore the following estimation is obtained.

$$V_n^{\frac{2-\tau}{2}}=(V_{b1}+\sum_{j=2}^{n}W_j)^{\frac{2-\tau}{2}}\leqslant \varLambda_{b1}(\zeta_1)|\zeta_1|^{2-\tau}+\sum_{j=2}^{n}|z_j|^{2-\tau}.\tag{39}$$

which together with (33) leads to

$$\dot V_n\leqslant-c\digamma_1V_n^{\frac{2-\tau}{2}}.\tag{40}$$

When $\digamma_1=\varGamma_1$, (40) indicates the domain $\varXi_1$ is prescribed-time attractive and the convergence time satisfies

$$T_a\leqslant T_{s1}(1-\exp(-\frac{2V_n^{\frac{\tau}{2}}(0)}{c\tau T_{s1}}))<T_{s1}.\tag{41}$$

Since V_n is a non-increasing function, it is easy to deduce from (40) that

$$V_{b1} = \frac{k_{b1}^2}{\pi}\tan(\frac{\pi\zeta_1^2}{2k_{b1}^2}) \leqslant V_n \leqslant V_n(0), \quad (42)$$

for all $\zeta \in \Omega_1 \times \mathbb{R}$. By a simple calculation, we can obtain

$$\frac{\pi\zeta_1^2}{2k_{b1}^2} \leqslant \tan^{-1}(\frac{\pi}{k_{b1}^2}V_n(0)) < \frac{\pi}{2},\ \forall t \geqslant 0, \quad (43)$$

and thereby the state ζ_1 remains in the set $|\zeta_1| < k_{b_1}$ (i.e., $-k_{11} < \zeta_1 < k_{12}$) and never violates the constraints.

Part 2 Local Prescribed/finite-time stable without violating constraints: When $\digamma_1 = 1$, let $C = \max_{\zeta\in\Xi_1} V_n(\zeta)$. Then (40) indicates the origin of the CLS is locally finite-time stable in the attraction domain Ξ_1 and the convergence time satisfies

$$T_l \leqslant \frac{2V_n^{\frac{\tau}{2}}(0)}{c\tau} \leqslant \frac{2C^{\frac{\tau}{2}}}{c\tau}. \quad (44)$$

Therefore, by selecting $c \geqslant (2C^{\frac{\tau}{2}})/(\tau T_1 - \tau T_{s1})$, one has $T_l \leqslant T_1 - T_{s1}$. Furthermore, similar to the argument in Part 2, it can be shown that the constraints $-k_{11} < p_1 < k_{12}$ is not violated as well.

Part 3 Stability analysis: The equation (40) indicates that the CLS is Lyapunov asymptotically convergent (stable) in both operational domains. Thanks to the properties of existence and continuation of the solutions, it is sure that the whole system is Lyapunov asymptotically stable. As a result, based on this and the results of Parts I and II, one has that the origin of the CLS is prescribed-time stable within $T_a + T_l < T_1$ without violating the constraints. Thus, the proof is completed. □

3.3 PTS of the ζ_0-subsystem

Since $\dot{\zeta}(t) \equiv 0$, then we have that $\zeta(t)$ keeps zero for all $t \geqslant T_1$. As a result, to achieve the PST task of system (1), we next only need to stabilize the ζ_0-subsystem in a prescribed time $T_2 \leqslant (1-\varepsilon)T_p$. Similar as that in Subsection 3.1, introduce

$$\digamma_2 = \begin{cases} \Gamma_2,\ \zeta_0 \in \{\mathbb{R} - \Xi_2\}\ \&\ t < T_{s2}, \\ 1, \quad \text{otherwise}, \end{cases} \quad (45)$$

where Ξ_2 is a small closed neighborhood of origin and

$$\Gamma_2 = \frac{T_{s2}}{T_{s2} - t}, \quad (46)$$

with the positive design parameter T_{s2} satisfying $0 < T_{s2} < T_2$.

Take the candidate Lyapunov function V_0 as $V_0 = V_{b0}$ and select

$$u_0 = -\digamma_2\beta_0\lceil\zeta_0\rfloor^{1-\omega}, \quad (47)$$

with $\beta_0 = \varphi_0 + \kappa$ and $\omega \in (0,1)$, κ being positive constants, one obtains

$$\dot{V}_0 \leqslant -\kappa\digamma_2\Lambda_{b0}(\zeta_0)|\zeta_0|^{2-\omega}. \quad (48)$$

Theorem 2 For the ζ_0-subsystem of (1) satisfying Assumption 1, the state feedback controller (47) drives the state ζ_0 to zero within the prescribed finite time T_2 without violating the constraints.

Proof This proof follows the same line of that of Theorem 1. □

Till now, the state feedback design for PTS of the system (1) is completed. Accordingly, the following theorem is stated to sum up the result.

Theorem 3 Consider the system (1) satisfying Assumptions 1–2. If the switching control strategy

$$u_0 = \begin{cases} u_0|_{(11)},\ t < \varepsilon T_p, \\ u_0|_{(47)},\ t \geqslant \varepsilon T_p, \end{cases} \quad (49)$$

$$u_1 = \zeta_{n+1}^*, \quad (50)$$

with a properly selection of the design parameters is applied, then the states of the CLS are driven to zero within any prescribed finite time T_p. while, at the same the constraints (2) are satisfied.

Proof The result holds readily from the results of Theorems 1–2. □

Remark 5 The idea of design procedure can be as summarized as:

1) For given prescribed-time $T_p > 0$, take $T_1 = \varepsilon T_p$ and $T_2 = (1-\varepsilon)T_p$ with $\varepsilon \in (0,1)$.

2) The designed controller $u_0 = u_0|_{(11)}$ ensures that the ζ_0-subsystem is well-defined in $[0, T_1)$. In this situation, by letting $T_{s1} < T_1$, the designed controller $u_1 = \zeta_{+1}^*$ with $\digamma_1 = \Gamma_1$ to a (small) pre-specified attraction domain Ξ_1 at some $T_a < T_{s1}$ without violating the constraints. In such way, the computational singularity of designed controller when $t \to T_{s1}$ is solved.

3) Appropriately selected parameter c guarantees that under designed controller $u_1 = \zeta_{+1}^*$ with $\digamma_1 = 1$, the system state $\zeta(t)$ once enters the attraction domain Ξ_1 then it converges to and stays at the origin $\zeta = 0$ for all $t \geqslant T_1 \geqslant T_a + T_l$, at the same time satisfying the constraints.

4) Switch the controller u_0 to $u_0 = u_0|_{(47)}$, which renders the state ζ to zero within the prescribed finite time T_2 without violating the constraints.

4 Simulation Example

Consider the following nonholonomic chained-form system:

$$\begin{cases} \dot{\zeta}_0 = u_0, \\ \dot{\zeta}_1 = u_0\zeta_2, \\ \dot{\zeta}_2 = u_1 + |\zeta_1|^\theta, \end{cases} \quad (51)$$

with $1/2 \leqslant \theta \leqslant 1$. Such system can be viewed as a perturbed version of unicycle-type mobile robot model [6]. When the robot works in a limited area, how to park the robot in a prescribed time turns into the problem of PTS of system (51) with output constraints (2).

It is clear that $|\zeta_1|^\theta$ renders that the system (51) is an essential nonsmooth system, to which the existing PTS designs of [33] and [34] are inapplicable. But, if the prescribed time $T_p = 5$ s, $k_{01} = k_{11} = 1 + 0.55\sin(10t)$ and $k_{02} = k_{12} = 1 + 0.4\sin(6t)$ is taken, it is not hard to check that such system satisfies the assumptions with $\underline{k}_{01} = \underline{k}_{11} = 0.45$, $\underline{k}_{02} = \underline{k}_{12} = 0.6$, $\bar{k}_{03} = \bar{k}_{13} = 0.55$, $\bar{k}_{04} = \bar{k}_{14} = 0.4$, $\tau = 1/3$, $\varphi_2 = 0.5\sqrt{1+\zeta_1^2}$ and $\varphi_1 = 0$. Therefore the prescribed-time controller designed as (32) and (47) with $h_1 = 0.1$, $\tilde{\varphi}_2 = \sqrt{1+\zeta_1^2}$, $\varepsilon = 3/5$, $c^* = 0.1$, $T_{s1} = 2$, $T_{s2} = 4$, $\varXi_1 = \{\zeta : \zeta_1^2 + \zeta_2^2 \leqslant 0.01\}$, $\varXi_2 = \{\zeta_0 : \zeta_0^2 \leqslant 0.01\}$, $\gamma = 6.25$, $\kappa = 1$, $\omega = 0.5$, $\varrho_{21} = 4.7216\varLambda_{b1}^{5/2}$, $\varrho_{22} = 0.7566\tilde{\varphi}_2^{5/4}$, $\varrho_{23} = 2.5198(1+z_2^2) + 12.1592(1+\zeta_1^2)^{5/3}\beta_1^{5/2}$ and $\varrho_{24} = 1.6133(1+\zeta_1^2)^{5/3}\beta_1^{5/2}$ can achieve the PTS of constrained system (51). For different initial conditions: (a) $(\zeta_0(0), \zeta_1(0), \zeta_2(0)) = (-0.1, 0.1, -1)$ and (b) $(\zeta_0(0), \zeta_1(0), \zeta_2(0)) = (-0.4, 0.9, -5)$, the simulation results depicted in Figs. 1–5 exhibit the appealing performance of the proposed prescribed-time control scheme.

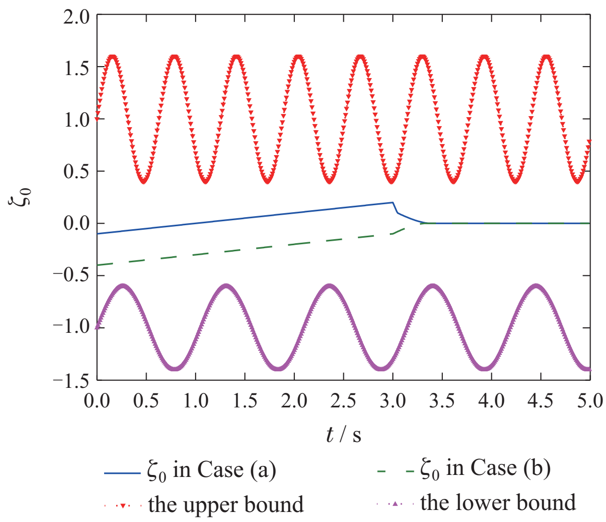

Fig. 1 System state ζ_0

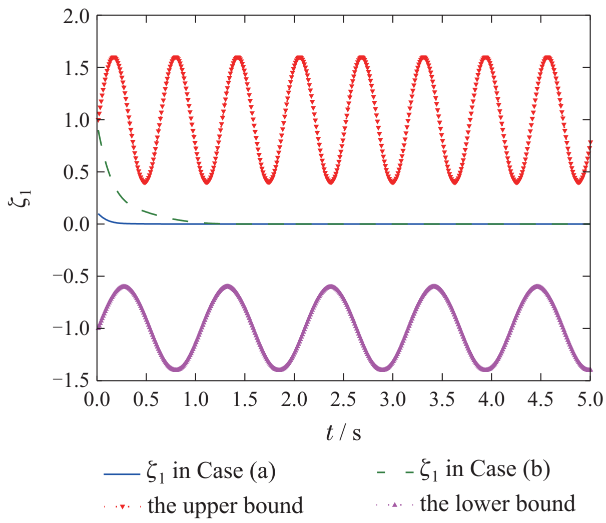

Fig. 2 System state ζ_1

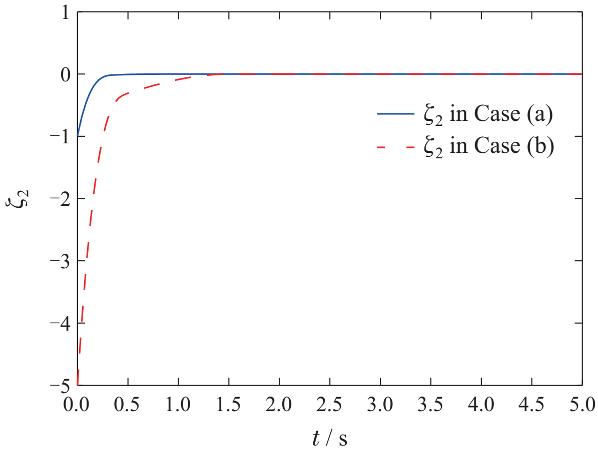

Fig. 3 System state ζ_2

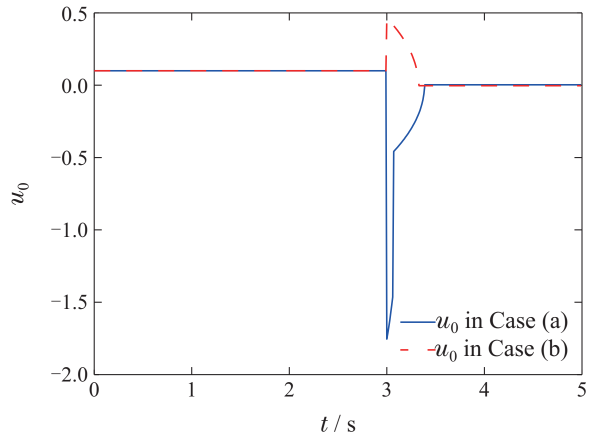

Fig. 4 Control input u_0

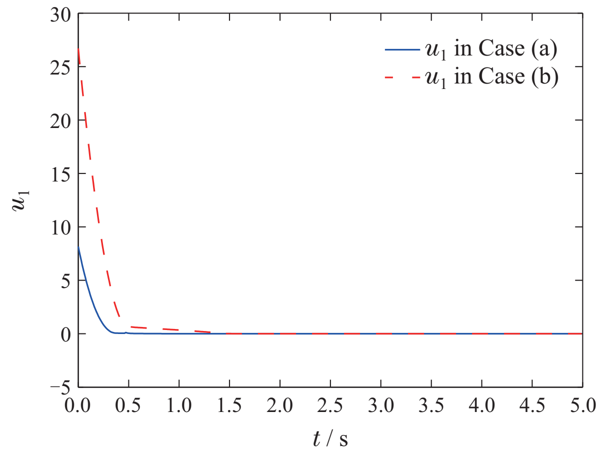

Fig. 5 System state u_1

5 Conclusions

By introducing the time-varying function into the virtual/actual controllers, a non-scaling design is developed for a kind of uncertain NSs with time-varying output constraints. The suitable switching mechanism makes the proposed control scheme achieving the prescribed-time stabilization, while solving the computationally singular problem effectively and leading to a simpler controller. Extension of this result with incomplete state information is one of our future research topics.

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