网络化不确定系统集中式融合鲁棒稳态估值器

陶贵丽[1], 李　爽[2], 刘文强[2†]

(1. 浙江传媒学院 媒体工程学院, 浙江 杭州 310018;

2. 浙江工商大学 信息与电子工程学院(萨塞克斯人工智能学院), 浙江 杭州 310018)

摘要: 对于一类在状态转移阵和系统观测阵中带相同的状态依赖乘性噪声、带噪声依赖乘性噪声、一步随机观测滞后、丢包和不确定噪声方差的多传感器网络化系统, 文章研究其鲁棒集中式融合稳态滤波问题. 应用增广方法将系统转换为带随机参数矩阵、相同过程和观测噪声的集中式融合系统. 应用去随机化方法和虚拟噪声技术, 系统进一步转化为仅带不确定噪声方差的集中式融合系统. 根据极大极小鲁棒估计原理, 本文提出了鲁棒集中式融合稳态Kalman估值器(预报器、滤波器和平滑器), 证明了所提出的集中式融合估值器的鲁棒性, 给出了鲁棒局部与集中式融合估值器之间的精度关系. 本文提出了应用于多传感器多通道滑动平均(MA)信号估计的一个实例, 给出了相应的鲁棒局部和集中式融合信号估值器. 仿真实验验证了所提出方法的有效性和正确性.

关键词: 集中式融合; 鲁棒稳态估值器; 一步随机滞后; 丢包; 不确定噪声方差; 极大极小鲁棒估计原理

引用格式: 陶贵丽, 李爽, 刘文强. 网络化不确定系统集中式融合鲁棒稳态估值器. 控制理论与应用, 2023, 40(8): 1466 – 1478

DOI: 10.7641/CTA.2022.20093

Centralized fusion robust steady-state estimators for networked uncertain systems

TAO Gui-li[1], LI Shuang[2], LIU Wen-qiang[2†]

(1. College of Media Engineering, Communication University of Zhejiang, Hangzhou Zhejiang 310018, China;
2. School of Information and Electronic Engineering (Sussex Artificial Intelligence Institute),
Zhejiang Gongshang University, Hangzhou Zhejiang 310018, China)

Abstract: The centralized fusion (CF) robust steady-state filtering problem is investigated for a class of multisensor networked systems with mixed uncertainties. The uncertainties include the same state-dependent multiplicative noises in state transition and system measurement matrices, noise-dependent multiplicative noises, one-step random delay, packet dropouts, and uncertain noise variances. By means of the augmentation approach, the system under study is converted into one with random parameter matrices and same process and measurement noises. Using the de-randomization approach and fictitious noise technique, the system is further converted into one with only uncertain noise variances. In the light of the minimax robust estimation principle, the robust CF steady-state Kalman estimators (predictor, filter, and smoother) are presented. The robustness of the proposed CF estimators is proved, the accuracy relations among the robust local and CF steady-state Kalman estimators are given. An example with application to multisensor multichannel moving average (MA) signal estimate is proposed, and the corresponding robust local and CF signal estimators are given. Simulation example verifies the effectiveness and correctness of the proposed method.

Key words: centralized fusion; robust steady-state estimators; one-step random delay; packet dropouts; uncertain noise variances; minimax robust estimation principle

Citation: TAO Guili, LI Shuang, LIU Wenqiang. Centralized fusion robust steady-state estimators for networked uncertain systems. *Control Theory & Applications*, 2023, 40(8): 1466 – 1478

收稿日期: 2022−02−02; 录用日期: 2022−08−02.

† 通信作者. E-mail: lwq@zjgsu.edu.cn; Tel.: +86 17367114907.

本文责任编委: 周杰.

黑龙江省自然科学基金项目(LH2019F035), 国家自然科学基金项目(61803148), 浙江省教育厅科研项目(Y202147323), 浙江传媒学院人才引进科研与创作项目(Z301B19539)资助.

Supported by the Natural Science Foundation of Heilongjiang Province (LH2019F035), the National Natural Science Foundation of China (61803148), the Scientific Research Project of Zhejiang Provincial Education Department (Y202147323) and the Scientific Research and Creative Project of Communication University of Zhejiang (Z301B19539).

1 引言

多传感器信息融合技术可以利用所有传感器的信息, 克服了单传感器受时间和空间限制的缺陷, 现已被广泛应用于目标跟踪、导航制导、信号处理等热门领域[1].

近年来, 网络化系统的滤波问题受到广泛关注[2–4]. 众所周知, 经典Kalman滤波方法要求系统的模型参数和噪声方差是精确已知的[5], 但在实际应用中, 由于建模误差或未建模动态以及随机扰动等原因, 导致系统模型是不确定的[6–17]. 当系统模型中存在不确定性时, Kalman滤波器的性能会严重变坏, 甚至导致滤波发散. 解决这一问题的方法之一是设计鲁棒Kalman滤波器, 即针对由不确定性所描述的一族系统模型来设计一个滤波器, 使得对所有容许的不确定性, 滤波器的实际滤波误差方差确保有一个最小上界[17].

在网络化系统中, 信道中会不可避免地存在不确定性, 例如随机观测滞后与丢包, 估值器的性能会受其影响[18]. 这类不确定性的存在会导致估值器接收到的数据出现偏差. 伯努利随机变量序列是描述这类不确定性的常用工具[19–21].

随机参数不确定性可以用乘性噪声来描述, 对确定性参数的随机扰动称为乘性噪声, 包括状态依赖和噪声依赖乘性噪声. 在系统状态和观测矩阵中的白噪声称为状态依赖乘性噪声, 在噪声转移矩阵中的白噪声称为噪声依赖乘性噪声. 噪声方差的不确定性可以通过确定的不确定性来描述, 即噪声方差是未知不确定的, 但有已知的保守上界[14–17]. 近年来, 对于带乘性噪声、不确定噪声方差、随机观测滞后和丢包的网络化系统, 鲁棒或最优状态估计问题已被广泛研究[15–17, 22–28]. 文献[22]中, 针对系统状态和观测矩阵中带乘性噪声并具有多步随机观测滞后和丢包的不确定网络化系统, 通过射影理论, 提出了最小方差意义下的最优线性估值器. 文献[23]中，针对带乘性噪声、一步随机观测滞后和丢包的网络化控制系统, 其中传感器到估值器和控制器到执行器的通道都受到乘性噪声影响, 利用射影理论推导出了最优估值器. 但文献[22–23]的结果都局限于单传感器系统.

对于在系统状态和观测矩阵中带乘性噪声, 并具有一步随机传输时滞和丢包的混合不确定多传感器网络化系统, 文献[24]基于矩阵加权融合算法提出了一种分布式融合滤波器. 对于一类在状态转移矩阵中存在乘性噪声, 并具有丢失观测、随机观测滞后和丢包的不确定多传感器网络化系统, 应用新息分析方法, 文献[25]提出了鲁棒集中式融合以及降维观测融合Kalman滤波器, 但文献[25]没有考虑系统观测矩阵中的乘性噪声. 文献[26]中, 针对带随机参数矩阵、一步随机观测滞后和丢包的多传感器网络化系统, 提出了局部最小二乘线性估值器(滤波器和固定点平滑器), 并利用矩阵加权融合算法得到了分布式融合滤波器和平滑器. 然而, 文献[22–26]均没有考虑噪声依赖乘性噪声, 且都假定系统噪声方差是精确已知的.

针对在系统状态和观测矩阵中带乘性噪声, 并带丢包和不确定噪声方差的多传感器网络化系统, 文献[27]提出了加权状态融合鲁棒Kalman估值器. 但文献[27]中没有考虑随机观测滞后不确定性. 对系统状态和观测矩阵中存在乘性噪声, 并具有一步随机观测滞后和不确定噪声方差的多传感器网络化系统, 文献[28]提出了综合协方差交叉融合鲁棒Kalman估值器. 文献[15]针对在系统状态与观测矩阵中存在相同乘性噪声, 以及带一步随机观测滞后、丢失观测、以及不确定噪声方差的多传感器网络化系统, 利用极大极小鲁棒估计准则, 提出了鲁棒矩阵加权和集中式融合稳态Kalman估值器. 文献[16]中, 针对带乘性噪声、两步随机观测滞后、丢失观测和不确定噪声方差的多传感器网络化系统, 提出了鲁棒集中式融合和加权观测融合稳态Kalman估值器. 文献[15–16]提出的鲁棒融合滤波方法可用于解决带有色观测噪声的多传感器单通道自回归信号的鲁棒融合滤波问题. 但文献[15–16, 27–28]中均没有考虑噪声依赖乘性噪声. 对于系统状态和过程噪声转移矩阵中带乘性噪声, 并具有不确定噪声方差、一步随机观测滞后和丢失观测的多传感器网络化系统, 文献[29]提出了鲁棒局部和5种融合时变Kalman估值器(预报器、滤波器和平滑器). 但文献 [29] 中没有考虑系统观测矩阵中的乘性噪声, 且文献[15–16, 28–29]都没有考虑丢包. 此外, 与文献[15–16]相比, 文献[28–29]中用到的增广方法会使系统状态转移矩阵具有较大维数. 对一类系统所有参数矩阵中带相同乘性噪声, 并带一步随机观测滞后和不连续丢包的多传感器网络化系统, 根据极大极小鲁棒估计原理, 文献[17]提出了鲁棒矩阵加权融合稳态Kalman估值器. 但文献[17]中没有考虑丢包补偿机制以及集中式融合滤波问题. 基于上述分析, 对于在系统状态和观测矩阵中带相同状态依赖乘性噪声, 并带噪声依赖乘性噪声、不确定噪声方差、一步随机观测滞后和丢包的多传感器网络化系统, 其鲁棒集中式融合稳态滤波问题尚未见报道. 因此, 本文对此进行研究.

本文的主要创新点如下: 1) 对于所研究的系统模型, 给出了其集中式融合鲁棒稳态估值器存在的充分条件, 并基于极大极小鲁棒估计原理提出了集中式融合鲁棒稳态Kalman估值器(预报器、滤波器和平滑器); 2) 应用增广方法、非负定矩阵分解方法和李雅普

诺夫方程方法, 证明了所提出的集中式融合估值器的鲁棒性; 3) 给出了一个应用于多传感器多通道滑动平均(moving average, MA)信号估计的例子, 解决了带随机参数矩阵的多通道MA信号的集中式融合估计问题, 得到了相应的鲁棒局部和集中式融合稳态信号估值器.

2 问题描述

考虑如下线性离散不确定多传感器网络化系统:

$$x(t+1)=(\varPhi+\sum_{k=1}^{n_\alpha}\alpha_k(t)\varPhi_k)x(t)+(\varGamma+\sum_{k=1}^{n_\beta}\beta_k(t)\varGamma_k)w(t),\tag{1}$$

$$z_i(t)=(H_i+\sum_{k=1}^{n_\alpha}\alpha_k(t)H_{ik})x(t)+v_i(t),\tag{2}$$

$$v_i(t)=D_iw(t)+\eta_i(t),\tag{3}$$

$$y_i(t)=\xi_i(t)z_i(t)+(1-\xi_i(t))\zeta_i(t)z_i(t-1)+(1-\xi_i(t))(1-\zeta_i(t))y_i(t-1),\quad i=1,\cdots,L,\tag{4}$$

其中: t是离散时间; $x(t)\in\mathbb{R}^n$是被估状态; $z_i(t)\in\mathbb{R}^{m_i}$是传感器接收到的观测; $y_i(t)\in\mathbb{R}^{m_i}$是估值器接收到的观测; $w(t)\in\mathbb{R}^r$是过程噪声; $v_i(t)\in\mathbb{R}^{m_i}$是观测噪声且线性相关于$w(t)$; $\alpha_k(t)\in\mathbb{R}^1, k=1,\cdots,n_\alpha$是状态依赖乘性噪声; $\beta_k(t)\in\mathbb{R}^1, k=1,\cdots,n_\beta$是噪声依赖乘性噪声; $\varPhi\in\mathbb{R}^{n\times n}$, $\varPhi_k\in\mathbb{R}^{n\times n}$, $\varGamma\in\mathbb{R}^{n\times r}$, $\varGamma_k\in\mathbb{R}^{n\times r}$, $H_i\in\mathbb{R}^{m_i\times n}$, $H_{ik}\in\mathbb{R}^{m_i\times n}$和$D_i\in\mathbb{R}^{m_i\times r}$是具有适当维数的已知常矩阵; $\varPhi_k,\varGamma_k$和H_{ik}是扰动方位矩阵; n_α, n_β是相应乘性噪声的数目; L是传感器的数目.

$\xi_i(t)\in\mathbb{R}^1$和$\zeta_i(t)\in\mathbb{R}^1, i=1,\cdots,L$是取值为0或1的各自不相关伯努利白噪声, 具有已知概率$\mathrm{Prob}(\xi_i(t)=1)=\pi_i$, $\mathrm{Prob}(\xi_i(t)=0)=1-\pi_i$, $\mathrm{Prob}(\zeta_i(t)=1)=\varsigma_i$, $\mathrm{Prob}(\zeta_i(t)=0)=1-\varsigma_i$, 其中$\pi_i$和$\varsigma_i$是已知的, 且$0\leqslant\pi_i\leqslant1, 0\leqslant\varsigma_i\leqslant1$. $\xi_i(t)$和$\zeta_i(t)$也不相关于其他随机信号. 容易得到以下结论:

$$\begin{cases}\mathrm{E}[\xi_i(t)]=\pi_i,\ \mathrm{E}[\xi_i^2(t)]=\pi_i,\\ \mathrm{E}[(\xi_i(t)-\pi_i)^2]=\pi_i(1-\pi_i),\\ \mathrm{E}[(\xi_i(t)-\pi_i)(\xi_j(t)-\pi_j)]=0,\ i\neq j,\\ \mathrm{E}[\zeta_i(t)]=\varsigma_i,\ \mathrm{E}[\zeta_i^2(t)]=\varsigma_i,\\ \mathrm{E}[(\zeta_i(t)-\varsigma_i)^2]=\varsigma_i(1-\varsigma_i),\\ \mathrm{E}[(\zeta_i(t)-\varsigma_i)(\zeta_j(t)-\varsigma_j)]=0,\ i\neq j.\end{cases}\tag{5}$$

假设 1 $w(t),\eta_i(t),\alpha_k(t)$和$\beta_k(t)$是具有零均值的互不相关白噪声, 它们的协方差为

$$\mathrm{E}[w(t)w^{\mathrm T}(u)]=\bar Q\delta_{tu},$$
$$\mathrm{E}[\eta_i(t)\eta_j^{\mathrm T}(u)]=\bar R_{\eta_i}\delta_{ij}\delta_{tu},$$
$$\mathrm{E}[\alpha_k(t)\alpha_h^{\mathrm T}(u)]=\bar\sigma_{\alpha_k}^2\delta_{kh}\delta_{tu},$$
$$\mathrm{E}[\beta_k(t)\beta_h^{\mathrm T}(u)]=\bar\sigma_{\beta_k}^2\delta_{kh}\delta_{tu},$$

其中: $\bar Q$, $\bar R_{\eta_i}$, $\bar\sigma_{\alpha_k}^2$和$\bar\sigma_{\beta_k}^2$分别是白噪声$w(t),\eta_i(t)$, $\alpha_k(t)$和$\beta_k(t)$的未知不确定实际(真实)方差, δ_{kj}表示Kronecker 函数, $\delta_{kk}=1, \delta_{kj}=0(k\neq j)$.

假设 2 初始状态$x(0)$不相关于$w(t),\eta_i(t)$, $\alpha_k(t),\beta_k(t),\xi_i(t)$和$\zeta_i(t)$, 且$\mathrm{E}[x(0)]=\mu_0$, 其中: μ_0是$x(0)$的均值, $\mathrm{E}[(x(0)-\mu_0)(x(0)-\mu_0)^{\mathrm T}]=\bar P_0$, $\bar P_0$是$x(0)$的未知不确定实际方差.

假设 3 $Q, R_{\eta_i}, \sigma_{\alpha_k}^2, \sigma_{\beta_k}^2$和$P_0$分别是$\bar Q, \bar R_{\eta_i}$, $\bar\sigma_{\alpha_k}^2, \bar\sigma_{\beta_k}^2$和$\bar P_0$的已知保守上界, 即满足

$$\bar Q\leqslant Q,\ \bar R_{\eta_i}\leqslant R_{\eta_i},\ \bar\sigma_{\alpha_k}^2\leqslant\sigma_{\alpha_k}^2,\ \bar\sigma_{\beta_k}^2\leqslant\sigma_{\beta_k}^2,\ \bar P_0\leqslant P_0.\tag{6}$$

注 1 带未知不确定实际方差$\bar Q, \bar R_{\eta_i}, \bar\sigma_{\alpha_k}^2, \bar\sigma_{\beta_k}^2$和$\bar P_0$的系统(1)–(4)称为实际系统, 带已知保守上界$Q, R_{\eta_i}, \sigma_{\alpha_k}^2$, $\sigma_{\beta_k}^2$和P_0的系统(1)–(4)称为最坏情形保守系统. “最坏情形”系统是指带最大噪声和初始状态方差的系统. 针对“最坏情形”系统的“最小”方差估值器称为极大极小鲁棒估值器, 这就是极大极小鲁棒估计原理.

问题是对不确定多传感器网络化系统(1)–(4)中的状态$x(t)$, 设计鲁棒集中式融合稳态Kalman估值器$\hat x_{\mathrm c}(t|t+N)$, 使得对于所有容许的不确定性, 它们的实际稳态估计误差方差$\bar P_{\mathrm c}(N)$有相应的最小上界$P_{\mathrm c}(N)$, 即$\bar P_{\mathrm c}(N)\leqslant P_{\mathrm c}(N)$, 下角标“c”表示集中式融合, $N=-1, N=0$和$N>0$分别表示一步预报器, 滤波器和平滑器.

注 2 在式(4)中, 白噪声$\xi_i(t)$和$\zeta_i(t)$被用来描述一步随机观测滞后和丢包. 如果$\xi_i(t)=1$, 则$y_i(t)=z_i(t)$(没有观测滞后和丢包); 如果$\xi_i(t)=0$且$\zeta_i(t)=1$, 则$y_i(t)=z_i(t-1)$(一步随机观测滞后); 如果$\xi_i(t)=0$且$\zeta_i(t)=0$, 则$y_i(t)=y_i(t-1)$(丢包), 即观测$z_i(t)$丢失, 但估值器在$t-1$时刻接收到的观测$y_i(t-1)$被用来作为t时刻的补偿, 这称为保持输入补偿机制. 这不同于文献[17]中的零输入补偿机制, 即如果当前时刻传感器观测丢失, 则估值器的观测将被置为0.

3 模型转换

3.1 增广集中式融合系统

合并式(2)给出的所有传感器输出向量, 得到如下增广观测输出方程:

$$z^{(\mathrm c)}(t)=(H^{(\mathrm c)}+\sum_{k=1}^{n_\alpha}\alpha_k(t)H_k^{(\mathrm c)})x(t)+v^{(\mathrm c)}(t),\tag{7}$$

其中: $z^{(\mathrm c)}(t)=[z_1^{\mathrm T}(t)\ \cdots\ z_L^{\mathrm T}(t)]^{\mathrm T}$, $H^{(\mathrm c)}=[H_1^{\mathrm T}\ \cdots\ H_L^{\mathrm T}]^{\mathrm T}$, $H_k^{(\mathrm c)}=[H_{1k}^{\mathrm T}\ \cdots\ H_{Lk}^{\mathrm T}]^{\mathrm T}$, $v^{(\mathrm c)}(t)=[v_1^{\mathrm T}(t)\ \cdots\ v_L^{\mathrm T}(t)]^{\mathrm T}$.

将式(2)代入式(4)中, 然后合并所有由式(4)给出

的估值器收到的观测向量可得

$$y^{(c)}(t)=\xi(t)(H^{(c)}+\sum_{k=1}^{n_\alpha}\alpha_k(t)H_k^{(c)})x(t)+\xi(t)\times v^{(c)}(t)+(I_m-\xi(t))\zeta(t)z^{(c)}(t-1)+(I_m-\xi(t))(I_m-\zeta(t))y^{(c)}(t-1),\tag{8}$$

其中: $y^{(c)}(t)=[y_1^{\rm T}(t)\ \cdots\ y_L^{\rm T}(t)]^{\rm T}, m=\sum_{i=1}^{L}m_i, \xi(t)={\rm diag}\{\xi_1(t)I_{m_1},\cdots,\xi_L(t)I_{m_L}\},\zeta(t)={\rm diag}\{\zeta_1(t)I_{m_1},\cdots,\zeta_L(t)I_{m_L}\},\ I_m={\rm diag}\{I_{m_1},\cdots,I_{m_L}\}$.

应用式(1)(7)–(8)可得如下增广集中式融合系统:

$$x_a(t+1)=\Phi_a(t)x_a(t)+\Gamma_a(t)w_a(t),\tag{9}$$

$$y^{(c)}(t)=H_a(t)x_a(t)+G_a(t)w_a(t),\tag{10}$$

其中:

$$\left\{\begin{aligned}
&x_a(t)=\begin{bmatrix}x(t)\\ z^{(c)}(t-1)\\ y^{(c)}(t-1)\end{bmatrix},\ w_a(t)=\begin{bmatrix}w(t)\\ v^{(c)}(t)\end{bmatrix},\\
&\Phi_a(t)=\begin{bmatrix}\Phi+\sum_{k=1}^{n_\alpha}\alpha_k(t)\Phi_k & (0)_{n\times m} & (0)_{n\times m}\\ H^{(c)}+\sum_{k=1}^{n_\alpha}\alpha_k(t)H_k^{(c)} & (0)_{m\times m} & (0)_{m\times m}\\ \xi(t)(H^{(c)}+\sum_{k=1}^{n_\alpha}\alpha_k(t)H_k^{(c)}) & (I_m-\xi(t))\zeta(t) & (I_m-\xi(t))(I_m-\zeta(t))\end{bmatrix},\\
&\Gamma_a(t)=\begin{bmatrix}\Gamma+\sum_{k=1}^{n_\beta}\beta_k(t)\Gamma_k & (0)_{n\times m}\\ (0)_{m\times r} & I_m\\ (0)_{m\times r} & \xi(t)\end{bmatrix},\\
&H_a(t)=[\xi(t)(H^{(c)}+\sum_{k=1}^{n_\alpha}\alpha_k(t)H_k^{(c)})\ \ (I_m-\xi(t))\zeta(t)\ \ (I_m-\xi(t))(I_m-\zeta(t))],\\
&G_a(t)=[(0)_{m\times r}\ \ \xi(t)].
\end{aligned}\right.\tag{11}$$

增广集中式融合系统(9)–(10)可进一步被转化为带常参数矩阵和乘性噪声的系统形式, 即

$$x_a(t+1)=(\Phi_a^m+\sum_{k=1}^{n_\alpha}\alpha_k(t)\Phi_a^{\alpha k}+\sum_{i=1}^{L}\xi_{iz}(t)\times\Phi_a^{\xi i}+\sum_{i=1}^{L}\zeta_{iz}(t)\Phi_a^{\zeta i}+\sum_{i=1}^{L}\gamma_{iz}(t)\Phi_a^{\gamma i}+\sum_{i=1}^{L}\xi_{iz}(t)\sum_{k=1}^{n_\alpha}\alpha_k(t)\Phi_a^{ki})x_a(t)+(\Gamma_a^m+\sum_{k=1}^{n_\beta}\beta_k(t)\Gamma_a^{\beta k}+\sum_{i=1}^{L}\xi_{iz}(t)\Gamma_a^{\xi i})w_a(t),\tag{12}$$

$$y^{(c)}(t)=(H_a^m+\sum_{k=1}^{n_\alpha}\alpha_k(t)H_a^{\alpha k}+\sum_{i=1}^{L}\xi_{iz}(t)H_a^{\xi i}+\sum_{i=1}^{L}\zeta_{iz}(t)H_a^{\zeta i}+\sum_{i=1}^{L}\gamma_{iz}(t)H_a^{\gamma i}+\sum_{i=1}^{L}\xi_{iz}(t)\times\sum_{k=1}^{n_\alpha}\alpha_k(t)H_a^{ki})x_a(t)+(G_a^m+\sum_{i=1}^{L}\xi_{iz}(t)G_a^{\xi i})w_a(t),\tag{13}$$

其中:

$$\Pi={\rm E}\,[\xi(t)]={\rm diag}\{\pi_1I_{m_1},\cdots,\pi_LI_{m_L}\},$$

$$\Xi={\rm E}[\zeta(t)]={\rm diag}\{\varsigma_1I_{m_1},\cdots,\varsigma_LI_{m_L}\},$$

$$\Phi_a^m=\begin{bmatrix}\Phi & (0)_{n\times m} & (0)_{n\times m}\\ H^{(c)} & (0)_{m\times m} & (0)_{m\times m}\\ \Pi H^{(c)} & (I_m-\Pi)\Xi & (I_m-\Pi)(I_m-\Xi)\end{bmatrix},$$

$$\Gamma_a^m=\begin{bmatrix}\Gamma & (0)_{n\times m}\\ (0)_{m\times r} & I_m\\ (0)_{m\times r} & \Pi\end{bmatrix},$$

$$H_a^m=[\Pi H^{(c)}\ \ (I_m-\Pi)\Xi\ \ (I_m-\Pi)(I_m-\Xi)],$$

$$G_a^m=[(0)_{m\times r}\ \ \Pi],$$

$$N_i={\rm diag}\{(0)_{m_1\times m_1},\cdots,(0)_{m_{i-1}\times m_{i-1}},I_{m_i},(0)_{m_{i+1}\times m_{i+1}},\ \cdots,\ (0)_{m_L\times m_L}\},$$

$$\Phi_a^{\alpha k}=\begin{bmatrix}\Phi_k & (0)_{n\times m} & (0)_{n\times m}\\ H_k^{(c)} & (0)_{m\times m} & (0)_{m\times m}\\ \Pi H_k^{(c)} & (0)_{m\times m} & (0)_{m\times m}\end{bmatrix},$$

$$\Phi_a^{\xi i}=\begin{bmatrix}(0)_{n\times n} & (0)_{n\times m} & (0)_{n\times m}\\ (0)_{m\times n} & (0)_{m\times m} & (0)_{m\times m}\\ N_iH^{(c)} & -N_i\Xi & -N_i+N_i\Xi\end{bmatrix},$$

$$\Phi_a^{\zeta i}=\begin{bmatrix}(0)_{n\times n} & (0)_{n\times m} & (0)_{n\times m}\\ (0)_{m\times n} & (0)_{m\times m} & (0)_{m\times m}\\ (0)_{m\times n} & N_i-\Pi N_i & -N_i+\Pi N_i\end{bmatrix},$$

$$\Phi_a^{\gamma i}=\begin{bmatrix}(0)_{n\times n} & (0)_{n\times m} & (0)_{n\times m}\\ (0)_{m\times n} & (0)_{m\times m} & (0)_{m\times m}\\ (0)_{m\times n} & -N_i & N_i\end{bmatrix},$$

$$\Phi_a^{ki}=\begin{bmatrix}(0)_{n\times n} & (0)_{n\times m} & (0)_{n\times m}\\ (0)_{m\times n} & (0)_{m\times m} & (0)_{m\times m}\\ N_iH_k^{(c)} & (0)_{m\times m} & (0)_{m\times m}\end{bmatrix},$$

$$\Gamma_a^{\beta k}=\begin{bmatrix}\Gamma_k & (0)_{n\times m}\\ (0)_{m\times r} & (0)_{m\times m}\\ (0)_{m\times r} & (0)_{m\times m}\end{bmatrix},$$

$$\Gamma_a^{\xi i}=\begin{bmatrix}(0)_{n\times r} & (0)_{n\times m}\\ (0)_{m\times r} & (0)_{m\times m}\\ (0)_{m\times r} & N_i\end{bmatrix},$$

$$H_a^{\alpha k}=[\Pi H_k^{(c)}\ \ (0)_{m\times m}\ \ (0)_{m\times m}],$$

$$H_a^{\xi i}=[N_iH^{(\mathrm{c})}\ \ -N_i\varXi\ \ -N_i+N_i\varXi],$$
$$H_a^{\zeta i}=[(0)_{m\times n}\ \ N_i-\varPi N_i\ \ -N_i+\varPi N_i],$$
$$H_a^{\gamma i}=[(0)_{m\times n}\ \ -N_i\ \ N_i],\ i=1,\cdots,L$$
$$H_a^{ki}=[N_iH_k^{(\mathrm{c})}\ \ (0)_{m\times m}\ \ (0)_{m\times m}],$$
$$G_a^{\xi i}=[(0)_{m\times r}\ \ N_i],\ \xi_{iz}(t)=\xi_i(t)-\pi_i,$$
$$\zeta_{iz}(t)=\zeta_i(t)-\varsigma_i,\ \gamma_{iz}(t)=\xi_{iz}(t)\zeta_{iz}(t).$$

此外, 利用式(5)可得

$$\begin{cases}\mathrm{E}[\xi_{iz}(t)]=0,\ \mathrm{E}[\xi_{iz}(t)\xi_{jz}^{\mathrm{T}}(t)]=0,\ i\neq j,\\ \sigma_{\xi_{iz}}^2=\mathrm{E}[\xi_{iz}(t)\xi_{iz}^{\mathrm{T}}(t)]=\pi_i(1-\pi_i),\\ \mathrm{E}[\zeta_{iz}(t)]=0,\ \mathrm{E}[\zeta_{iz}(t)\zeta_{jz}^{\mathrm{T}}(t)]=0,\ i\neq j,\\ \sigma_{\zeta_{iz}}^2=\mathrm{E}[\zeta_{iz}(t)\zeta_{iz}^{\mathrm{T}}(t)]\varsigma_i(1-\varsigma_i),\\ \mathrm{E}[\gamma_{iz}(t)]=0,\ \mathrm{E}[\gamma_{iz}(t)\gamma_{jz}^{\mathrm{T}}(t)]=0,\ i\neq j,\\ \sigma_{\gamma_{iz}}^2=\mathrm{E}[\gamma_{iz}(t)\gamma_{iz}^{\mathrm{T}}(t)]=\pi_i(1-\pi_i)\varsigma_i(1-\varsigma_i).\end{cases}\tag{14}$$

容易证得$\xi_{iz}(t)$, $\zeta_{iz}(t)$和$\gamma_{iz}(t)$是互不相关的白噪声.

注 3 相比于文献[28–29], 由式(11)给出的增广状态$x_a(t)$具有较小的维数, 如果采用文献[28–29]中的方法, 增广状态的维数为$2n+2m$, 而式(11)给出的增广状态$x_a(t)$的维数仅为$n+2m$. 可见, 本文方法可减少计算量.

引理 1[30] 令R_i是$m_i\times m_i$的半正定矩阵, 即$R_i\geqslant 0$, 则块对角矩阵$R_\delta=\mathrm{diag}\{R_1,\cdots,R_L\}\geqslant 0$.

式(7)中$v^{(\mathrm{c})}(t)$可表示为$v^{(\mathrm{c})}(t)=D^{(\mathrm{c})}w(t)+\eta^{(\mathrm{c})}(t)$, 且$D^{(\mathrm{c})}=[D_1^{\mathrm{T}}\ \cdots\ D_L^{\mathrm{T}}]^{\mathrm{T}}$, $\eta^{(\mathrm{c})}(t)=[\eta_1^{\mathrm{T}}(t)\ \cdots\ \eta_L^{\mathrm{T}}(t)]^{\mathrm{T}}$. $\eta^{(\mathrm{c})}(t)$的实际和保守方差分别为$\bar{R}_{\eta_{\mathrm{c}}}=\mathrm{diag}\{\bar{R}_{\eta_1},\cdots,\bar{R}_{\eta_L}\}$和$R_{\eta_{\mathrm{c}}}=\mathrm{diag}\{R_{\eta_1},\cdots,R_{\eta_L}\}$. 用$R_{\eta_{\mathrm{c}}}$减$\bar{R}_{\eta_{\mathrm{c}}}$, 并应用式(6)和引理1可得: $\bar{R}_{\eta_{\mathrm{c}}}\leqslant R_{\eta_{\mathrm{c}}}$. 由式(11)可得$w_a(t)=F_{\mathrm{c}}w(t)+G_{\mathrm{c}}\eta^{(\mathrm{c})}(t)$, 其中$F_{\mathrm{c}}=[I_r\ \ D^{(\mathrm{c})\mathrm{T}}]^{\mathrm{T}}$, $G_{\mathrm{c}}=[(0)_{m\times r}\ \ I_m]^{\mathrm{T}}$. 白噪声$w_a(t)$的实际和保守方差分别为

$$\begin{cases}\bar{Q}_a=F_{\mathrm{c}}\bar{Q}F_{\mathrm{c}}^{\mathrm{T}}+G_{\mathrm{c}}\bar{R}_{\eta_{\mathrm{c}}}G_{\mathrm{c}}^{\mathrm{T}},\\ Q_a=F_{\mathrm{c}}QF_{\mathrm{c}}^{\mathrm{T}}+G_{\mathrm{c}}R_{\eta_{\mathrm{c}}}G_{\mathrm{c}}^{\mathrm{T}}.\end{cases}\tag{15}$$

用Q_a减$\bar{Q}_a$, 并应用$\bar{Q}\leqslant Q$和$\bar{R}_{\eta_{\mathrm{c}}}\leqslant R_{\eta_{\mathrm{c}}}$可得

$$\bar{Q}_a\leqslant Q_a.\tag{16}$$

3.2 实际和保守状态二次非中心距

由式(12)可得$x_a(t)$的保守二次非中心距为

$$\begin{aligned}X_a(t+1)=&\varPhi_a^mX_a(t)\varPhi_a^{m\mathrm{T}}+\sum_{k=1}^{n_\alpha}\sigma_{\alpha_k}^2\varPhi_a^{\alpha k}X_a(t)\varPhi_a^{\alpha k\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\varPhi_a^{\xi i}X_a(t)\varPhi_a^{\xi i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\zeta_{iz}}^2\varPhi_a^{\zeta i}X_a(t)\varPhi_a^{\zeta i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\gamma_{iz}}^2\varPhi_a^{\gamma i}X_a(t)\varPhi_a^{\gamma i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\sum_{k=1}^{n_\alpha}\sigma_{\alpha_k}^2\varPhi_a^{ki}\times X_a(t)\varPhi_a^{ki\mathrm{T}}+\\&\varGamma_a^mQ_a\varGamma_a^{m\mathrm{T}}+\sum_{k=1}^{n_\beta}\sigma_{\beta_k}^2\varGamma_a^{\beta k}Q_a\varGamma_a^{\beta k\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\varGamma_a^{\xi i}Q_a\varGamma_a^{\xi i\mathrm{T}}.\end{aligned}\tag{17}$$

带初值$X_a(0)=\mathrm{diag}\{X(0),(0)_{m\times m},(0)_{m\times m}\}$, $X(0)=P_0+\mu_0\mu_0^{\mathrm{T}}$. 在式(17)中, 分别用$\bar{X}_a(t)$, $\bar{\sigma}_{\alpha_k}^2$, $\bar{\sigma}_{\beta_k}^2$和$\bar{Q}_a$代替$X_a(t)$, $\sigma_{\alpha_k}^2$, $\sigma_{\beta_k}^2$和Q_a, 可得增广状态$x_a(t)$的实际二次非中心距$\bar{X}_a(t+1)$, 带初值$\bar{X}_a(0)=\mathrm{diag}\{\bar{X}(0),(0)_{m\times m},(0)_{m\times m}\}$, $\bar{X}(0)=\bar{P}_0+\mu_0\mu_0^{\mathrm{T}}$.

引理 2 在假设3条件下, 可得

$$\bar{X}_a(t)\leqslant X_a(t),\ t\geqslant 0.\tag{18}$$

证 完全类似于文献[15]中引理5的证明, 容易证得引理2成立. 证毕.

引理 3 对于多传感器网络化系统(1)–(4), 在假设1–3条件下, 如果$\rho(\bar{\varPhi}_{\mathrm{c}})<1$, $\bar{\varPhi}_{\mathrm{c}}=\varPhi_a^m\otimes\varPhi_a^{m\mathrm{T}}+\sum_{k=1}^{n_\alpha}\bar{\sigma}_{\alpha_k}^2\varPhi_a^{\alpha k}\otimes\varPhi_a^{\alpha k}+\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\varPhi_a^{\xi i}\otimes\varPhi_a^{\xi i}+\sum_{i=1}^{L}\sigma_{\zeta_{iz}}^2\varPhi_a^{\zeta i}\otimes\varPhi_a^{\zeta i}+\sum_{i=1}^{L}\sigma_{\gamma_{iz}}^2\varPhi_a^{\gamma i}\otimes\varPhi_a^{\gamma i}+\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\sum_{k=1}^{n_\alpha}\bar{\sigma}_{\alpha_k}^2\varPhi_a^{ki}\otimes\varPhi_a^{ki}$, 则有如下收敛性:

$$\lim_{t\to\infty}X_a(t)=X_a,$$
$$\begin{aligned}X_a=&\varPhi_a^mX_a\varPhi_a^{m\mathrm{T}}+\sum_{k=1}^{n_\alpha}\sigma_{\alpha_k}^2\varPhi_a^{\alpha k}X_a\varPhi_a^{\alpha k\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\varPhi_a^{\xi i}X_a\varPhi_a^{\xi i\mathrm{T}}+\sum_{i=1}^{L}\sigma_{\zeta_{iz}}^2\varPhi_a^{\zeta i}X_a\varPhi_a^{\zeta i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\gamma_{iz}}^2\varPhi_a^{\gamma i}X_a\varPhi_a^{\gamma i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\sum_{k=1}^{n_\alpha}\sigma_{\alpha_k}^2\varPhi_a^{ki}X_a\varPhi_a^{ki\mathrm{T}}+\varGamma_a^mQ_a\varGamma_a^{m\mathrm{T}}+\\&\sum_{k=1}^{n_\beta}\sigma_{\beta_k}^2\varGamma_a^{\beta k}Q_a\varGamma_a^{\beta k\mathrm{T}}+\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\varGamma_a^{\xi i}Q_a\varGamma_a^{\xi i\mathrm{T}}.\end{aligned}$$

证 如果$\rho(\bar{\varPhi}_{\mathrm{c}})<1$, 则类似于文献[24,31]中的证明过程, 通过直接应用文献[32–33]中的结果可证明引理3成立. 证毕.

在X_a的表达式中, 分别用$\bar{X}_a$, $\bar{\sigma}_{\alpha_k}^2$, $\bar{\sigma}_{\beta_k}^2$和$\bar{Q}_a$代替X_a, $\sigma_{\alpha_k}^2$, $\sigma_{\beta_k}^2$和Q_a, 可得实际稳态二次非中心距$\bar{X}_a$, 且有收敛性: $\lim_{t\to\infty}\bar{X}_a(t)=\bar{X}_a$. 应用引理3, 当$t\to\infty$时, 对式(18)取极限可得如下稳态矩阵不等式关系:

$$\bar{X}_a\leqslant X_a.\tag{19}$$

3.3　虚拟过程和观测噪声

增广状态方程(12)可被改写为

$$x_a(t+1)=\varPhi_a^m x_a(t)+w_f(t),\tag{20}$$

其中$w_f(t)$是虚拟过程噪声,

$$\begin{aligned}w_f(t)=&(\sum_{k=1}^{n_\alpha}\alpha_k(t)\varPhi_a^{\alpha k}+\sum_{i=1}^{L}\xi_{iz}(t)\varPhi_a^{\xi i}+\\&\sum_{i=1}^{L}\zeta_{iz}(t)\varPhi_a^{\zeta i}+\sum_{i=1}^{L}\gamma_{iz}(t)\varPhi_a^{\gamma i}+\\&\sum_{i=1}^{L}\xi_{iz}(t)\sum_{k=1}^{n_\alpha}\alpha_k(t)\varPhi_a^{ki})x_a(t)+\\&(\varGamma_a^m+\sum_{k=1}^{n_\beta}\beta_k(t)\varGamma_a^{\beta k}+\sum_{i=1}^{L}\xi_{iz}(t)\varGamma_a^{\xi i})w_a(t).\end{aligned}$$

容易证得$w_f(t)$是零均值白噪声, 且它的保守稳态方差为

$$\begin{aligned}Q_f=&\sum_{k1}^{n_\alpha}\sigma_{\alpha_k}^2\varPhi_a^{\alpha k}X_a\varPhi_a^{\alpha k\mathrm{T}}+\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\varPhi_a^{\xi i}X_a\varPhi_a^{\xi i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\zeta_{iz}}^2\varPhi_a^{\zeta i}X_a\varPhi_a^{\zeta i\mathrm{T}}+\sum_{i=1}^{L}\sigma_{\gamma_{iz}}^2\varPhi_a^{\gamma i}X_a\varPhi_a^{\gamma i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\sum_{k=1}^{n_\alpha}\sigma_{\alpha_k}^2\varPhi_a^{ki}X_a\varPhi_a^{ki\mathrm{T}}+\varGamma_a^mQ_a\varGamma_a^{m\mathrm{T}}+\\&\sum_{k=1}^{n_\beta}\sigma_{\beta_k}^2\varGamma_a^{\beta k}Q_a\varGamma_a^{\beta k\mathrm{T}}+\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\varGamma_a^{\xi i}Q_a\varGamma_a^{\xi i\mathrm{T}}.\end{aligned}\tag{21}$$

式(21)中分别用$\bar{X}_a$, $\bar{\sigma}_{\alpha_k}^2$, $\bar{\sigma}_{\beta_k}^2$和$\bar{Q}_a$代替X_a, $\sigma_{\alpha_k}^2$, $\sigma_{\beta_k}^2$和Q_a, 可得$w_f(t)$的实际稳态方差$\bar{Q}_f$. 令$\sigma_{\alpha_k}^2=\bar{\sigma}_{\alpha_k}^2+\Delta\sigma_{\alpha_k}^2$和$\sigma_{\beta_k}^2=\bar{\sigma}_{\beta_k}^2+\Delta\sigma_{\beta_k}^2$, 用$Q_f$减$\bar{Q}_f$, 并应用式(16)(19), 容易证得如下矩阵不等式:

$$\bar{Q}_f\leqslant Q_f.\tag{22}$$

增广集中式融合观测方程(13)可被改写为

$$y^{(\mathrm{c})}(t)=H_a^m x_a(t)+v_f(t),\tag{23}$$

其中$v_f(t)$是零均值虚拟观测白噪声, 且$v_f(t)$可表示为

$$\begin{aligned}v_f(t)=&(\sum_{k=1}^{n_\alpha}\alpha_k(t)H_a^{\alpha k}+\sum_{i=1}^{L}\xi_{iz}(t)H_a^{\xi i}+\\&\sum_{i=1}^{L}\zeta_{iz}(t)H_a^{\zeta i}+\sum_{i=1}^{L}\gamma_{iz}(t)H_a^{\gamma i}+\\&\sum_{i=1}^{L}\xi_{iz}(t)\sum_{k=1}^{n_\alpha}\alpha_k(t)H_a^{ki})x_a(t)+\\&(G_a^m+\sum_{i=1}^{L}\xi_{iz}(t)G_a^{\xi i})w_a(t).\end{aligned}$$

它的保守稳态方差为

$$\begin{aligned}R_f=&\sum_{k=1}^{n_\alpha}\sigma_{\alpha_k}^2H_a^{\alpha k}X_aH_a^{\alpha k\mathrm{T}}+\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2H_a^{\xi i}X_aH_a^{\xi i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\zeta_{iz}}^2H_a^{\zeta i}X_aH_a^{\zeta i\mathrm{T}}+\sum_{i=1}^{L}\sigma_{\gamma_{iz}}^2H_a^{\gamma i}X_aH_a^{\gamma i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\sum_{k=1}^{n_\alpha}\sigma_{\alpha_k}^2H_a^{ki}X_aH_a^{ki\mathrm{T}}+G_a^mQ_aG_a^{m\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2G_a^{\xi i}Q_aG_a^{\xi i\mathrm{T}}.\end{aligned}\tag{24}$$

在式(24)中, 分别用$\bar{X}_a$, $\bar{\sigma}_{\alpha_k}^2$和$\bar{Q}_a$代替$X_a$, $\sigma_{\alpha_k}^2$和Q_a, 可得$v_f(t)$的实际稳态方差$\bar{R}_f$. 用R_f减$\bar{R}_f$, 并应用式(16)(19), 容易证得

$$\bar{R}_f\leqslant R_f.\tag{25}$$

虚拟噪声$w_f(t)$和$v_f(t)$的保守稳态相关矩阵为

$$\begin{aligned}S_f=&\sum_{k=1}^{n_\alpha}\sigma_{\alpha_k}^2\varPhi_a^{\alpha k}X_aH_a^{\alpha k\mathrm{T}}+\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\varPhi_a^{\xi i}X_aH_a^{\xi i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\zeta_{iz}}^2\varPhi_a^{\zeta i}X_aH_a^{\zeta i\mathrm{T}}+\sum_{i=1}^{L}\sigma_{\gamma_{iz}}^2\varPhi_a^{\gamma i}X_aH_a^{\gamma i\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\sum_{k=1}^{n_\alpha}\sigma_{\alpha_k}^2\varPhi_a^{ki}X_aH_a^{ki\mathrm{T}}+\varGamma_a^mQ_aG_a^{m\mathrm{T}}+\\&\sum_{i=1}^{L}\sigma_{\xi_{iz}}^2\varGamma_a^{\xi i}Q_aG_a^{\xi i\mathrm{T}},\end{aligned}\tag{26}$$

式中分别用$\bar{X}_a$, $\bar{\sigma}_{\alpha_k}^2$和$\bar{Q}_a$代替$X_a$, $\sigma_{\alpha_k}^2$和Q_a, 可得$w_f(t)$和$v_f(t)$的实际稳态相关矩阵$\bar{S}_f$.

假设 4　假设$(\varPhi_a^m, H_a^m)$是完全能检对, $(\bar{\varPhi}, \varUpsilon)$是完全能稳对, 其中: $\bar{\varPhi}=\varPhi_a^m-SH_a^m$, $S=S_fR_f^{-1}$, $\varUpsilon\varUpsilon^{\mathrm{T}}=Q_f-S_fR_f^{-1}S_f^{\mathrm{T}}$.

4　鲁棒集中式融合稳态Kalman估值器

4.1　鲁棒集中式融合稳态Kalman预报器

对于由式(20)(23)给出的带已知保守噪声统计Q_f, R_f和S_f的最坏情形时不变集中式融合系统, 在假设1–4条件下, 基于极大极小鲁棒估计原理[30], 应用标准Kalman滤波算法[5], 可得保守最优集中式融合稳态一步Kalman预报器为

$$\hat{x}_a^{(\mathrm{c})}(t+1|t)=\varPsi^{(\mathrm{c})}\hat{x}_a^{(\mathrm{c})}(t|t-1)+K^{(\mathrm{c})}y^{(\mathrm{c})}(t),\tag{27}$$

$$\varPsi^{(\mathrm{c})}=\varPhi_a^m-K^{(\mathrm{c})}H_a^m,\tag{28}$$

$$\begin{aligned}K^{(\mathrm{c})}=&(\varPhi_a^mP_a^{(\mathrm{c})}(-1)H_a^{m\mathrm{T}}+S_f)\times\\&(H_a^mP_a^{(\mathrm{c})}(-1)H_a^{m\mathrm{T}}+R_f)^{-1}.\end{aligned}\tag{29}$$

带初值$\hat{x}_a^{(\mathrm{c})}(0|-1)=[\mu_0^{\mathrm{T}}\quad((0)_{m\times1})^{\mathrm{T}}\quad((0)_{m\times1})^{\mathrm{T}}]^{\mathrm{T}}$, 且$\varPsi^{(\mathrm{c})}$是稳定矩阵.

保守融合稳态一步预报误差方差$P_a^{(\mathrm{c})}(-1)$满足如下稳态Riccati方程:

$$\begin{aligned}P_a^{(\mathrm{c})}(-1)=&\varPhi_a^mP_a^{(\mathrm{c})}(-1)\varPhi_a^{m\mathrm{T}}-(\varPhi_a^mP_a^{(\mathrm{c})}(-1)H_a^{m\mathrm{T}}+\\&S_f)(H_a^mP_a^{(\mathrm{c})}(-1)H_a^{m\mathrm{T}}+R_f)^{-1}\times\\&(\varPhi_a^mP_a^{(\mathrm{c})}(-1)H_a^{m\mathrm{T}}+S_f)^{\mathrm{T}}+Q_f.\end{aligned}\tag{30}$$

注 4　由带保守上界Q, R_{η_i}, $\sigma_{\alpha_k}^2$, $\sigma_{\beta_k}^2$和P_0的最坏情形系统(1)–(4)所产生的局部观测$y_i(t)$称为保守局部观测, 它

是不可利用的(未知的). 于是, 由保守局部观测$y_i(t)$组成的式(8)中给出的保守集中式融合观测$y^{(c)}(t)$也是不可利用的. 由带实际方差$\bar{Q}, \bar{R}_{\eta_i}, \bar{\sigma}^2_{\alpha_k}, \bar{\sigma}^2_{\beta_k}$和$\bar{P}_0$的实际系统(1)–(4)所产生的局部观测$y_i(t)$称为实际局部观测, 它是通过传感器得到的, 是可利用的(已知的). 进而, 由实际局部观测$y_i(t)$组成的式(8)中给出的实际集中式融合观测$y^{(c)}(t)$是可利用的. 在式(27)中, 用实际融合观测$y^{(c)}(t)$代替保守融合观测$y^{(c)}(t)$, 可得实际集中式融合Kalman预报器.

定义集中式融合稳态预报误差为$\tilde{x}^{(c)}_a(t+1|t) = x_a(t+1) - \hat{x}^{(c)}_a(t+1|t)$, 由式(20)减式(27)可得

$$\begin{aligned}\tilde{x}^{(c)}_a(t+1|t) = {} & \varPsi^{(c)}\tilde{x}^{(c)}_a(t|t-1) + w_f(t) - K^{(c)}v_f(t) = \\ & \varPsi^{(c)}\tilde{x}^{(c)}_a(t|t-1) + [I_{n+2m} - K^{(c)}] \times \\ & \lambda_f(t),\end{aligned} \tag{31}$$

其中: 符号I_{n+2m}表示维数为$(n+2m)\times(n+2m)$的单位矩阵, 增广噪声$\lambda_f(t) = [w^{\mathrm{T}}_f(t) \ \ v^{\mathrm{T}}_f(t)]^{\mathrm{T}}$. 容易得到$\lambda_f(t)$的实际和保守稳态方差分别为

$$\bar{\varLambda}_f = \begin{bmatrix}\bar{Q}_f & \bar{S}_f \\ \bar{S}^{\mathrm{T}}_f & \bar{R}_f\end{bmatrix},\ \varLambda_f = \begin{bmatrix}Q_f & S_f \\ S^{\mathrm{T}}_f & R_f\end{bmatrix}. \tag{32}$$

应用式(31)可得保守集中式融合稳态Kalman一步预报误差方差, 也满足如下李雅普诺夫方程:

$$\begin{aligned}P^{(c)}_a(-1) = {} & \varPsi^{(c)}P^{(c)}_a(-1)\varPsi^{(c)\mathrm{T}} + [I_{n+2m} - K^{(c)}] \times \\ & \varLambda_f[I_{n+2m} - K^{(c)}]^{\mathrm{T}},\end{aligned} \tag{33}$$

式中分别用$\bar{P}^{(c)}_a(-1)$和$\bar{\varLambda}_f$代替$P^{(c)}_a(-1)$和$\varLambda_f$, 可得实际融合稳态一步预报误差方差$\bar{P}^{(c)}_a(-1)$的稳态李雅普诺夫方程.

引理 4[30] 令$\varLambda \geqslant 0$, $\varLambda \in \mathbb{R}^{r\times r}$, 设$\varLambda_\delta = (\varLambda_{ij})_{rL\times rL}$, $\varLambda_{ij} = \varLambda, i,j = 1,\cdots,L$, 则$\varLambda_\delta \geqslant 0$.

引理 5 在假设3条件下, 可得如下矩阵不等式关系: $\bar{\varLambda}_f \leqslant \varLambda_f$.

证明过程详见附录A.

引理 6[34] 考虑如下李雅普诺夫方程: $U = CUC^{\mathrm{T}} + V$, 其中: U, C和V是$n\times n$矩阵, V是对称矩阵, C是稳定矩阵(即它的所有特征值都在单位圆内). 如果$V \geqslant 0$, 则U是对称并唯一的, 且$U \geqslant 0$.

定理 1 在假设1–4条件下, 由式(27)给出的实际集中式融合稳态Kalman预报器具有鲁棒性, 即对于所有容许的不确定性, 实际预报误差方差$\bar{P}^{(c)}_a(-1)$, 满足如下关系:

$$\bar{P}^{(c)}_a(-1) \leqslant P^{(c)}_a(-1), \tag{34}$$

且$P^{(c)}_a(-1)$是$\bar{P}^{(c)}_a(-1)$的最小上界.

证明过程详见附录B.

由式(27)给出的实际融合稳态Kalman预报器为鲁棒融合稳态Kalman预报器, 由式(34)给出的矩阵不等式关系称为它的鲁棒性.

4.2 鲁棒集中式融合稳态Kalman滤波器和平滑器

基于实际集中式融合稳态Kalman一步预报器$\hat{x}^{(c)}_a(t|t-1)$, 可得实际集中式融合稳态Kalman滤波器($N=0$)和平滑器($N>0$)$\hat{x}^{(c)}_a(t|t+N)$为[35]

$$\begin{aligned}\hat{x}^{(c)}_a(t|t+N) = {} & \hat{x}^{(c)}_a(t|t-1) + \sum_{k=0}^{N} K^{(c)}(k) \times \\ & \varepsilon^{(c)}(t+k),\ N \geqslant 0,\end{aligned} \tag{35}$$

$$\begin{aligned}K^{(c)}(k) = {} & P^{(c)}_a(-1)\varPsi^{(c)\mathrm{T}k}H^{m\mathrm{T}}_a[H^m_aP^{(c)}_a(-1) \times \\ & H^{m\mathrm{T}}_a + R_f]^{-1},\ k \geqslant 0,\end{aligned} \tag{36}$$

$$\varepsilon^{(c)}(t) = y^{(c)}(t) - H^m_a\hat{x}^{(c)}_a(t|t-1). \tag{37}$$

类似于文献[35]中的推导, 可得集中式融合稳态滤波和平滑误差$\tilde{x}^{(c)}_a(t|t+N) = x_a(t) - \hat{x}^{(c)}_a(t|t+N)$为

$$\begin{aligned}\tilde{x}^{(c)}_a(t|t+N) = {} & \varPsi^{(c)}_N\tilde{x}^{(c)}_a(t|t-1) + \\ & \sum_{\rho=0}^{N}[K^{\mathrm{cw}}_{N\rho}, K^{\mathrm{cv}}_{N\rho}]\lambda_f(t+\rho),\end{aligned} \tag{38}$$

其中: $\varPsi^{(c)}_N = I_{(n+2m)\times(n+2m)} - \sum_{k=0}^{N} K^{(c)}(k)H^m_a\varPsi^{(c)\mathrm{k}}$; $K^{\mathrm{cw}}_{N\rho} = -\sum_{k=\rho+1}^{N} K^{(c)}(k)H^m_a\varPsi^{(c)\mathrm{k}-\rho-1}$, $N>0$, $\rho = 0,\cdots,N-1$; $K^{\mathrm{cw}}_{NN} = 0$, $N \geqslant 0$, $\rho = N$; $K^{\mathrm{cv}}_{N\rho} = \sum_{k=\rho+1}^{N} K^{(c)}(k)H^m_a\varPsi^{(c)\mathrm{k}-\rho-1}K^{(c)} - K^{(c)}(\rho)$, $N>0$, $\rho = 0,\cdots,N-1$; $K^{\mathrm{cv}}_{NN} = -K^{(c)}(N)$, $N \geqslant 0$, $\rho = N$.

利用式(38)可得保守稳态估计误差方差为

$$\begin{aligned}P^{(c)}_a(N) = {} & \varPsi^{(c)}_NP^{(c)}_a(-1)\varPsi^{(c)\mathrm{T}}_N + \sum_{\rho=0}^{N}\left[K^{\mathrm{cw}}_{N\rho}\ \ K^{\mathrm{cv}}_{N\rho}\right] \times \\ & \varLambda_f\left[K^{\mathrm{cw}}_{N\rho}\ \ K^{\mathrm{cv}}_{N\rho}\right]^{\mathrm{T}},\ N \geqslant 0,\end{aligned} \tag{39}$$

式中分别用$\bar{P}^{(c)}_a(N)$, $\bar{P}^{(c)}_a(-1)$和$\bar{\varLambda}_f$代替$P^{(c)}_a(N)$, $P^{(c)}_a(-1)$和$\varLambda_f$, 可得实际估计误差方差$\bar{P}^{(c)}_a(N)$.

定理 2 在假设1–4条件下, 由式(35)给出的实际集中式融合稳态Kalman滤波器和平滑器具有鲁棒性, 即对于所有容许的不确定性, 相应的所有实际估计误差方差$\bar{P}^{(c)}_a(N)$满足如下关系:

$$\bar{P}^{(c)}_a(N) \leqslant P^{(c)}_a(N),\ N \geqslant 0, \tag{40}$$

且$P^{(N)}_a(-1)$是$\bar{P}^{(N)}_a(-1)$的最小上界.

证明过程详见附录C.

由式(35)给出的实际集中式融合稳态Kalman滤波器和平滑器被称为鲁棒集中式融合稳态Kalman滤波器和平滑器, 由式(40)给出的矩阵不等式关系称为它

们的鲁棒性.

推论 1 由$x_a(t)=[x^{\mathrm{T}}(t)\ z^{(\mathrm{c})\mathrm{T}}(t-1)\ y^{(\mathrm{c})\mathrm{T}}(t-1)]^{\mathrm{T}}$, 可得原始系统(1)–(4)的鲁棒集中式融合稳态Kalman估值器$\hat{x}^{(\mathrm{c})}(t|t+N)=[I_n\ \ (0)_{n\times m}\ \ (0)_{n\times m}]\hat{x}_a^{(\mathrm{c})}(t|t+N),\ N=-1,\ N\geqslant 0$, 它们的实际和保守融合误差方差分别为

$$\bar{P}^{(\mathrm{c})}(N)=[I_n\ \ (0)_{n\times m}\ \ (0)_{n\times m}]\bar{P}_a^{(\mathrm{c})}(N)\times[I_n\ \ (0)_{n\times m}\ \ (0)_{n\times m}]^{\mathrm{T}},\tag{41}$$

$$P^{(\mathrm{c})}(N)=[I_n\ \ (0)_{n\times m}\ \ (0)_{n\times m}]P_a^{(\mathrm{c})}(N)\times[I_n\ \ (0)_{n\times m}\ \ (0)_{n\times m}]^{\mathrm{T}}.\tag{42}$$

对所有容许的不确定性, 集中式融合稳态Kalman估值器$\hat{x}^{(\mathrm{c})}(t|t+N)$具有鲁棒性, 即

$$\bar{P}^{(\mathrm{c})}(N)\leqslant P^{(\mathrm{c})}(N),\ N=-1,\ N\geqslant 0,\tag{43}$$

且$P^{(\mathrm{c})}(N)$是$\bar{P}^{(\mathrm{c})}(N)$的最小上界.

推论 2 完全类似于式(9)–(43)的推导, 容易得到原始系统(1)–(4)的鲁棒局部稳态Kalman估值器$\hat{x}_i(t|t+N),\ N=-1,\ N\geqslant 0,\ i=1,\cdots,L$, 且它们的实际估计误差方差$\bar{P}_i(N)$有相应的最小上界$P_i(N)$, 即

$$\bar{P}_i(N)\leqslant P_i(N),\ N=-1,\ N\geqslant 0.\tag{44}$$

注 5 利用射影理论可以证得

$$P^{(\mathrm{c})}(N)\leqslant P_i(N),\ N=-1,\ N\geqslant 0,\ i=1,\cdots,L,\tag{45}$$

$$P^{(\mathrm{c})}(N)<P^{(\mathrm{c})}(N-1)<\cdots<P^{(\mathrm{c})}(1)<P^{(\mathrm{c})}(0)<P^{(\mathrm{c})}(-1),\ N\geqslant 1.\tag{46}$$

对式(43)–(46)中的矩阵求迹, 得到如下矩阵迹不等式关系:

$$\begin{cases}\mathrm{tr}\bar{P}^{(\mathrm{c})}(N)\leqslant\mathrm{tr}P^{(\mathrm{c})}(N),\ \mathrm{tr}\bar{P}_i(N)\leqslant\mathrm{tr}P_i(N),\\ \mathrm{tr}P^{(\mathrm{c})}(N)\leqslant\mathrm{tr}P_i(N),\ N=-1,\ N\geqslant 0,\ i=1,\cdots,L,\end{cases}\tag{47}$$

$$\mathrm{tr}P^{(\mathrm{c})}(N)<\mathrm{tr}P^{(\mathrm{c})}(N-1)<\cdots<\mathrm{tr}P^{(\mathrm{c})}(1)<\mathrm{tr}P^{(\mathrm{c})}(0)<\mathrm{tr}P^{(\mathrm{c})}(-1),\ N\geqslant 1.\tag{48}$$

注 6 在注5中, $\mathrm{tr}\bar{P}^{(\mathrm{c})}(N)$和$\mathrm{tr}\bar{P}_i(N)$被称为相应鲁棒Kalman估值器的实际精度, $\mathrm{tr}P^{(\mathrm{c})}(N)$和$\mathrm{tr}P_i(N)$被称为鲁棒精度(或者全局精度). 迹的值越小意味着精度越高. 注5表明, 估值器的实际精度都高于或等于它的鲁棒精度, 鲁棒精度是最低的实际精度. 集中式融合器的鲁棒精度高于各局部估值器.

5 带有色观测噪声和混合不确定性的多通道MA信号估计应用实例

自回归滑动平均(autoregressive MA, ARMA)信号滤波问题经常发生在信号处理、状态估计、目标跟踪、反卷积以及时间序列分析等领域. 考虑如下带有色观测噪声和混合不确定性的多传感器多通道MA信号:

$$s(t)=B_t(q^{-1})u(t),\tag{49}$$

$$z_i(t)=s(t)+r(t)+\eta_i(t),\ i=1,\cdots,L,\tag{50}$$

$$A_t(q^{-1})r(t)=e(t),\tag{51}$$

$$y_i(t)=\xi_i(t)z_i(t)+(1-\xi_i(t))\zeta_i(t)z_i(t-1)+(1-\xi_i(t))(1-\zeta_i(t))y_i(t-1),\tag{52}$$

其中:

$$\begin{cases}A_t(q^{-1})=I_m+A_1(t-1)q^{-1}+\cdots+A_p(t-p)q^{-p},\\ A_k(t)=(a_k^{(\mathrm{gh})}(t))_{m\times m},\ a_k^{(\mathrm{gh})}(t)=a_k^{(\mathrm{gh})}+\alpha_k^{(\mathrm{gh})}(t),\\ \quad g,h=1,\cdots,m,\ 1\leqslant k\leqslant p,\end{cases}\tag{53}$$

$$\begin{cases}B_t(q^{-1})=B_1(t-1)q^{-1}+\cdots+B_p(t-p)q^{-p},\\ B_k(t)=(b_k^{(\mathrm{gh})}(t))_{m\times m},\ b_k^{(\mathrm{gh})}(t)=b_k^{(\mathrm{gh})}+\beta_k^{(\mathrm{gh})}(t),\\ \quad g,h=1,\cdots,m,\ 1\leqslant k\leqslant p,\end{cases}\tag{54}$$

其中: $s(t)\in\mathbb{R}^m$是被估的多通道信号; $u(t)\in\mathbb{R}^m$是输入噪声; $z_i(t)\in\mathbb{R}^m$是第i个传感器收到的观测; $y_i(t)\in\mathbb{R}^m$是估值器接收到的观测; $\xi_i(t)\in\mathbb{R}^1$和$\zeta_i(t)\in\mathbb{R}^1$, $i=1,\cdots,L$是满足式(5)的相互独立的伯努利白噪声; $r(t)\in\mathbb{R}^m$是满足式(51)的有色观测噪声; $\eta_i(t)\in\mathbb{R}^m$是观测白噪声, $e(t)\in\mathbb{R}^m$是白噪声. $A_t(q^{-1})$和$B_t(q^{-1})$是q^{-1}的p阶多项式; q^{-1}是单位滞后算子, 即$q^{-1}s(t)=s(t-1)$; $A_k(t)\in\mathbb{R}^{m\times m}$和$B_k(t)\in\mathbb{R}^{m\times m}$, $k=1,\cdots,p$是随机参数矩阵; $a_k^{(\mathrm{gh})}(t)$, $b_k^{(\mathrm{gh})}(t)$分别是$A_k(t)$和$B_k(t)$中第g行第h列位置的元素; $a_k^{(gh)}(t)\in\mathbb{R}^1$是带已知均值$a_k^{(gh)}$和随机扰动$\alpha_k^{(\mathrm{gh})}(t)$的标量随机元素; $b_k^{(\mathrm{gh})}(t)\in\mathbb{R}^1$是带已知均值$b_k^{(\mathrm{gh})}$和随机扰动$\beta_k^{(\mathrm{gh})}(t)$的标量随机元素.

$u(t), e(t), \eta_i(t), \alpha_k^{(\mathrm{gh})}(t)$和$\beta_k^{(\mathrm{gh})}(t)$是各自不相关的零均值白噪声, $\bar{Q}_u, \bar{Q}_e, \bar{R}_{\eta_i}, \bar{\sigma}^2_{\alpha_k^{(\mathrm{gh})}}$和$\bar{\sigma}^2_{\beta_k^{(\mathrm{gh})}}$分别是它们的未知不确定实际(真实)方差. $Q_u, Q_e, R_{\eta_i}, \sigma^2_{\alpha_k^{(\mathrm{gh})}}$和$\sigma^2_{\beta_k^{(\mathrm{gh})}}$分别是$\bar{Q}_u, \bar{Q}_e, \bar{R}_{\eta_i}, \bar{\sigma}^2_{\alpha_k^{(\mathrm{gh})}}$和$\bar{\sigma}^2_{\beta_k^{(\mathrm{gh})}}$的已知保守上界, 即$\bar{Q}_u\leqslant Q_u$, $\bar{Q}_e\leqslant Q_e, \bar{R}_{\eta_i}\leqslant R_{\eta_i}$, $\bar{\sigma}^2_{\alpha_k^{(\mathrm{gh})}}\leqslant\sigma^2_{\alpha_k^{(\mathrm{gh})}}, \bar{\sigma}^2_{\beta_k^{(\mathrm{gh})}}\leqslant\sigma^2_{\beta_k^{(\mathrm{gh})}}$.

目的是为多传感器多通道MA信号$s(t)$设计鲁棒集中式融合稳态Kalman估值器.

由式(53)–(54)可得

$$A_k(t)=A_k+\Lambda_k(t),\ B_k(t)=B_k+\Upsilon_k(t),\ 1\leqslant k\leqslant p.\tag{55}$$

其中: $A_k=(a_k^{(\mathrm{gh})})_{m\times m}, \Lambda_k(t)=(\alpha_k^{(\mathrm{gh})}(t))_{m\times m}, B_k=(b_k^{(\mathrm{gh})})_{m\times m}, \Upsilon_k(t)=(\beta_k^{(\mathrm{gh})}(t))_{m\times m},\ g,\ h=1,\cdots,m,$

$1 \leqslant k \leqslant p$. $\Lambda_k(t)$和$\Upsilon_k(t)$可被改写为

$$\begin{cases} \Lambda_k(t) = \sum_{g=1}^{m}\sum_{h=1}^{m} \alpha_k^{(\mathrm{gh})}(t)\Lambda_k^{(\mathrm{gh})}, \\ \Upsilon_k(t) = \sum_{g=1}^{m}\sum_{h=1}^{m} \beta_k^{(\mathrm{gh})}(t)\Upsilon_k^{(\mathrm{gh})}, \end{cases} \tag{56}$$

其中: $\Lambda_k^{(\mathrm{gh})}$和$\Upsilon_k^{(\mathrm{gh})}$都是$m\times m$矩阵, 它们的第$g$行第$h$列元素等于1, 其余元素均为0.

带随机参数矩阵的多通道MA信号模型(49)可被转换为如下等价的状态空间模型[5]:

$$x_s(t+1) = \Phi_s x_s(t) + \Gamma_s(t)u(t), \tag{57}$$

$$s(t) = H_s x_s(t), \tag{58}$$

其中:

$$\Phi_s = \begin{bmatrix} (0)_{m\times m} & & I_{m(p-1)} & \\ (0)_{m\times m} & & & \\ \vdots & & & \\ (0)_{m\times m} & (0)_{m\times m} & \cdots & (0)_{m\times m} \end{bmatrix},$$

$$\Gamma_s(t) = \begin{bmatrix} B_1(t) \\ B_2(t) \\ \vdots \\ B_p(t) \end{bmatrix},\ H_s = [I_m\ \ (0)_{m\times m}\ \ \cdots\ \ (0)_{m\times m}].$$

将$B_k(t) = B_k + \Upsilon_k(t)$代入$\Gamma_s(t)$, 并应用式(56)得

$$\Gamma_s(t) = \Gamma_s + \sum_{k=1}^{p}\sum_{g=1}^{m}\sum_{h=1}^{m} \beta_k^{(\mathrm{gh})}(t)\Gamma_{sk}\Upsilon_k^{(\mathrm{gh})}, \tag{59}$$

其中: $\Gamma_s = [B_1^{\mathrm{T}}\ \ B_2^{\mathrm{T}}\ \ \cdots\ \ B_p^{\mathrm{T}}]^{\mathrm{T}}$, $\Gamma_{sk} = [(0)_{m\times m}\ \cdots\ (0)_{m\times m}\ \ I_m\ \ (0)_{m\times m}\ \cdots\ (0)_{m\times m}]^{\mathrm{T}}$, 这里$\Gamma_{sk}$, $k = 1,\cdots,p$是$mp\times m$矩阵, 它的第$(k,1)$个块矩阵为单位矩阵I_m, 其余位置均为$(0)_{m\times m}$.

有色观测噪声$r(t)$有等价的状态空间模型为[5]

$$x_r(t+1) = \Phi_r(t)x_r(t) + \Gamma_r e(t), \tag{60}$$

$$r(t) = H_r(t)x_r(t) + e(t), \tag{61}$$

其中:

$$\Phi_r(t) = \begin{bmatrix} -A_1(t) & -A_2(t) & \cdots & -A_p(t) \\ & & & (0)_{m\times m} \\ & I_{m(p-1)} & & \vdots \\ & & & (0)_{m\times m} \end{bmatrix},$$

$$\Gamma_r = \begin{bmatrix} I_m \\ (0)_{m\times m} \\ \vdots \\ (0)_{m\times m} \end{bmatrix},$$

$$H_r(t) = [-A_1(t)\ \ -A_2(t)\ \ \cdots\ \ -A_p(t)].$$

将$A_k(t) = A_k + \Lambda_k(t)$代入$\Phi_r(t)$和$H_r(t)$, 并应用式(56)得

$$\begin{cases} \Phi_r(t) = \Phi_r + \sum_{k=1}^{p}\sum_{g=1}^{m}\sum_{h=1}^{m} \alpha_k^{(\mathrm{gh})}(t)\Phi_{rI}\Lambda_k^{(\mathrm{gh})}\Phi_{rk}, \\ H_r(t) = H_r + \sum_{k=1}^{p}\sum_{g=1}^{m}\sum_{h=1}^{m} \alpha_k^{(\mathrm{gh})}(t)\Lambda_k^{(\mathrm{gh})}H_{rk}. \end{cases} \tag{62}$$

其中:

$$\Phi_r = \begin{bmatrix} -A_1 & -A_2 & \cdots & -A_p \\ & & & (0)_{m\times m} \\ & I_{m(p-1)} & & \vdots \\ & & & (0)_{m\times m} \end{bmatrix},$$

$$\Phi_{rI} = \begin{bmatrix} -I_m \\ (0)_{m\times m} \\ \vdots \\ (0)_{m\times m} \end{bmatrix},$$

$$\Phi_{rk} = [(0)_{m\times m}\ \cdots\ (0)_{m\times m}\ \ I_m\ \ (0)_{m\times m}\ \cdots\ (0)_{m\times m}],$$

$$H_r = [-A_1\ \ -A_2\ \ \cdots\ \ -A_p],$$

$$H_{rk} = [(0)_{m\times m}\ \cdots\ (0)_{m\times m}\ \ -I_m\ \ (0)_{m\times m}\ \cdots\ (0)_{m\times m}].$$

这里Φ_{rk}, $k = 1,\cdots,p$是$m\times mp$矩阵, 它的第$(1,k)$个块矩阵为单位矩阵I_m, 其余位置均为$(0)_{m\times m}$. H_{rk}, $k = 1,\cdots,p$是$m\times mp$矩阵, 它的第$(1,k)$个块矩阵为$-I_m$, 其余位置均为$(0)_{m\times m}$.

定义

$$x(t) = \begin{bmatrix} x_s(t) \\ x_r(t) \end{bmatrix},\ \Phi(t) = \begin{bmatrix} \Phi_s & (0)_{mp\times mp} \\ (0)_{mp\times mp} & \Phi_r(t) \end{bmatrix},$$

$$\Gamma(t) = \begin{bmatrix} \Gamma_s(t) & (0)_{mp\times m} \\ (0)_{mp\times m} & \Gamma_r \end{bmatrix},\ w(t) = \begin{bmatrix} u(t) \\ e(t) \end{bmatrix},$$

$$H(t) = [H_s,\ \ H_r(t)].$$

于是可得如下增广系统模型:

$$x(t+1) = \Phi(t)x(t) + \Gamma(t)w(t), \tag{63}$$

$$z_i(t) = H(t)x(t) + v_i(t),\ i = 1,\cdots,L, \tag{64}$$

$$v_i(t) = e(t) + \eta_i(t), \tag{65}$$

其中: 零均值白噪声$w(t)$的实际和保守方差分别为$\bar{Q} = \mathrm{diag}\{\bar{Q}_u, \bar{Q}_e\}$, $Q = \mathrm{diag}\{Q_u, Q_e\}$, 用$Q$减去$\bar{Q}$, 并应用引理1可得$\bar{Q} \leqslant Q$.

应用式(59)和式(62)可将增广系统(63)–(65)转换为如下带常参数的系统:

$$\begin{aligned} x(t+1) = {} & (\Phi + \sum_{k=1}^{p}\sum_{g=1}^{m}\sum_{h=1}^{m} \alpha_k^{(\mathrm{gh})}(t)\Phi_{\alpha k}^{(\mathrm{gh})})x(t) + \\ & (\Gamma + \sum_{k=1}^{p}\sum_{g=1}^{m}\sum_{h=1}^{m} \beta_k^{(\mathrm{gh})}(t)\Gamma_{\beta k}^{(\mathrm{gh})})w(t), \end{aligned} \tag{66}$$

$$z_i(t)=(H+\sum_{k=1}^{p}\sum_{g=1}^{m}\sum_{h=1}^{m}\alpha_k^{(\mathrm{gh})}(t)H_{\alpha k}^{(\mathrm{gh})})x(t)+ v_i(t),\ i=1,\cdots,L, \tag{67}$$

$$v_i(t)=Dw(t)+\eta_i(t), \tag{68}$$

其中: $\varPhi=\mathrm{diag}\{\varPhi_s,\varPhi_r\}$, $\varGamma=\mathrm{diag}\{\varGamma_s,\varGamma_r\}$, $\varPhi_{\alpha k}^{(\mathrm{gh})}=\mathrm{diag}\{0,\varPhi_{rI}\varLambda_k^{(\mathrm{gh})}\varPhi_{rk}\}$, $\varGamma_{\beta k}^{(\mathrm{gh})}=\mathrm{diag}\{\varGamma_{sk}\varUpsilon_k^{(\mathrm{gh})},0\}$, $H=[H_s\ \ H_r]$, $H_{\alpha k}^{(\mathrm{gh})}=[(0)_{m\times mp}\ \ \varLambda_k^{(\mathrm{gh})}H_{rk}]$, $D=[(0)_{m\times m}\ \ I_m]$.

因此, 应用状态空间方法和增广方法, 多通道MA信号模型(49)–(54)被转换成带乘性噪声、一步随机观测时滞、丢包和不确定噪声方差的状态空间模型式(52)(66)–(68).

由式(58)可得$s(t)=[H_s\ \ (0)_{m\times mp}]x(t)$, 于是, 鲁棒融合信号估计问题可以通过鲁棒融合状态估计来解决, 应用射影理论[5]可得

$$\hat{s}^{(\mathrm{c})}(t|t+N)=[H_s\ \ (0)_{m\times mp}]\hat{x}^{(\mathrm{c})}(t|t+N),\quad N=-1,\ N\geqslant 0. \tag{69}$$

信号$s(t)$的实际和保守集中式融合稳态估计误差方差分别为

$$\begin{cases}\bar{P}^{(\mathrm{c})\mathrm{s}}(N)=[H_s\ \ (0)_{m\times mp}]\bar{P}^{(\mathrm{c})}(N)[H_s\ \ (0)_{m\times mp}]^{\mathrm{T}},\\ P^{(\mathrm{c})\mathrm{s}}(N)=[H_s\ \ (0)_{m\times mp}]P^{(\mathrm{c})}(N)[H_s\ \ (0)_{m\times mp}]^{\mathrm{T}}.\end{cases} \tag{70}$$

这里, 上角标“s”表示信号.

注 7 对系统(52)(66)–(68), 当取$m=1$时, MA信号$s(t)$是单通道信号, 此时系统(52)(66)–(68)可视为系统(1)–(4)的一种特殊情况, 其中: $n_\alpha=n_\beta=p$, $\varPhi_k=\varPhi_{\alpha k}^{(11)}$, $\varGamma_k=\varGamma_{\beta k}^{(11)}$, $H_i=H$, $H_{ik}=H_{\alpha k}^{(11)}$, $D_i=D$.

类似式(69)–(70), 可得鲁棒局部稳态信号估值器$\hat{s}_i(t|t+N)$, $N=-1$, $N\geqslant 0$, $i=1,\cdots,L$和它的实际和保守局部稳态估计误差方差$\bar{P}_i^s(N)$和$P_i^s(N)$. 容易证得定理1和定理2对信号$s(t)$成立.

6 仿真实例

在仿真实验中, 考虑多传感器单通道MA信号模型(49)–(54), 设$m=1$, $p=2$, $L=3$, $a_1^{(11)}=0.8$, $a_2^{(11)}=-0.09$, $b_1^{(11)}=-0.5$, $b_2^{(11)}=-0.36$, $\bar{Q}_u=3$, $Q_u=4$, $\bar{Q}_e=0.05$, $Q_e=0.1$, $\bar{R}_{\eta_1}=0.25$, $R_{\eta_1}=0.35$, $\bar{R}_{\eta_2}=1$, $R_{\eta_2}=1.2$, $\bar{R}_{\eta_3}=0.4$, $R_{\eta_3}=0.6$, $\bar{\sigma}^2_{\alpha_1^{(11)}}=0.01$, $\sigma^2_{\alpha_1^{(11)}}=0.02$, $\bar{\sigma}^2_{\alpha_2^{(11)}}=0.02$, $\sigma^2_{\alpha_2^{(11)}}=0.03$, $\bar{\sigma}^2_{\beta_1^{(11)}}=0.03$, $\sigma^2_{\beta_1^{(11)}}=0.04$, $\bar{\sigma}^2_{\beta_2^{(11)}}=0.04$, $\sigma^2_{\beta_2^{(11)}}=0.05$, $\pi_1=0.9$, $\pi_2=\pi_3=0.85$, $\varsigma_1=0.85$, $\varsigma_2=0.85$, $\varsigma_3=0.85$. 仿真结果如下.

由于信号$s(t)\in R^1$, 所以估计误差方差的迹值等于相应的估计误差方差值. 表1给出了实际和保守局部与集中式融合稳态估计误差方差的比较, 这验证了由式(43)–(46)给出的稳态精度关系. 表1中$N=-1$表示预报器, $N=0$表示滤波器, $N=1$表示一步平滑器, $N=2$表示两步平滑器.

表 1 信号$s(t)$的稳态鲁棒和实际精度比较

Table 1 Comparison of steady-state robustness and actual accuracy of signal $s(t)$

	$N=-1$	$N=0$	$N=1$	$N=2$
$P_1^{\mathrm{s}}(N)$	1.7146	0.6788	0.6081	0.5990
$\bar{P}_1^{\mathrm{s}}(N)$	1.2113	0.4412	0.3966	0.3904
$P_2^{\mathrm{s}}(N)$	1.7691	1.0428	0.9655	0.9626
$\bar{P}_2^{\mathrm{s}}(N)$	1.2627	0.7509	0.6978	0.6961
$P_3^{\mathrm{s}}(N)$	1.7456	0.8749	0.7936	0.7884
$\bar{P}_3^{\mathrm{s}}(N)$	1.2360	0.5760	0.5210	0.5171
$P^{(\mathrm{c})\mathrm{s}}(N)$	1.6849	0.4886	0.4249	0.4113
$\bar{P}^{(\mathrm{c})\mathrm{s}}(N)$	1.1874	0.3099	0.2727	0.2641

图1给出了鲁棒局部和集中式融合稳态一步平滑器$\hat{s}_i(t|t+1)$, $i=1,2,3$和$\hat{s}^{(\mathrm{c})}(t|t+1)$的跟踪效果. 从图1可看到, 鲁棒集中式融合稳态一步平滑器$\hat{s}^{(\mathrm{c})}(t|t+1)$的跟踪性能更好.

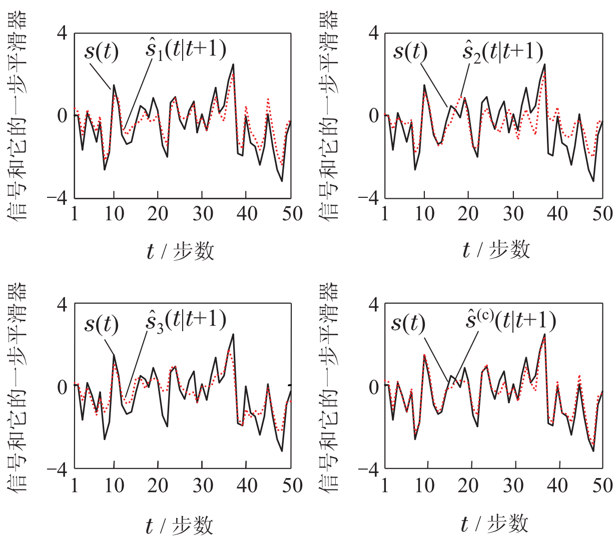

图 1 信号$s(t)$和它的实际局部和集中式融合一步平滑器

Fig. 1 Signal $s(t)$ and its actual local and centralized fusion one-step smoothers

图2给出了鲁棒集中式融合稳态信号估值器$\hat{s}^{(\mathrm{c})}(t|t-1)$, $\hat{s}^{(\mathrm{c})}(t|t)$和$\hat{s}^{(\mathrm{c})}(t|t+1)$的跟踪效果. 可看到, 它们的实际估计误差依次减小, 这与式(46)给出的精度关系一致.

为了说明状态依赖乘性噪声$\alpha_k^{(11)}(t)$, $k=1,2$和噪声依赖乘性噪声$\beta_k^{(11)}(t)$, $k=1,2$对鲁棒信号滤波器$\hat{s}^{(\mathrm{c})}(t|t)$的精度影响, 令$[\sigma^2_{\alpha_1^{(11)}}\ \ \sigma^2_{\alpha_2^{(11)}}]=\lambda_1[1\ \ 1]$, $[\sigma^2_{\beta_1^{(11)}}\ \ \sigma^2_{\beta_2^{(11)}}]=\lambda_2[1\ \ 1]$, λ_1和λ_2按步长0.01从0.01增加至0.09, 图3给出了鲁棒精度$P^{(\mathrm{c})\mathrm{s}}(0)$(即$\mathrm{tr}P^{(\mathrm{c})\mathrm{s}}(0)$)

随着λ_1和λ_2的增加的变化情况. 可看到, 当两类噪声方差增加时, $P^{(c)s}(0)$的值也增加, 这意味着集中式融合稳态信号滤波器$\hat{s}^{(c)}(t|t)$的鲁棒精度降低.

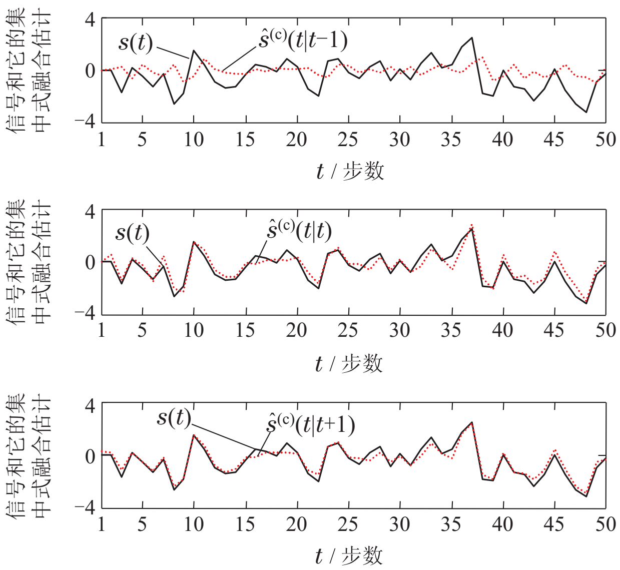

图 2 信号$s(t)$和它的集中式融合估值器

Fig. 2 Signal $s(t)$ and its centralized fusion estimator

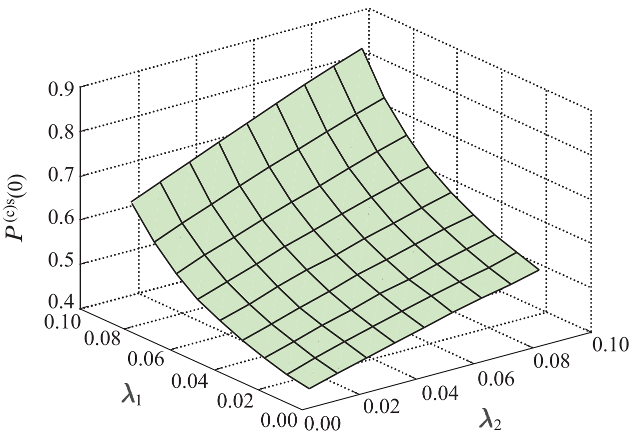

图 3 $P^{(c)s}(0)$随着λ_1和λ_2增加时的变化情况

Fig. 3 Changes of $P^{(c)s}(0)$ with the increase of λ_1 and λ_2

7 结论

针对在系统状态转移矩阵和观测矩阵中带相同状态依赖乘性噪声, 并带噪声依赖乘性噪声、不确定噪声方差、一步随机观测滞后和丢包的多传感器网络化系统, 应用增广方法、去随机化方法和虚拟噪声技术将该系统转换为仅带不确定噪声方差的集中式融合系统. 转换之后系统的过程噪声和观测噪声是相同的, 这可避免求解它们的相关矩阵. 根据极大极小鲁棒估计原理, 提出了鲁棒集中式融合稳态Kalman估值器(预报器、滤波器和平滑器). 应用增广噪声方法、非负定矩阵分解方法和李雅普诺夫方程方法, 证明了估值器的鲁棒性. 所提出的方法可用于解决带随机参数矩阵、不确定噪声方差和网络化随机不确定性的多传感器多通道MA信号的鲁棒融合Kalman滤波问题. 仿真实验证明了所提出方法的可应用性与正确性.

参考文献:

[1] LIGGINS M E, HALL D L, LLINAS J. *Handbook of Multisensor Data Fusion: Theory and Practice, Second Edition*. New York: CRC Press, 2009.

[2] WANG Z D, NIU Y G. Distributed estimation and filtering for sensor networks. *International Journal of Systems Science*, 2011, 42(9): 1421 – 1425.

[3] HU J, WANG Z D, CHEN D Y, et al. Estimation, filtering and fusion for networked systems with network-induced phenomena: New progress and prospects. *Information Fusion*, 2016, 31: 65 – 75.

[4] SUN S L, LIN H L, MA J, et al. Multi-sensor distributed fusion estimation with applications in networked systems: A review paper. *Information Fusion*, 2017, 38: 122 – 134.

[5] ANDERSON B D O, MOORE J B. *Optimal Filtering*. NJ: Prentice-Hall, Englewood Cliffs, 1979.

[6] GAO M, HU J, CHEN D Y, et al. Resilient state estimation for time-varying uncertain dynamical networks with datapacket dropouts and switching topology: Anevent-triggered method. *IET Control Theory & Applications*, 2020, 14(3): 367 – 377.

[7] CHEN B, HU G Q, HO D W C, et al. A new approach to linear/nonlinear distributed fusion estimation problem. *IEEE Transactions on Automatic Control*, 2019, 64(3): 1301 – 1308.

[8] CHEN B, HU G Q. Nonlinear state estimation under bounded noises. *Automatica*, 2018, 98: 159 – 168.

[9] WANG S Y, WANG Z D, DONG H L, et al. Recursive state estimation for linear systems with lossy measurements under time-correlated multiplicative noises. *Journal of the Franklin Institute*, 2020, 357(3): 1887 – 1908.

[10] WANG S Y, FANG H J, TIAN X G. Robust estimator design for networked uncertain systems with imperfect measurements and uncertain-covariance noises. *Neurocomputing*, 2017, 230: 40 – 47.

[11] LIU W, XIE X P, QIAN W, et al. Optimal linear filtering for networked control systems with random matrices, correlated noises, and packet dropouts. *IEEE Access*, 2020, 8: 59987 – 59997.

[12] LIU W. Optimal estimation for discrete-time linear systems in the presence of multiplicative and time-correlated additive measurement noises. *IEEE Transactions on Signal Processing*, 2015, 63(17): 4583 – 4593.

[13] LIU W. Optimal filtering for discrete-time linear systems with time-correlated multiplicative measurement noises. *IEEE Transactions on Automatic Control*, 2016, 61(7): 1972 – 1978.

[14] LIU W Q, WANG X M, DENG Z L. Robust centralized and weighted measurement fusion Kalman estimators for uncertain multisensor systems with linearly correlated white noises. *Information Fusion*, 2017, 35: 11 – 25.

[15] LIU W Q, TAO G L, FAN Y J, et al. Robust fusion steady-state filtering for multisensor networked systems with one-step random delay, missing measurements, and uncertain-variance multiplicative and additive white noises. *International Journal of Robust & Nonlinear Control*, 2019, 29(14): 4716 – 4754.

[16] LIU W Q, TAO G L, SHEN C. Robust measurement fusion steady-state estimator design for multisensor networked systems with random two-step transmission delays and missing measurements. *Mathematics & Computers in Simulation*, 2021, 181: 242 – 283.

[17] LIU W Q, DENG Z L. Weighted fusion robust steady-state estimators for multisensor networked systems with one-step random delay and inconsecutive packet dropouts. *International Journal of Adaptive Control & Signal Processing*, 2020, 34(2): 151 – 182.

[18] LEWIS F L, XIE L H, POPA D. *Optimal and Robust Estimation: With an Introduction to Stochastic Control Theory, Second Edition*. New York: CRC Press, 2008.

[19] SUN S L, XIE L H, XIAO W D, et al. Optimal linear estimationfor systems with multiple packet dropouts. *Automatica*, 2008, 44(5): 1333 – 1342.

[20] MA J, SUN S L. Optimal linear estimators for systems with random sensor delays, multiple packet dropouts and uncertain observations. *IEEE Transactions on Signal Processing*, 2011, 59(11): 5181 – 5192.

[21] MA J, SUN S L. Centralized fusion estimators for multisensor systems with random sensor delays, multiple packet dropouts and uncertain observations. *IEEE Sensors Journal*, 2013, 13(4): 1228 – 1235.

[22] WANG S Y, FANG H J, TIAN X G. Minimum variance estimation for linear uncertain systems with one-step correlated noises and incomplete measurements. *Digital Signal Processing*, 2016, 49: 126 – 136.

[23] LI M Y, ZHANG L, CHU D S. Optimal estimation for systems with multiplicative noises, random delays and multiple packet dropouts. *IET Signal Process*, 2016, 10(8): 880 – 887.

[24] MA J, SUN S L. Distributed fusion filter for networked stochastic uncertain systems with transmission delays and packet dropouts. *Signal Processing*, 2017, 130: 268 – 278.

[25] CHEN B, ZHANG W A, HU G Q, et al. Networked fusion Kalman filtering with multiple uncertainties. *IEEE Transactions on Aerospace & Electronic Systems*, 2015, 51(3): 2332 – 2349.

[26] ÁGUILA R C, CARAZO A H, PÉREZ J L. Networked distributed fusion estimation under uncertain outputs with random transmission delays, packet losses and multi-packet processing. *Signal Processing*, 2019, 156: 71 – 83.

[27] YANG C S, YANG Z B, DENG Z L. Robust weighted state fusion Kalman estimators for networked systems with mixed uncertainties. *Information Fusion*, 2019, 45: 246 – 265.

[28] RAN C J, DENG Z L. Robust integrated covariance intersection fusion Kalman estimators for networked systems with random measurement delays, multiplicative noises, and uncertain noise variances. *International Journal of Adaptive Control & Signal Processing*, 2020, 34(11): 1697 – 1725.

[29] RAN C J, DENG Z L. Robust fusion Kalman estimators for networked mixed uncertain systems with random one-step measurement delays, missing measurements, multiplicative noises and uncertain noise variances. *Information Sciences*, 2020, 534: 27 – 52.

[30] QI W J, ZHANG P, DENG Z L. Robust weighted fusion Kalman filters for multisensor time-varying systems with uncertain noise variances. *Signal Processing*, 2014, 99: 185 – 200.

[31] LI N, SUN S L, MA J. Multi-sensor distributed fusion filtering for networked systems with different delay and loss rates. *Digital Signal Processing*, 2014, 34: 29 – 38.

[32] LIU Guangming, SU Weizhou. Survey of linear discrete-time stochastic controls systems with multiplicative noises. *Control Theory & Applications*, 2013, 30(8): 929 – 946.
(刘光明, 苏为洲. 具有乘性噪声的线性离散时间随机控制系统综述. 控制理论与应用, 2013, 30(8): 929 – 946.)

[33] WANG Z D, HO D W C, LIU X H. Robust filtering under randomly varying sensor delay with variance constraints. *Circuits and Systems II: Express Briefs, IEEE Transactions on*, 2004, 51(6): 320 – 326.

[34] KAILATH T, SAYED A H, HASSIBI B. *Linear Estimation*. NJ: Prentice-Hall, Englewood Cliffs, 2000.

[35] SUN X J, GAO Y, DENG Z L, et al. Multi-model information fusion Kalman filtering and white noise deconvolution. *Information. Fusion*, 2010, 11(2): 163 – 173.

附录A

证　定义$\Delta\Lambda_f=\Lambda_f-\bar{\Lambda}_f$, $\Delta Q_f=Q_f-\bar{Q}_f$, $\Delta S_f=S_f-\bar{S}_f$, $\Delta R_f=R_f-\bar{R}_f$, $\Delta Q_a=Q_a-\bar{Q}_a$和$\Delta X_a=X_a-\bar{X}_a$, 且令$\sigma^2_{\alpha_k}=\bar{\sigma}^2_{\alpha_k}+\Delta\sigma^2_{\alpha_k}$和$\sigma^2_{\beta_k}=\bar{\sigma}^2_{\beta_k}+\Delta\sigma^2_{\beta_k}$, 由式(21), 式(24)(26)(32)可得

$$\Delta\Lambda_f=\Delta\Lambda_f^{(1)}+\Delta\Lambda_f^{(2)}+\cdots+\Delta\Lambda_f^{(10)}+\Delta\Lambda_f^{(11)},$$

其中:

$$\Delta\Lambda_f^{(1)}=\sum_{k=1}^{n_\alpha}\bar{\sigma}^2_{\alpha_k}M_a^{\alpha k}\Delta X_gM_a^{\alpha k\mathrm{T}},$$
$$\Delta\Lambda_f^{(2)}=\sum_{k=1}^{n_\alpha}\Delta\sigma^2_{\alpha_k}M_a^{\alpha k}X_gM_a^{\alpha k\mathrm{T}},$$
$$\Delta\Lambda_f^{(3)}=\sum_{i=1}^{L}\sigma^2_{\xi_{iz}}M_a^{\xi i}\Delta X_gM_a^{\xi i\mathrm{T}},$$
$$\Delta\Lambda_f^{(4)}=\sum_{i=1}^{L}\sigma^2_{\zeta_{iz}}M_a^{\zeta i}\Delta X_gM_a^{\zeta i\mathrm{T}},$$
$$\Delta\Lambda_f^{(5)}=\sum_{i=1}^{L}\sigma^2_{\gamma_{iz}}M_a^{\gamma i}\Delta X_gM_a^{\gamma i\mathrm{T}},$$
$$\Delta\Lambda_f^{(6)}=\sum_{i=1}^{L}\sigma^2_{\xi_{iz}}\sum_{k=1}^{n_\alpha}\bar{\sigma}^2_{\alpha_k}M_a^{ki}\Delta X_gM_a^{ki\mathrm{T}},$$
$$\Delta\Lambda_f^{(7)}=\sum_{i=1}^{L}\sigma^2_{\xi_{iz}}\sum_{k=1}^{n_\alpha}\Delta\sigma^2_{\alpha_k}M_a^{ki}X_gM_a^{ki\mathrm{T}},$$
$$\Delta\Lambda_f^{(8)}=N_a^m\Delta Q_gN_a^{m\mathrm{T}},$$
$$\Delta\Lambda_f^{(9)}=\sum_{i=1}^{L}\sigma^2_{\xi_{iz}}N_a^{\xi i}\Delta Q_gN_a^{\xi i\mathrm{T}},$$
$$\Delta\Lambda_f^{(10)}=\sum_{k=1}^{n_\beta}\bar{\sigma}^2_{\beta_k}N_a^{\beta k}\Delta Q_gN_a^{\beta k\mathrm{T}},$$
$$\Delta\Lambda_f^{(11)}=\sum_{k=1}^{n_\beta}\Delta\sigma^2_{\beta_k}N_a^{\beta k}Q_gN_a^{\beta k\mathrm{T}},$$

这里:

$$M_a^{\alpha k}=\mathrm{diag}\{\Phi_a^{\alpha k},H_a^{\alpha k}\},\ M_a^{\xi i}=\mathrm{diag}\{\Phi_a^{\xi i},H_a^{\xi i}\},$$
$$M_a^{\zeta i}=\mathrm{diag}\{\Phi_a^{\zeta i},H_a^{\zeta i}\},\ M_a^{\gamma i}=\mathrm{diag}\{\Phi_a^{\gamma i},H_a^{\gamma i}\},$$
$$M_a^{ki}=\mathrm{diag}\{\Phi_a^{ki},H_a^{ki}\},\ N_a^{m}=\mathrm{diag}\{\Gamma_a^{m},G_a^{m}\},$$
$$N_a^{\xi i}=\mathrm{diag}\{\Gamma_a^{\xi i},G_a^{\xi i}\},\ N_a^{\beta k}=\mathrm{diag}\{\Gamma_a^{\beta k},0\},$$
$$\Delta X_g=\begin{bmatrix}\Delta X_a & \Delta X_a\\ \Delta X_a & \Delta X_a\end{bmatrix},\ X_g=\begin{bmatrix}X_a & X_a\\ X_a & X_a\end{bmatrix},$$
$$\Delta Q_g=\begin{bmatrix}\Delta Q_a & \Delta Q_a\\ \Delta Q_a & \Delta Q_a\end{bmatrix},\ Q_g=\begin{bmatrix}Q_a & Q_a\\ Q_a & Q_a\end{bmatrix}.$$

由式(19)和引理4得$\Delta X_g\geqslant 0$, 从而得到$\Delta\Lambda_f^{(k)}\geqslant 0$, $k=1,3,4,5,6$. 根据方差矩阵的半正定性, 应用引理4可得$X_g\geqslant 0$, 进而可得$\Delta\Lambda_f^{(2)}\geqslant 0$, $\Delta\Lambda_f^{(7)}\geqslant 0$. 此外, 应用式(16)和引理4可得$\Delta Q_g\geqslant 0$, 进而可得$\Delta\Lambda_f^{(k)}\geqslant 0$, $k=8,9,10$. 根据方差矩阵的半正定性, 应用引理4可得$Q_g\geqslant 0$, 进而可得$\Delta\Lambda_f^{(11)}\geqslant 0$. 综上可得$\Delta\Lambda_f=\Delta\Lambda_f^{(1)}+\Delta\Lambda_f^{(2)}+\cdots+\Delta\Lambda_f^{(10)}+\Delta\Lambda_f^{(11)}\geqslant 0$, 即$\bar{\Lambda}_f\leqslant\Lambda_f$.　证毕.

附录B

证　令$\Delta P_a^{(\mathrm{c})}(-1)=P_a^{(\mathrm{c})}(-1)-\bar{P}_a^{(\mathrm{c})}(-1)$, 由式(33)得

$$\Delta P_a^{(\mathrm{c})}(-1)=\Psi^{(\mathrm{c})}\Delta P_a^{(\mathrm{c})}(-1)\Psi^{(\mathrm{c})\mathrm{T}}+\Delta^{(\mathrm{c})},$$

$$\Delta^{(\mathrm{c})} = [I_{n+2m} \ \ -K^{(\mathrm{c})}]\Delta\Lambda_f[I_{n+2m} \ \ -K^{(\mathrm{c})}]^{\mathrm{T}}.$$

由引理5得$\Delta^{(\mathrm{c})} \geqslant 0$. 由于$\Psi^{(\mathrm{c})}$是稳定矩阵, 则由引理6有$\Delta P_a^{(\mathrm{c})}(-1) \geqslant 0$, 即式(34)成立. 取$\bar{Q}=Q$, $\bar{R}_{\eta_i}=R_{\eta_i}$, $\bar{\sigma}^2_{\alpha_k}=\sigma^2_{\alpha_k}$, $\bar{\sigma}^2_{\beta_k}=\sigma^2_{\beta_k}$和$\bar{P}_0=P_0$, 则假设3仍成立. 由$\bar{R}_{\eta_i}=R_{\eta_i}$可得$\bar{R}_{\eta_c}=R_{\eta_c}$, 于是由式(15)可得$\bar{Q}_a=Q_a$. 从$\bar{X}(0)=\bar{P}_0+\mu_0\mu_0^{\mathrm{T}}$和$X(0)=P_0+\mu_0\mu_0^{\mathrm{T}}$可得$\bar{X}(0)=X(0)$, 于是有$\bar{X}_a(0)=X_a(0)$. 通过迭代容易证得$X_a(t)=\bar{X}_a(t)$. 由引理3可得$X_a=\bar{X}_a$. 由式(21)可得$Q_f=\bar{Q}_f$; 由式(24)可得$R_f=\bar{R}_f$; 由式(26)可得$S_f=\bar{S}_f$. 由式(32)可得$\Lambda_f=\bar{\Lambda}_f$, 于是有$\Delta^{(\mathrm{c})}=0$. 应用引理6可得$\Delta P_a^{(\mathrm{c})}(-1)=0$, 即$\bar{P}_a^{(\mathrm{c})}(-1)=P_a^{(\mathrm{c})}(-1)$. 如果$P_a^*$是$\bar{P}_a^{(\mathrm{c})}(-1)$的任意一个其他上界, 则有$P_a^{(\mathrm{c})}(-1)=\bar{P}_a^{(\mathrm{c})}(-1)\leqslant P_a^*$, 这意味着$P_a^{(\mathrm{c})}(-1)$是$\bar{P}_a^{(\mathrm{c})}(-1)$的最小上界. 证毕.

附录C

证 令$\Delta P_a^{(\mathrm{c})}(N)=P_a^{(\mathrm{c})}(N)-\bar{P}_a^{(\mathrm{c})}(N)$, 由式(39)可得

$$\Delta P_a^{(\mathrm{c})}(N)=\Psi_N^{(\mathrm{c})}\Delta P_a^{(\mathrm{c})}(-1)\Psi_N^{(\mathrm{c})\mathrm{T}}+\sum_{\rho=0}^{N}[K_{N\rho}^{\mathrm{cw}} \ \ K_{N\rho}^{\mathrm{cv}}]\times \Delta\Lambda_f[K_{N\rho}^{\mathrm{cw}} \ \ K_{N\rho}^{\mathrm{cv}}]^{\mathrm{T}}.$$

应用引理5和式(34)可得$\Delta P_a^{(\mathrm{c})}(N)\geqslant 0$, 即式(40)成立. 类似于定理1的证明, 可证得$P_a^{(\mathrm{c})}(N)$是$\bar{P}_a^{(\mathrm{c})}(N)$的最小上界, 细节在此省略. 证毕.

作者简介:

陶贵丽 博士, 讲师, 目前研究方向为多传感器信息融合、鲁棒Kalman滤波和广义系统状态估计, E-mail: taoguili_5605@163.com;

李 爽 硕士研究生, 目前研究方向为多传感器信息融合和鲁棒Kalman滤波, E-mail: 1320731630@qq.com;

刘文强 博士, 副教授, 硕士生导师, 目前研究方向为多传感器信息融合、鲁棒Kalman滤波和网络化系统建模, E-mail: lwq@zjgsu.edu.cn.