基于扰动观测器的水面无人船自适应模糊控制器设计

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摘要: 为了使水面无人船(USV)获得更好的跟踪性能, 本文设计了基于扰动观测器和命令滤波器的自适应模糊控制器. 对于该系统存在建模不确定性和外部环境的扰动, 采用模糊逻辑系统(FLS)和一个新的扰动观测器对其进行逼近和补偿. 在扰动观测器和控制器中加入了一个新的自适应参数, 用来改善控制精度. 基于此, 本文设计了命令滤波反步控制方法, 可以保证系统在所有状态下都是有界的, 且跟踪误差在有限时间内小于规定的精度. 仿真结果显示该方法有效, 且可以满足给定的控制精度.

关键词: 模糊逻辑; 水面无人船; 扰动观测器; Lyapunov稳定性定理; 自适应控制系统

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Design of adaptive fuzzy controller for unmanned surface vessel based on disturbance observer

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Abstract: In order to achieve the better tracking performance of unmanned surface vehicle (USV), an adaptive fuzzy controller based on the disturbance observer and command filter is designed in this paper. The fuzzy logic system (FLS) and a new disturbance observer are used to approximate and compensate the system with modeling uncertainty and disturbance of external environment. A new adaptive parameter is added to the disturbance observer and controller to improve the control accuracy. Based on this, a command-filter-based backstepping control method is designed to ensure that all the states of the system is bounded and the tracking errors are less than a prescribed accuracy in finite time. The simulation results show that this method is effective and can satisfy the given control precision.

Key words: fuzzy logic; unmanned surface vessel; disturbance observer; Lyapunov stability theorem; adaptive control system

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1 引言

随着科技的进步, 水面无人船(unmanned surface vehicle, USV)能够以智能和安全的优势承担海洋任务, 因此, 长期以来在军事和商业领域得到广泛应用[1–4], 如海洋勘探和采集、海上救援、环境检查等. 然而, 由于系统中存在建模不确定性和外部扰动等因素, 使得USV轨迹跟踪在控制精度方面难以得到保证. 而精确的轨迹跟踪是保证任务高效完成的重要保证, 因此, 轨迹跟踪控制成为现阶段USV研究领域的主要研究方向.

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在风、波浪、海流等复杂工况下, 非线性USV已经成为现阶段的研究热点, 然而在实现精确的轨迹跟踪方面仍然面临着很大的挑战. 为了解决轨迹跟踪中控制精度的问题, 本文对USV非线性动力学模型进行了线性化和相应的线性控制处理, 这样可以使其在特定的平衡点附近具有良好的跟踪性能. 例如在文献[5]中提出了一种基于可辨识非线性动态模型的USV自适应轨迹跟踪控制算法, 其中虚拟控制输入、动态面和自适应项应用于处理匹配和不匹配的不确定性. 文献[6]提出了一种基于有限时间观测器的时变大滑移角和未知外部扰动下的USV跟踪控制方法. 虽然这些利用非线性USV模型的研究对特定平衡点具有良好的跟踪能力, 但它们只适用于某些特定轨迹(例如直线路径). 然而在实际中, USV并不只是在平衡点上工作. 由于存在来自复杂海洋条件的外部未知扰动, 使得USV远离平衡点, 甚至会使其跟踪性能变差. 为了实现USV更好的跟踪性能, 文献[7–8]解决了速度和航向控制的问题, 优化了USV的电力续航能力. 在非线性系统[9–13]的轨迹跟踪问题上提出了滑模控制[14]、自适应反步控制[15–19]和模型预测控制[20]等控制算法, 其中反步法通过反向设计使Lyapunov函数和控制器的设计过程系统化、结构化, 而且还可以控制相对阶为n的非线性系统.

为了提高系统的观测性能并减轻模型不确定性的不利影响, 文献[21–24]设计了自适应模糊逻辑系统和神经网络(neural networks, NNs)来估计USV的运动学状态, 其中模型的不确定性通过模糊逻辑系统 (fuzzy logic system, FLS)或 NNs 来逼近. 针对外界环境扰动的影响, 文中采用观测器[25–31]对其进行估计和补偿. 文献[32]提出了一个基于扰动观测器的快速非奇异模糊终端滑模控制器, 用于USV的轨迹跟踪控制, 但是文中设计的扰动观测器只适用于估计慢时变干扰, 且假设总扰动的导数为零的情况, 然而这种情况在实际中是不合理的. 文献[27]提出了一种基于扰动观测器的滑模控制设计, 采用扰动观测器对外界干扰进行估计和补偿. 以上的方法虽能获得良好的跟踪性能, 但是并不能很好掌握其控制精度.

基于上述问题, 针对非线性USV的动力学模型, 本文设计了一种新的基于扰动观测器和命令滤波器的自适应模糊控制算法. 主要贡献如下:

1) 由于时变扰动是由风、浪和海流引起的, 因此幅值经常是剧烈变化的. 基于此, 本文设计了一个新的扰动观测器, 用来估计和补偿外部环境扰动的影响;

2) 设计了一个命令滤波器来降低反步设计中的复杂性, 同时可以有效的削弱对虚拟控制信号的需求, 巧妙地抑制了抖振现象. 对于命令滤波所产生的误差影响, 本文引入补偿信号来消除;

3) 该控制方法的调节参数相较于其他自适应模糊控制方法并不敏感, 方便了实际应用;

4) 为了解决控制精度不好掌握的问题, 本文引入了一个新的自适应参数a, 不仅保证系统的跟踪误差在有限时间内小于规定的精度, 还使其具有较快的响应速度. 最后, 基于Lyapunov稳定性定理, 保证了闭环系统状态的有界性, 实现了USV的高精度跟踪控制.

2 系统的数学模型

USV的数学模型可以描述为式(1)–(2), 分别为运动学模型和动力学模型, 即

$$\begin{cases} \dot{x} = u\cos\psi - v\sin\psi, \\ \dot{y} = u\sin\psi + v\cos\psi, \\ \dot{\psi} = r, \end{cases} \tag{1}$$

$$\begin{cases} \dot{u} = d_u + \dfrac{1}{m_u}f_u(\nu) + \omega_u(t), \\ \dot{v} = d_v + \dfrac{1}{m_v}f_v(\nu) + \omega_v(t), \\ \dot{r} = d_r + \dfrac{1}{m_r}f_r(\nu) + \omega_r(t), \end{cases} \tag{2}$$

其中: x, y为以地球为固定坐标系的位置坐标; ψ表示USV的航向角; $\nu = [u\ \ v\ \ r]^{\mathrm{T}}$为定身坐标系中的速度向量, 其中$u, v, r$分别为浪涌速度、摇摆速度和偏航速度; $m_i (i = u, v, r$, 如不作特殊说明, 下同)为船舶的惯性参数, 本文认为是已知的; d_u, d_v, d_r为模型中的控制输入; $\omega_i(t)$为由波浪、风和海流产生的环境扰动, 是外部不确定性. 式(2)船舶动力学中的不确定性函数为

$$\begin{aligned} f_u(\nu) &= -X_u u + m_v vr - X_{|u|u}|u|u, \\ f_v(\nu) &= -Y_v v - m_u ur - Y_{|v|v}|v|v, \\ f_r(\nu) &= -N_r r + (m_u - m_v)uv - N_{|r|r}|r|r, \end{aligned}$$

其中带不同下标的X, Y, N表示水动力参数.

从式(1)中的运动学回路中可以看出, 虚拟船舶在没有动力学回路的情况下可以生成参考路径式(3), 即

$$\begin{cases} \dot{x}_{\mathrm{d}} = u_{\mathrm{d}}\cos\psi_{\mathrm{d}} - v_{\mathrm{d}}\sin\psi_{\mathrm{d}}, \\ \dot{y}_{\mathrm{d}} = u_{\mathrm{d}}\sin\psi_{\mathrm{d}} + v_{\mathrm{d}}\cos\psi_{\mathrm{d}}, \\ \dot{\psi}_{\mathrm{d}} = r_{\mathrm{d}}, \end{cases} \tag{3}$$

其中带下标"d"的变量表示系统(1)对应的虚拟船的状态.

此外, 考虑到以下假设.

假设 1 假设$\omega(t)$和$\dot{\omega}(t)$是有界的, 因此存在ℓ和λ满足$|\omega_i(t)| \leqslant \ell, |\dot{\omega}(t)| \leqslant \lambda$(为了化简写法, 在下文中省略$t$).

假设 2 期望速度$u_{\mathrm{d}}, v_{\mathrm{d}}, r_{\mathrm{d}}$都是有界的, 且有界导数$\dot{u}_{\mathrm{d}}, \dot{v}_{\mathrm{d}}, \dot{r}_{\mathrm{d}}$可微.

3 位置跟踪器的设计

3.1 模糊逻辑系统

本文采用FLS[32]来逼近式(2)中的不确定性函数, FLS由4部分组成, 即模糊规则库、模糊器、模糊推理机和解模糊器. 模糊规则库是对系统的经验总结, 将这些经验用模糊条件语句IF-THEN表达出来. 本文考虑模糊规则库由N条规则组成的情况, 其中第l条的规则为

$$R^l: \text{IF } \beta_1 \text{ is } F_1^l,\ \beta_2 \text{ is } F_2^l,\ \cdots,\ \beta_n \text{ is } F_n^l,$$
$$\text{THEN } \vartheta \text{ is } G^l,$$

其中: $l=1,2,\cdots,N$, $\beta=[\beta_1\ \ \beta_2\ \ \cdots\ \ \beta_n]^{\mathrm T}$为系统的输入; ϑ为系统的输出; $F_i^l(i=1,2,\cdots,n)$和G^l分别为定义在β和ϑ论域上的模糊集; $\mu_{F_i^l(\beta_i)}, \mu_{G^l(\vartheta)}$表示这些模糊集的隶属度函数. 利用单点模糊化、乘积推理和中心平均去模糊化, FLS可以表示为

$$\vartheta=\frac{\sum_{l=1}^{N}\bar\vartheta_l\prod_{i=1}^{n}\mu_{F_i^l}(\beta_i)}{\sum_{l=1}^{N}\prod_{i=1}^{n}\mu_{F_i^l}(\beta_i)}, \tag{4}$$

其中$\bar\vartheta_l=\max\limits_{\vartheta\in\mathbb R}\mu_{G^l}(\vartheta)$, 在简化形式下模糊基函数定义为

$$\varPhi_l(\beta)=\frac{\prod_{i=1}^{n}\mu_{F_i^l}(\beta_i)}{\sum_{l=1}^{N}\prod_{i=1}^{n}\mu_{F_i^l}(\beta_i)}, \tag{5}$$

定义式中$\varPhi(\beta)$=$[\varPhi_1(\beta)\ \ \varPhi_2(\beta)\ \ \cdots\ \ \varPhi_N(\beta)]^{\mathrm T}$, 将FLS的权值表示为$\varTheta=\left[\bar\vartheta_1\ \ \bar\vartheta_2\ \ \cdots\ \ \bar\vartheta_N\right]^{\mathrm T}$, 对式(4)进一步简化为$\vartheta=\varTheta^{\mathrm T}\varPhi(\beta)$. 本文引入以下引理[33].

引理 1 $f(\beta)$是一个定义在紧集$\varOmega$上的连续函数, 对于任意给定常数$\varepsilon>0$, 存在FLS满足

$$\sup_{\beta\in\varOmega}\left|f(\beta)-\varTheta^{\mathrm T}\varPhi(\beta)\right|\leqslant\varepsilon. \tag{6}$$

系统(2)中的不确定性函数$f_i(\nu)$可以用FLS逼近, 其中FLS中的输入β为系统(2)中的速度向量ν, 定义模糊最优参数为

$$\varTheta_i^*=\arg\min_{\varTheta_i\in\mathbb R_i}[\sup|\hat f_i(\nu\,|\varTheta_i)-f_i(\nu)|], \tag{7}$$

式中

$$\hat f_i(\nu\,|\varTheta_i)=\varTheta_i^{\mathrm T}\varPhi_i(\nu). \tag{8}$$

模糊最小逼近误差ε_i定义为

$$\varepsilon_i=f_i(\nu)-\hat f_i(\nu\,|\varTheta_i^*), \tag{9}$$

且满足$|\varepsilon_i|\leqslant\varepsilon_i^*$, 其中$\varepsilon_i^*>0$是一个未知常数, 它表示误差$\varepsilon_i$的界限. 此外, 由于$\varTheta_i^*$是未知的, 令$\hat\varTheta_i$是对$\varTheta_i^*$的估计, 其误差向量$\tilde\varTheta_i=\varTheta_i^*-\hat\varTheta_i$.

3.2 控制器的设计

定义系统的误差变量为

$$\begin{cases}x_{\mathrm e}=(x-x_{\mathrm d})\cos\psi+(y-y_{\mathrm d})\sin\psi,\\ y_{\mathrm e}=-(x-x_{\mathrm d})\sin\psi+(y-y_{\mathrm d})\cos\psi,\\ \psi_{\mathrm e}=\psi-\psi_{\mathrm d},\end{cases} \tag{10}$$

对上式求导, 并结合式(1)(3)可以得到

$$\begin{cases}\dot x_{\mathrm e}=u-u_{\mathrm d}\cos\psi_{\mathrm e}-v_{\mathrm d}\sin\psi_{\mathrm e}+y_{\mathrm e}r,\\ \dot y_{\mathrm e}=v+u_{\mathrm d}\sin\psi_{\mathrm e}-v_{\mathrm d}\cos\psi_{\mathrm e}-x_{\mathrm e}r,\\ \dot\psi_{\mathrm e}=r-r_{\mathrm d},\end{cases} \tag{11}$$

对上式求导可得

$$\begin{cases}\ddot x_{\mathrm e}=\dot u-\dot u_{\mathrm d}\cos\psi_{\mathrm e}+u_{\mathrm d}\sin\psi_{\mathrm e}\dot\psi_{\mathrm e}-\\ \quad\dot v_{\mathrm d}\sin\psi_{\mathrm e}-v_{\mathrm d}\cos\psi_{\mathrm e}\dot\psi_{\mathrm e}+\\ \quad\dot y_{\mathrm e}r+y_{\mathrm e}\ddot\psi_{\mathrm e}+y_{\mathrm e}\dot r_{\mathrm d},\\ \ddot y_{\mathrm e}=\dot v+\dot u_{\mathrm d}\sin\psi_{\mathrm e}+u_{\mathrm d}\cos\psi_{\mathrm e}\dot\psi_{\mathrm e}-\\ \quad\dot v_{\mathrm d}\cos\psi_{\mathrm e}+v_{\mathrm d}\sin\psi_{\mathrm e}\dot\psi_{\mathrm e}-\\ \quad\dot x_{\mathrm e}r-x_{\mathrm e}\ddot\psi_{\mathrm e}-x_{\mathrm e}\dot r_{\mathrm d},\\ \ddot\psi_{\mathrm e}=\dot r-\dot r_{\mathrm d}.\end{cases} \tag{12}$$

根据式(2)(11)–(12)可以得到

$$\begin{cases}\frac{\mathrm d}{\mathrm dt}(\dot x_{\mathrm e}-y_{\mathrm e}\dot\psi_{\mathrm e})=\\ d_u+\frac{1}{m_u}f_u(\nu)+\omega_u-\dot u_{\mathrm d}\cos\psi_{\mathrm e}-\dot v_{\mathrm d}\sin\psi_{\mathrm e}+\\ u_{\mathrm d}\sin\psi_{\mathrm e}(r-r_{\mathrm d})-v_{\mathrm d}\cos\psi_{\mathrm e}(r-r_{\mathrm d})+\\ y_{\mathrm e}\dot r_{\mathrm d}+(v+u_{\mathrm d}\sin\psi_{\mathrm e}-v_{\mathrm d}\cos\psi_{\mathrm e}-x_{\mathrm e}r)\,r_{\mathrm d},\\ \frac{\mathrm d}{\mathrm dt}(\dot y_{\mathrm e}+x_{\mathrm e}\dot\psi_{\mathrm e})=\\ d_v+\frac{1}{m_v}f_v(\nu)+\omega_v+\dot u_{\mathrm d}\sin\psi_{\mathrm e}-\dot v_{\mathrm d}\cos\psi_{\mathrm e}+\\ u_{\mathrm d}\cos\psi_{\mathrm e}(r-r_{\mathrm d})+v_{\mathrm d}\sin\psi_{\mathrm e}(r-r_{\mathrm d})-\\ x_{\mathrm e}\dot r_{\mathrm d}-(u-u_{\mathrm d}\cos\psi_{\mathrm e}-v_{\mathrm d}\sin\psi_{\mathrm e}+y_{\mathrm e}r)\,r_{\mathrm d},\\ \frac{\mathrm d}{\mathrm dt}\dot\psi_{\mathrm e}=d_r+\frac{1}{m_r}f_r(\nu)+\omega_r-\dot r_{\mathrm d},\end{cases} \tag{13}$$

即

$$\frac{\mathrm d}{\mathrm dt}(M_0\dot\eta_{\mathrm e})=G+F+\omega, \tag{14}$$

其中: $G=[d_u-\dot u_{\mathrm d}\cos\psi_{\mathrm e}+u_{\mathrm d}\sin\psi_{\mathrm e}(r-r_{\mathrm d})+(v+u_{\mathrm d}\sin\psi_{\mathrm e}-v_{\mathrm d}\cos\psi_{\mathrm e}-x_{\mathrm e}r)r_{\mathrm d}-\dot v_{\mathrm d}\sin\psi_{\mathrm e}-v_{\mathrm d}\cos\psi_{\mathrm e}(r-r_{\mathrm d})+y_{\mathrm e}\dot r_{\mathrm d}\quad d_v+\dot u_{\mathrm d}\sin\psi_{\mathrm e}-(u-u_{\mathrm d}\cos\psi_{\mathrm e}-v_{\mathrm d}\sin\psi_{\mathrm e}+y_{\mathrm e}r)r_{\mathrm d}-\dot v_{\mathrm d}\cos\psi_{\mathrm e}+u_{\mathrm d}\cos\psi_{\mathrm e}(r-r_{\mathrm d})+v_{\mathrm d}\sin\psi_{\mathrm e}(r-r_{\mathrm d})-x_{\mathrm e}\dot r_{\mathrm d}\quad d_r-\dot r_{\mathrm d}]^{\mathrm T}$, $\omega=[\omega_u\ \ \omega_v\ \ \omega_r]^{\mathrm T}$, $\dot\eta_{\mathrm e}=[\dot x_{\mathrm e}\ \ \dot y_{\mathrm e}\ \ \dot\psi_{\mathrm e}]^{\mathrm T}$, $M_0=[1,0,-y_{\mathrm e};\ \ 0,1,x_{\mathrm e};\ \ 0,0,1]$, $F=[\frac{1}{m_u}f_u(\nu)\ \ \ \frac{1}{m_u}f_v(\nu)\ \ \ \frac{1}{m_u}f_r(\nu)]^{\mathrm T}$.

为了估计和补偿USV由风、波、流引起的外部扰动, 设计如下扰动观测器:

$$\dot S=-L(G+\hat F)-aL\hat\omega, \tag{15}$$

其中: a是一个新的调节参数, $L=\mathrm{diag}\,\{l_u,l_v,l_r\}(l_u, l_v, l_r$为正实数), $\hat\omega=[\hat\omega_u\ \ \hat\omega_v\ \ \hat\omega_r]^{\mathrm T}$, $\hat F=[\frac{1}{m_u}\hat\varTheta_u^{\mathrm T}\varPhi_u$

$(\nu)\ \dfrac{1}{m_v}\hat{\Theta}_v^{\mathrm{T}}\varPhi_v(\nu)\ \ \dfrac{1}{m_r}\hat{\Theta}_r^{\mathrm{T}}\varPhi_r(\nu)]^{\mathrm{T}}$为非线性函数$F$的FLS逼近函数.

注 1 所提出扰动观测器的设计方法不同于现有的文献[24–30]等, 本文首次在扰动观测器中加入了一个自适应参数a, 目的是为了削弱系统中扰动的影响. 在接下来的设计中, 它的数值可以通过一个动态方程来改变. 正是因为该自适应参数的加入, 才能够在理论上确保整个闭环系统的跟踪精度.

设环境扰动的估计值为

$$\hat{\omega}=\frac{S+N}{a}, \tag{16}$$

其中$N=LM_0\dot{\eta}_{\mathrm{e}}$, 假设

$$\tilde{\omega}=\frac{\omega}{a}-\hat{\omega}, \tag{17}$$

对式(17)两边同时乘以a, 并求导可得

$$\dot{a}\tilde{\omega}+a\dot{\tilde{\omega}}=\dot{\omega}-\dot{S}-\dot{N}, \tag{18}$$

移项, 得

$$\begin{aligned}\dot{\tilde{\omega}}=&-\frac{\dot{a}}{a}\tilde{\omega}+\frac{\dot{\omega}}{a}-\frac{\dot{S}+\dot{N}}{a}=\\&-\frac{L}{a}(-G-\hat{F}-a\hat{\omega}+\frac{\mathrm{d}}{\mathrm{d}t}(M_0\dot{\eta}_{\mathrm{e}}))-\\&\frac{\dot{a}}{a}\tilde{\omega}+\frac{\dot{\omega}}{a}=\\&-\frac{\dot{a}}{a}\tilde{\omega}+\frac{\dot{\omega}}{a}-\frac{L}{a}(F-\hat{F})-L\tilde{\omega}.\end{aligned} \tag{19}$$

引理 2 定义命令滤波器[34]为

$$\begin{cases}\dot{\eta}_{i1}=\eta_{i2},\\ \dot{\eta}_{i2}=\dfrac{1}{\varpi^2}\left(-\zeta_{i1}\arctan(\eta_{i1}-\alpha_i)-\right.\\ \qquad\left.\zeta_{i2}\arctan(\varpi\eta_{i2})\right),\end{cases} \tag{20}$$

其中: η_{i1},η_{i2}为系统的状态变量; α_i为虚拟控制信号; $\varpi,\zeta_{i1},\zeta_{i2}$为大于零的可调节参数, 对于$\tau>0$和$\rho\tau>2$, 有

$$\eta_\gamma-\alpha_i^{(\gamma-1)}=_{\mathrm{a}}(\varpi^{\rho\tau-\gamma+1}), \tag{21}$$

其中: $\gamma=1,2,\cdots,n$, $_{\mathrm{a}}(\varpi^{\rho\tau-\gamma+1})$是$\eta_\gamma$与$\alpha_i^{(\gamma-1)}$之间的$\varpi^{\rho\tau-\gamma+1}$阶逼近误差; $\tau=\dfrac{1-\hbar}{\hbar}$, $\hbar\in(0,\min\{\dfrac{\rho}{\rho+2},\dfrac{1}{2}\})$.

注 2 在本文中, 笔者引用了有限时间命令滤波器, 它可以更快地逼近虚拟控制信号的导数, 且可以有效的削弱对虚拟控制信号的需求, 巧妙地抑制了抖振现象. 由于命令滤波器可能会导致滤波误差, 这将会增加获取小跟踪误差的难度. 在本文中, 将设计补偿信号, 以消除由命令滤波器引起的误差的影响.

对于$\lambda=x,y,\psi,u,v,r$(如不作特殊说明, 下同), 补偿信号ξ_λ定义为

$$\begin{cases}\dot{\xi}_x=-k_x\xi_x+\xi_u+\eta_{u1}-\alpha_u,\\ \dot{\xi}_y=-k_y\xi_y+\xi_v+\eta_{v1}-\alpha_v,\\ \dot{\xi}_\psi=-k_\psi\xi_\psi+\xi_r+\eta_{r1}-\alpha_r,\end{cases} \tag{22}$$

$$\begin{cases}\dot{\xi}_u=-(k_u+a^3)\xi_u-\xi_x,\\ \dot{\xi}_v=-(k_v+a^3)\xi_v-\xi_y,\\ \dot{\xi}_r=-(k_r+a^3)\xi_r-\xi_\psi.\end{cases} \tag{23}$$

其中k_λ为大于零的可调节参数.

定义虚拟控制信号α_i为

$$\begin{cases}\alpha_u=-k_xx_{\mathrm{e}}+u_{\mathrm{d}}\cos\psi_{\mathrm{e}}+v_{\mathrm{d}}\sin\psi_{\mathrm{e}}-\\ \qquad y_{\mathrm{e}}(\xi_r+\eta_{r1}),\\ \alpha_v=-k_yy_{\mathrm{e}}-u_{\mathrm{d}}\sin\psi_{\mathrm{e}}+v_{\mathrm{d}}\cos\psi_{\mathrm{e}}+\\ \qquad x_{\mathrm{e}}(\xi_r+\eta_{r1}),\\ \alpha_r=-k_\psi\psi_{\mathrm{e}}+r_{\mathrm{d}}.\end{cases} \tag{24}$$

接下来对定义的跟踪误差进行坐标变换

$$\begin{cases}P_x=x_{\mathrm{e}}-\xi_x,P_y=y_{\mathrm{e}}-\xi_y,P_\psi=\psi_{\mathrm{e}}-\xi_\psi,\\ z_u=u-\eta_{u1},z_v=v-\eta_{v1},z_r=r-\eta_{r1},\\ P_u=z_u-\xi_u,P_v=z_v-\xi_v,P_r=z_r-\xi_r.\end{cases} \tag{25}$$

选取自适应控制律

$$\dot{\hat{\Theta}}_i=ac_iP_i\varPhi_i(\nu)-b_i\hat{\Theta}_i, \tag{26}$$

其中: c_i和b_i为可调节参数, 自适应参数a定义如下:

$$\begin{aligned}\dot{a}=q\max\{&|x-x_{\mathrm{d}}|-\delta,|y-y_{\mathrm{d}}|-\delta,\\&|\psi-\psi_{\mathrm{d}}|-\delta,0\},\end{aligned} \tag{27}$$

式中: δ表示允许的最大跟踪误差, q表示大于零的可调节参数. 可以看出随着误差的增长, a是一个不减函数.

考虑到整个闭环系统, 选取以下Lyapunov函数:

步骤 1 考虑到误差P_x, 选取第1个Lyapunov函数为

$$V_{11}=\frac{1}{2}P_x^2, \tag{28}$$

对上式求导得

$$\begin{aligned}\dot{V}_{11}=&P_x\dot{P}_x=P_x(\dot{x}_{\mathrm{e}}-\dot{\xi}_x)=\\&P_x(u-u_{\mathrm{d}}\cos\psi_{\mathrm{e}}-v_{\mathrm{d}}\sin\psi_{\mathrm{e}}+y_{\mathrm{e}}r-\\&(-k_x\xi_x+\xi_u+\eta_{u1}-\alpha_u))=\\&P_x(P_u-k_xP_x+y_{\mathrm{e}}r-y_{\mathrm{e}}\xi_r-y_{\mathrm{e}}\eta_{r1})=\\&-k_xP_x^2+P_xP_u+P_xy_{\mathrm{e}}P_r.\end{aligned} \tag{29}$$

步骤 2 考虑到误差$P_u,\tilde{\omega}_u,\tilde{\Theta}_u$, 选取第2个Lyapunov函数为

$$V_{21}=\frac{1}{2}P_u^2+\frac{1}{2}\tilde{\omega}_u^{\mathrm{T}}\tilde{\omega}_u+\frac{1}{2ac_um_u}\tilde{\Theta}_u^{\mathrm{T}}\tilde{\Theta}_u, \tag{30}$$

对其求导得

$\dot{V}_{21}=$

$$
\begin{aligned}
&P_u\dot{P}_u+\tilde{\omega}_u^{\mathrm{T}}\dot{\tilde{\omega}}_u-\frac{1}{ac_um_u}\tilde{\Theta}_u^{\mathrm{T}}\dot{\tilde{\Theta}}_u-\\
&\frac{\dot{a}}{2a^2c_um_u}\tilde{\Theta}_u^{\mathrm{T}}\tilde{\Theta}_u=\\
&P_u(d_u+\frac{1}{m_u}f_u(\nu)+\omega_u-\dot{\eta}_{u1}+\\
&(k_u+a^3)\xi_u+\xi_x)+\tilde{\omega}_u(-\frac{\dot{a}}{a}\tilde{\omega}_u+\frac{\dot{\omega}_u}{a}-\\
&\frac{l_u}{a}(\frac{1}{m_u}f_u(\nu)-\frac{1}{m_u}\hat{\Theta}_u^{\mathrm{T}}\varPhi_u(\nu))-l_u\tilde{\omega}_u)-\\
&\frac{1}{ac_um_u}\tilde{\Theta}_u^{\mathrm{T}}\dot{\tilde{\Theta}}_u-\frac{\dot{a}}{2a^2c_um_u}\tilde{\Theta}_u^{\mathrm{T}}\tilde{\Theta}_u.
\end{aligned}\tag{31}
$$

选取实际控制输入

$$
\begin{aligned}
d_u=&-(k_u+a^3)z_u-\frac{1}{m_u}\hat{\Theta}_u^{\mathrm{T}}\varPhi_u(\nu)-\\
&a\hat{\omega}_u+\dot{\eta}_{u1}-x_{\mathrm{e}}.
\end{aligned}\tag{32}
$$

另外, 根据Young不等式有

$$
\begin{cases}
\dfrac{P_i}{m_i}\left(f_i\left(\nu\right)-\Theta^{*\mathrm{T}}\varPhi_i\left(\nu\right)\right)\leqslant\dfrac{a^3}{2}P_i^2+\dfrac{\varepsilon_i^2}{2a^3},\\
P_i\left(\omega_i-a\hat{\omega}_i\right)\leqslant\dfrac{a^3}{2}P_i^2+\dfrac{\tilde{\omega}_i^2}{2a},\\
\tilde{\Theta}_i^{\mathrm{T}}\hat{\Theta}_i\leqslant-\dfrac{1}{2}\tilde{\Theta}_i^{\mathrm{T}}\tilde{\Theta}_i+\dfrac{1}{2}\Theta_i^{*\mathrm{T}}\Theta_i^*,
\end{cases}\tag{33}
$$

将式(32)代入式(31), 并结合式(33)最终可以得到

$$
\begin{aligned}
&\dot{V}_{21}=\\
&P_u(-(k_u+a^3)z_u+\frac{1}{m_u}(f_u(\nu)-\hat{\Theta}_u^{\mathrm{T}}\varPhi_u(\nu))+\\
&\omega_u-a\hat{\omega}_u+(k_u+a^3)\xi_u-P_x)-(\frac{\dot{a}}{a}+l_u)\tilde{\omega}_u^2+\\
&\frac{\tilde{\omega}_u\dot{\omega}_u}{a}-\frac{l_u}{am_u}\tilde{\omega}_u(f_u(\nu)-\hat{\Theta}_u^{\mathrm{T}}\varPhi_u(\nu))-\\
&\frac{1}{c_um_u}\tilde{\Theta}_u^{\mathrm{T}}(P_u\varPhi_u(\nu)-\frac{b_u}{a}\hat{\Theta}_u)-\\
&\frac{\dot{a}}{2a^2c_um_u}\tilde{\Theta}_u^{\mathrm{T}}\tilde{\Theta}_u=\\
&\frac{P_u}{m_u}(f_u(\nu)-\hat{\Theta}_u^{\mathrm{T}}\varPhi_u(\nu)-\tilde{\Theta}_u^{\mathrm{T}}\varPhi_u(\nu))+\\
&P_u(\omega_u-a\hat{\omega}_u)+P_u(k_u+a^3)(\xi_u-z_u)-\\
&\frac{l_u}{am_u}\tilde{\omega}_u(f_u(\nu)-\Theta_u^{*\mathrm{T}}\varPhi_u(\nu)+\tilde{\Theta}_u^{\mathrm{T}}\varPhi_u(\nu))-\\
&(\frac{\dot{a}}{a}+l_u)\tilde{\omega}_u^2+\frac{\tilde{\omega}_u\dot{\omega}_u}{a}+\frac{b_u}{ac_um_u}\tilde{\Theta}_u^{\mathrm{T}}\hat{\Theta}_u-\\
&\frac{\dot{a}}{2a^2c_um_u}\tilde{\Theta}_u^{\mathrm{T}}\tilde{\Theta}_u-P_uP_x\leqslant\\
&\frac{\varepsilon_u^2}{2a^3}+\frac{\dot{\omega}_u^2-\dfrac{l_u}{m_u}\varepsilon_u^2}{2a}-(\frac{\dot{a}-1}{a}+l_u+\frac{l_u}{am_u})\tilde{\omega}_u^2-\\
&k_uP_u^2-P_uP_x-(\frac{l_u}{2am_u}+\frac{b_u}{2ac_um_u})\tilde{\Theta}_u^{\mathrm{T}}\tilde{\Theta}_u+\\
&\frac{b_u}{2ac_um_u}\Theta_u^{*\mathrm{T}}\Theta_u^*.
\end{aligned}\tag{34}
$$

步骤 3 考虑到误差P_y, 选取第3个Lyapunov函数为

$$
V_{12}=\frac{1}{2}P_y^2,\tag{35}
$$

对其求导得

$$
\begin{aligned}
\dot{V}_{12}=&P_y\dot{P}_y=P_y(\dot{y}_{\mathrm{e}}-\dot{\xi}_y)=\\
&P_y(v+u_{\mathrm{d}}\sin\psi_{\mathrm{e}}-v_{\mathrm{d}}\cos\psi_{\mathrm{e}}-x_{\mathrm{e}}r-\\
&(-k_y\xi_y+\xi_v+\eta_{v1}-\alpha_v))=\\
&P_y\left(P_v-k_yP_y-x_{\mathrm{e}}r+x_{\mathrm{e}}\xi_r+x_{\mathrm{e}}\eta_{r1}\right)=\\
&-k_yP_y^2+P_yP_v-P_yx_{\mathrm{e}}P_r.
\end{aligned}\tag{36}
$$

步骤 4 考虑到误差$P_v,\tilde{\omega}_v,\tilde{\Theta}_v$, 选取第4个Lyapunov函数为

$$
V_{22}=\frac{1}{2}P_v^2+\frac{1}{2}\tilde{\omega}_v^{\mathrm{T}}\tilde{\omega}_v+\frac{1}{2ac_vm_v}\tilde{\Theta}_v^{\mathrm{T}}\tilde{\Theta}_v,\tag{37}
$$

对其求导得

$$
\begin{aligned}
&\dot{V}_{22}=\\
&P_v\dot{P}_v+\tilde{\omega}_v^{\mathrm{T}}\dot{\tilde{\omega}}_v-\frac{1}{ac_vm_v}\tilde{\Theta}_v^{\mathrm{T}}\dot{\tilde{\Theta}}_v-\\
&\frac{\dot{a}}{2a^2c_vm_v}\tilde{\Theta}_v^{\mathrm{T}}\tilde{\Theta}_v=\\
&P_v(d_v+\frac{1}{m_v}f_v(\nu)+\omega_v-\dot{\eta}_{v1}+\\
&(k_v+a^3)\xi_v+\xi_y)+\tilde{\omega}_v(-\frac{\dot{a}}{a}\tilde{\omega}_v+\frac{\dot{\omega}_v}{a}-\\
&\frac{l_v}{a}(\frac{1}{m_v}f_v(\nu)-\frac{1}{m_v}\hat{\Theta}_v^{\mathrm{T}}\varPhi_v(\nu))-l_v\tilde{\omega}_v)-\\
&\frac{1}{ac_vm_v}\tilde{\Theta}_v^{\mathrm{T}}\dot{\tilde{\Theta}}_v-\frac{\dot{a}}{2a^2c_vm_v}\tilde{\Theta}_v^{\mathrm{T}}\tilde{\Theta}_v.
\end{aligned}\tag{38}
$$

选取实际控制输入

$$
\begin{aligned}
d_v=&-\left(k_v+a^3\right)z_v-\frac{1}{m_v}\hat{\Theta}_v^T\varPhi_v\left(\nu\right)-\\
&a\hat{\omega}_v+\dot{\eta}_{v1}-y_{\mathrm{e}}.
\end{aligned}\tag{39}
$$

将式(39)代入式(38), 并结合式(33)可得

$$
\begin{aligned}
&\dot{V}_{22}=\\
&P_v(-(k_v+a^3)z_v+\frac{1}{m_v}(f_v\left(\nu\right)-\hat{\Theta}_v^{\mathrm{T}}\varPhi_v\left(\nu\right))+\\
&\omega_v-a\hat{\omega}_v+(k_v+a^3)\xi_v-P_y)-(\frac{\dot{a}}{a}+l_v)\tilde{\omega}_v^2+\\
&\frac{\tilde{\omega}_v\dot{\omega}_v}{a}-\frac{l_v}{am_v}\tilde{\omega}_v(f_v(\nu)-\hat{\Theta}_v^{\mathrm{T}}\varPhi_v\left(\nu\right))-\\
&\frac{1}{c_vm_v}\tilde{\Theta}_v^{\mathrm{T}}(P_v\varPhi_v(\nu)-\frac{b_v}{a}\hat{\Theta}_v)-\frac{\dot{a}}{2a^2c_vm_v}\tilde{\Theta}_v^{\mathrm{T}}\tilde{\Theta}_v=\\
&\frac{P_v}{m_v}(f_v(\nu)-\hat{\Theta}_v^{\mathrm{T}}\varPhi_v(\nu)-\tilde{\Theta}_v^{\mathrm{T}}\varPhi_v(\nu))+\\
&P_v(\omega_v-a\hat{\omega}_v)+P_v(k_v+a^3)(\xi_v-z_v)-\\
&\frac{l_v}{am_v}\tilde{\omega}_v(f_v(\nu)-\Theta_v^{*\mathrm{T}}\varPhi_v(\nu)+\tilde{\Theta}_v^{\mathrm{T}}\varPhi_v(\nu))-
\end{aligned}
$$

$$(\frac{\dot{a}}{a}+l_v)\tilde{\omega}_v^2+\frac{\tilde{\omega}_v\dot{\omega}_v}{a}-P_vP_y+\frac{b_v}{ac_vm_v}\tilde{\Theta}_v^{\mathrm{T}}\hat{\Theta}_v-\frac{\dot{a}}{2a^2c_vm_v}\tilde{\Theta}_v^{\mathrm{T}}\tilde{\Theta}_v\leqslant\frac{\varepsilon_v^2}{2a^3}+\frac{\dot{\omega}_v^2-\dfrac{l_v}{m_v}\varepsilon_v^2}{2a}-(\frac{\dot{a}-1}{a}+l_v+\frac{l_v}{am_v})\tilde{\omega}_v^2-k_vP_v^2-P_vP_y-(\frac{l_v}{2am_v}+\frac{b_v}{2ac_vm_v})\tilde{\Theta}_v^{\mathrm{T}}\tilde{\Theta}_v+\frac{b_v}{2ac_vm_v}\Theta_v^{*\mathrm{T}}\Theta_v^*.\tag{40}$$

步骤 5 考虑到误差P_ψ, 选取第5个Lyapunov函数为

$$V_{13}=\frac{1}{2}P_\psi^2,\tag{41}$$

对其求导得

$$\dot{V}_{13}=P_\psi\dot{P}_\psi=P_\psi(\dot{\psi}_{\mathrm{e}}-\dot{\xi}_\psi)=P_\psi(r-r_{\mathrm{d}}+k_\psi\xi_\psi-\xi_r-\eta_{r1}+\alpha_r)=P_\psi(P_r-k_\psi P_\psi)=-k_\psi P_\psi^2+P_\psi P_r.\tag{42}$$

步骤 6 考虑到误差$P_r,\tilde{\omega}_r,\tilde{\Theta}_r$, 选取第6个Lyapunov函数为

$$V_{23}=\frac{1}{2}P_r^2+\frac{1}{2}\tilde{\omega}_r^{\mathrm{T}}\tilde{\omega}_r+\frac{1}{2ac_rm_r}\tilde{\Theta}_r^{\mathrm{T}}\tilde{\Theta}_r,\tag{43}$$

对其求导得

$$\dot{V}_{23}=P_r\dot{P}_r+\tilde{\omega}_r^{\mathrm{T}}\dot{\tilde{\omega}}_r-\frac{1}{ac_rm_r}\tilde{\Theta}_r^{\mathrm{T}}\dot{\hat{\Theta}}_r-\frac{\dot{a}}{2a^2c_rm_r}\tilde{\Theta}_r^{\mathrm{T}}\tilde{\Theta}_r=P_r(d_r+\frac{1}{m_r}f_r(\nu)+\omega_r-\dot{\eta}_{r1}+(k_r+a^3)\xi_r+\xi_\psi)+\tilde{\omega}_r(-\frac{\dot{a}}{a}\tilde{\omega}_r+\frac{\dot{\omega}_r}{a}-\frac{l_r}{a}(\frac{1}{m_r}f_r(\nu)-\frac{1}{m_r}\hat{\Theta}_r^{\mathrm{T}}\varPhi_r(\nu))-l_r\tilde{\omega}_r)-\frac{1}{ac_rm_r}\tilde{\Theta}_r^{\mathrm{T}}\dot{\hat{\Theta}}_r-\frac{\dot{a}}{2a^2c_rm_r}\tilde{\Theta}_r^{\mathrm{T}}\tilde{\Theta}_r.\tag{44}$$

选取实际控制输入

$$d_r=-(k_r+a^3)z_r-\frac{1}{m_r}\hat{\Theta}_r^{\mathrm{T}}\varPhi_r(\nu)-a\hat{\omega}_r+\dot{\eta}_{r1}-\psi_{\mathrm{e}}+P_yx_{\mathrm{e}}-P_xy_{\mathrm{e}}.\tag{45}$$

将式(45)代入式(44), 并结合式(33)可得

$$\dot{V}_{23}=P_r(-(k_r+a^3)z_r+\frac{1}{m_r}(f_r(\nu)-\hat{\Theta}_r^{\mathrm{T}}\varPhi_r(\nu))+\omega_r-a\hat{\omega}_r+(k_r+a^3)\xi_r-P_\psi+P_yx_{\mathrm{e}}-P_xy_{\mathrm{e}})-(\frac{\dot{a}}{a}+l_r)\tilde{\omega}_r^2-\frac{l_r}{am_r}\tilde{\omega}_r(f_r(\nu)-\hat{\Theta}_r^{\mathrm{T}}\varPhi_r(\nu))-\frac{1}{c_rm_r}\tilde{\Theta}_r^{\mathrm{T}}(P_r\varPhi_r(\nu)-\frac{b_r}{a}\hat{\Theta}_r)+\frac{\tilde{\omega}_r\dot{\omega}_r}{a}-\frac{\dot{a}}{2c_ra^2m_r}\tilde{\Theta}_r^{\mathrm{T}}\tilde{\Theta}_r=\frac{P_r}{m_r}(f_r(\nu)-\hat{\Theta}_r^{\mathrm{T}}\varPhi_r(\nu)-\tilde{\Theta}_r^{\mathrm{T}}\varPhi_r(\nu))+P_r(\omega_r-a\hat{\omega}_r)+P_r(k_r+a^3)(\xi_r-z_r)+P_rP_yx_{\mathrm{e}}-P_rP_xy_{\mathrm{e}}-(\frac{\dot{a}}{a}+l_r)\tilde{\omega}_r^2+\frac{\tilde{\omega}_r\dot{\omega}_r}{a}-\frac{l_r}{am_r}\tilde{\omega}_r(f_r(\nu)-\Theta_r^{*\mathrm{T}}\varPhi_r(\nu)+\tilde{\Theta}_r^{\mathrm{T}}\varPhi_r(\nu))+\frac{b_r}{ac_rm_r}\tilde{\Theta}_r^{\mathrm{T}}\hat{\Theta}_r-\frac{\dot{a}}{2c_ra^2m_r}\tilde{\Theta}_r^{\mathrm{T}}\tilde{\Theta}_r-P_rP_\psi\leqslant-k_rP_r^2-P_rP_\psi+P_rP_yx_{\mathrm{e}}-P_rP_xy_{\mathrm{e}}+\frac{\varepsilon_r^2}{2a^3}+\frac{\dot{\omega}_r^2-\dfrac{l_r}{m_r}\varepsilon_r^2}{2a}-(\frac{\dot{a}-1}{a}+l_r+\frac{l_r}{am_r})\tilde{\omega}_r^2-(\frac{l_r}{2am_r}+\frac{b_r}{2ac_rm_r})\tilde{\Theta}_r^{\mathrm{T}}\tilde{\Theta}_r+\frac{b_r}{2ac_rm_r}\Theta_r^{*\mathrm{T}}\Theta_r^*.\tag{46}$$

4 稳定性证明

根据上述控制器的设计过程, 选取如下形式的Lyapunov函数:

$$V=V_{11}+V_{21}+V_{12}+V_{22}+V_{13}+V_{23},\tag{47}$$

对式(47)求导得

$$\dot{V}=\dot{V}_{11}+\dot{V}_{21}+\dot{V}_{12}+\dot{V}_{22}+\dot{V}_{13}+\dot{V}_{23}.\tag{48}$$

由式(29)(34)(36)(40)(42)(46)得

$$\dot{V}\leqslant-k_xP_x^2-k_yP_y^2-k_\psi P_\psi^2-k_uP_u^2-k_vP_v^2-k_rP_r^2+\frac{\dot{\omega}_u^2+\dot{\omega}_v^2+\dot{\omega}_r^2-\dfrac{l_u}{m_u}\varepsilon_u^2-\dfrac{l_v}{m_v}\varepsilon_v^2-\dfrac{l_r}{m_r}\varepsilon_r^2}{2a}+\frac{\varepsilon_u^2+\varepsilon_v^2+\varepsilon_r^2}{2a^3}-(\frac{\dot{a}-1}{a}+l_u+\frac{l_u}{am_u})\tilde{\omega}_u^2-(\frac{\dot{a}-1}{a}+l_v+\frac{l_v}{am_v})\tilde{\omega}_v^2-(\frac{l_u}{2am_u}+\frac{b_u}{2ac_um_u})\tilde{\Theta}_u^{\mathrm{T}}\tilde{\Theta}_u-(\frac{\dot{a}-1}{a}+l_r+\frac{l_r}{am_r})\tilde{\omega}_r^2-(\frac{l_v}{2am_v}+\frac{b_v}{2ac_vm_v})\tilde{\Theta}_v^{\mathrm{T}}\tilde{\Theta}_v-(\frac{l_r}{2am_r}+\frac{b_r}{2ac_rm_r})\tilde{\Theta}_r^{\mathrm{T}}\tilde{\Theta}_r+\frac{b_u}{2ac_um_u}\Theta_u^{*\mathrm{T}}\Theta_u^*+\frac{b_v}{2ac_vm_v}\Theta_v^{*\mathrm{T}}\Theta_v^*+\frac{b_r}{2ac_rm_r}\Theta_r^{*\mathrm{T}}\Theta_r^*.\tag{49}$$

假设$A_0=\min\{(1+\dfrac{1}{am_u})l_u-1, b_u+c_ul_u, k_x, k_y, k_\psi, k_u, k_v, k_r, b_v+c_vl_v, b_r+c_rl_r, (1+\dfrac{1}{am_v})l_v-1, (1+\dfrac{1}{am_r})l_r-1\}$为大于零的实数,

$$B_0=\frac{b_u}{2c_um_u}\Theta_u^{*\mathrm{T}}\Theta_u^*-\frac{\frac{l_u}{m_u}\varepsilon_u^2+\frac{l_v}{m_v}\varepsilon_v^2+\frac{l_r}{m_r}\varepsilon_r^2}{2}+\frac{b_v}{2c_vm_v}\Theta_v^{*\mathrm{T}}\Theta_v^*+\frac{b_r}{2c_rm_r}\Theta_r^{*\mathrm{T}}\Theta_r^*+\frac{\dot\omega_u^2+\dot\omega_v^2+\dot\omega_r^2}{2}+\frac{\varepsilon_u^2+\varepsilon_v^2+\varepsilon_r^2}{2a^2}$$

为常数. 那么式(49)可变换为

$$\dot V\leqslant -A_0V+\frac{B_0}{a}.\tag{50}$$

定理 1　对于具有模型不确定性和时变扰动的系统(1)(2)和由式(3)产生的跟踪信号, 设计了扰动观测器(15)、命令滤波器(20)、补偿信号(22)–(23)、虚拟控制信号(24)、自适应控制律(26)、自适应参数(27), 则式(50)成立, 即对于有界初始条件$x, y, \psi, \omega_i, \Theta_i$的闭环系统来说有唯一解, 并且满足: 1) 闭环系统所有信号在有限时间内是有界的; 2) 存在有限时间$T>0$, 使$\sup|y-y_{\mathrm d}|\leqslant\delta$.

证　式(50)两边同时乘以$\mathrm e^{A_0t}$并对t积分得

$$V\leqslant (V(0)-\frac{B_0}{aA_0})\mathrm e^{-A_0t}+\frac{B_0}{aA_0},\tag{51}$$

存在一个正实数$\kappa>\dfrac{B_0}{aA_0}$使

$$(V(0)-\frac{B_0}{aA_0})\mathrm e^{-A_0t}+\frac{B_0}{aA_0}=\kappa.\tag{52}$$

证毕.

由定理1的证明过程可以得到V在稳定区间内所需的时间为

$$t=\frac{1}{A_0}\ln(\frac{V_0(0)-\frac{B_0}{aA_0}}{\kappa-\frac{B_0}{aA_0}}),\tag{53}$$

因此, 根据Lyapunov稳定性定理, 通过调节扰动观测器和自适应控制律的参数, 可以使得闭环系统的信号$\{P_\lambda, \tilde\omega, \tilde\Theta\}$在有限时间内有界. 下面证明$a$也是有界的.

证　因为a具有单调非递减性质, 所以$\lim\limits_{t\to\infty}a=+\infty$. 因此存在一个时间$T_1$满足

$$a\geqslant\frac{8B_0}{A_0^2P_\lambda^2},\ \forall t>T_1.\tag{54}$$

由式(51)(54)可得

$$V\leqslant (V(0)-\frac{A_0P_\lambda^2}{8})\mathrm e^{-A_0t}+\frac{A_0P_\lambda^2}{8}.\tag{55}$$

对于一个确定的时间$T_2>T_1$, 则有

$$\sup_{t\geqslant T_2}|P_\lambda|\leqslant\sqrt{\frac{2V}{A_0}}=\frac{\delta}{2}.\tag{56}$$

这时a的导数等于0, a不再继续增加, 证明a是有界的.　证毕.

注 3　在扰动观测器和自适应模糊输出反馈控制器中, 文中增加了一个有界不减的自适应参数a. 且在a的帮助下, 闭环系统的所有信号都是有界的, 并且在有限时间内跟踪误差小于规定的精度, 因此, 该扰动观测器和控制器对系统的不确定性和时变扰动具有鲁棒性.

5 仿真实验

在本节中, 为了验证本文所提出自适应模糊控制方法的有效性, 将其分别与系统中不含参数a和系统中不含模糊项的跟踪性能进行了比较. USV的初始值设置为$[x\ \ y\ \ \psi\ \ u\ \ v\ \ r]=[0.1\,\mathrm m\ \ 0.1\,\mathrm m\ \ 0\,\mathrm{rad}\ \ 0\,\mathrm{m/s}\ \ 0\,\mathrm{m/s}\ \ 0\,\mathrm{rad/s}]$, 模型参数为$m_u=25.8\,\mathrm{kg}$, $m_v=33.8\,\mathrm{kg}$, $m_r=2.76\,\mathrm{kg\cdot m^2}$, $X_u=12\,\mathrm N$, $Y_v=17\,\mathrm N$, $N_r=0.5\,\mathrm{N\cdot m}$, $\omega_u=10+10\sin(2t+\dfrac{\pi}{3})\,\mathrm N$, $\omega_v=10\sin(2t+\dfrac{\pi}{3})\,\mathrm N$, $\omega_r=10\cos(2t+\dfrac{\pi}{3})\,\mathrm{N/s}$, $X_{|u|u}=2.5\,\mathrm N$, $Y_{|v|v}=4.5\,\mathrm N$, $N_{|r|r}=0.1\,\mathrm N$.

选取的隶属度函数如下:

$$\mu_{F_i^1}=\exp(\frac{-(i+3)^2}{4}),\ \mu_{F_i^2}=\exp(\frac{-(i+2)^2}{4}),$$
$$\mu_{F_i^3}=\exp(\frac{-(i+1)^2}{4}),\ \mu_{F_i^4}=\exp(\frac{-(i+0)^2}{4}),$$
$$\mu_{F_i^5}=\exp(\frac{-(i-1)^2}{4}),\ \mu_{F_i^6}=\exp(\frac{-(i-2)^2}{4}),$$
$$\mu_{F_i^7}=\exp(\frac{-(i-3)^2}{4}),\ \mu_{F_r^1}=\exp(\frac{-(r+1.5)^2}{2}),$$
$$\mu_{F_r^2}=\exp(\frac{-(r+1)^2}{2}),\ \mu_{F_r^3}=\exp(\frac{-(r+0.5)^2}{2}),$$
$$\mu_{F_r^4}=\exp(\frac{-(r+0)^2}{2}),\ \mu_{F_r^5}=\exp(\frac{-(r-0.5)^2}{2}),$$
$$\mu_{F_r^6}=\exp(\frac{-(r-1)^2}{2}),\ \mu_{F_r^7}=\exp(\frac{-(r-1.5)^2}{2}),$$

其中: $i=u,v$, 控制精度$\delta=0.1$, 期望速度如下:

当$0\,\mathrm s<t\leqslant 10\,\mathrm s$时, $u_{\mathrm d}=0.8\,\mathrm{m/s}$, $v_{\mathrm d}=0.5\,\mathrm{m/s}$, $r_{\mathrm d}=0.4\mathrm e^{-0.5t}\,\mathrm{rad/s}$. 当$10\,\mathrm s\leqslant t\leqslant 20\,\mathrm s$时, $u_{\mathrm d}=0.8\,\mathrm{m/s}$, $v_{\mathrm d}=0\,\mathrm{m/s}$, $r_{\mathrm d}=0\,\mathrm{rad/s}$. 当$20\,\mathrm s\leqslant t\leqslant 45\,\mathrm s$时, $u_{\mathrm d}=1\,\mathrm{m/s}$, $v_{\mathrm d}=0\,\mathrm{m/s}$, $r_{\mathrm d}=0.5\,\mathrm{rad/s}$. 当$45\,\mathrm s\leqslant t\leqslant 60\,\mathrm s$时, $u_{\mathrm d}=1\,\mathrm{m/s}$, $v_{\mathrm d}=0\,\mathrm{m/s}$, $r_{\mathrm d}=-0.5\,\mathrm{rad/s}$.

当选取的调节参数为$k_x=4$, $k_y=4$, $k_\psi=4$, $k_u=1$, $k_v=2$, $k_r=1$, $l_u=1$, $l_v=1$, $l_r=1$, $b_u=1$, $b_v=1$,

$b_r = 1, c_u = 1, c_v = 1, c_r = 1, \varpi = 0.025$, $\zeta_{u1} = 10$, $\zeta_{u2} = 10, \zeta_{v1} = 10, \zeta_{v2} = 10, \zeta_{r1} = 10, \zeta_{r2} = 10, q = 8$, 系统中带模糊项时, 扰动观测器和控制器中含自适应参数a与不含自适应a的仿真结果如图1–7. 图1为时变扰动下跟踪位置和航向角的响应曲线. 图2为浪涌速度、摇摆速度和偏航速度的响应曲线. 图3为USV的平面跟踪轨迹. 从图中可以看出, 他们都能够成功地跟踪虚拟船所生成的参考轨迹, 但从局部放大视图下可以明显的看出, 通过引入自适应参数a, 可以使USV在期望轨迹上更加快速的跟踪, 且控制效果更好. 图4为系统的跟踪误差. 从图中可以看出, 在扰动发生后, 系统的跟踪误差越来越小, 稳定后含自适应参数a的跟踪误差能满足所要求的控制精度, 而不含自适应参数a的跟踪误差不能满足要求, 且控制效果较差. 图5为系统的控制输入. 从图中可以看出, 在速度发生变化的瞬间, 控制输入会随之发生变化. 图6为自适应参数a的响应曲线. 从图中可以看出, 自适应参数a在开始时不断增大, 这是由于系统的误差大于所规定的的误差精度造成的. 此时通过增大a来调节系统的控制精度, 当系统的误差稳定在规定的控制精度内时, a保持稳定. 图7为系统自适应范数$\|\Theta_i\|$的曲线, 从图中可以看出, 自适应范数在经过一段时间学习后趋于稳定.

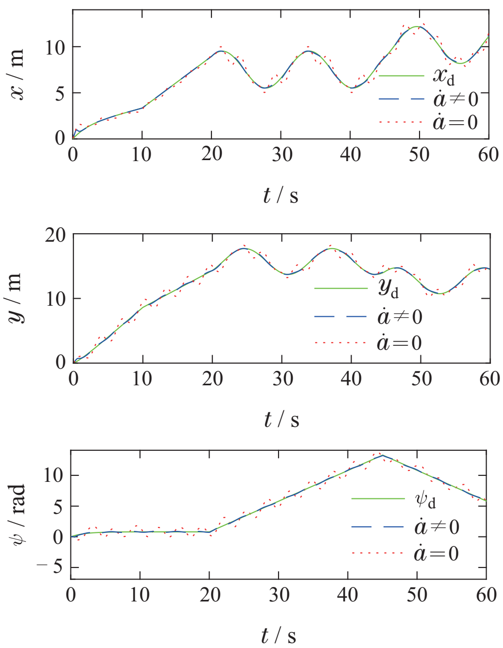
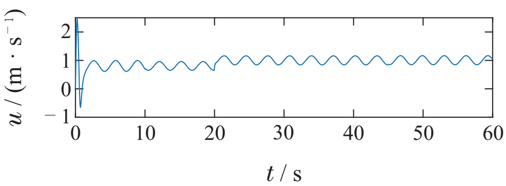


图 1 时变扰动下的跟踪位置和航向角

Fig. 1 Tracking positions and heading angle under time-varying disturbances



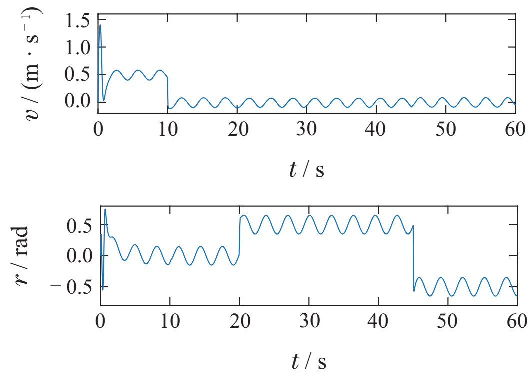


图 2 速度u, v, r ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

Fig. 2 Speed u, v, r ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

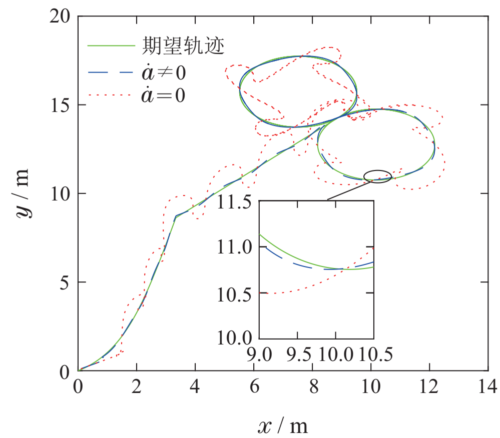


图 3 USV的平面轨迹 ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

Fig. 3 Planer trajectories of USV ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

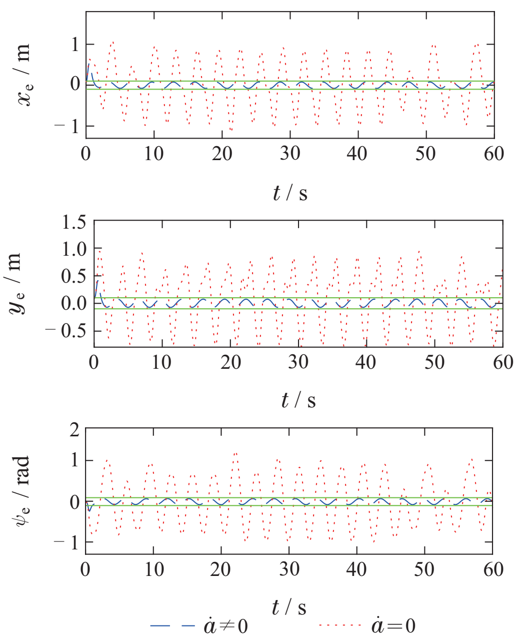


图 4 误差x_e, y_e, ψ_e ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

Fig. 4 Errors x_e, y_e, ψ_e ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

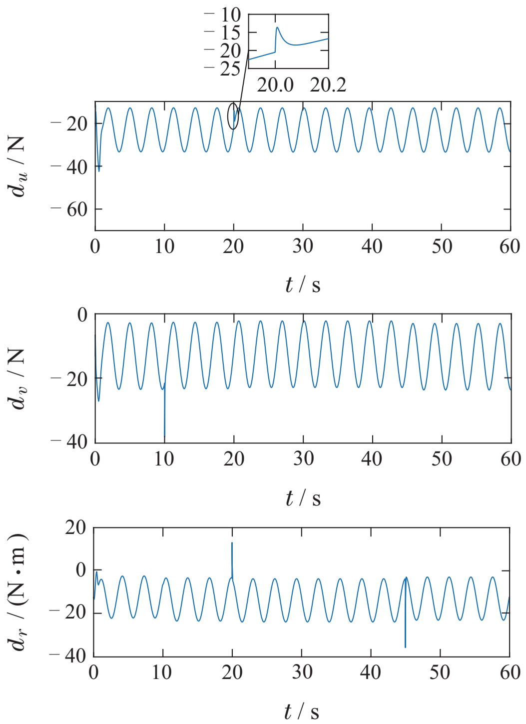


图 5 控制输入 ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

Fig. 5 Control inputs ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

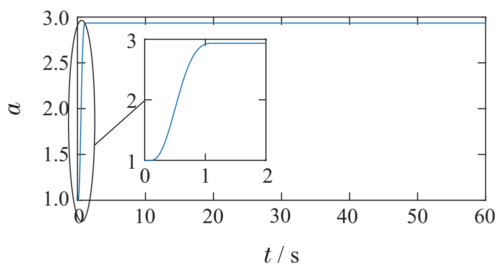


图 6 自适应参数a ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

Fig. 6 Adaptive parameter a ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

当选取的调节参数为$k_x=4$, $k_y=2$, $k_\psi=2$, $k_u=0.2$, $k_v=0.1$, $k_r=0.1$, $l_u=1$, $l_v=1$, $l_r=1$, $b_u=120$, $b_v=20$, $b_r=10$, $c_u=120$, $c_v=30$, $c_r=30$, $\varpi=0.025$, $\zeta_{u1}=10$, $\zeta_{u2}=10$, $\zeta_{v1}=10$, $\zeta_{v2}=10$, $\zeta_{r1}=10$, $\zeta_{r2}=10$, $q=8$时, 在此调节参数下, 若系统中不含自适应参数a, 输出信号发散, 系统不稳定. 若系统中含自适应参数a, 跟踪误差依旧可以满足所要求的控制精度. 图8–10描述了系统中带自适应参数a时, 扰动观测器和控制器中含模糊项与不含模糊项的系统的跟踪轨迹、跟踪误差以及自适应参数a的曲线. 从图8–10中可以看出, 含模糊项的系统控制精度更高. 特别是从图10可以看出, 含模糊项的系统a值要小于不含模糊项的系统, 这说明, 控制系统中含模糊项时, 主要调节参数取值不需要增长太多, 闭环系统的跟踪误差就可以满足控制精度的要求. 另外还可以从图8–9中看出, 加模糊项的闭环控制系统超调量要小于不加模糊项的闭环控制系统.

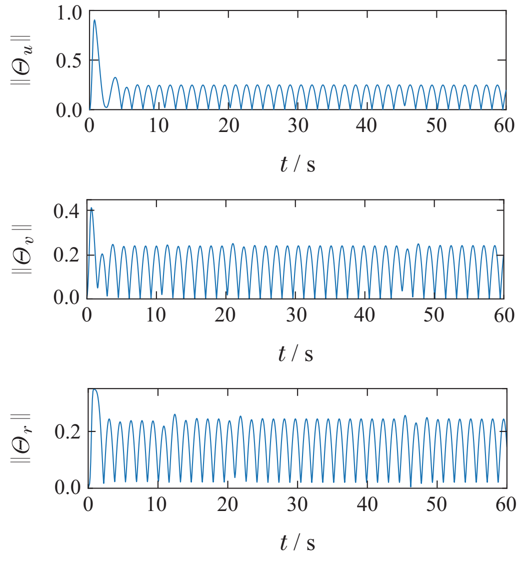


图 7 含参数a的自适应范数$\|\Theta_i\|$($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

Fig. 7 Adaptive norm $\|\Theta_i\|$ with parameter a ($k_x = 4, k_y = 4, k_\psi = 4, k_u = 1, k_v = 2, k_r = 1$)

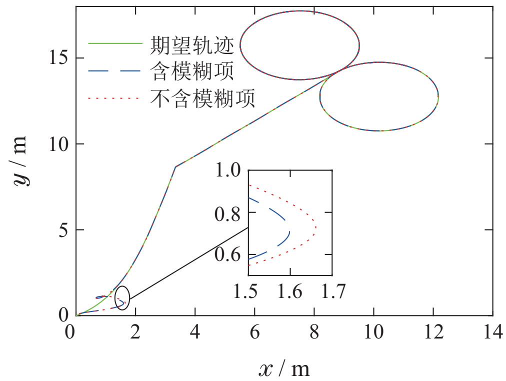
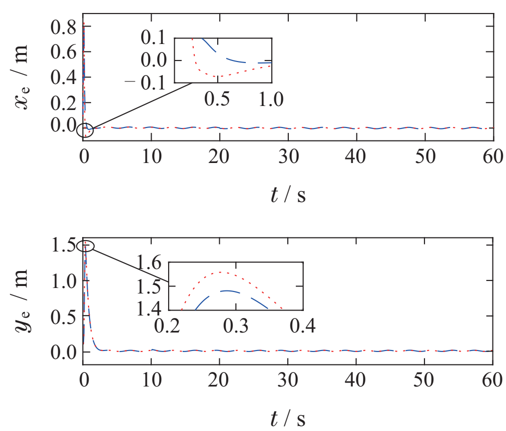


图 8 USV的平面轨迹($k_x = 4, k_y = 2, k_\psi = 2, k_u = 0.2, k_v = 0.1, k_r = 0.1$)

Fig. 8 Planer trajectories of USV ($k_x = 4, k_y = 2, k_\psi = 2, k_u = 0.2, k_v = 0.1, k_r = 0.1$)



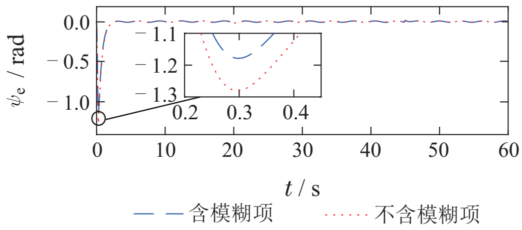


图 9 误差x_e, y_e, ψ_e ($k_x = 4, k_y = 2, k_\psi = 2, k_u = 0.2, k_v = 0.1, k_r = 0.1$)

Fig. 9 Errors x_e, y_e, ψ_e ($k_x = 4, k_y = 2, k_\psi = 2, k_u = 0.2, k_v = 0.1, k_r = 0.1$)

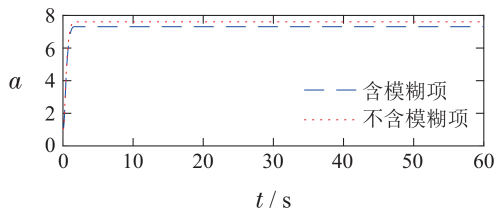


图 10 自适应参数a ($k_x = 4, k_y = 2, k_\psi = 2, k_u = 0.2, k_v = 0.1, k_r = 0.1$)

Fig. 10 Adaptive parameter a ($k_x = 4, k_y = 2, k_\psi = 2, k_u = 0.2, k_v = 0.1, k_r = 0.1$)

选取调节参数$k_x = 40$, $k_y = 40$, $k_\psi = 40$, $k_u = 40$, $k_v = 40$, $k_r = 40$, 当系统中含模糊项时, 扰动观测器和控制器中含自适应参数a与不含自适应参数a的跟踪轨迹和跟踪误差如图11–12所示. 此时二者都能够满足系统所要求的的控制精度, 且跟踪效果一致. 图13–14为系统中含模糊项和自适应参数a时, 后两组参数的轨迹和误差x_e对比图, 从图中可以看出, 当参数设置较小时, 系统初始的跟踪误差比较大, 且响应速度较慢, 此时自适应参数a增加, 系统的控制强度增大, 跟踪误差变小. 而参数设置较大时, 系统的响应速度较快, 在1 s后二者的跟踪误差区别不大. 因此, 引入自适应参数a的控制器对调节参数的大小不敏感, 在调节参数设置较小的情况下, 本文提出的方法也可以满足系统所要求的控制精度.

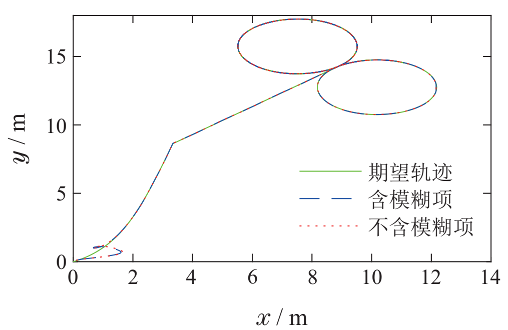


图 11 USV的平面轨迹($k_x = 40, k_y = 40, k_\psi = 40, k_u = 40, k_v = 40, k_r = 40$)

Fig. 11 Planer trajectories of USV ($k_x = 40, k_y = 40, k_\psi = 40, k_u = 40, k_v = 40, k_r = 40$)

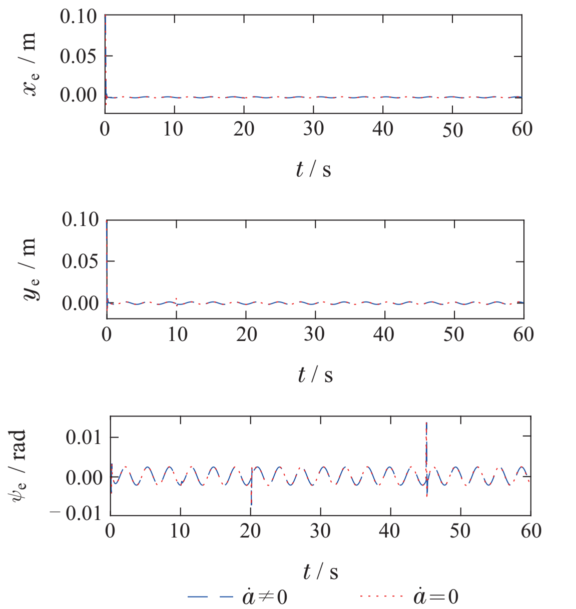


图 12 误差x_e, y_e, ψ_e ($k_x = 40, k_y = 40, k_\psi = 40, k_u = 40, k_v = 40, k_r = 40$)

Fig. 12 Errors x_e, y_e, ψ_e ($k_x = 40, k_y = 40, k_\psi = 40, k_u = 40, k_v = 40, k_r = 40$)

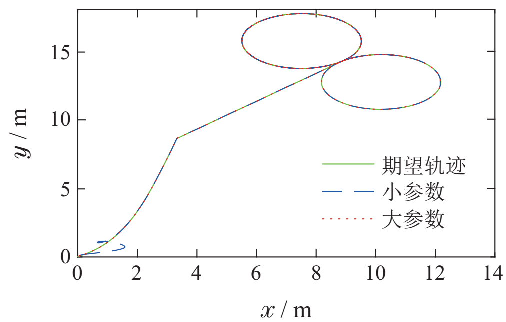


图 13 两组参数轨迹对比

Fig. 13 Comparison of two sets of parameter about trajectories

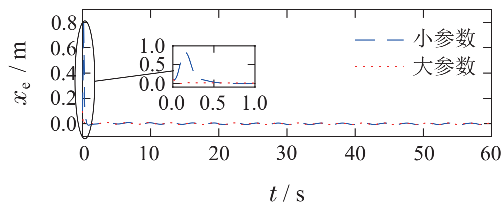


图 14 两组参数的x_e对比

Fig. 14 Comparison of two sets of parameters about x_e

6 结论

本文主要研究了USV的路径跟踪控制问题, 采用模糊逻辑系统和扰动观测器对系统中的建模不确定性和外部时变扰动进行估计和补偿, 结合命令滤波反步控制方法降低了计算过程的复杂性, 此外, 还引入了补偿信号来消除命令滤波所产生的误差. 在此基础上, 在控制器和扰动观测器中加入了一个新的参数a, 通过调节a的大小来调节系统的控制精度, 使其在规定的时间内实现有效且快速的跟踪, 控制效果优于传

统方法. 最后通过Lyapunov方法, 证明了整个闭环系统的稳定性, 仿真结果显示系统的所有信号都是有界的, 且具有良好的跟踪性能, 验证了本文中所提出自适应模糊控制器的有效性.

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