基于自适应事件触发牵制控制的多时滞随机耦合神经网络簇同步

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摘要: 本文通过自适应事件触发牵制控制策略, 研究了多时滞的随机耦合神经网络在均方意义下以指数速率进行簇同步的问题. 在耦合神经网络中, 同一簇中的节点只需与相应的孤立节点同步, 而对于不同簇中节点之间的同步状态没有要求. 首先, 本文提出了一种事件触发牵制控制方法来解决耦合神经网络中节点数量众多、通讯复杂的问题. 该方法不仅能减少耦合神经网络中控制器的数量, 还可以减少控制信号的传输次数、减轻网络传输压力. 然后根据M矩阵方法, 建立了随机耦合神经网络均方指数稳定的充分条件. 同时, 利用自适应控制策略, 给出了反馈增益的更新规律. 最后, 通过一个数值例子验证了所提出的自适应事件触发牵制控制策略的有效性和适用性.

关键词: 多时滞; 事件触发; 随机耦合神经网络; 簇同步

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Cluster synchronization of multi-delayed stochastic coupled neural networks via adaptive event-triggered pinning control

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Abstract: In this paper, the cluster synchronization of multi-delayed stochastic coupled neural networks at exponential rate in the sense of mean square is studied by the adaptive event-triggered pinning control strategy. In coupled neural networks, nodes in the same cluster only need to synchronize with the corresponding isolated nodes, but there is no requirement for the synchronization state between nodes in different clusters. Firstly, an event-triggered pinning control method is proposed to solve the problems of large number of nodes and complex communication in the coupled neural networks. This method not only reduce the number of controllers in the coupled neural networks, but also reduce the transmission times of control signals and the transmission pressure of the network. Then, according to the M-matrix method, a sufficient condition for the mean square exponential stability of stochastic coupled neural networks is established. At the same time, the update law of feedback gain is given by the adaptive control strategy. Finally, a numerical example is given to verify the effectiveness and applicability of the proposed adaptive event-triggered pinning control strategy.

Key words: multi-delay; event-triggering; stochastic coupled neural networks; cluster synchronization

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1 引言

在过去的几十年中, 一些具有单个或多个时滞的耦合神经网络[1–3]得到了广泛的研究, 尤其是在其稳定性分析方面. 随着对耦合神经网络更加深入的研究, 需要考虑到系统的干扰问题, 例如跳变[4]和随机扰动[5]. 耦合神经网络中的组件故障或子系统互连可能

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会导致网络参数的突然变化. 因此引入了马尔可夫切换[6]. 这可以有效地反映一种模式跳转到另一种模式时参数的变化. 此外, 实际中耦合神经网络的参数具有随机性, 或会受到外部随机噪声的干扰. 其中高斯白噪声[7]就常被用作耦合神经网络中的随机扰动. 在文献[8]中, 针对具有多时滞和马尔可夫切换随机神经网络, 利用M矩阵方法, 保证了自适应控制下的指数稳定.

近年来, 耦合神经网络的同步因其在不同学科和领域的潜在应用而得到了进一步发展[9–10]. 耦合神经网络的同步是使驱动网络和响应网络的状态保持一致. 然而, 对于一些实际问题, 需要将耦合网络分成许多簇. 其中, 同一簇中的网络需要同步, 而对不同簇之间的动态行为没有要求, 这种行为就被称为簇同步[11–13].

然而, 耦合网络通常由大量的节点组成. 因此, 通过将控制器添加到所有的节点上来控制这些复杂的网络是非常困难和不现实的. 为了节省控制成本, 可以将耦合神经网络划分为多个子网络, 然后只对每个子网络中的一小部分节点进行控制操作, 从而迫使整个网络达到同步. 这就是通过牵制控制[14–15]在多个子网络上进行所谓的簇同步. 在文献[14]中, 对于非线性耦合子网络上的簇同步问题, 采用了合适的自适应控制方法来处理牵制节点与其领导节点之间的未知反馈增益. 然而, 在连续或周期性的采样通信过程中, 会执行一些不必要的控制任务, 这将导致存储或计算的浪费. 为了避免这种情况, 提出了事件触发牵制控制[16–17]的方法, 该方法具有减少更新信息数量和减少网络资源浪费的优点. 与文献[16–17]相比, 本文考虑到现实环境中的噪声与多时滞的影响, 不仅稳定性得到了提高, 而且应用的场景广泛.

本文的创新如下:

1) 构建了带有马尔可夫切换、多时滞和随机噪声的耦合神经网络模型, 此模型更具有普遍性.

2) 通过自适应事件触发机制和牵制控制实现了耦合神经网络的簇同步. 与控制所有节点方法相比, 不仅能使耦合神经网络快速同步, 而且可以减少控制信号的传输次数、减轻网络传输压力.

2 问题描述

耦合神经网络中节点间的相互作用关系可以用一个有向图来描述. $\boldsymbol{G}=(\mathcal{V},\mathcal{E},\mathcal{B})$表示一个$n$阶加权有向图. $\mathcal{V}=\{1,2,\cdots,N\}$是耦合神经网络中节点的集, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$是网络中边界的集. 耦合连接矩阵是$\mathcal{B}=(b_{ij})_{N\times N}\in\mathbb{R}^{N\times N}$. 在耦合神经网络中, 节点划分为$m$个簇, 表示为$\mathcal{V}=\{\mathcal{G}_1,\mathcal{G}_2,\cdots,\mathcal{G}_m\}$, $\mathcal{G}_p\cap\mathcal{G}_q\neq 0$, $p,q\in\{1,\cdots,m\}$. 节点在每个簇中表示为$\mathcal{G}_1=\{\mathfrak{L}_0+1,\cdots,\mathfrak{L}_1\},\cdots,\mathcal{G}_m=\{\mathfrak{L}_{m-1}+1,\cdots,\mathfrak{L}_m\}$, $\mathfrak{L}_0=0$, $\mathfrak{L}_m=N$. 牵制控制的节点为$\mathfrak{L}=\{\mathfrak{L}_0+1,\cdots,\mathfrak{L}_{m-1}+1\}$.

$\{r(t)\}_{t\geqslant 0}$是概率空间$(\varXi,\boldsymbol{F},\boldsymbol{P})$上的一个右连续马尔可夫链, 在有限状态集$S=\{1,2,\cdots,n\}$, 生成子$\varUpsilon=(\gamma_{ij})_{N\times N}$由以下公式给出:

$$\mathrm{P}\{r(t+\varDelta)=j|r(t)=i\}=\begin{cases}\gamma_{ij}\varDelta+o(\varDelta), & i\neq j,\\ 1+\gamma_{ii}\varDelta+o(\varDelta), & i=j,\end{cases}$$

其中: $\varDelta>0$, $\gamma\geqslant 0$是从i到j的传输速率. 如果$i\neq j$, 则$\gamma_{ii}=-\sum\limits_{j\neq i}\gamma_{ij}$.

耦合神经网络由N个节点的组成, 其第i个节点的动态行为表示为

$$\begin{aligned}\mathrm{d}\phi_i(t)=&\ [\alpha_i(\phi_i(t),\phi_{i\tau}(t),r(t))+U_i(r(t))+\\&\sum_{j=1}^{N}b_{ij}\varGamma\beta(\phi_j(t))]\mathrm{d}t+\\&\varrho(t,\phi_i(t),\sum_{\pi=1}^{N}\phi_i(t-\tau_\pi(t),r(t)))\mathrm{d}\omega(t),\end{aligned}\tag{1}$$

其中: $\phi_i(t)\in\mathbb{R}^n$是耦合网络中第i个节点的状态向量; $\alpha_i(\phi_i(t),\ \phi_{i\tau}(t),\ r(t))=-A_i(r(t))\phi_i(t)+C_i(r(t))\times f(\phi_i(t))+\sum\limits_{\pi=1}^{N}W_\pi(r(t))f(\phi_i(t-\tau_\pi(t)))$, 其中$f(\phi_i(t))$是神经元的激活函数; $\beta(\phi_j(t))$是非线性耦合函数; 耦合连接矩阵$\mathcal{B}=[b_{ij}]_{m\times m}$; $\varGamma$ 是网络的内耦合矩阵; $U_i(r(t))$是网络中牵制节点的控制器. 为方便起见, 令$r(t)=r$, $A_i(r(t))=A_i^r$, $C_i(r(t))=C_i^r$和$W_\pi(r(t))=W_\pi^r$. τ_π是时滞且$0<\tau_\pi\leqslant\check{\tau}_\pi$和$\dot{\tau}_\pi\leqslant\check{\tau}_\pi<1$, $\check{\tau}_\pi$和$\check{\tau}_\pi$都是常数. $\varrho:\mathbb{R}^n\times\mathbb{R}^n\times S\to\mathbb{R}^{n\times m}$是随机噪声强度矩阵. $\omega(t)$是定义在完备概率空间$(\varXi,\boldsymbol{F},\boldsymbol{P})$上的$n$维布朗运动.

孤立节点可以表示为

$$\begin{aligned}\mathrm{d}\varphi_l(t)=&\ \hat{\alpha}_l(\varphi_l(t),\varphi_{l\tau}(t),r)\mathrm{d}t+\\&\varrho(t,\varphi_l(t),\sum_{\pi=1}^{N}\varphi_l(t-\tau_\pi),r)\mathrm{d}\omega(t),\\&l=1,2,\cdots,m,\end{aligned}\tag{2}$$

其中$\hat{\alpha}_l(\varphi_l(t),\varphi_{l\tau}(t),r)=-A_l^r\varphi_l(t)+C_l^rf(\varphi_l(t))+\sum\limits_{\pi=1}^{N}W_{\pi l}^rf(\varphi_l(t-\tau_\pi(t)))$.

如果有

$$\begin{aligned}&\lim_{t\to+\infty}\|\phi_i(t)-\varphi_l(t)\|=0,\\&i=\mathcal{G}_d,\ d=1,2,\cdots,m,\end{aligned}\tag{3}$$

则耦合网络(1)中的节点与孤立节点(2)实现簇同步.

注 1 耦合网络(1)可以分成m个簇, 即网络中存在m个领导者. 相应地, m个孤立节点可以被称为m个跟随者. 因此, 簇同步可以看作是多智能体网络[18]中多领导者–多跟随者的协同问题.

当耦合神经网络节点(1)和孤立节点(2)满足触发条件时,数据才能被传输到控制器中. 网络触发条件为

$$\begin{cases} t_{k+1} = t_k + \min\{t|(\varepsilon_i(t) - \varepsilon_i(t_k))^{\mathrm{T}} \times \\ \qquad \varepsilon_i(t) - \varepsilon_i(t_k)) \geqslant \vartheta \varepsilon_i^{\mathrm{T}}(t)\varepsilon_i(t), \\ \varepsilon_i(t) = \phi_i(t) - \varphi_l(t),\ \vartheta > 0. \end{cases} \tag{4}$$

节点i的触发时刻序列为$\{t_{k1}^i, \cdots, t_{kd}^i, \cdots\}$. 事件触发控制器为

$$U_i^r(t) = \wp(r(t))\kappa_i\delta_i(\phi_i(t_k^i) - \varphi_l(t_k^i) + \phi_i(t) - \varphi_l(t)), \tag{5}$$

其中: $\wp(r(t))$为控制器$U_i^r(t)$中的自适应更新参数,

$$\kappa_i = \begin{cases} 1,\ i \in \mathfrak{L}, \\ 0,\ i \notin \mathfrak{L}. \end{cases}$$

注 2　为了实现簇同步, 网络中的节点需要通过耦合结构与相邻节点交换状态信息. 在事件触发算法中, 节点i在触发时刻$t_{k_d}^i$更新其耦合值, 并立即将$\phi_j(t_{k_d}^i)$和$U_i^r(t)$的信息发送给它的所有邻居. 在所提出的事件触发算法下, 网络中的每个节点不需要在每个时刻验证事件触发条件. 因此, 事件触发算法可以减少耦合神经网络的计算负载, 节省更多的能量.

当$i \in \mathfrak{L}$时, 且$t \in [t_{k_d}^i, t_{k_{d+1}}^i)$, 误差网络为

$$\begin{aligned} \mathrm{d}\varepsilon_i(t) = {} & [-A_l^r\varepsilon_i^{\mathrm{T}}(t) - (A_i^r - A_l^r)\phi_i(t) + \\ & C_l^r g(\varepsilon_i(t)) + (C_i^r - C_l^r)f(\phi_i(t)) + \\ & \sum_{\pi=1}^N W_{\pi l}^r g(\varepsilon_i(t - \tau_\pi(t))) + \\ & \sum_{\pi=1}^N (W_{\pi i}^r - W_{\pi l}^r)f(\phi_i(t - \tau_\pi(t))) + \\ & \sum_{j=1}^N b_{ij}\Gamma\beta(\phi_j(t_{k_d}^i)) + U_i^r(t)]\mathrm{d}t + \\ & [\varrho(t,\ \phi_i(t), \sum_{\pi=1}^N \phi_i(t - \tau_\pi(t), r) - \\ & \varrho(t, \varphi_l(t), \sum_{\pi=1}^N \varphi_l(t - \tau_\pi(t)), r)]\mathrm{d}\omega(t). \end{aligned} \tag{6}$$

当$i \notin \mathfrak{L}$时, 且$t \in [t_{k_d}^i, t_{k_{d+1}}^i)$, 误差网络为

$$\begin{aligned} \mathrm{d}\varepsilon_i(t) = {} & [-A_l^r\varepsilon_i^{\mathrm{T}}(t) - (A_i^r - A_l^r)\phi_i(t) + \\ & C_l^r g(\varepsilon_i(t)) + (C_i^r - C_l^r)f(\phi_i(t)) + \\ & \sum_{\pi=1}^N W_{\pi l}^r g(\varepsilon_i(t - \tau_\pi(t))) + \\ & \sum_{\pi=1}^N (W_{\pi i}^r - W_{\pi l}^r)f(\phi_i(t - \tau_\pi(t))) + \\ & \sum_{j=1}^N b_{ij}\Gamma\beta(\phi_j(t_{k_d}^i))]\mathrm{d}t + \\ & [\varrho(t, \phi_i(t), \sum_{\pi=1}^N \phi_i(t - \tau_\pi(t), r) - \\ & \varrho(t, \varphi_l(t), \sum_{\pi=1}^N \varphi_l(t - \tau_\pi(t)), r)]\mathrm{d}\omega(t). \end{aligned} \tag{7}$$

图1给出了误差网络的结构图. 误差网络首先根据事件触发机制判断网络中的节点触发时刻, 输出的$\varepsilon_i(t_k)$利用自适应更新率更新网络中的参数, 再通过牵制控制器解决误差网络节点的同步问题.

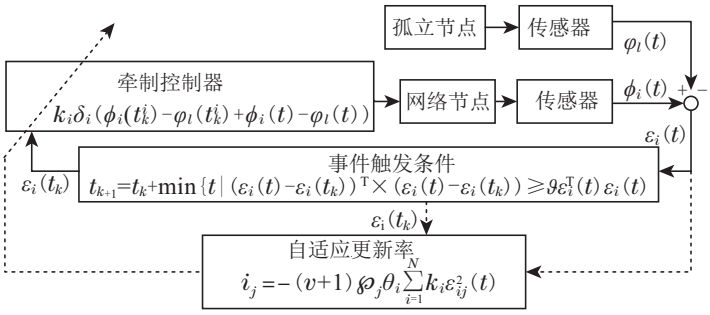

图 1　网络结构图

Fig. 1　Structure diagram of network

为了简化, 设

$$\begin{aligned} \bar{\alpha}_i = {} & -(A_i^r - A_l^r)\phi_i(t) + (C_i^r - C_l^r)f(\phi_i(t)) + \\ & C_l^r g(\varepsilon_i(t)) + \sum_{\pi=1}^N W_{\pi l}^r g(\varepsilon_i(t - \tau_\pi(t))) + \\ & \sum_{\pi=1}^N (W_{\pi i}^r - W_{\pi l}^r)f(\phi_i(t - \tau_\pi(t))) + \\ & \kappa_i U_i^r(t) + \sum_{j=1}^N b_{ij}\Gamma\beta(\phi_j(t_{k_d}^i)), \end{aligned} \tag{8}$$

$$\begin{aligned} \bar{\varrho} = {} & \varrho(t, \phi_i(t), \sum_{\pi=1}^N \phi_i(t - \tau_\pi(t), r) - \\ & \varrho(t, \varphi_l(t), \sum_{\pi=1}^N \varphi_l(t - \tau_\pi(t)), r). \end{aligned} \tag{9}$$

简化后, 误差网络为

$$\mathrm{d}\varepsilon_i(t) = \bar{\alpha}_i\mathrm{d}t + \bar{\varrho}\mathrm{d}\omega(t). \tag{10}$$

误差网络的初始状态为$\varepsilon_i(s) = \psi_i(s),\ s \in [-\check{\tau},\ 0]$, $\psi(s) \in L_{\boldsymbol{F}_0}^2([-\check{\tau},\ 0],\ \mathbb{R}^n)$.

定义 1[19]　存在$\mathcal{B} = [b_{ij}]_{N_1 \times N_2}, i = 1, 2, \cdots, N_1$, $j = 1, 2, \cdots, N_2$.

如果存在

$$b_{ij}>0(i\neq j),$$

$$b_{ij}=-\sum_{i=1,j\neq i}^{N_1}b_{ij}=-\sum_{j=1,j\neq i}^{N_2}b_{ji}(N_1=N_2),$$

且$\mathcal{B}$是不可约的, 则$\mathcal{B}$就称为属于B_1类, 表示为$\mathcal{B}\in B_1$.

如果$\mathcal{B}\in B_1$且$\mathcal{B}$是对称的, 则$\mathcal{B}\in B_2$.

如果$\mathcal{B}=[b_{ij}]_{N_1\times N_2}(N_1\neq N_2)$和$\sum\limits_{j=1}^{N_2}b_{ij}=0(i=1,\cdots,N_1)$, 则$\mathcal{B}\in B_3$.

定义 2[19] 矩阵$\mathcal{B}\in\mathbb{R}^{N\times N}$, 将$\mathcal{B}$划分成$m$个簇.

$$\mathcal{B}=\begin{bmatrix}\mathcal{B}_{11} & \mathcal{B}_{12} & \cdots & \mathcal{B}_{1m}\\ \mathcal{B}_{21} & \mathcal{B}_{22} & \cdots & \mathcal{B}_{2m}\\ \vdots & \vdots & & \vdots\\ \mathcal{B}_{m1} & \mathcal{B}_{m2} & \cdots & \mathcal{B}_{mm}\end{bmatrix},$$

其中$\mathcal{B}_{ij}\in\mathbb{R}^{(\mathcal{L}_i-\mathcal{L}_{i-1})\times(\mathcal{L}_j-\mathcal{L}_{j-1})}(i,\ j\in1,\cdots,m)$. 如果$\mathcal{B}_{ii}$和$\mathcal{B}_{ij}$满足零行和准则且$\mathcal{B}_{ii}\in B_1,\mathcal{B}_{ij}\in B_3$, 则$\mathcal{B}\in B_4$.

注 3 $\mathcal{B}\in B_4$表示同一簇中节点的连接模式是一致的, 但不同簇中节点的连接模式不同. 不同簇中的连接模式对耦合神经网络的同步状态没有影响. 即$\mathcal{B}\in B_4$时,

$$\lim_{t\to\infty}\|\varepsilon_p-\varepsilon_q\|=0,\ (p,\ q\in g_d,\ d=1,\cdots,m),$$

$$\lim_{t\to\infty}\|\varepsilon_p-\varepsilon_q\|\neq0,\ (p,\ q\notin g_d,\ d=1,\ \cdots,\ m).$$

假设 1 假设激活函数$f(\cdot)$满足线性增长条件, 则存在常数$L>0$, 使得

$$|f(x)-f(y)|\leqslant L|x-y|,\ (x,y\in\mathbb{R}^n).$$

假设 2 假设连续噪声强度矩阵$\varrho(\cdot)$满足Lipschitz条件, 则存在常数$\eta>0,\lambda_1>0,\cdots,\lambda_n>0$, 使得

$$\begin{aligned}&\mathrm{tr}\{\varrho(t,\varepsilon_i(t),\varepsilon_i(t-\tau_1(t)),\cdots,\varepsilon_i(t-\tau_n(t)))\}^{\mathrm{T}}\times\\&\{\varrho(t,\varepsilon_i(t),\varepsilon_i(t-\tau_1(t)),\cdots,\varepsilon_i(t-\tau_n(t)))\}\leqslant\\&\eta|\varepsilon_i(t)|^2+\lambda_1|\varepsilon_i(t-\tau_1)|^2+\cdots+\lambda_n|\varepsilon_i(t-\tau_n)|^2.\end{aligned}$$

定义 3[2] 如果有以下的不等式:

$$\lim_{t\to\infty}\sup\frac{1}{t}\log(\mathcal{E}|\varepsilon_i(t,\psi(s))|^2)<0,\tag{11}$$

其中$\psi(s)\in L^2_{\boldsymbol{F}_0}([-\check{\tau},\ 0],\ \mathbb{R}^n)$, 则误差网络(10)是均方指数稳定的.

为了分析误差网络(10)的稳定性, 给出了以下的引理.

引理 1[17] 令$u,v\in\mathbb{R}^n$, 则$u^{\mathrm{T}}v\leqslant u^{\mathrm{T}}u+v^{\mathrm{T}}v$.

引理 2[20] 考虑一个具有马尔可夫切换和时滞的n维随机微分方程

$$\begin{aligned}\mathrm{d}\phi_i(t)=&[\alpha_i(\phi_i(t),\ \phi_{i\tau}(t),\ r(t))]\mathrm{d}t+\\&\varrho(t,\ \phi_i(t),\phi_{i\tau}(t),\ r(t))\mathrm{d}\omega(t).\end{aligned}$$

在$t\in[0,\ \infty)$上, 初始状态如下:

$$\{\psi(s):\ -\check{\tau}\leqslant s\leqslant0\}\in L^2_{\boldsymbol{F}_0}([-\check{\tau},\ 0],\ \mathbb{R}^n).$$

对于$V(\cdot)\in\mathcal{C}^{2,1}(\mathbb{R}_+\times S\times\mathbb{R}^n;\ \mathbb{R}_+)$, 定义算子$\pounds$为

$$\begin{aligned}\pounds V=&V_t+V_{\phi_i}\bar{\alpha}_i+\frac{1}{2}\mathrm{tr}(\bar{\varrho}^{\mathrm{T}}V_{\phi_i\phi_i}\bar{\varrho})+\\&\sum_{j=1}^{N}\gamma_{ij}V(t,\ j,\ \phi_i(t)),\end{aligned}$$

其中:

$$V_t(t,\phi_i(t),\ r(t))=\frac{\partial V(t,\ \phi_i(t),\ r(t))}{\partial t},$$

$$\begin{aligned}V_{\phi_i}(t,\phi_i(t),r(t))=&(\frac{\partial V(t,\phi_i(t),\ r(t))}{\partial\phi_{i_1}},\\&\cdots,\ \frac{\partial V(t,\ \phi_i(t),\ r(t))}{\partial\phi_{i_n}}),\end{aligned}$$

$$V_{\phi_i\phi_i}(t,\phi_i(t),\ r(t))=(\frac{\partial^2V(t,\phi_i(t),\ r(t))}{\partial\phi_{i_i}\partial\phi_{i_j}})_{n\times n}.$$

设$V(\cdot)\in\mathcal{C}^{2,1}(\mathbb{R}_+\times S\times\mathbb{R}^n;\ \mathbb{R}_+)$和$\tau_1,\tau_2$是有界停时. 如果$V(t,\phi(t))$和$\pounds V(t,\phi(t))$的边界为$t\in[\tau_1,\tau_2]$ $(0\leqslant\tau_1\leqslant\tau_2)$, 其概率为1, 则

$$\begin{aligned}&\mathcal{E}V(\tau_2,\phi(\tau_2))=\\&\mathcal{E}V(\tau_1,\phi(\tau_1))+\mathcal{E}\int_{\tau_1}^{\tau_2}\pounds V(s,\phi(s))\mathrm{d}s.\end{aligned}$$

引理 3[21] 设$T>0$和$u(\cdot)$是$[0,T]$上的一个Borel可测的有界非负函数. 如果

$$u(t)\leqslant\varOmega+\varPi\int_0^t u(s)\mathrm{d}s,$$

则$u(t)\leqslant\varOmega\exp(\varPi t),\ \forall\,0\leqslant t\leqslant T$.

引理 4[21] 如果$M=(m_{ij})_{n\times n}\in\mathbb{R}^{n\times n}$, 其中$m_{ij}<0(i\neq j)$, 则以下条件是成立的:

1) M是一个非奇异的M矩阵;

2) M的每个实特征值都是正的;

3) M是正稳定的. 即M^{-1}是存在的且$M^{-1}>0$ ($M^{-1}\geqslant0$和M^{-1}中至少有一个元素为正).

3 主要结论

在这部分, 通过自适应事件触发牵制控制，建立了耦合神经网络的均方指数稳定准则.

定理 1 假设$M:=-\mathrm{diag}\{\underbrace{\varLambda,\cdots,\varLambda}_{N}\}-\varUpsilon$是一个非奇异的$M$矩阵, 其中, $\varLambda=\mu+\varPhi+L_1^2+\sum\limits_{\pi=1}^{N}\varPsi_\pi+\eta+\sum\limits_{j=1}^{N}b_{ij}L_2\varGamma\varGamma^{\mathrm{T}}+\sum\limits_{j=1}^{N}b_{ij}L_2\upsilon$. 设$m>0$, $\overrightarrow{m}=(\underbrace{m,m,\cdots,m}_{N})^{\mathrm{T}}$ $(\theta_1,\theta_2,\cdots,\theta_N)^{\mathrm{T}}:=M^{-1}\overrightarrow{m}\gg0$, 所有的元素$M^{-1}\overrightarrow{m}$是正的). 假设

$$\min_{i\in S}\max_{1\leqslant\pi\leqslant n}\{\frac{1}{1-\check{\tau}_\pi}\}\sum_{\pi=1}^{n}\varTheta_\pi\leqslant$$

$$-(\Lambda\theta_i+\sum_{s=1}^{N}\gamma_{is}q_s), \tag{12}$$

其中$\Theta_\pi=\lambda_\pi+L_1^2$. 在假设1和假设2下, 选择控制器(5)中$\wp(r(t))$的参数更新律为

$$\dot{l}_j=-(\upsilon+1)\wp_j\theta_i\sum_{i=1}^{N}\kappa_i\varepsilon_{ij}^2(t), \tag{13}$$

且选取A_l^r, C_l^r和W_l^r的参数更新律为

$$\dot{\tilde{a}}_{lj}=\jmath_{lj}\theta_i\varepsilon_{ij}^{\mathrm{T}}(t)\phi_{i\gamma}(t), \tag{14}$$

$$\dot{\tilde{c}}_{ij\gamma}=-\Phi_{ij\gamma}\theta_i\varepsilon_{ij}^{\mathrm{T}}(t)f(\phi_{i\gamma}(t)), \tag{15}$$

$$\dot{\tilde{w}}_{ij\gamma\pi}=-\Psi_{ij\gamma\pi}\theta_i\varepsilon_{ij}^{\mathrm{T}}(t)f(\varphi_{l\gamma}(t-\tau_\pi)), \tag{16}$$

则误差网络(10)是均方指数稳定的.

证 考虑如下Lyapunov函数:

$$V=\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)\theta_i\varepsilon_i(t)+\sum_{j=1}^{n}((\frac{1}{\wp_j})l_j^2+\sum_{i=1}^{N}\frac{1}{\jmath_{lj}}\tilde{a}_{lj}^2+\sum_{i=1}^{N}\sum_{\gamma=1}^{n}\frac{1}{\Phi_{ij\gamma}}\tilde{c}_{ij\gamma}^2+\sum_{i=1}^{N}\sum_{\gamma=1}^{n}\sum_{\pi=1}^{n}\frac{1}{\Phi_{ij\gamma}}\tilde{w}_{ij\gamma\pi}^2).$$

根据误差网络(10)和自适应更新律(13)–(16), 得

$$\begin{aligned}\pounds V=&2\sum_{l=1}^{m}(\sum_{i\in\mathfrak{L}}+\sum_{i\notin\mathfrak{L}})\varepsilon_i^{\mathrm{T}}(t)\theta_i[-A_l^r\varepsilon_i(t)+\\&C_l^rg(\varepsilon_i(t))-(A_i^r-A_l^r)\phi_i(t)+(C_i^r-C_l^r)\times\\&f(\phi_i(t))+\sum_{\pi=1}^{N}W_{\pi l}^rg(\varepsilon_i(t-\tau_\pi(t)))+\\&\sum_{\pi=1}^{N}(W_{\pi i}^r-W_{\pi l}^r)f(\phi_i(t-\tau_\pi(t)))+\\&\kappa_i\delta_i(\phi_i(t_k^i)-\varphi_l(t_k^i)+\phi_i(t)-\varphi_l(t))+\\&\sum_{j=1}^{N}b_{ij}\Gamma\beta(\phi_j(t_{k_d}^i))]\mathrm{d}t+\mathrm{tr}[\bar{\varrho}^{\mathrm{T}}\theta_i\bar{\varrho}]+\\&\sum_{s=1}^{N}\gamma_{js}\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)\theta_i\varepsilon_i(t)+2\sum_{j=1}^{n}((\frac{1}{\wp_j})\dot{l}_jl_j+\\&\sum_{i=1}^{N}\frac{1}{\jmath_{lj}}\dot{\tilde{a}}_{lj}\tilde{a}_{lj}+\sum_{i=1}^{N}\sum_{\gamma=1}^{n}\frac{1}{\Phi_{ij\gamma}}\dot{\tilde{c}}_{ij\gamma}\tilde{c}_{ij\gamma}+\\&\sum_{i=1}^{N}\sum_{\gamma=1}^{n}\sum_{\pi=1}^{n}\tfrac{1}{\Psi_{ij\gamma\pi}}\dot{\tilde{w}}_{ij\gamma\pi}\tilde{w}_{ij\gamma\pi})=\\&2\sum_{l=1}^{m}(\sum_{i\in\mathfrak{L}}+\sum_{i\notin\mathfrak{L}})\varepsilon_i^{\mathrm{T}}(t)\theta_i[-A_l^r\varepsilon_i^{\mathrm{T}}(t)+\\&C_l^rg(\varepsilon_i(t))+\sum_{\pi=1}^{N}W_{\pi l}^rg(\varepsilon_i(t-\tau_\pi(t)))+\\&\kappa_i\delta_i(\phi_i(t_k^i)-\varphi_l(t_k^i)+\phi_i(t)-\varphi_l(t))+\\&\sum_{j=1}^{N}b_{ij}\Gamma\beta(\phi_j(t_{k_d}^i))]\mathrm{d}t+\mathrm{tr}[\bar{\varrho}^{\mathrm{T}}\theta_i\bar{\varrho}]+\\&\sum_{s=1}^{N}\gamma_{js}\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)\theta_i\varepsilon_i(t)+2\sum_{j=1}^{n}(\tfrac{1}{\wp_j})\dot{l}_jl_j.\end{aligned} \tag{17}$$

根据假设1, 得

$$-\varepsilon_i^{\mathrm{T}}(t)A_l^r\varepsilon_i(t)\leqslant-\mu\left|\varepsilon_i(t)\right|^2, \tag{18}$$

根据假设1和引理1, 可得

$$\begin{aligned}&\varepsilon_i^{\mathrm{T}}(t)C_l^rg(\varepsilon_i(t))\leqslant\\&\frac{1}{2}\varepsilon_i^{\mathrm{T}}(t)C_l^r(C_l^r)^{\mathrm{T}}\varepsilon_i(t)+\frac{1}{2}g^{\mathrm{T}}(\varepsilon_i(t))g(\varepsilon_i(t))\leqslant\\&\frac{1}{2}(\Phi+L_1^2)\left|\varepsilon_i(t)\right|^2,\end{aligned} \tag{19}$$

同理,

$$\begin{aligned}&\sum_{\pi=1}^{N}\varepsilon_i^{\mathrm{T}}(t)W_{\pi l}^rg\big(\varepsilon_i(t-\tau_\pi(t))\leqslant\\&\frac{1}{2}\sum_{\pi=1}^{N}\varepsilon_i^{\mathrm{T}}(t)W_{\pi l}^r(W_{\pi l}^r)^{\mathrm{T}}\varepsilon_i(t)+\\&\frac{1}{2}\sum_{\pi=1}^{N}g^{\mathrm{T}}(\varepsilon_i(t-\tau_\pi))g(\varepsilon_i(t-\tau_\pi))\leqslant\\&\frac{1}{2}\sum_{\pi=1}^{N}(\Psi_\pi\left|\varepsilon_i(t)\right|^2+L_1^2\left|\varepsilon_i(t-\tau_\pi)\right|^2).\end{aligned} \tag{20}$$

根据假设2, 得

$$\begin{aligned}\mathrm{tr}[\bar{\varrho}^{\mathrm{T}}\theta_i\bar{\varrho}]\leqslant&\eta\left|\varepsilon_i(t)\right|^2+\lambda_1\left|\varepsilon_i(t-\tau_1)\right|^2+\\&\cdots+\lambda_n\left|\varepsilon_i(t-\tau_n)\right|^2.\end{aligned} \tag{21}$$

根据式(4), 得

$$\begin{aligned}&\varepsilon_i^{\mathrm{T}}(t)\delta_i(\phi_i(t_k^i)-\varphi_l(t_k^i)-\phi_i(t)+\varphi_l(t))\leqslant\\&\varepsilon_i^{\mathrm{T}}(t)l_j(\varepsilon_i(t_k^i)-\varepsilon_i(t))\leqslant\\&l_j[\varepsilon_i^{\mathrm{T}}(t)\varepsilon_i(t)+(\varepsilon_i(t_k^i)-\varepsilon_i(t))^{\mathrm{T}}(\varepsilon_i(t_k^i)-\varepsilon_i(t))]\leqslant\\&l_j\varepsilon_i^{\mathrm{T}}(t)\varepsilon_i(t)+\upsilon l_j\varepsilon_i^{\mathrm{T}}(t)\varepsilon_i(t)\leqslant\\&(\upsilon+1)l_j\left|\varepsilon_i(t)\right|^2.\end{aligned} \tag{22}$$

根据假设1和$\mathcal{B}\in B_4$, 计算出

$$\begin{aligned}&\sum_{l=1}^{m}(\sum_{i\in\mathfrak{L}}+\sum_{i\notin\mathfrak{L}})\varepsilon_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Gamma\beta(\phi_j(t_{k_d}^i))\leqslant\\&\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Gamma\beta(\phi_j(t_{k_d}^i))\leqslant\\&\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)L_2\sum_{j=1}^{N}b_{ij}\Gamma\phi_j(t_{k_d}^i)\leqslant\\&\sum_{l=1}^{m}\sum_{i\notin\mathfrak{L}}\varepsilon_i^{\mathrm{T}}(t)L_2\sum_{j=1}^{N}b_{ij}\Gamma(\phi_j(t_{k_d}^i)-\\&\varphi_l(t_{k_d}^i)+\varphi_l(t_{k_d}^i))=\\&\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)L_2\sum_{j=1}^{N}b_{ij}\Gamma(\varepsilon_j(t_{k_d}^i)+\varphi_l(t_{k_d}^i))=\\&\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)L_2\sum_{j=1}^{N}b_{ij}\Gamma(\varepsilon_j(t_{k_d}^i)).\end{aligned} \tag{23}$$

根据事件触发机制(4), 式(23)可简化为

$$\begin{aligned}&\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)L_2\sum_{j=1}^{N}b_{ij}\Gamma(\varepsilon_j(t_{k_d}^i))=\\&\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)L_2\sum_{j=1}^{N}b_{ij}\Gamma(\varepsilon_j(t_{k_d}^i)-\varepsilon_j(t))+\end{aligned}$$

$$\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)L_2\sum_{j=1}^{N}b_{ij}\Gamma(\varepsilon_j(t))\leqslant$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}L_2\big(\varepsilon_i^{\mathrm{T}}(t)\Gamma\Gamma^{\mathrm{T}}\varepsilon_i(t)+$$
$$(\varepsilon_j(t_{k_d}^i)-\varepsilon_j(t))^{\mathrm{T}}(\varepsilon_j(t_{k_d}^i)-\varepsilon_j(t))\big)+$$
$$\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(t)L_2\sum_{j=1}^{N}b_{ij}\Gamma(\varepsilon_j(t))\leqslant$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}L_2\Gamma\Gamma^{\mathrm{T}}|\varepsilon_i(t)|^2+$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}L_2\upsilon|\varepsilon_j(t)|^2. \tag{24}$$

将式(18)–(24)代入式(17), 得

$$\begin{aligned}\pounds V=&-2\sum_{i=1}^{N}[\theta_i(\mu+\varPhi+L_1^2+\sum_{\pi=1}^{N}\varPsi_\pi+\eta+\\&\sum_{j=1}^{N}b_{ij}L_2\Gamma\Gamma^{\mathrm{T}}+\sum_{j=1}^{N}b_{ij}L_2\upsilon)+\\&\sum_{s=1}^{N}\gamma_{js}\theta_s]|\varepsilon_i(t)|^2+L_1^2\sum_{\pi=1}^{N}|\varepsilon_i(t-\tau_\pi)|^2+\\&\lambda_1|\varepsilon_i(t-\tau_1)|^2+\cdots+\lambda_n|\varepsilon_i(t-\tau_n)|^2=\\&-\varLambda|\varepsilon_i(t)|^2+\varTheta_1|\varepsilon_i(t-\tau_1)|^2+\\&\cdots+\varTheta_n|\varepsilon_i(t-\tau_n)|^2.\end{aligned} \tag{25}$$

对于函数$V(\cdot)$, 根据引理1和式(12), 可得

$$\begin{aligned}\mathcal{E}|\varepsilon_i(t)|^2\leqslant&\mathcal{E}V(0,\psi(0))+\mathcal{E}\int_0^{\mathrm{T}}\pounds V(s,\\&\varepsilon_i(s-\tau_1),\cdots,\varepsilon_i(s-\tau_n))\mathrm{d}s\leqslant\\&\mathcal{E}V(0,\psi(0))+\mathcal{E}\int_0^{\mathrm{T}}(-\varLambda|\varepsilon_i(t)|^2+\\&\varTheta_1|\varepsilon_i(t-\tau_1)|^2+\cdots+\\&\varTheta_n|\varepsilon_i(t-\tau_n)|^2)\mathrm{d}s.\end{aligned} \tag{26}$$

对于$\int_0^{\mathrm{T}}|\varepsilon_i(s-\tau_\pi)|^2\mathrm{d}s$, 令$u=s-\tau_\pi(s)$, $\mathrm{d}u=(1-\dot{\tau}_\pi(s))\mathrm{d}s$, 则

$$\begin{aligned}&\int_0^t|\varepsilon_i(s-\tau_\pi)|^2\mathrm{d}s\leqslant\\&\int_{-\tau_\pi(0)}^{t-\tau_\pi(t)}\frac{1}{1-\dot{\tau}_\pi}|\varepsilon_i(s-\tau_\pi)|^2\mathrm{d}s\leqslant\\&\frac{1}{1-\check{\tau}_\pi}\int_{-\check{\tau}_\pi}^{t}|\varepsilon_i(s-\tau_\pi)|^2\mathrm{d}s+\\&\frac{1}{1-\check{\tau}_\pi}\int_0^{t}|\varepsilon_i(s-\tau_\pi)|^2\mathrm{d}s\leqslant\\&\frac{\check{\tau}_\pi}{1-\check{\tau}_\pi}\max_{\check{\tau}_\pi\leqslant s\leqslant 0}|\psi(s)|^2+\\&\frac{1}{1-\check{\tau}_\pi}\int_0^{t}|\varepsilon_i(s-\tau_\pi)|^2\mathrm{d}s\leqslant\\&\max_{1\leqslant\pi\leqslant n}\{\frac{\check{\tau}_\pi}{1-\check{\tau}_\pi}\}\max_{\check{\tau}_\pi\leqslant s\leqslant 0}|\psi(s)|^2+\\&\max_{1\leqslant\pi\leqslant n}\{\frac{1}{1-\check{\tau}_\pi}\}\int_0^{t}|\varepsilon_i(s-\tau_\pi)|^2\mathrm{d}s.\end{aligned} \tag{27}$$

因此,

$$\begin{aligned}&\mathcal{E}|\varepsilon_i(t)|^2\leqslant\\&\mathcal{E}V(0,\psi(0))+\\&\max_{1\leqslant\pi\leqslant n}\{\frac{\check{\tau}_\pi}{1-\check{\tau}_\pi}\}\sum_{\pi=1}^{n}\varTheta_\pi\max_{\check{\tau}_\pi\leqslant s\leqslant 0}|\psi(s)|^2\leqslant\\&\int_0^t[-\varDelta+\max_{1\leqslant\pi\leqslant n}\{\frac{\check{\tau}_\pi}{1-\check{\tau}_\pi}\}\sum_{\pi=1}^{n}\varTheta_\pi]\mathcal{E}|\varepsilon_i(s)|^2\mathrm{d}s.\end{aligned} \tag{28}$$

根据引理3, 可得

$$\mathcal{E}\left|\phi_i\right|^2\leqslant\varOmega+\int_0^t\varPi\mathcal{E}\left|\phi_i\right|^2\mathrm{d}s, \tag{29}$$

其中:

$$\begin{aligned}\varOmega=&\mathcal{E}V(0,\psi(0))+\\&\max_{1\leqslant\pi\leqslant n}\{\frac{\check{\tau}_\pi}{1-\check{\tau}_\pi}\}\sum_{\pi=1}^{n}\varTheta_\pi\max_{\check{\tau}_\pi\leqslant s\leqslant 0}|\psi(s)|^2,\\\varPi=&-\varLambda+\max_{1\leqslant\pi\leqslant n}\{\frac{\check{\tau}_\pi}{1-\check{\tau}_\pi}\}\sum_{\pi=1}^{n}\varTheta_\pi.\end{aligned}$$

很明显, $\varOmega>0$, $\varPi<0$. 通过引理3, 得$\mathcal{E}|\varepsilon_i(t)|^2\leqslant\varOmega\times\exp(\varPi t)$. $\lim\limits_{t\to\infty}\sup\frac{1}{t}\log(\mathcal{E}\left|\varepsilon_i(t,\psi(s))\right|^2)\leqslant\varOmega<0$. 因此, 误差网络(10)是均方指数稳定的. 证毕.

注 4 本文通过在式(10)中使用自适应牵制控制方法来实现簇同步. 从牵制节点$\mathfrak{L}=\{\mathfrak{L}_0+1,\cdots,\mathfrak{L}_{m-1}+1\}$可以看出, 每个簇中的第1个节点是具有自适应更新律$\dot{l}_j=-(\upsilon+1)\wp_j\theta_i\sum\limits_{i=1}^{N}\varepsilon_{ij}^2(t)$的, 其他节点与牵制节点通讯.

4 仿真

为了验证本文所获得的多时滞耦合神经网络簇同步的自适应事件触发牵制控制器的有效性. 选取一个具有6个节点, 2个簇的耦合神经网络, 如图2所示, 其中$s1$, $s2$表示两个孤立节点. $\mathcal{G}_1=\{1,2\}$, $\mathcal{G}_2=\{3,4,5,6\}$和牵制节点为$\mathfrak{L}=\{1,3\}$. 令$S=\{1,2\}$, 生成子$\varUpsilon=\begin{bmatrix}-1&1\\0.5&-0.5\end{bmatrix}$. 网络参数选择如下: $L_1=L_2=1$, $\tau=0.5$, $[\theta_1\ \ \theta_2]^{\mathrm{T}}=[0.2667\ \ 0.2667]^{\mathrm{T}}$, $\phi_i(t)=\begin{bmatrix}\phi_{i1}(t)\\\phi_{i2}(t)\end{bmatrix}$, $\mathcal{B}=\begin{bmatrix}\mathcal{B}_{11}&\mathcal{B}_{12}\\\mathcal{B}_{21}&\mathcal{B}_{22}\end{bmatrix}$, $\mathcal{B}_{11}=\begin{bmatrix}-4&4\\3&-3\end{bmatrix}$,

$$\mathcal{B}_{12}=\begin{bmatrix}0&0.5&-0.2&-0.3\\0.5&0&0.5&-1\end{bmatrix},$$

$$\mathcal{B}_{21}=\begin{bmatrix}0.1&-0.1\\-0.2&0.2\\0&0\\0.1&-0.1\end{bmatrix},$$

$$\mathcal{B}_{22}=\begin{bmatrix}-0.5&0.1&0.5&-0.1\\0&-0.5&0.3&0.2\\-0.2&0&-0.3&0.5\\0&0.5&-0.3&-0.2\end{bmatrix},$$

$$A_i^1=\begin{bmatrix}7&0\\0&6\end{bmatrix},\ A_i^2=\begin{bmatrix}6&0\\0&6\end{bmatrix},\ i\in\mathcal{G}_1,$$

$$A_i^1=\begin{bmatrix}5&0\\0&5\end{bmatrix},\ A_i^2=\begin{bmatrix}4&0\\0&5\end{bmatrix},\ i\in\mathcal{G}_2,$$

$$C_i^1=\begin{bmatrix}-2&2\\-2&1\end{bmatrix},\ C_i^2=\begin{bmatrix}1&2\\-2&-2\end{bmatrix},\ i\in\mathcal{G}_1,$$

$$C_i^1=\begin{bmatrix}1&2\\-2&1\end{bmatrix},\ C_i^2=\begin{bmatrix}-3&2\\1&-1\end{bmatrix},\ i\in\mathcal{G}_2,$$

$$W_i^1=\begin{bmatrix}-2&1.8\\-1.2&0.8\end{bmatrix},W_i^2=\begin{bmatrix}1.1&2\\-0.8&1\end{bmatrix},\ i\in\mathcal{G}_1,$$

$$W_i^1=\begin{bmatrix}1.2&1.2\\-1&0.8\end{bmatrix},W_i^2=\begin{bmatrix}1&1\\-1.8&2\end{bmatrix},\ i\in\mathcal{G}_2,$$

$$\delta_i^1=\begin{bmatrix}-1.2&0\\0&-1\end{bmatrix},\delta_i^2=\begin{bmatrix}-1.1&0\\0&1.4\end{bmatrix},\ i\in\mathcal{G}_1,$$

$$\delta_i^1=\begin{bmatrix}1.5&0\\0&1.5\end{bmatrix},\delta_i^2=\begin{bmatrix}-1&0\\0&-2\end{bmatrix},\ i\in\mathcal{G}_2,$$

$$\alpha(\phi_i(t))=\begin{bmatrix}-1.8\sin(\phi_{i1}(t))+0.8\tanh(\phi_{i2}(t))\\0.5\cos(\phi_{i2}(t))-\tanh(\phi_{i1}(t))\end{bmatrix},$$

$$\beta(\phi_i(t))=\begin{bmatrix}5\phi_{i1}(t)+\sin(2\phi_{i2}(t))\\0.5\phi_{i2}(t)+\cos(\phi_{i1}(t))\end{bmatrix},$$

$$\varrho(t,1,\phi_i(t),\phi_i(t-\tau))=[\phi_{i1}(t-\tau),\ \phi_{i2}(t)],$$

$$\varrho(t,2,\phi_i(t),\phi_i(t-\tau))=[0.5\phi_{i1}(t),0.5\phi_{i2}(t-\tau)].$$

初始状态为

$$\phi_1(t)=[1.5\ \ -1.2]^{\mathrm{T}},\ \phi_2(t)=[-1.8\ \ -1]^{\mathrm{T}},$$

$$\phi_3(t)=[-1.5\ \ -1]^{\mathrm{T}},\ \phi_4(t)=[-1\ \ -0.5]^{\mathrm{T}},$$

$$\phi_5(t)=[1\ \ -1.5]^{\mathrm{T}},\ \phi_6(t)=[-1.8\ \ 1.8]^{\mathrm{T}},$$

$$s_1(t)=[1.5\ \ -1.2]^{\mathrm{T}},\ s_2(t)=[-1.5\ \ 1.5]^{\mathrm{T}}.$$

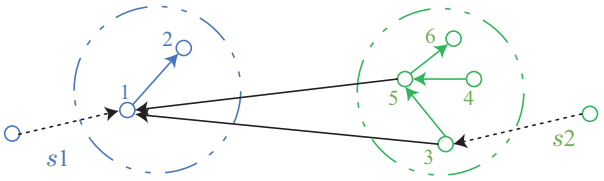

图 2　具有6个节点, 2个簇的网络

Fig. 2　A network with 2 clusters and 6 nodes

经验证, 以上的参数是满足定理1的. 误差网络(10)是均方指数稳定的. 仿真结果如图3–7所示. 其中, 图3表示网络中的马尔可夫链. 图4表示网络中的布朗运动. 图5表示事件触发机制, 网络节点在4 s内触发5次. 事件触发控制器$U_1(t)$和$U_2(t)$的输出如图6所示. 图7表示网络中两个簇的误差轨迹, 可以看到两个簇中的6个节点的误差轨迹最终都能趋于稳定. 与文献[16]相比, 网络节点的误差轨迹图(图7)可以更快地达到同步.

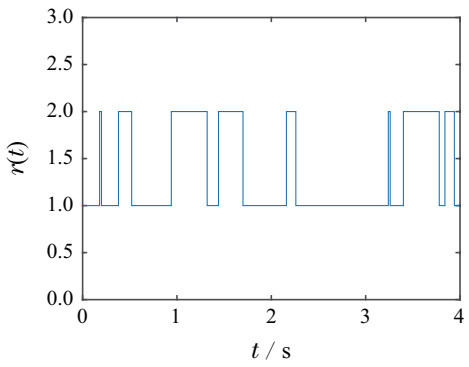

图 3　马尔可夫链

Fig. 3　Markov chain

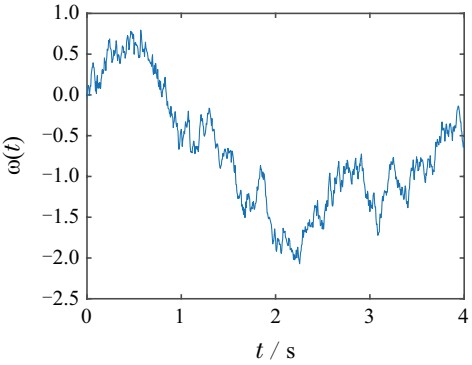

图 4　布朗运动

Fig. 4　Brownian motion

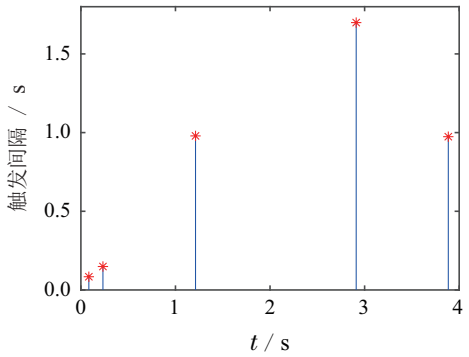

图 5　事件触发机制

Fig. 5　Event triggered mechanism

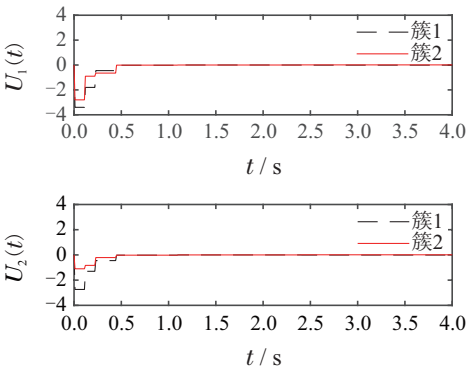

图 6　事件触发控制器$U_1(t)$和$U_2(t)$的输出

Fig. 6　Output of event triggered controller $U_1(t)$ and $U_2(t)$

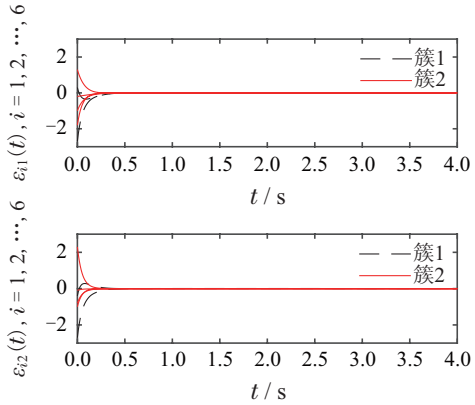

图 7 网络误差轨迹

Fig. 7 Error trajectory of networks

5 结论

本文通过自适应事件触发牵制控制方法研究了多时滞随机耦合神经网络的簇同步. 在耦合神经网络中, 同一簇中的节点可以与相应的孤立节点保持一致, 而不同的簇具有不同的同步状态. 对于多时滞随机耦合神经网络, 使用自适应事件触发牵制策略可以在均方意义上以指数速率实现簇同步. 最后, 通过一个仿真数例验证了所得理论结果的有效性.

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